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Autores / Autors	Peñarrocha Alós, Ignacio Dolz Algaba, Daniel Aparicio Marín, Néstor Sanchis Llopis, Roberto
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Synthesis of nonlinear controller for wind turbines stability when providing grid support

I. Peñarrocha, D. Dolz, N. Aparicio and R. Sanchis*

Departament d'Enginyeria de Sistemes Industrials i Disseny, Universitat Jaume I, 12071 Castelló, Spain.

SUMMARY

This paper presents a new nonlinear polynomial controller for wind turbines that assures stability and maximizes the energy produced while imposing a bound in the generated power derivative in normal operation (guarantees a smooth operation against wind turbulence). The proposed controller structure also allows eventually producing a transient power increase to provide grid support, in response to a demand from a frequency controller. The controller design uses new optimization over polynomials techniques, leading to a tractable semidefinite programming problem.
The ability of the wind turbine to increase its power under partial load operation has been analysed. The above optimization techniques have allowed quantifying the maximum transient overproduction that can be demanded to the wind turbine without violating minimum speed constraints (that could lead to unstable behaviour), as well as the total generated energy loss. The ability to evaluate this shortfall has permitted the development of an optimization procedure in which wind farm overproduction requirements are divided into individual turbines, assuring that the total energy loss in the wind farm is minimum, while complying with the maximum demanded power constraints.

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KEY WORDS: Power generation control; wind power generation; transient grid support; polynomial control; sum of squares.

1. INTRODUCTION

Wind energy penetrations have reached significant levels in many power systems. It has forced many system operators to change their grid codes in order to ask wind generators for additional duties, including grid support to improve frequency control [1]. These new requirements take into account the specific characteristics of wind energy.

Therefore, for the case of contributing to frequency control, it is typical to require the provision of downward regulation through the implementation of an asymmetrical droop control that only acts

*Correspondence to: Departament d'Enginyeria de Sistemes Industrials i Disseny, Universitat Jaume I, 12071 Castelló, Spain. E-mail: rsanchis@uji.es

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during over-frequency events [2, 3]. This is always possible for any wind turbine technology since all of them can curtail the generated power when ordered to do so.

On the contrary, contribution to upward regulation presents some limitations depending on the operating point of the wind generators. It does not present problems when working either at full load (i.e. the pitch angle is not at its optimum value) because there is extra energy available in the wind that can be extracted just by changing the pitch angle; or deliberately deloaded because there is an amount of reserves available [4]. At partial load, however, only variable speed wind generators can increase the generated power beyond its mechanical input since this is only possible by increasing the electromagnetic torque, and that can only be achieved through the use of power electronics, allowing access to power controller reference. The extra power needed is extracted from the kinetic energy stored in the blades. Thus, the wind generator starts to decelerate leaving the optimum speed. The new electromagnetic torque reference is usually the result of the sum of the reference torque that comes from the speed controller (looks for the maximum production speed) plus an additional term fixed by the frequency controller [5]. Before reaching the minimum speed, the generated power must be reduced below the captured power in order to accelerate the machine so it recovers its original operating point.

The most widely used configuration found in the literature is a PI controller for speed control and a PD controller for frequency control [5,6]. In the latter, the proportional gain is just a droop control, needed for the provision of primary frequency regulation, whereas the derivative gain is for inertia emulation, needed in variable speed wind generators to provide frequency response as their power converters decouple machine and grid frequencies.

Many authors have shown how this, or similar configurations, can contribute to the reduction in frequency variations; the methods differ primarily in how to deal with the wind generator speed reduction. The decrease in speed produces a significant reduction in the power extracted from the wind, leading to instability if the electromagnetic reference is not changed before reaching the minimum speed. In [7] it is assumed that the speed control helps in maintaining stability since it tries to keep wind generator speed within the limits. However, it recognizes that it is not possible for all the cases and gives an example of a wind generator becoming unstable after providing support for frequency control. In [8] a speed controller is designed to act slow enough to minimize the variation of its output during the initial transient of an under-frequency event. The response of the wind generator only during the transient is guaranteed by adding a washout filter before the droop control. A washout filter is known as a transient droop in hydro turbines. Wind turbines from General Electric offer primary regulation and inertia emulation separately [9]. Primary regulation is only possible if the wind turbine is previously deloaded. Inertia emulation is always possible but is only used during under-frequency. When inertia emulation is enabled, the speed control is programmed far slower and a first order filter is added at its output. There are grid codes that require inertial emulation, which is met for instance by defining a given amount and duration of the extra power that must be generated [10].

Neither of the previously proposed solutions demonstrates global closed loop system stability. Furthermore, the performance is only evaluated ex post facto. This means that the controller designs are not developed in an optimal way.

More complex controllers have been recently proposed for wind turbines [11–13], using a linear parameter varying controller that is designed using a linearized system model, and the wind speed as

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time varying parameter. They have produced some improvement in performance as their controllers are computed with an optimization procedure trying to fulfill some given constraints on the machine operation (via $\mathcal{H}_\infty$ controller design). Nonetheless, there is still the need to find a control scheme that optimizes the generated power at the same time that it provides grid support to contribute to system frequency control, as it is stated in [14].

The present work develops a new strategy for nonlinear polynomial controller design that translates the goals and restrictions that a wind turbine must satisfy, including limitations in both ramp rates and stresses and fatigue in mechanical components, into a computationally tractable optimization problem. The proposed strategy allows us to determine ex ante the machine behaviour when it provides grid support (transient overproduction), and the limit of the operating conditions in which the stability is assured. Furthermore, it allows us to predict on line the total generated energy loss after overproduction transients (including the subsequent recovery). Using these predictions, a wind farm controller is proposed to dispatch the power demands via an optimization procedure that minimizes the total energy loss. This proposal clearly improves other simpler strategies, like proportional ones [15], that may result in a higher total energy loss and may cause instability in the wind turbines.

The structure of the work is as follows. First, Section 2 presents the models considered for both the wind generator and the wind farm. Section 3 develops the proposed state and wind observer. Section 4 presents the proposed controller design including strategies to compute the admissible bounds on the overproduction demand, and the total energy loss during overproduction transients. Section 5 presents a power dispatch function, while in Section 6 the proposed strategies are tested and some simulation results are shown. Finally, Section 7 summarizes the main conclusions.

LIST OF SYMBOLS

List of Parameters	Meaning
H_t	Wind turbine inertia constant
H_g	Generator inertia constant
D_{tg}	Friction constant
K_{tg}	Elastic constant
c_{ij}	Coefficients of the aerodynamical torque polynomial
τ_{em}	Electromagnetic constant time of the generator
τ_v	Constant time of the wind generator model
$\omega_{g,\min},\ \omega_{g,\max}$	Limits of the allowed speed
$\bar{v}_{\min},\ \bar{v}_{\max}$	Considered limits on the mean wind speed
$K(\tilde{x}, \tilde{\omega}_g^\star)$	Controller function
Q	Tuning gain matrix for Kalman filtering
R	Measurement noise covariance matrix

List of Variables	Meaning
T_t	Torque due to wind action
T_{em}	Generator's electromagnetic torque

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$T^{\star}_{em}$	Desired generator's electromagnetic torque
ω_t	Slow shaft (blades) rotational speed
ω_g	Fast shaft (generator) rotational speed
$\omega^{\star}_g$	Desired fast shaft (generator) rotational speed
$\omega^{\star}_g(\bar{v})$	Rotational speed that maximizes generation
$\bar{\omega}^{\star}_g$	$\omega^{\star}_g$ in steady state
$\tilde{\omega}^{\star}_g$	Variations of $\omega^{\star}_g$ around the steady state
θ	Angular difference between equivalent masses
β	Blade pitch angle
v	Wind speed
$\bar{v}$	Mean wind speed
$\hat{\bar{v}}$	Mean wind speed estimation
$\tilde{v}$	Wind's turbulence component
P	Generated electric power
P_t	Available wind power
$\Delta P^{\star}$	Desired transient power generation increase
$\Delta T^{\star}$	Incremental electromagnetic torque
$\Delta T^{\star}_{\max}$	Maximum allowed incremental electromagnetic torque
x_k	Discrete time state for Kalman filtering
I	Controller integral error
w	White noise for wind modelling
x	Continuous time state for controller design
$\bar{x}$	State at the equilibrium point
$\tilde{x}$	State variations around the equilibrium point
$\tilde{T}^{\star}_{em}$	Incremental control action (the output of the controller)
$V(\tilde{x}),\ W(\tilde{x})$	Lyapunov functions
$e(t)$	Electrical energy deviation from the optimal production
L	Total energy loss
$L(\Delta P^{\star})$	Total energy loss function depending on incremental power demand
i	Grid point for computational issues
j	Number of turbine in the wind plant

2. PROBLEM STATEMENT

2.1. Wind turbine mathematical model

Doubly fed induction generators (DFIG) are the most widely used until now [16] so the analysis is focused on these machines. A mathematical model for a DFIG wind turbine connected to an electrical grid will be developed including the drive train, the aerodynamic effects, and the electronic converter. This is a simplified but complex enough model to achieve with sufficient accuracy the proposed goals (performance quantification). The drive train is modeled by means of two inertias connected through a spring and a shock absorber [9], leading to equations (the dependence on time

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t is omitted for brevity):

$$H_t\dot{\omega}_t = T_t - D_{tg}\,(\omega_t - \omega_g) - K_{tg}\,\theta \tag{1}$$

$$H_g\dot{\omega}_g = D_{tg}\,(\omega_t - \omega_g) + K_{tg}\,\theta - T_{em} \tag{2}$$

$$\dot{\theta} = \omega_t - \omega_g \tag{3}$$

where ω_t is the slow shaft rotational speed (i.e., the wind turbine), ω_g the rotational speed of the fast shaft connected to the generator rotor, H_t the wind turbine inertia constant, H_g the generator inertia constant, T_t the torque developed by the wind turbine due to wind action and T_{em} the electromagnetic torque of the generator. Speeds and torques are expressed in p.u. units (i.e. relative values with respect to their nominal value, meaning 1 the nominal value and 0.5 half the nominal value). θ is the angular difference between equivalent masses. The torque developed by the wind turbine can be expressed approximately by means of static functions of the wind speed, v, rotor rotational speed and blade pitch angle, β. Several functions for this torque can be found in the literature. In this work, this torque has been approximated by a polynomial function as

$$T_t = \sum_{i=0}^{4}\sum_{j=0}^{4} c_{ij} v^i \omega_t^j, \tag{4}$$

where the pitch blade angle is assumed to be zero, as, in this work, only low and medium wind speeds (the more probable ones) are assumed.

The wind can be characterized by means of its mean value and a turbulence component, as stated in the IEC-standard [17–19]

$$v(t) = \bar{v}(t) + \tilde{v}(t).$$

The mean value of the wind $\bar{v}(t)$ is assumed to change slowly in time (in a scale of hours) and it can be modelled by Van der Hoven's spectral model plus a Weibull probability distribution. For the turbulence part a Kaimal model can be used. In the short-term a periodic variation due to tower shadow can also be added. Those models have been implemented in order to generate the wind used in the simulation verification of the proposed analysis and prediction methods explained later, but the details are omitted for brevity (they can be found in [18, 19]).

The electromagnetic torque is achieved by means of a current control loop in the power electronics converter that presents a much faster dynamics than the one being analysed. It can be approximately modelled by a first order model that depends on the generator speed (as in [11]) as

$$\dot{T}_{em} = \frac{\omega_g}{\tau_{em}}(T^{\star}_{em} - T_{em}). \tag{5}$$

The generated electric power is given by

$$P = T_{em}\omega_g. \tag{6}$$

2.2. *Wind turbine and farm control objectives*

The wind turbine operation requires a controller that decides the electromagnetic torque to be applied at each instant of time. The objectives of the controller are divided into two groups. First, the

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goals related to the normal operation and the behaviour with respect wind variations, and second, the goals related to the transient grid support capabilities.

Objectives related to normal operation (behaviour with respect wind variations):

- Maximize the generated electrical power, tracking as fast as possible the optimal generation speed as a function of the mean wind speed.
- Bound the generated power derivative through all the operating range. For example according to standard 61400-21 [20], i.e. a rate limitation of 10% of the rated nominal power in one minute ($|\dot{P}| \leq 0.1/60$ p.u./s).
- Guarantee that the generator speed remains in a safe range ($\omega_g \in [\omega_{g,\min}, \omega_{g,\max}]$, usually $\omega_{g,\min} = 0.8$ p.u., and $\omega_{g,\max} = 1.2$ p.u.).

Objectives for the wind turbines related to grid support demand events:

- To be able to produce a transient increase in the generated power, with a prescribed peak value, to provide grid support.
- To be able to compute off line, for the designed controller, the maximum power increase that can be demanded to the wind turbine, as a function of the mean wind speed, while guaranteeing that the generator speed remains inside the safe operating range.

Objectives for the wind farm related to grid support demand events:

- To minimize the total energy loss, distributing the transient power demand between the wind turbines in an optimal way.

About the ability of producing a transient increase in the generated power, the idea is to help the grid to restore faster its nominal frequency when some failure occurs in any conventional electrical source. The power generated by the wind turbine can be transiently increased by means of decelerating the machine and injecting its stored kinetic energy.

The following considerations must be taken into account with respect to transient overproduction demands. First, in this work, the considered situations are those in which the wind speed is equal or below rated. Second, the power increment is achieved by means of a machine deceleration, and it must be assured that the machine does not exceed the limits of the operating range, or to an unstable behaviour. Also, the higher transient production is (for wind speeds equal or below rated) followed by a recovery transient in which the machine is restored to its normal operation and, during that recovery, needs to capture wind power to accelerate [6,21]. During this process, the total amount of electrical energy produced is always lower than the one generated if no power overproduction were demanded. Both the injected energy during the initial power overproduction transient and the total energy loss depend on the demanded power, but the relation is nonlinear (as it will be shown later in Section 4.4 and numerically in Section 6). Therefore, an optimization procedure is proposed to assure that the demanded overproduction is satisfied by the wind farm at the same time that the total electrical energy loss is minimized.

2.3. Wind turbine and farm control structure

The proposed controller for each wind turbine and the wind farm controller have the structure shown in Fig. 1. An existing grid frequency control system (that is outside of the scope of the paper) is

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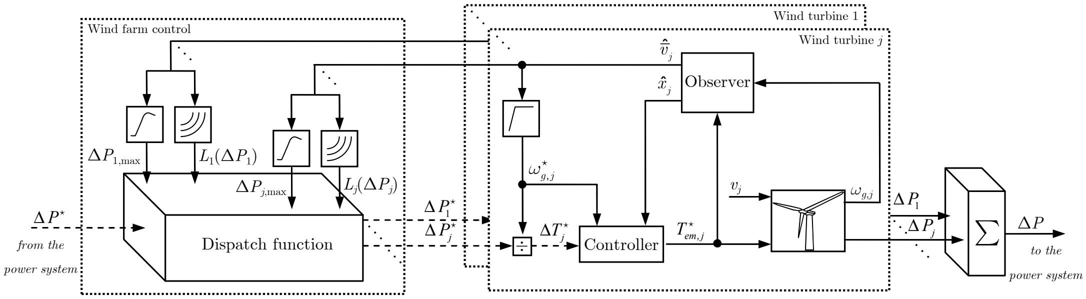

Figure 1. Proposed control structure.

assumed to eventually ask the wind farm for a transient increase in the generated power, defined by a given peak value, $\Delta P^{\star}$. The dashed arrows indicate signals that are not continuous, but instead, are only defined in the discrete instant when a transient increase event is produced by the frequency controller. The wind turbine controller has the following structure (for each of the $j = 1, \ldots, N$ wind turbines in a farm). First, a wind and state observer is implemented using the measurement of the generator speed and the applied electromagnetic torque. The observed wind speed is filtered in order to obtain a soft mean wind speed estimation $\hat{\bar{v}}_j$. The wind mean speed is then used to obtain the speed generator reference $\omega_g^{\star}$ that leads to the maximum power generation. This is obtained by maximizing the available wind power (given by $P_t = T_t\, \omega_t$), within the allowed range of w_t, and taking into account that in steady state $\omega_t = \omega_g$, leading to an static function $\omega_g^{\star}(\bar{v})$.

The optimal generator speed reference and the estimated state are the only inputs of the speed controller during normal operation. This controller computes the control action $T_{em}^{\star}$ by means of a polynomial controller (see Section 4), whose structure includes a PI controller as a particular case (i.e., traditional PI controller can also be handled with this structure).

In order to be able to respond to the eventual grid support demand, the peak incremental power demand received by the wind farm from the frequency controller, $\Delta P^{\star}$, is split into several incremental overproduction demands that are dispatched to each wind turbine controller, $\Delta P_j^{\star}$, that, divided by the speed reference, results in a peak value of incremental electromagnetic torque, $\Delta T_j^{\star}$:

$$\Delta T_j^{\star} = \frac{\Delta P_j^{\star}}{\omega_g^{\star}} \tag{7}$$

This torque is an eventual signal that takes a value different from zero only at the instant when the overproduction is demanded. The controller incorporates this eventual signal such that a transient incremental torque (and hence generated power) with a peak value of $\Delta T_j^{\star}$ is generated, vanishing with time as it is compensated by the controller (due to the integral action), finally recovering the previous steady state value. A detailed description of how the controller deals with this signal can be found in Section 4.3.

The incremental power demand must be bounded by a function of the wind speed, $\Delta P_{j,\max}(\bar{v})$, in order to assure that the machine is not taken out of the safe operating range at any instant, or destabilized, as it will be shown in Section 4.3, where a procedure to compute that bound is developed.

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A power dispatch function is proposed, as shown in Fig. 1, whose objective is to satisfy the total demanded overproduction with the minimum possible energy loss. In order to make the assignment decision, the maximum possible overproduction demand ($\Delta P_{i,\max}$) as well as the energy loss related to the production demand ($L_j(\Delta P_j)$) are assumed to be known for each turbine (the procedures to compute them are developed in Section 4.3 and 4.4).

In the following sections, the observer, the controller and the dispatch function design procedures are detailed. In Section 3 and 4 one single turbine is considered and, therefore, the index numbering the turbine inside the wind farm will be omitted.

3. STATE AND WIND OBSERVER

For the state and wind observer, a simple random walk is used for the wind generation model $\dot{v} = w_v$, where w_v is white Gaussian noise. With this wind model, a forward difference approximation of the DFIG model is defined with a sufficiently small period T, leading to

$$x_k = \begin{bmatrix} v_k \\ \omega_{t,k} \\ \omega_{g,k} \\ T_{em,k} \\ \theta_k \end{bmatrix} = \begin{bmatrix} v_{k-1} + T\, w_{v,k-1} \\ \omega_{t,k-1} + \frac{T}{H_t}\left(T_{t,k-1} - D_{tg}\Delta\omega_{k-1} - K_{tg}\theta_{k-1}\right) \\ \omega_{g,k-1} + \frac{T}{H_g}\left(-T_{em,k-1} + D_{tg}\Delta\omega_{k-1} + K_{tg}\theta_{k-1}\right) \\ T_{em,k-1} + \frac{T\,\omega_{g,k-1}}{\tau_{em}}(T^{\star}_{em,k-1} - T_{em,k-1}) \\ \theta_{k-1} + T\,\Delta\omega_{k-1} \end{bmatrix}$$
$$\omega_{g,k} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix} x_k \tag{8}$$

where $\Delta\omega_{k-1} = \omega_{t,k-1} - \omega_{g,k-1}$, and $T_{t,k-1} = \sum_{i=0}^{3}\sum_{j=0}^{3} c_{ij} v^i_{k-1}\omega^j_{t,k-1}$. Let us now express the previous model as

$$x_k = f(x_{k-1}, T^{\star}_{em,k-1}) + w_{k-1} \tag{9}$$
$$\omega_{g,k} = C\,x_k + \nu_k, \tag{10}$$

where w_{k-1} is assumed to be a white noise disturbance vector taking into account the wind speed variations and possible model errors, and ν_k is the measurement noise, assumed to be a white noise signal with known variance $\mathcal{E}\{\nu_k^2\} = R$. The proposed algorithm, based on the Extended Kalman Filter, that must be computed at each sampling period to estimate the wind speed, its mean and root

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mean square value is

$$
\begin{aligned}
\hat{x}_k^- &= f(\hat{x}_{k-1}, T^\star_{em,k-1}) && \text{(11a)}\\
P_k^- &= F_{k-1} P_{k-1} F_{k-1}^T + Q && \text{(11b)}\\
L_k &= P_k^- C^T (C\, P_k^-\, C^T + R)^{-1} && \text{(11c)}\\
\hat{x}_k &= \hat{x}_k^- + L_k (w_{g,k} - C\, \hat{x}_k^-) && \text{(11d)}\\
P_k &= (I - L_k\, C) P_k^- && \text{(11e)}\\
\hat{\bar{v}}_k &= p \cdot \hat{\bar{v}}_k + (1-p) \cdot \hat{v}_k && \text{(11f)}\\
\hat{\sigma}_{v2,k} &= p \cdot \hat{\sigma}_{v2,k-1} + (1-p) \cdot (\hat{\bar{v}}_k - \hat{v}_k)^2 && \text{(11g)}\\
\hat{\sigma}_{v,k} &= \sqrt{\hat{\sigma}_{v2,k}} && \text{(11h)}
\end{aligned}
$$

where Q is used as a tuning parameter (see Section 6), p is a slow discrete time pole ($0 < p \lesssim 1$) chosen to be the discrete-time equivalent of a continuous-time pole similar to the model that generates the mean wind speed variations (about $1/600s^{-1}$, hence $p \approx e^{-\frac{T}{600}}$). The matrix F_{k-1} is given by

$$
F_{k-1} = \left.\frac{\partial f}{\partial x}\right|_{\hat{x}_{k-1}, T^\star_{em,k-1}}.
$$

Note that this algorithm is useful for both wind estimation and state observation, and can be used to implement wind control algorithms based on polynomial state feedback control, or to address optimization procedures that depend on the working conditions of different wind turbines. This idea is explored in the following sections. From now on, it will be assumed that the wind and state observer has been tuned properly and, therefore proper estimates of the state and wind are available. For that reason, and in order to avoid an abuse of notation, the estimated state ($\hat{x}$) will be rewritten as x, and the estimated mean wind speed ($\hat{\bar{v}}$) as $\bar{v}$.

4. CONTROLLER DESIGN

4.1. *Control system modeling*

In this section, the procedure to obtain the speed controller for normal operation is explained. The design strategy has been developed by using Lyapunov methods and applying optimization techniques over polynomials (see [22] for the details on the technique, and [23–27] for other recent applications). For these techniques a dynamical polynomial model of the system is assumed to be available, fulfilling

$$
\dot{x} = f(x) + g(x) w, \qquad f(0) = 0 \tag{12}
$$

where x is the state vector, w are the inputs, and $f(x)$ and $g(x)$ are given polynomial vectorial functions. For design purposes, the wind will be modelled as a slowly time varying mean value $\bar{v}$ plus a signal generated by a bounded white noise w filtered by a first order system with a low time

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constant, leading to

$$\dot{\tilde{v}} = \frac{1}{\tau_v}(w - \tilde{v}), \tag{13}$$

$$v = \bar{v} + \tilde{v}. \tag{14}$$

The system model used for the design procedure is defined as

$$\underbrace{\begin{bmatrix} \dot{\tilde{v}} \\ \dot{\omega}_t \\ \dot{\omega}_g \\ \dot{\theta} \\ \dot{I} \\ \dot{T}_{em} \end{bmatrix}}_{\dot{x}} = \begin{bmatrix} \frac{1}{\tau_v}(w - \tilde{v}) \\ \frac{1}{H_t}\left(T_t(v,\omega_t) + D_{tg}(\omega_t - \omega_g) - K_t\theta\right) \\ \frac{1}{H_g}\left(-T_{em} - D_{tg}(\omega_t - \omega_g) + K_t\theta\right) \\ \omega_t - \omega_g \\ \omega_g^\star - \omega_g \\ \frac{\omega_g}{\tau_{em}}(T_{em}^\star - T_{em}) \end{bmatrix} \tag{15}$$

where $T_t(v, \omega_t)$ is the polynomial defined in (4), I is the integral of the generator speed tracking error, and $T_{em}^\star = K(x, \omega_g^\star)$ is the control action to be defined. The controller can be a polynomial function of the state x (including the integral error), and reference input $\omega_g^\star$. This is a polynomial dynamic model with inputs w and $\omega_g^\star$. The proposed control scheme fixes the speed reference as a function of the estimated mean wind speed. In order to analyse the tracking behaviour of the controlled system with respect to changes in the speed reference, it will be written as $\omega_g^\star = \omega_g^\star(\bar{v}) + \tilde{\omega}_g^\star$, where $\omega_g^\star(\bar{v})$ is the optimal speed reference, and $\tilde{\omega}_g^\star$ represents a possible change in this reference. Introducing this concept into dynamics equation it leads to

$$\dot{x} = f(x, \bar{v}) + g_w\, w + g_P\tilde{\omega}_g^\star + g_T(x)T_{em}^\star, \tag{16}$$

where $\bar{v}$ can be considered as a time varying parameter whose slow variation will be neglected (note that the dynamics of the mean wind speed is much slower than the rest of the dynamics considered in the model). As model (16) does not fulfill the condition $f(0, \bar{v}) = 0$, as needed, new incremental variables $\tilde{x}$ must be defined fulfilling $x = \bar{x} + \tilde{x}$, where $\bar{x}$ is the value that makes $f(\bar{x}, \bar{v}) = 0$. From this equation it is easy to derive the following expressions for the equilibrium points

$$\bar{\omega}_t = \bar{\omega}_g = \omega_g^\star(\bar{v}),\ \bar{T}_{em} = T_t(\bar{v}, \omega_g^\star(\bar{v})) = T_t(\bar{v}),\ \bar{\theta} = \frac{T_t(\bar{v})}{K_t}.$$

This has two consequences. The first one is that the electromagnetic reference torque (the control action) must take a non-zero value at the equilibrium point ($\bar{T}_{em}^\star = \bar{T}_{em}$)[†] that must be taken into account in the change of variables, that now will be expressed as $T_{em}^\star = \bar{T}_{em}^\star + \tilde{T}_{em}^\star$, with $\tilde{T}_{em}^\star = K(\tilde{x}, \tilde{\omega}_g^\star)$ and $\tilde{\omega}_g^\star = \omega_g^\star - \bar{\omega}_g^\star$. With these changes of variables, the model can be expressed as

$$\dot{\tilde{x}} = f(\tilde{x}, \bar{v}) + g_w w + g_P\tilde{\omega}_g^\star + g_T(\tilde{x})\tilde{T}_{em}^\star, \tag{17}$$

[†]In practice, this value will be achieved by the controller thanks to the integral error term I.

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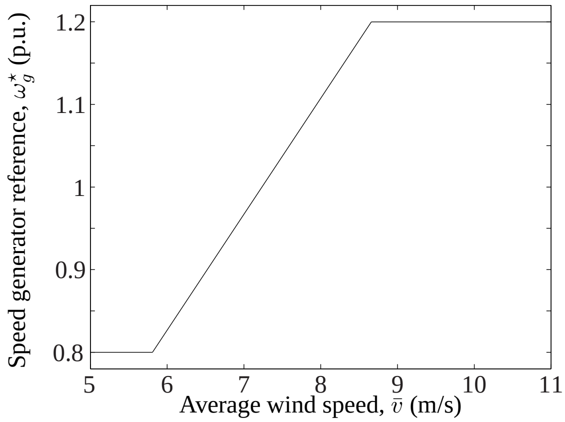

Figure 2. Optimal speed generator reference as a function of mean wind speeds.

The second one is that the non-polynomial relationship $\omega_g^\star(\bar{v})$ (introduced in Section 2.3 and Fig. 1, and detailed in Section 4.2 and Fig. 2) appears, and, therefore the previous model cannot be written with a polynomial dependency of $\bar{v}$ (if $\omega_g^\star(\bar{v})$ were a polynomial, the result would be a polynomial parameter varying system). For this reason, an approximative method is proposed using a grid of n different mean wind speeds $\bar{v}_i$ between $\bar{v}_{\min}$ and $\bar{v}_{\max}$ (normally $\bar{v}_{\min} = 6$, $\bar{v}_{\max} = 10$ for low wind applications) fulfilling $\bar{v}_{\min} = \bar{v}_1 < \bar{v}_2 < \cdots < \bar{v}_n = \bar{v}_{\max}$. The number of points in the grid is a trade-off between fitting the nonlinear behaviour with sufficient precision and the required computational cost for the algorithms to come. With this grid, we assume from now on that we have a set of n possible polynomial models

$$\dot{\tilde{x}} = f_i(\tilde{x}) + g_w\, w + g_P\, \tilde{\omega}_g^\star + g_{T,i}(\tilde{x})\tilde{T}_{em}^\star, \tag{18}$$

where w is the wind turbulence, and $\tilde{x} = [\tilde{v}, \tilde{\omega}_t, \tilde{\omega}_g, \tilde{\theta}, \tilde{I}, \tilde{T}_{em}]^T$ and $\tilde{\omega}_g^\star$ are the variations of the state and reference from the equilibrium point, defined as a function of the mean wind speed $\bar{v}_i$ by

$$\bar{\omega}_{t,i} = \bar{\omega}_{g,i} = \omega_g^\star(\bar{v}_i), \quad \bar{T}_{em,i} = T_t(\bar{v}_i, \omega_g^\star(\bar{v}_i)), \quad \bar{\theta}_i = \frac{T_t(\bar{v}_i)}{K_t}.$$

4.2. *Optimization based controller design*

The previous controller depends on the mean wind speed by means of the optimal speed reference $\omega_g^\star(\bar{v})$. Then, before computing the controller, the optimal speed reference function must be obtained assuring that the operation of the machine in steady state generates the maximum power. This can be obtained as a result of the following optimization problem, that must be solved for each mean wind speed ($i = 1, \ldots, n$):

$$\begin{aligned} \omega_g^\star(\bar{v}_i) = \arg \max_{T_{em}^\star, \omega_g} \quad & T_t\, \omega_t \\ \text{s.t.} \quad & f(x, \bar{v}_i) + g_T(x) T_{em}^\star = 0, \\ & \omega_{t,\min} \le \omega_t \le \omega_{t,\max}. \end{aligned}$$

Figure 2 shows the function $\omega_g^\star(\bar{v})$ obtained for the wind turbine analyzed in Section 6.

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As previously described in Section 2.2, the objectives of the controller during normal operation are to smooth the generated power (i.e., to assure that $|\dot{P}|$ is lower than the imposed limit for a wind with a given turbulence), at the same time that the optimal generation speed is tracked as fast as possible (when a change on the mean wind speed is detected). In order to attain these objectives, one must take into account that in a given wind farm, the turbulence intensities can be bounded. This bound will lead to different covariance of the fast wind variations depending on the mean wind speed, as stated in standard IEC 61400-1 [17]. With that covariance, a maximum value of the turbulence can be obtained with the 3σ confidence interval. Let us call that bound $\bar{w}_i$ for each mean wind speed.

Also, it must be noted that the nonlinear polynomial model has been obtained for a given operation range in which the controller must work. For this reason, the problem of finding a controller that attains the stated goals will be solved locally using state region constraints. For the case of the wind turbine generator, these are the known limits of the speeds and electromagnetic torque. This region will be defined in terms of the incremental variables $\tilde{x}$ and, therefore it will depend on the mean wind speed. It will be denoted as $\mathcal{D}_i$ and defined as follows:

$$\mathcal{D}_i = \left\{ \tilde{x} : \begin{array}{l} \omega_{g,\min} \leq \bar{\omega}_{g,i} + \tilde{\omega}_g \leq \omega_{g,\max} \\ \omega_{t,\min} \leq \bar{\omega}_{t,i} + \tilde{\omega}_t \leq \omega_{t,\max} \\ T_{em,\min} \leq \bar{T}_{em,i} + \tilde{T}_{em} \leq T_{em,\max} \end{array} \right\}.$$

The following theorem is useful to find a controller that assures that the power variations do not violate the established constraints at each of the mean wind speeds ($|\dot{P}| < \bar{\dot{P}}$), and that the integral speed tracking error is minimized.

Theorem 4.1

For all $i = 1, \ldots, n$ points in the gridding, if there exist a positive real number γ, $2n$ Lyapunov functions $V_i(\tilde{x})$, $W_i(\tilde{x})$ and a function $K(\tilde{x}, \tilde{\omega}_g^\star)$ fulfilling,

$$V_i(\tilde{x}) > 0,\ \tilde{x} \neq 0,\ V_i(0) = 0,\ \forall \tilde{x} \in \mathcal{D}_i \tag{19a}$$
$$\dot{V}_i(\tilde{x}) \leq 0,\ \forall w^2 < \bar{w}_i^2, \forall\, \tilde{x} \in \{\tilde{x} | V_i(\tilde{x}) = 1\},\ \forall \tilde{x} \in \mathcal{D}_i \tag{19b}$$
$$\dot{P}_i^2 \leq \bar{\dot{P}}^2,\ \forall\, \tilde{x} \in \{\tilde{x} | V_i(\tilde{x}) \leq 1\},\ \forall \tilde{x} \in \mathcal{D}_i \tag{19c}$$
$$W_i(\tilde{x}) > 0,\ \tilde{x} \neq 0,\ W_i(0) = 0,\ \forall \tilde{x} \in \mathcal{D}_i \tag{19d}$$
$$\dot{W}_i(\tilde{x}) \leq \tilde{\omega}_g^{\star\,2},\ \forall\, \tilde{x} \in \{\tilde{x} | W_i(\tilde{x}) \leq 1\},\ \forall \tilde{x} \in \mathcal{D}_i \tag{19e}$$
$$I^2 < \gamma,\ \forall\, \tilde{x} \in \{\tilde{x} | W_i(\tilde{x}) \leq 1\},\ \forall \tilde{x} \in \mathcal{D}_i \tag{19f}$$

with

$$\dot{V}_i(\tilde{x}) = \frac{\partial V_i(\tilde{x})}{\partial \tilde{x}} \left(f_i(\tilde{x}) + g_w\, w + g_{T,i}(\tilde{x}) K(\tilde{x}, \tilde{\omega}_g^\star) \right),$$
$$\dot{W}_i(\tilde{x}) = \frac{\partial W_i(\tilde{x})}{\partial x} \left(f_i(\tilde{x}) + g_P(x) \tilde{\omega}_g^\star + g_{T,i}(\tilde{x}) K(\tilde{x}, \tilde{\omega}_g^\star) \right),$$
$$\dot{P}_i = \dot{\omega}_g\, T_{em} + \omega_g\, \dot{T}_{em} = \dot{\tilde{\omega}}_g (\bar{T}_{em,i} + \tilde{T}_{em}) + (\bar{\omega}_{g,i} + \tilde{\omega}_g) \dot{\tilde{T}}_{em},$$

then, under null initial conditions, a constant speed reference and wind disturbances bounded by $\|w\|_\infty < \bar{w}_i$, the power derivative is bounded by $\|\dot{P}\|_\infty < \bar{\dot{P}}$. Furthermore, under null wind

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disturbances, the reference tracking error is bounded by $\|I\|_\infty < \sqrt{\gamma}$ for reference changes bounded by $\|\tilde{\omega}_g^\star\|_2 < 1$.

Proof

Constraints (19a) and (19b) indicate that $V_i(\tilde{x})$ is a Lyapunov function such that decreases on the boundary defined by $V_i(\tilde{x}) = 1$ for all inputs fulfilling $|w| \leq \bar{w}_i$, and, therefore, the state will always be contained in the set $V_i(\tilde{x}) \leq 1$ under null initial conditions. Condition (19c) indicates that the inclusion of the state in that set implies $\|\dot{P}_i\|_\infty < \bar{\dot{P}}$.

Now, if null initial conditions are assumed ($\tilde{x}(0) = 0$), and both sides of inequality (19e) are integrated assuming that $\int_0^T \omega_g^{\star 2}\, \mathrm{d}t \leq 1$, it leads to $W_i(\tilde{x}(T)) \leq 1$ (as $W_i(\tilde{x}(0)) = 0$). On the other hand, under null reference input ($\tilde{\omega}_g^\star = 0$), constraint (19e) leads to $\dot{W}_i(\tilde{x}) \leq 0,\ \forall W_i(\tilde{x}) \leq 1$, i.e., the Lyapunov function decreases for all $\tilde{x}$ inside the set defined by $W_i(\tilde{x}) \leq 1$. Therefore, $W_i(\tilde{x}) \leq 1$ is the reachable set for all reference inputs $\tilde{\omega}_g^\star$ bounded in energy by 1. Finally, constraint (19f) states that the integral error is bounded by $\|I\|_\infty < \sqrt{\gamma}$ for all $\tilde{x}$ inside the mentioned set. □

Remark 4.1

The previous theorem allows us to find a controller fulfilling the proposed constraints, and, furthermore, if a minimization of γ is addressed, the fastest feasible controller can be obtained. However, it is difficult to solve the minimization problem if the Lyapunov function, controller functions, and set membership functions ($\mathcal{D}_i$) are not first restricted to a predefined structure. If these functions are forced to be polynomials of a given order, then the previous problem can be converted to an optimization over polynomials one. Besides, that optimization can be further simplified to reach a computationally tractable numerical problem, if the positivity of the polynomial functions over the different sets is restricted to sum of squares constraints using the Positivstellensatz result [22] that can be found in the appendix. This simplified problem (sum of squares optimization) can be reduced to a semidefinite program problem that can be efficiently solved with well-known interior-point algorithms. Of course, the price paid is the introduction of some degree of conservativeness.

To find the (local) controller that assures that the reference tracking speed is maximized and that the power derivative is bounded, the following optimization problem is proposed:

$$
\begin{aligned}
\min_{V_i, W_i, K, \gamma} \quad & \gamma \qquad (20)\\
\text{s.t.} \quad & \forall i = 1, \dots, n: \\
& V_i(\tilde{x}) - \epsilon \tilde{x}^T \tilde{x} - p_{1,i}(\tilde{x}) \in \Sigma \\
& -\dot{V}_i(\tilde{x}) - s_i(\tilde{x})(\bar{w}_i^2 - w^2) + q_i(\tilde{x})(V_i(\tilde{x}) - 1) - p_{2,i}(\tilde{x}) \in \Sigma \\
& \bar{\dot{P}}^2 - \dot{P}^2 - r_i(\tilde{x})(1 - V_i(\tilde{x})) - p_{3,i}(\tilde{x}) \in \Sigma \\
& W_i(\tilde{x}) - \epsilon \tilde{x}^T \tilde{x} - p_{4,i}(\tilde{x}) \in \Sigma \\
& \omega_g^{\star 2} - \dot{W}_i(\tilde{x}) - l_i(\tilde{x})(1 - W_i(\tilde{x})) - p_{5,i}(\tilde{x}) \in \Sigma \\
& \gamma - I^2 - t_i(\tilde{x})(1 - W_i(\tilde{x})) - p_{6,i}(\tilde{x}) \in \Sigma \\
& s_i(\tilde{x}),\ r_i(\tilde{x}),\ t_i(\tilde{x}),\ l_i(\tilde{x})\ \in \Sigma, \quad \gamma > 0,
\end{aligned}
$$

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where $V_i(\tilde{x})$, $W_i(\tilde{x})$, $q_i(\tilde{x})$, $r_i(\tilde{x})$, $s_i(\tilde{x})$, $t_i(\tilde{x})$ and $K(\tilde{x}, \tilde{\omega}_g^\star)$, are polynomials to be obtained during optimization problem, Σ is the set of sum of squares polynomials, ϵ is a small positive constant, and where the derivative functions $\dot{V}_i(\tilde{x})$, $\dot{W}_i(\tilde{x})$, $\dot{P}_i$ must be expressed as indicated in theorem 4.1. Polynomials $p_{k,i}(\tilde{x})$ ($k = 1, \ldots, 6$) are formed as in

$$\begin{aligned} p_{k,i}(\tilde{x}) =& \alpha_{k,i,1}(\tilde{x})(\tilde{\omega}_g - (\bar{\omega}_{g,i} - \omega_{g,\min}))((\omega_{g,\max} - \bar{\omega}_{g,i}) - \tilde{\omega}_g) \\ &+ \alpha_{k,i,2}(\tilde{x})(\tilde{\omega}_t - (\bar{\omega}_{t,i} - \omega_{t,\min}))((\omega_{t,\max} - \bar{\omega}_{t,i}) - \tilde{\omega}_t) \\ &+ \alpha_{k,i,3}(\tilde{x})(\tilde{T}_{em} - (\bar{T}_{em,i} - T_{em,\min}))((T_{em,\max} - \bar{T}_{em,i}) - \tilde{T}_{em}) \end{aligned} \tag{21}$$

where $\alpha_{k,i,l} \in \Sigma$, $l = \{1, 2, 3\}$ are polynomials to be obtained during optimization problem. These polynomials $p_{k,i}(\tilde{x})$ are used to solve the problem locally, allowing us to find a feasible solution on the previously defined set $\mathcal{D}_i$.

Note that this optimization problem is non-convex as products of decision variables appear. For that reason, a solver for bilinear matrix inequalities, an iterative procedure, or more recent strategies (see [28]) are required. The numerical aspects of this optimization problem are out of the scope of this paper.

4.3. Bounding the overproduction demand

The controller designed in the previous section for normal operation maximizes the tracking speed while bounding the power derivative. However, this controller must also be capable of responding to eventual grid support demands defined as an incremental power peak, $\Delta P^\star$. To cope with this requirement, the incremental peak power demand received by the wind generator, $\Delta P^\star$, is converted into a peak incremental electromagnetic torque signal of constant value $\Delta T^\star$ that is an eventual input of the speed controller. The simplest way of generating a peak of value $\Delta T^\star$ in the torque is to add this value as a constant (step) disturbance to the output of the previously designed speed controller. The controller reacts producing a transient incremental torque (and hence generated power) with a peak value of $\Delta T^\star$, that vanishes with time after a transient, as it is compensated by the controller (due to the integral action), finally recovering the previous steady state value. The behaviour during this transient can be observed in the Fig. 10. During the initial part of this transient, the machine is decelerated transferring the blades' kinetic energy to the grid. Then, the machine is accelerated again, reducing the generated power, until the equilibrium point is recovered. This strategy has a minor implementation problem: each time an overproduction event occurs, the disturbance signal is incremented, and as a result, the integral term of the speed controller grows. This could produce numerical issues in the long term. This problem can be easily solved by implementing a mathematically equivalent strategy: instead of adding a step disturbance of $\Delta T^\star$ at the controller output, an impulse signal of the necessary value should be added once to the integral term of the controller. This would produce an instantaneous increase in the controller output, that would be compensated after a transient, returning the controller to its previous steady state. The exact value of the impulse to be added to the integral term can be easily computed from the controller equation, as the value that produces an instantaneous increase of $\Delta T^\star$ on the controller output. As both strategies are equivalent, in the sequel, the first one is assumed to be applied for analysis purposes.

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During the transient, the generator speed reaches a minimum value that depends on the value of incremental torque, $\Delta T^{\star}$. It will be useful to compute which is the maximum incremental torque that can be added to the controller output while guaranteeing that the generator speed remains inside the safe operating range (it is not decelerated too much, i.e., $\omega_g > \omega_{g,\min}$). In this way, the incremental torque added as a disturbance can be limited to remain inside the safe speed range. Obviously, the maximum torque will depend on the operating point (i.e. on the wind mean speed). Another aspect that should be analysed when studying the behaviour under overproduction demands is the effect of the transient overproduction on the overall electrical energy generation, as a function of the operating point.

Once the controller has been obtained ($\tilde{T}^{\star}_{em} = K(\tilde{x}, \tilde{\omega}^{\star}_g)$), some assumptions are taken on the modelled dynamics, in order to formulate the closed loop system model. First, the dynamics of the wind turbulence signal is assumed to be negligible ($\tilde{v} = 0$), and, therefore, $v = \bar{v}$. As a consequence, it is also assumed that the speed reference $\omega^{\star}_g(\bar{v})$ will not change during this transient and, therefore, $\tilde{\omega}_g = 0$. Moreover, it is assumed that the incremental torque signal will appear when the wind turbine is operating on its optimal point $\bar{\omega}_g = \omega^{\star}_g(\bar{v})$ and, therefore, the difference between the present speed and the limit one ($\omega_g = \omega_{g,\min}$) is given by the known quantity $\omega^{\star}_g(\bar{v}) - \omega_{g,\min}$ (i.e., the incremental speed $|\tilde{\omega}_g|$ must be bounded by $|\omega^{\star}_g(\bar{v}) - \omega_{g,\min}|$). Finally, the electromagnetic reference torque is assumed to include the external incremental signal $\Delta T^{\star}$. With these assumptions, the system model (18) with the controller is reformulated in closed loop for each point of the grid, taking into account the disturbance input $\Delta T^{\star}$, leading to

$$\dot{\tilde{x}} = f_{CL,i}(\tilde{x}) + g_{T,i}(\tilde{x})\Delta T^{\star}, \tag{22}$$

The following theorem allows us to bound the incremental speed $\tilde{\omega}_g$ signal under a step input input on the incremental torque signal.

Theorem 4.2
For each point in the grid, $i = 1, \ldots, n$, if there exists a Lyapunov function $V_i(\tilde{x})$ fulfilling,

$$V_i(\tilde{x}) > 0,\ \tilde{x} \neq 0,\ \forall \tilde{x} \in \mathcal{D}_i \tag{23a}$$
$$\dot{V}_i(\tilde{x}) \leq 0,\ \forall \Delta T^{\star 2} < {\Delta T^{\star}_{i,\max}}^2, \forall\, \tilde{x} \in \{\tilde{x} | V_i(\tilde{x}) = 1\},\ \forall \tilde{x} \in \mathcal{D}_i \tag{23b}$$
$$\tilde{\omega}^2_g \leq (\bar{\omega}_{g,i} - \omega_{g,\min})^2,\ \forall\, \tilde{x} \in \{\tilde{x} | V_i(\tilde{x}) \leq 1\},\ \forall \tilde{x} \in \mathcal{D}_i \tag{23c}$$

with

$$\dot{V}_i(\tilde{x}) = \frac{\partial V_i(\tilde{x})}{\partial x} \left(f_{CL,i}(\tilde{x}) + g_{T,i}(x)\Delta T^{\star} \right), \tag{24}$$

then, under incremental torque demands bounded by $\|\Delta T^{\star}\| < \Delta T^{\star}_{i,\max}$, the rotor speed fulfills $\omega_g \geq \omega_{g,\min}$.

Proof
Similar to Theorem 4.1, the first two constraints indicate that the state will always be contained in the set $V_i(\tilde{x}) \leq 1$ under null initial conditions and input torque signals bounded by $|\Delta T^{\star}| < \Delta T^{\star}_{i,\max}$. Condition (23c) implies the following bound on the incremental speed $\|\tilde{\omega}_g\|_{\infty} < \bar{\omega}_{g,i} - \omega_{g,\min}$, i.e. the absolute speed will fulfill $\omega_g > \omega_{g,\min}$. □

Remark 4.2

If one finds the maximum $\Delta T^{\star}_{i,\max}$ such that conditions (23) are fulfilled, then the maximum admissible $\Delta T^{\star}$ is obtained. As explained in Remark 4.1, this problem can be simplified to a numerically tractable one if it is reformulated as a sum of squares problem. In this case, the following optimization problem leads to the maximum torque step value that assures that the generator speed does not go below $\omega_{g,\min}$ p.u. for a given mean wind speed $\bar{v}_i$ ($i = 1, \ldots, n$):

$$\begin{aligned} &\max_{V_i, \tau_i} \; \Delta T^{\star}_{i,\max} \\ &\text{s.t. } V_i(\tilde{x}) - \epsilon \tilde{x}^T \tilde{x} - p_{1,i}(\tilde{x}) \in \Sigma \\ &\quad -\dot{V}(\tilde{x}) - s_i(\tilde{x})({\Delta T^{\star}_{i,\max}}^2 - \Delta T^{\star 2}) + q_i(\tilde{x})(V_i(\tilde{x}) - 1) - p_{2,i}(\tilde{x}) \in \Sigma \\ &\quad ((\bar{\omega}_{g,i} - \omega_{g,\min})^2 - \tilde{\omega}_g^2) - t_i(\tilde{x})(1 - V(\tilde{x})) - p_{3,i}(\tilde{x}) \in \Sigma, \\ &\quad s_i(\tilde{x}),\; t_i(\tilde{x}) \; \in \Sigma, \qquad \Delta T^{\star}_{i,\max} > 0, \end{aligned} \tag{25}$$

where $V_i(\tilde{x})$, $s_i(\tilde{x})$, $q_i(\tilde{x})$ and $t_i(\tilde{x})$ are polynomials to be obtained during optimization problem, Σ is the set of sum of squares polynomials, ϵ is a small positive constant, and where the derivative function $\dot{V}_i(\tilde{x})$, must be expressed as indicated in Theorem 4.2. Polynomials $p_{k,i}(\tilde{x})$ ($k = 1, \ldots, 3$) are included to restrict the search on the set $\mathcal{D}_i$, and they are formed as in (21), where $\alpha_{k,i,l} \in \Sigma$, $l = \{1, 2, 3\}$ are polynomials to be obtained during optimization problem.

After computation of this optimization problem over the n gridding points, a smooth function is proposed to be defined by interpolating the obtained values, leading to a bound of the maximum allowable torque as a function of the estimated wind mean speed

$$\Delta T^{\star} \leq \Delta T^{\star}_{\max}(\bar{v}). \tag{26}$$

If the incremental electromagnetic torque is obtained as proposed in equation (7) (as a result of a demanded incremental power $\Delta P^{\star}$), the maximum allowable value for the incremental power as a function of the wind mean speed is given by

$$\Delta P^{\star}_{\max}(\bar{v}) = \Delta T^{\star}_{\max}(\bar{v})\, \omega^{\star}_g(\bar{v}). \tag{27}$$

This function is represented in Fig. 1 above the dispatch function block, and in Fig. 6 in the numerical example section.

4.4. Evaluating the total energy loss

As shown in Fig. 10, when an incremental torque disturbance is added to the output of the speed controller (or an equivalent impulse is added to the integral term of the controller), while working in steady state on an optimum generation equilibrium point, the electrical power is increased during a short period, but then the power decreases, falling below the optimum point, and finally the power increases again until the machine recovers its optimum equilibrium point. Due to the nonlinear nature of the wind power capture (that depends on the turbine speed), during this operation, the total produced electrical energy is lower than the one that would have been obtained if no incremental power had been required.

Let us introduce a new variable e defined as

$$e(t) = \int_0^t \left(P(\tau) - \bar{P}\right) \mathrm{d}\tau, \tag{28}$$

that is the integral along time of the difference between the real generated power and the optimal generated power in steady state for a given mean wind speed (with no transient overproduction). This variable can be viewed as the total electrical energy deviation from the optimal production. If, at a given instant, the quantity e is positive, it means that the produced electrical energy is larger than the one that would be obtained if operating in steady state in the optimal equilibrium point. Note that this situation ($e(t) > 0$) is only possible during the transient time in which the machine is being decelerated and the electrical power is increased thanks to transformation of the stored kinetic energy on electrical one. Let us define the total injected energy as the value of $e(t)$ when $P(\tau) - \bar{P} = 0$, i.e. when $e(t)$ reaches its maximum positive value (see Fig. 7). If at a given instant of time t, $e(t) < 0$, then the total generated electrical power until time t is lower than the one that would have been obtained if operating at the optimal regime.

Let us also define the total energy loss as the quantity

$$L = -\lim_{t\to\infty} e(t) = -\lim_{t\to\infty} \int_0^t \left(P(\tau) - \bar{P}\right) \mathrm{d}\tau \tag{29}$$

In order to analyse the benefits of attending overproduction demands, the total energy loss should be evaluated for different operating conditions and incremental power signals $\Delta P^\star$ (bounded by $\Delta P^\star_{\max}(\bar{v})$ if speeds under the lower allowed limit must be avoided), leading to an evaluation function $L(\bar{v}, \Delta P^\star)$. In order to find this function, let us consider the state space nonlinear differential equation formed by (22) plus state equation

$$\dot{e} = P - \bar{P} = \bar{\omega}_{g,i}\tilde{T}_{em} + \tilde{\omega}_g \bar{T}_{em,i} + \tilde{\omega}_g \tilde{T}_{em}, \tag{30}$$

where $\bar{\omega}_{g,i}$ and $\bar{T}_{em,i}$ are the equilibrium speed and torque values for a given wind mean speed $\bar{v}_i$ of the previously defined grid.

The following numerical approximation is proposed. For each mean wind speed in the grid $\bar{v}_i$, m different incremental power values $\delta_{i,k}$ (for $k = 1, \ldots, m$) fulfilling

$$0 < \delta_{i,1} < \cdots < \delta_{i,m} = \Delta P^\star_{i,\max}$$

are taken (with $\Delta P^\star_{i,\max}$ the resulting maximum power demand obtained from optimization problem (25) and equation(27)). For this grid of the incremental power values, the loss function is calculated through numerical approximate integration[‡] of the resulting system of differential equations including e, leading to a matrix of loss function values $L_{i,k} = L(\bar{v}_i, \delta_{i,k})$ for $i = 1, \ldots, n$, $k = 1, \ldots, m$. The total energy loss function is proposed to be a smooth function $L(\bar{v}, \Delta P^\star)$ that

[‡]Note that this value cannot be obtained analytically due to the nonlinear behaviour of the system of differential equations

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approximates and upper bounds the previous points with a polynomial form as

$$L(\bar{v}, \Delta P^{\star}) = a(\bar{v}) + b(\bar{v})\Delta P^{\star} + c(\bar{v})\Delta P^{\star 2}, \tag{31}$$

where $a(\bar{v})$, $b(\bar{v})$ and $c(\bar{v}) > 0$ are polynomials functions on $\bar{v}$ (see Section 5 for the necessity of the positivity condition on $c(\bar{v})$). In order to obtain this upper bounding function, the following optimization problem is proposed

$$\begin{aligned} \min_{a(\bar{v}),b(\bar{v}),c(\bar{v})} \quad & \gamma \\ \text{s.t.} \quad & \gamma \geq 0, \; \sum_{i=0}^{n}\sum_{k=0}^{m} \epsilon_{i,k} \leq \gamma, \quad \epsilon_{i,k} \geq 0, \; \forall i,k \\ & c(\bar{v}) - s(\bar{v})(\bar{v} - \bar{v}_{\min})(\bar{v}_{\max} - \bar{v}) \in \Sigma, \quad s(\bar{v}) \in \Sigma \end{aligned} \tag{32}$$

where $a(\bar{v})$, $b(\bar{v})$ and $c(\bar{v})$ are polynomials to be obtained during the optimization (see Fig. 9) and where

$$\epsilon_{i,k} = a(\bar{v}_i) + b(\bar{v}_i)\Delta P^{\star}_{i,k} + c(\bar{v}_i)\Delta P^{\star}_{i,k} - L_{i,k}, \quad i = 1, \ldots, n; \; k = 1, \ldots, m$$

is the residual error of the polynomial approximation that wants to be minimized. Note that for a given mean wind speed, the loss function can be represented as a function of the incremental demanding power (see Fig. 8, leading to $L(\Delta P^{\star})$ (this is the right block above the dispatch function block in Fig. 1, representing a different second order polynomial for each value of the mean wind speed).

5. POWER DISPATCH FUNCTION

The wind farm eventually receives from the frequency controller a desired total incremental power demand $\Delta P^{\star}$ that must be achieved adding the powers of all wind turbines in the farm. Let us assume that a wind farm is formed by N wind turbines, and let us use the index j to enumerate each of the turbines. The dispatch function objective in Fig. 1 is to decide the optimal overproduction demand (in the sense of minimum total energy loss) to be sent to each wind generator controller. To achieve this goal, an optimization problem is proposed, taking into account that the following quantities or functions are available for the present operating point (the procedure to obtain them has been explained on previous sections): the maximum possible overproduction demand that can be supported by each generator ($\Delta P^{\star}_{j,\max}$), and the total energy loss evaluation function due to the overproduction demand ($L_j(\Delta P^{\star}_j)$).

For the given operating equilibrium point, as the mean wind speed is assumed to be known, the loss function $L_j(\Delta P^{\star}_j)$ is upper bounded by the second order polynomial

$$L_j(\Delta P^{\star}_j) = a_j + b_j\Delta P^{\star}_j + c_j\Delta P^{\star 2}_j$$

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where a_j, b_j, c_j are the evaluations of polynomials $a(\bar{v})$, $b(\bar{v})$ and $c(\bar{v})$ at the incident mean wind speed over turbine j, and where c_j is always a positive constant as $c(\bar{v})$ has been restricted to be positive.

The optimum overproduction demand for each one of the N turbines that are present in a wind farm is obtained by solving the following optimization problem:

$$\min_{\Delta P_j} \sum_{j=1}^{N} L_j(\Delta P_j^\star) \tag{33}$$

$$\text{s.t. } 0 \leq \Delta P_j^\star \leq \Delta P_{j,\max}^\star, \quad \sum_{j=1}^{N} \Delta P_j^\star = \Delta P^\star$$

where $\Delta P_j^\star$ are the decision variables, $L_j(\Delta P_j^\star)$ is the loss function and $\Delta P_{j,\max}^\star$ is the maximum admissible overproduction demand for wind generator j that depends on the estimated mean wind speed for that turbine (defined in Section 4.3). With this, the previous optimization problem can be rewritten as

$$\min_{x} \quad x^T H\, x + f\, x$$
$$\text{s.t. } A\, x \leq b, \quad C\, x = d$$

that is a standard quadratic optimization problem [29] in which matrix[§]

$$H = \mathrm{diag}\{c_1, \ldots, c_N\}$$

is positive definite, and therefore, a finite optimal solution can be found for the decision variables $x_j = \Delta P_j^\star$ ($j = 1, \ldots, N$), with standard optimization tools[¶].

This optimization problem should be solved each time a new transient incremental power is required from the frequency controller. The computational time needed to solve this kind of standard optimization problems depends polynomially on the number of decision variables (i.e. the number of wind turbines), and on the processor capacity. After several experiments with an Intel i5 processor it has been determined that the computational time is in the order of tenths of a second when the number of decision variables is below one hundred (hence for the number of wind turbines in usual wind farms).

6. SIMULATION RESULTS

The techniques developed in the previous sections have been tested for a given wind turbine whose parameters are as follows. First, the coefficients of the function (4) that generates the mechanical

[§] "diag" refers to a diagonal matrix with the indicated entries in its main diagonal.
[¶] As H is a positive definite matrix, this problem is convex, and so the minimum is a global minimum.

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torque as a function of the rotational and wind speed are

$$c_{00} = -0.02239, \quad c_{01} = 0.0939, \quad c_{02} = -0.4654, \quad c_{03} = 0.5032, \quad c_{10} = -0.0046,$$

$$c_{11} = 0.0545, \quad c_{12} = -0.1784, \quad c_{20} = 0.0075, \quad c_{21} = 0.0219, \quad c_{30} = -9.0580 \cdot 10^{-4}$$

The parameters of the mechanical model and the actuators are

$$H_t = 4.66\,\text{s}, \quad H_g = 1.92\,\text{s}, \quad K_{tg} = 218\,\text{p.u.}, \quad D_{tg} = 2.3\,\text{p.u.}, \quad \tau_v = \tau_{em} = 0.02\,\text{s}$$

First, a simulation model has been implemented, including all the effects considered in this work and also a wind generation model as stated in Section 2. Then, a wind and state observer has been implemented as explained in Section 3. The tuning parameters of the observer (elements of matrix Q) have been obtained as follows. First, a noise measurement with a variance of $R = 0.01$ has been assumed. As an starting point, the disturbance covariance matrix has been initially set at $Q = Q_0$:

$$Q_0 = \text{diag}\{4^2, 0.4^2, 0.4^2, 1.2^2, 0.01^2\},$$

where each diagonal entry has been initially fixed to the square of the range of the corresponding variable (mean wind speed: $v \in [6, 10]$, wind turbine and generator speeds: $\omega_t, \omega_g \in [0.8, 1.2]$, electromagnetic torque: $T_t \in [0, 1.2]$, and angular difference: $\theta \in [-0.005, 0.005]$). Then, the Kalman filter has been tested with matrix Q_0 and the elements corresponding to the states that presented a lower a priori error have been decreased accordingly to that a priori error. With this, a new matrix gain has been obtained:

$$Q_1 = \text{diag}\{4^2, 0.4^2, 0.2^2, 0.01^2, 0.01^2\}.$$

Note that the decreased elements are the ones related to the generator speed and electromagnetic torque because their equations are not directly related to the wind speed or the wind captured torque, the elements more responsible for the a priori error. Finally matrix Q_1 has been scaled leading to $Q = k \cdot Q_1$, where the parameter k has been set by trial and error to achieve a compromise between settling time of the observer due to wind variations, and covariance error on steady state, leading to $k = 0.3$, i.e.:

$$Q = \text{diag}\{4.8, 0.048, 0.012, 3 \cdot 10^{-5}, 3 \cdot 10^{-5}\}, \qquad R = 0.01.$$

Fig. 3 shows a five-hour wind speed simulation where the two components can be appreciated: a slowly time varying mean, and the turbulence. On the left is illustrated the mean wind speed estimate, while on the right a zoom in a reduced time interval is shown in order to appreciate the instantaneous wind speed estimation. As it can be seen, the estimation error is low, leading to a estimation error covariance of $1.9 \cdot 10^{-4}$.

Then, using the Yalmip parser [30] for defining optimization problems subject to sum of squares constraints, and the solver PENBMI[‖] [31–33], a polynomial controller has been found via the

[‖]PENBMI can handle optimization problems with bilinear matrix inequalities, as required by the proposed optimization problem.

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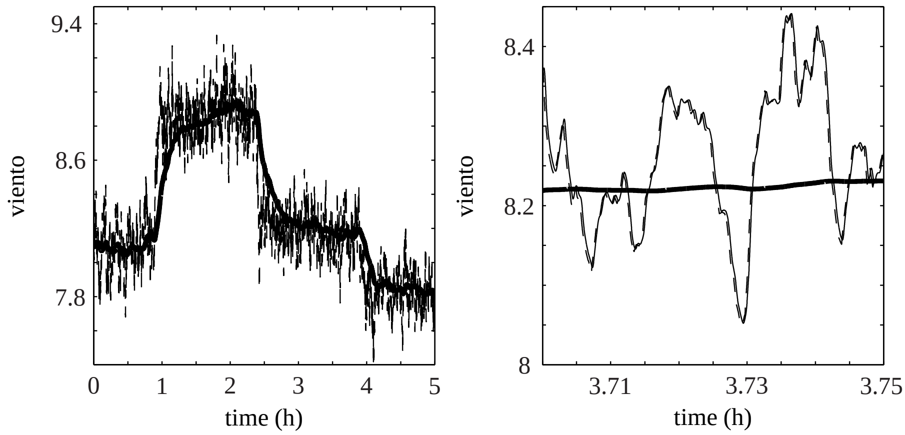

Figure 3. Five-hour wind speed estimation, where ('−−'): wind speed, ('-'): estimated wind speed, ('-'): estimated mean wind seed.

optimization problem (20) in order to fulfill the condition that the power derivatives do not exceed the maximum allowable value of $0.1/60$ p.u./s, leading to an optimization index $\gamma = 20.25$. The control action (including the polynomial controller) is implemented as**:

$$\begin{aligned} \omega_g^\star(t) &= \omega_g^\star(\bar{v}(t)), \\ \tilde{\omega}_g(t) &= \omega_g^\star(t) - \omega_g(t), \\ T_{em}^\star(t) &= -0.0872\tilde{\omega}_g(t) + 0.0018\tilde{\omega}_g(t)^2 - 0.0249\int_0^t \tilde{\omega}_g(\tau)d\tau. \end{aligned} \tag{34}$$

If a PI controller is designed instead of a polynomial one, via the optimization problem (20), with the same restrictions, the following PI controller is obtained

$$T_{em}^\star(t) = -0.0136\tilde{\omega}_g(t) - 0.0078\int_0^t \tilde{\omega}_g(\tau)d\tau, \tag{35}$$

that leads to the optimization index $\gamma = 39.69$. As it can be appreciated, using a polynomial controller improves significantly the performance of the control system.

The observer plus controller scheme has been tested in the developed simulation model, showing its effectiveness, as it can be observed in Fig. 4, where a simulation of the wind turbine with a mean wind speed of 8 m/s in a normal operation controlled by the polynomial controller (34) is shown. The fulfillment of the bound in the power derivative can be observed and the maximum value of the quadratic estimation error ($(x - \hat{x})^T(x - \hat{x})$) is 0.003. Moreover, for the same wind speed, Fig 5 shows the performance comparison between the polynomial controller in (34) and the PI controller in (35). As it can be observed, the polynomial controller leads to a less conservative result as the power derivative values are higher than for the PI controller, i.e., closer to the imposed bounds. The polynomial controller leads to an 1% increase of generated power due to the more efficient optimal generator speed tracking.

**We assume that the observer is properly tuned, and, therefore, both the mean wind speed ($\bar{v}$), and the generator rotational speed ω_g are available.

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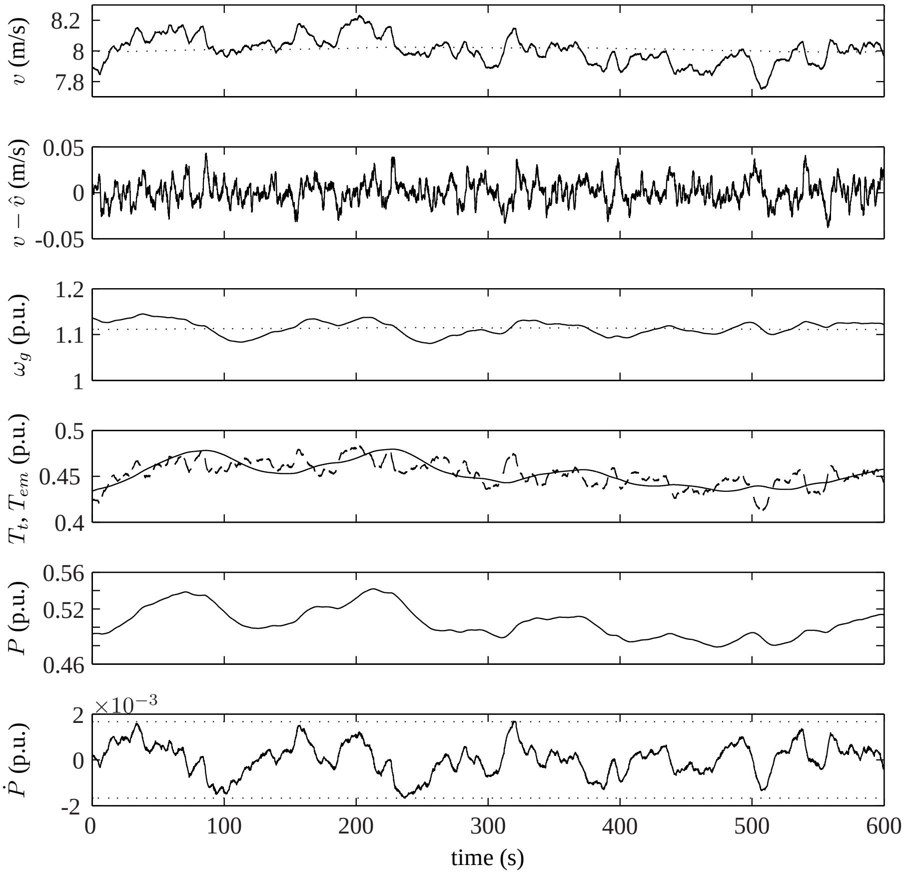

Figure 4. Wind turbine generator behavior in normal operation when $\bar{v} = 8$ m/s for the polynomial controller (34) showing: the wind speed and its mean ('··'), the estimation wind speed error, the generator rotational speed and its reference ('··'), the electromagnetic and aerodynamic ('−−') torques, the generated power and its derivative with the imposed bounds. Maximum value of the quadratic estimation error $((x-\hat{x})^T(x-\hat{x}))$ of 0.003.

With the obtained polynomial controller and the resulting closed loop system model, the bound on the maximum allowable incremental power demand inputs has been obtained as a function of the mean wind speed with the help of optimization problem (25) and equation (27), leading to the bounds shown in Fig. 6. It can be appreciated that the theoretical obtained bounds are close to bounds obtained in simulation and always below them. Then, the total injected energy and energy loss have been obtained as a function of the mean wind speed and demanding power as explained in Section 4.4, leading to the respective curves that can be observed in Fig. 7 and Fig. 8 for different mean wind values. Moreover, in Fig 9 the parameters $a(\bar{v})$, $b(\bar{v})$, $c(\bar{v})$ obtained in (32) of the second order polynomial proposed in (31) that model the loss energy function $L(\bar{v}, \Delta P^{\star})$ are shown.

A simulation of the system behaviour, starting at the equilibrium point defined by a constant wind speed of $\bar{v} = 8m/s$, under an impulse input $\Delta P^{\star} = 0.2$ (the maximum allowed power increase that has been predicted) is shown in Fig. 10. It can be appreciated how the previous functions have been able to predict that for the maximum allowed power overproduction: (i) the rotor speed reaches a minimum value close to $\omega_{g,\min} = 0.8$ p.u., (ii) the total injected energy is 2.2 p.u.·s (see Fig. 7) and (iii) the total energy loss is 4 p.u.·s (see Fig. 8).

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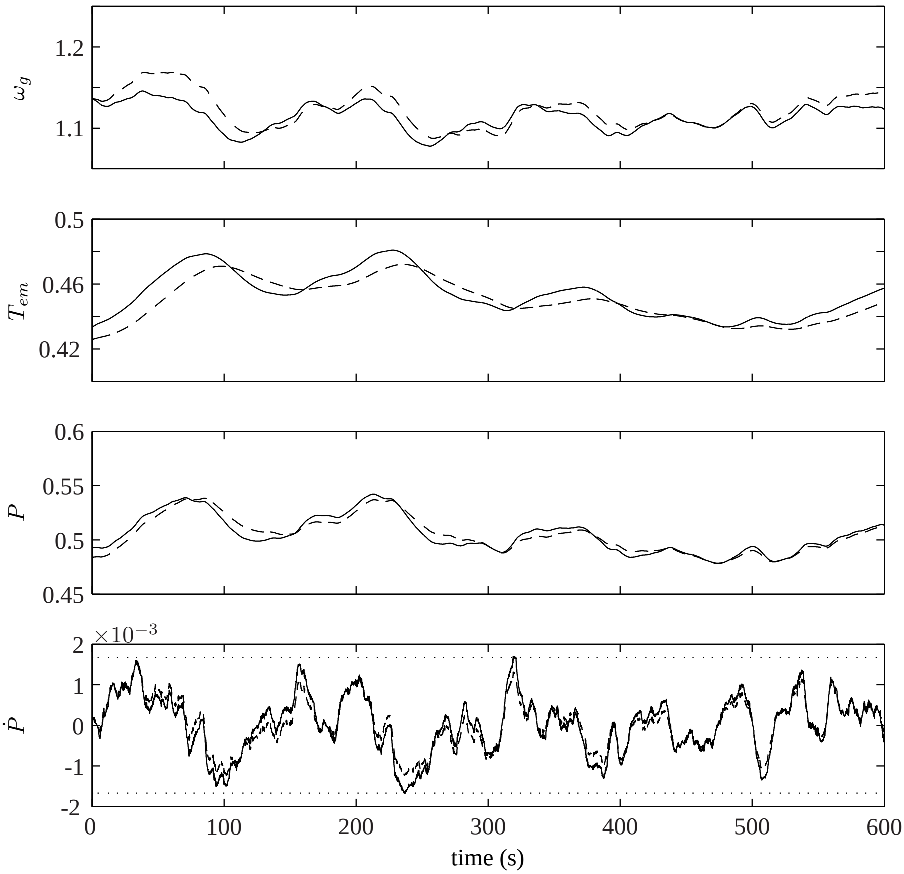

Figure 5. Performance comparison between the polynomial controller (34) ('-'), and the PI controller (35) ('−−') showing: the generator rotational speed, the electromagnetic torque, the generated power and its derivative with the imposed bounds.

With these functions, the proposed dispatch strategy explained in Section 5 has been tested for the following case. Let us assume that a wind farm is formed by 25 wind turbines ($N = 25$) controlled with the previously computed controller. Let us also assume that the turbines are being affected by winds with a different mean speed depending on their location on the farm and those mean wind speeds are equally spaced within the range $\bar{v} \in [6, 10]$. Consider that, initially, the wind turbines are operating in steady state at their equilibrium point. Consider now that an incremental power demand $\Delta P^{\star}$ of 10% of the actual total generated power in the farm is required by a frequency controller, and has to be dispatched within the farm. Let us compare the proposed dispatch strategy (solving optimization problem (33)) with a proportional approach [15] in which each wind turbine j receives an incremental power demand proportional to its contribution to the global power production in the farm as

$$\Delta P_j^{\star} = \frac{P_j}{\sum_{k=1}^{N} P_k} \Delta P^{\star}. \tag{36}$$

The comparative results of the power dispatch strategies are shown in Fig. 11, where it can be observed that the optimization approach proposes a lower power increase to the wind turbines turning faster as compared to the proportional approach. Furthermore, the proposed optimization approach fits the power contribution of the slow wind turbines to its maximum allowable value

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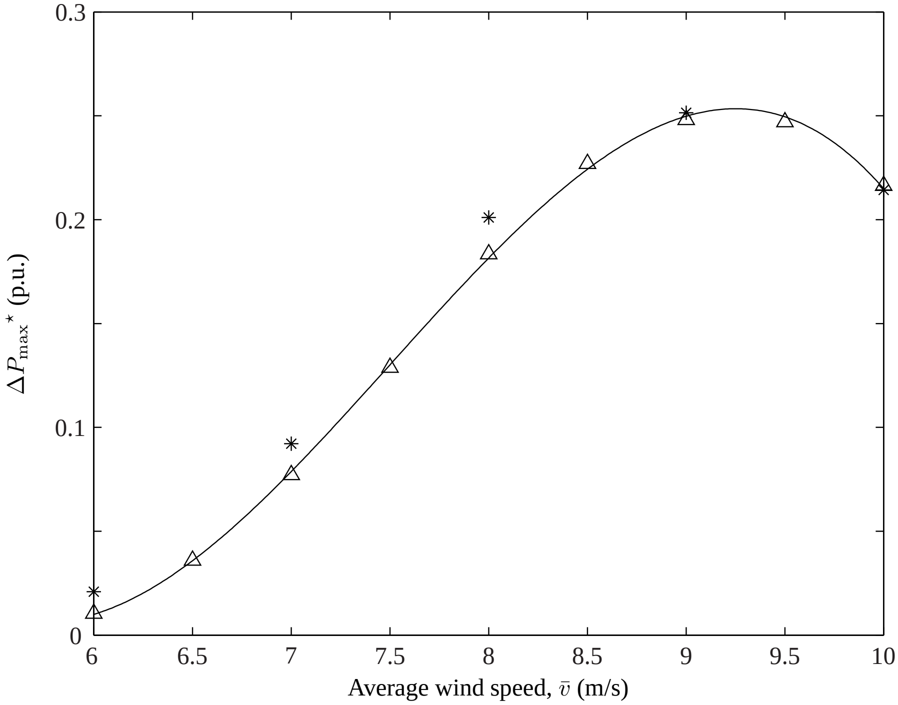

Figure 6. Maximum power overproduction that can be sent to a wind generator, where Δ represents the values calculated using the optimization problem (25) and equation (27), and $*$ represents the values obtained from simulations.

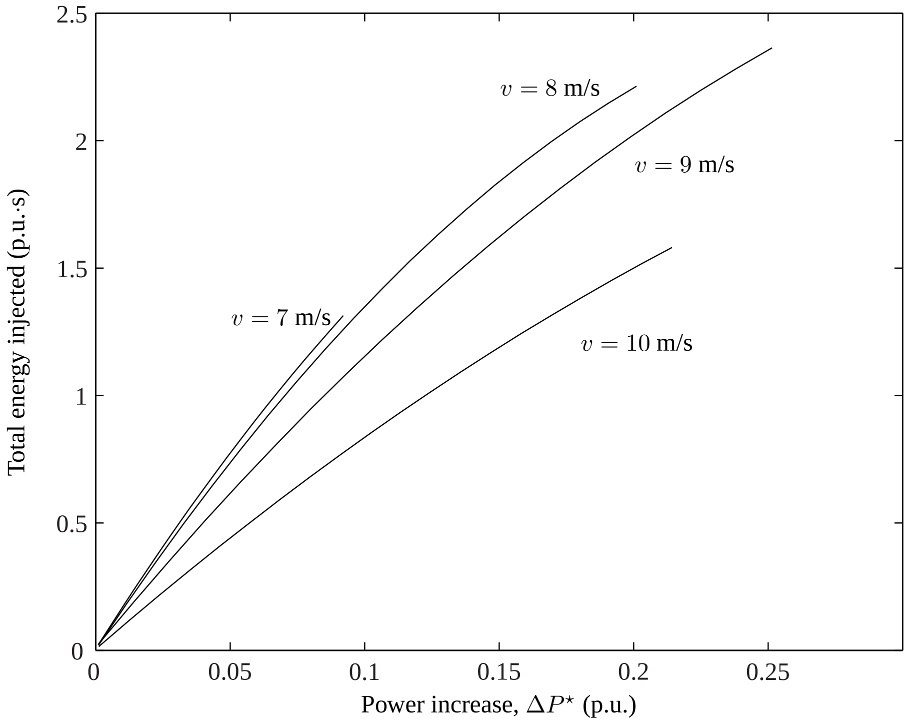

Figure 7. Injected energy as a function of the power overproduction demand for different mean wind speeds.

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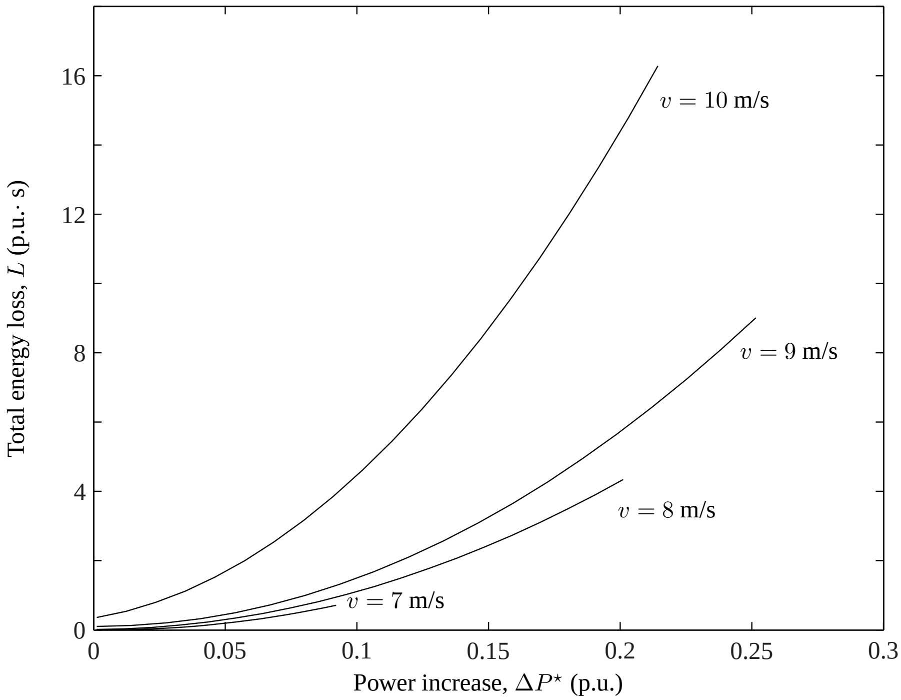

Figure 8. Total energy loss as a function of the power overproduction demand for different mean wind speeds.

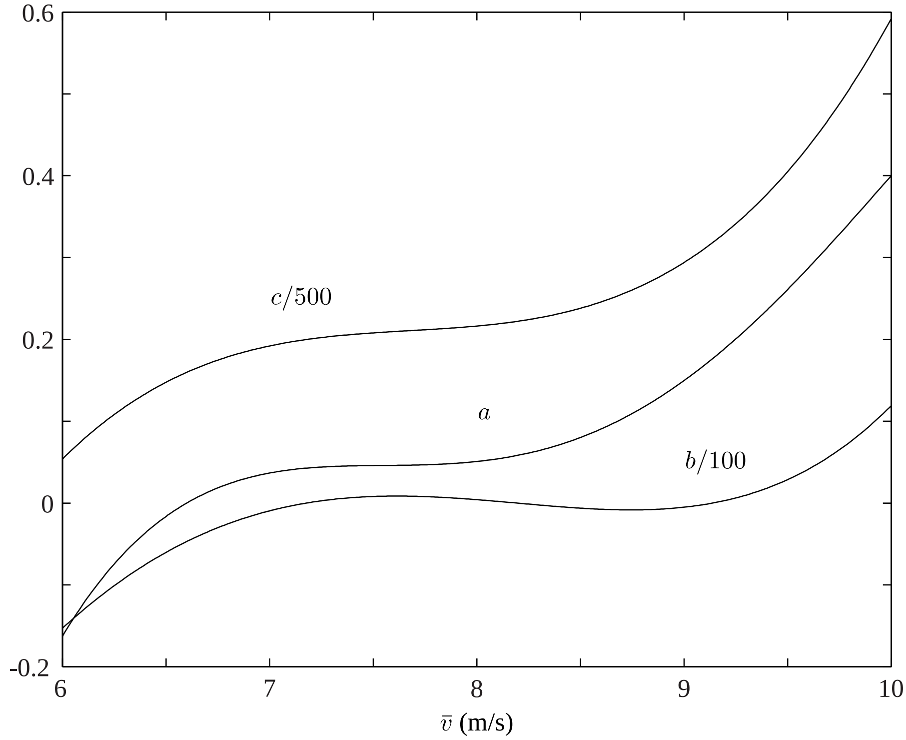

Figure 9. Parameters $a(\bar{v})$, $b(\bar{v})$, $c(\bar{v})$ of the total energy loss modeled by a second order polynomial $L(\bar{v}, \Delta P^{\star})$ (see (31)), obtained from the optimization problem (32).

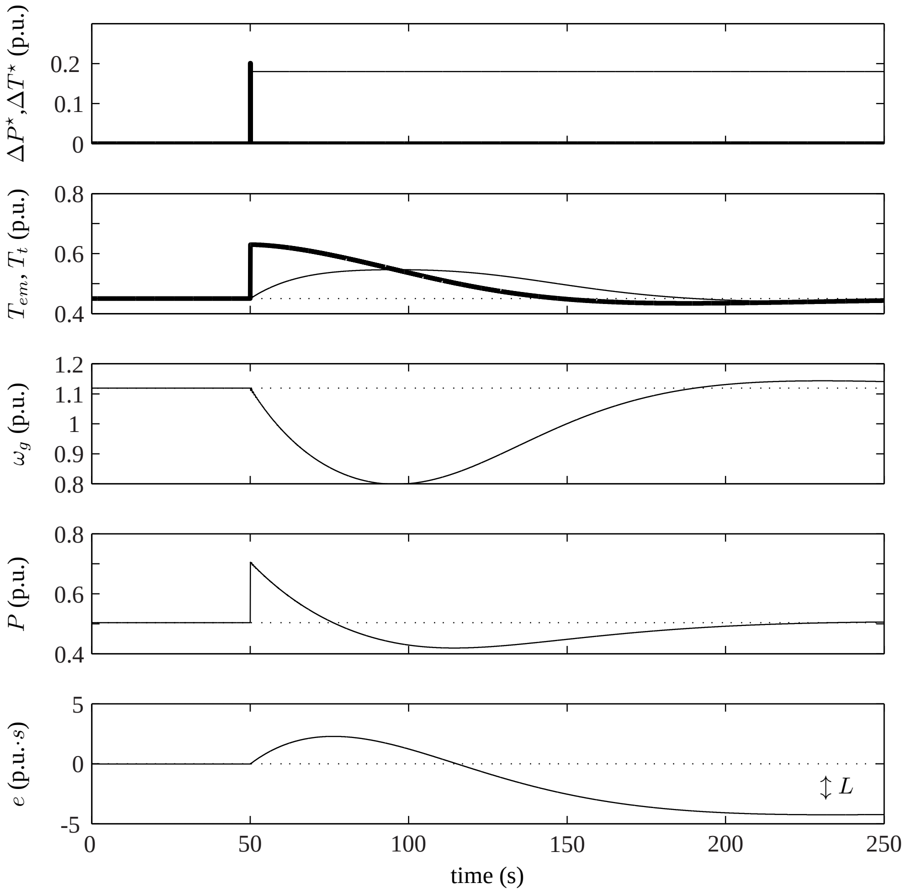

Figure 10. Wind turbine generator behaviour after a power increase demand of $\Delta P^\star = 0.2$ starting from the operating point defined by a constant wind speed $\bar{v} = 8m/s$, showing: the peak power overproduction demand ('**-**') and the resulting incremental torque, the electromagnetic ('**-**') and aerodynamic torque, the generator rotational speed and its optimal value ('··'), the generated power and the generated energy.

compatible with the required speed range, while the proportional approach leads to an excessive deceleration of the slowest turbines (leading to rotational speed below the minimum allowed speed). Finally the total loss function has been evaluated with both strategies, leading to a total loss of 12.88 p.u.·s in the proposed approach, while the total loss in the proportional dispatch approach [15] is 19.22 p.u.·s. A save of the 33% of the total energy loss produced by the required overproduction transient is achieved by the proposed approach with respect to proportional approaches.

7. CONCLUSION

In this work, an advanced control strategy for wind turbines has been proposed maximizing the electrical power generation while bounding the power variations in normal operation. The proposed controller structure allows eventually providing grid support by producing transient power increases (using the stored kinetic energy) in response to an eventual demand from a frequency controller.

A polynomial model for a doubly fed induction generator has been first developed. Based on that model, a wind and state observer has been proposed, leading to an algorithm that estimates the

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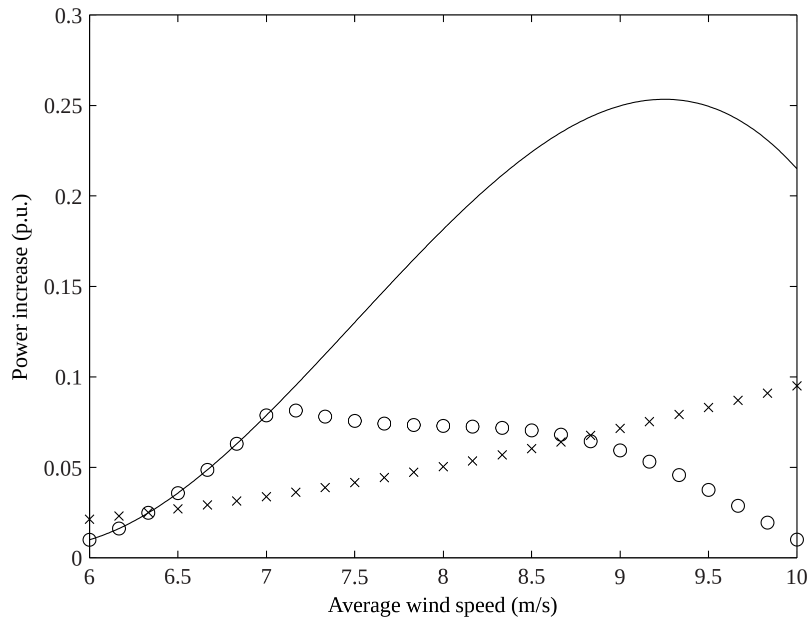

Figure 11. Power overproduction distribution for 25 turbines in a farm to respond to a 10% incremental power demand using two strategies: ∘ for the proposed approach solving the optimization problem (33), and × for the proportional approach in [15] using (36). The continuous line represents the maximum allowable power increment without violating minimum velocity constraints.

mean and variance of the wind speed in real-time during wind turbine operation. For the proposed model, a polynomial controller guaranteeing some given constraints on the variations with time on the generated power has been designed. With the designed controller, bounds on the maximum non destabilizing allowable incremental torque or power demand for transient overproduction operation have been obtained. A total electrical loss function has also been defined, allowing us to quantify the negative effect that overproduction transients cause on the total produced energy.

Finally, this loss function has been used to develop a dispatch strategy that decides the power increment that is demanded to each wind turbine on a farm when a frequency controller demands a transient effort to the plant to provide grid support. The proposed dispatch strategy minimizes the total electrical loss on the wind farm due to the overproduction transient.

All the proposed strategies have been translated to numerically tractable problems and their effectiveness have been demonstrated through several simulations.

APPENDIX

The following result justifies the simplifications of the optimization problems presented in sections 4.2 and 4.3, needed to obtain the controller and the quantities that allow us to describe the behaviour. They can be derived from the called Positivstellensatz result [22] which states that feasibility conditions over polynomials can be dealt with by looking for some sum of squares. On the other hand, it can be demonstrated that the feasibility problem of expressing a polynomial as

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a sum of squares is equivalent to solve a Semidefinite Programming Problem with Linear Matrix Inequalities constraints.

Lemma 7.1
If there exist sum of squares polynomials $s_i(x)$ ($i = 1, \ldots, n$, $x \in \mathbb{R}^n$) and polynomial $q(x)$ such that

$$f(x) - \sum_{i=1}^{n} s_i(x)\, g_i(x) + q(x)\, l(x) \in \Sigma,$$

being Σ the set of sum of squares polynomials in $\mathbb{R}^n$, then, the following condition holds

$$f(x) \geq 0, \quad \forall\, g_i(x) \geq 0,\ l(x) = 0.$$

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