Noname manuscript No.
(will be inserted by the editor)

State-Space Time Series modelling in Panels: A TVP application to the Feldstein-Horioka Puzzle

Mariam Camarero · **Juan Sapena** · **Cecilio Tamarit**

Received: date / Accepted: date

Abstract In this paper, we develop a State-Space framework for modeling panel time series data. Our research extends the simple canonical model generally employed in the literature, into a panel-data time-varying parameters framework, combining both fixed (either common and country-specific) and varying components. Under some specific circumstances, this setting can be understood as a mean-reverting panel time-series model, where the mean fixed parameter can, at the same time, include a deterministic trend. Regarding the transition equation, our structure allows for the estimation of different autoregressive alternatives, and include control instruments, whose coefficients can be set-up either common or idiosyncratic. This is particularly useful to detect asymmetries among individuals (countries) to common shocks. We develop a GAUSS code that allows for the introduction of restrictions regarding the variances of both the transition and measurement equations. Finally, we use this empirical framework to test for the Feldstein-Horioka puzzle in a 17-country panel. The results show its usefulness to solving complexities in macroeconomic empirical research.
JEL Classification: C23, F32, F36.

Keywords Feldstein-Horioka puzzle · panel unit root tests · multiple structural breaks · common factors · Kalman Filter · Time varying parameters

Authors gratefully acknowledge the financial support from Ministerio de Economía, Industria y Competitividad (MINEIC) and FEDER project ECO2017-83255-C3-3-P. Juan Sapena also acknowledges the financial support from the Generalitat Valenciana (project GV/2017/052) and the Chair "Betelgeux" for a Sustainable Economic Development. This paper has been developed within the research thematic network financed by MINEIC. The usual disclaimer applies.

Mariam Camarero
Universitat Jaume I, INTECO Joint Research Unit. Department of Economics. Campus del Riu Sec, E-12071 Castellon de la Plana, Spain
Tel.: +34-964728590
E-mail: camarero@uji.es

Juan Sapena
Catholic University of Valencia, Faculty of Economics & Management. 34 Calle Corona, E-46003 Valencia, Spain
Tel.: +34-963637412 - Ext 4360
E-mail: juan.sapena@ucv.es

Cecilio Tamarit
University of Valencia, INTECO Joint Research Unit. Department of Applied Economics II. PO Box 22.006 - E-46071 Valencia, Spain
Tel.: +34-963828361
E-mail: cecilio.tamarit@uv.es

1 Introduction

In this paper, we develop a state-space framework for modelling panel time series. Since its origin in Systems Theory, and famous early applications in the Apollo and Polaris aerospace program, State-space models have been extensively used in Economics and Finance literature, along with their estimation through the popular Kalman filter algorithm (Kalman (1960); Kalman and Bucy (1961)). State-space models can be usefully employed to address macroeconomic problems, either because some variables are unobservable, or just because they contain coefficients that are inherently time-varying, making economic relationships potentially unstable.
During the last decades, State-space modelling has been increasingly used in Economics, proving to be a very useful tool providing simple representations of relatively complex problems. Nevertheless there are very few contributions that discuss how those models can be used in practice and why they are such a powerful tool for practitioners. On the contrary, most of the empirical research in the area of Economics and Finance that employs a state-space framework, has restricted the general model by introducing simplifying assumptions to the general framework (i.e. assuming random-walk type state equations, no stochastic control instruments, restrictions on the variance of measurement errors, etc.).
Conversely, our research contributes to this strand of the literature by extending the simplest canonical framework into a panel-data time-varying frame, that combines both fixed and varying parameters in the measurement equation. Under some specific circumstances (see, for example, Hamilton 1994a), this setting can be understood as a mean-reverting panel time-series model. We depart from James Hamilton's univariate code in several respects[1]. We have programmed a new GAUSS code extending the seminal code to admit the introduction of restrictions regarding the variances of both the transition and measurement equations. First, our empirical model is setup with the addition of a potential deterministic trend in the fixed parameter. Second, the panel modelization has been enriched allowing for the inclusion of either common and country-specific fixed parameters, and a common unobservable factor with idiosyncratic fixed parameters in the measurement equation. Third, regarding the transition equation, our structure enables the estimation of different autoregressive alternatives, where control instruments are also granted. Besides, their coefficients can be set-up either common or idiosyncratic, what is particularly interesting for detecting asymmetries among individuals (countries) to common shocks. Finally, in order to check its adequacy when solving complexities in macroeconomics empirical research, we conduct an empirical application of the proposed statistical frame, reexamining the so-called Feldstein-Horioka (FH hereafter) puzzle, that is, the "unexpected" correlation of domestic investment to domestic savings in a context of apparent deregulation and liberalization of international financial markets. Hence we estimate the model for a 17-country panel including both 12 Eurozone member countries and 5 industrialized countries, using the previous dynamic framework for the period 1970-2016. The main advantage of our empirical approach is that it captures the dynamics of the FH coefficient, highly consistent with the increased financial integration. Global risk and country size are relevant elements to unpuzzle the savings-investment correlation. The inclusion of time-varying estimates reveals certain heterogeneity across EMU countries and shows how asymmetric shocks, both idiosyncratic or commons, affect the circumstances under which domestic investment might be constrained by savings retention.
Our research extends the existing literature in several directions. First, we address the FH puzzle (by departing from both the customary cross-country averaging or single-parameter time series estimation) and introduce a time-varying-parameter (TVP) model to uncover the potential dynamic evolution of the FH coefficient. We also contribute to the literature by trying to ascertain the role of global risk aversion measures on the savings-investment regression, allowing for different time-varying responses across countries after the Great Recession.
The rest of the article is organised as follows: Section 2 introduces the state-space framework and the Kalman filter algorithm, Section 3 adapts it into a time-varying parameters model and proposes a strategy for multi-country panel. Section 4 presents the empirical application for the Feldstein-Horioka puzzle, introduces the dataset and formulates the model in the state-space framework; Section 5 presents and discusses the results, and Section 6 concludes.

2 State-Space Models

The state-space representation of a linear system models the dynamics of an observed $(n \times 1)$ vector y_t, in terms of a possibly unobserved (or state) $(r \times 1)$ vector ξ_t. [2]. The Kalman Filter is, essentially, an algorithm for generating

[1] The code can be downloaded at http://econweb.ucsd.edu/ jhamilto/KALMAN.ZIP.

[2] Excellent textbook treatments of state-space models are provided in Harvey (1993, 1989), Hamilton (1994a,b), West and Harrison (1997), or Kim and Nelson (1999), among others. They all use different conventions, but the notation used here is based on James Hamilton's, with slight variations.

minimum mean square error forecasts in a state-space model [3]. In state-space time-series models, (each of) the unobserved components determining the time-series behavior has a stochastic nature, which is governed by a linear stochastic difference equation (or state-transition equation):

$$\underset{(r\times1)}{\xi_{t+1}} = \underset{(r\times r)}{F}\underset{(r\times1)}{\xi_t} + \underset{(r\times s)}{B}\underset{(s\times1)}{Z_{t+1}} + \underset{(r\times1)}{\nu_{t+1}} \tag{1}$$

where F denotes an $(r\times r)$ state-transition matrix for the unobserved component, and ν_t represents the $(r\times 1)$ vector of random variables containing the process noise terms for each parameter in the state vector and is assumed to be i.i.d. $N(0,Q)$, where

$$E(\nu_{t+1}\nu_{t+1}^{'}) = \underset{(r\times r)}{Q} \tag{2}$$

and variance-covariance equal to,

$$E(\nu_{t+1}\nu_{\tau+1}^{'}) = \begin{cases} \mathrm{Q} & for \quad t=\tau \\ 0 & for \quad t\neq\tau \end{cases} \tag{3}$$

Finally, Z_t is a vector containing any control inputs, either deterministic (drift and/or deterministic trend) or stochastic. If present, control inputs affect the state through the $(r\times 1)$ control input matrix, B, which applies the effect of each control input parameter in the vector on the state vector. The introduction of stochastic control inputs is not frequently employed in empirical economic research, but it is common practice in engineering and control theory [4]

The measurement or observation equation models the dynamics of the observable variables that are assumed to be related to the state vector, providing information on ξ_t. It takes the following general form:

$$\underset{(n\times1)}{y_t} = \underset{(n\times k)}{\mathbf{A}'}\underset{(k\times1)}{x_t} + \underset{(n\times r)}{\mathbf{H}'}\underset{(r\times1)}{\xi_t} + \underset{(n\times1)}{w_t} \tag{4}$$

where y_t represents an $(n\times 1)$ vector of variables that are observed at date t, x_t represents a $(k\times 1)$ vector of exogenous determinants, their coefficients being included in the $(k\times n)$ matrix A. H is an $(r\times n)$ matrix of coefficients for the $(r\times 1)$ vector of unobserved components ξ_t.
The measurement error, w_t is an (n x 1) vector assumed to be i.i.d. $N(0,R)$, independent of ξ_t and ν_t and for $t=1,2,...$, where

$$E(w_t w_t^{'}) = \underset{(n\times n)}{R} \tag{5}$$

and variance covariance equal to,

$$E(w_t w_\tau^{'}) = \begin{cases} \mathrm{R} & for \quad t=\tau \\ 0 & for \quad t\neq\tau \end{cases} \tag{6}$$

Finally, (7) represents the initial condition of the system:

$$\xi_1 \sim N\left(\xi_{1|0}, P_{1|0}\right). \tag{7}$$

Writing a model in state-space form means imposing certain values (such as zero or one) on some of the elements of F, Q, B, A, H and R, and interpreting the other elements as particular parameters of interest. Typically we will not know the values of these other elements, but need to estimate them on the basis of observation of $(y_1,y_2,...,y_T)$ and $(x_1,x_2,...,x_T)$. In its basic form, the model assumes that the values of F, (and eventually B), Q, A, H and R are all fixed and known, but (some of them) could be functions of time.
This kind of models is particularly useful for measuring expectations that cannot be observed directly. If these expectations are formed rationally, there are certain implications for the time-series behavior of the observed series that can help to modelize them.
According to Harvey (1989), it does not exist a unique representation of a state-space formulation of a model. That is why the state variables obtained internally in the system have to be specified according to the nature of the problem with the ultimate goal of containing all the information necessary to determine the behavior of the period-to-period system with the minimum number of parameters.

[3] See Appendix 6 for a presentation of the rationale and the elements of the Kalman filter, which mainly consist of the sequence of predicting and updating equations.

[4] See Faragher (2012).

3 A Time-Varying Parameters State-Space framework for panel data

In this section we explain in detail different alternatives to estimate time-varying coefficient regression models and their implications, going from simpler specifications to more complex ones that have been covered by our Gauss code for computation.

3.1 Time-varying coefficient regression models

Unlike in standard linear econometrics, the time-varying parameters approach takes as its point of departure the idea that there may exist an underlying changing economy, where some shocks lead to permanent changes in the driving parameters of the econometric models. Swamy and Tavlas (2003) discuss how this technique, particularly in the ”second generation models” that include a set of ‘driving’ variables, are able to overcome a wide range of model misspecification and produce consistent estimates of the parameters. All in all, any econometric model is almost certainly a misspecified version of truth, and this misspecification may take the form of omitted variables, endogeneity problems, measurement errors, and incorrect functional form. In fact, as stated by Granger (2008), the true model tends to be highly non-linear. These problems are expected to lead to coefficients that will be unstable and time-varying $y_t = x_t \beta_t + \omega_t$.
Although an important number of econometric models with time-varying parameters for panel data have been proposed in the past[5], these models have not become popular in empirical research mostly because of computational difficulties. Wells (1996) distinguishes four different categories of models: Ordinary Least Squares (OLS), random coefficients (RCF), random walks (RW) and mean reverting (MRV) models. In traditional OLS, the regression coefficients remain constant for all periods:

$$\beta_t = \bar{\beta} \tag{8}$$

Hildreth and Houck (1968) and Swamy (1970) proposed Random Coefficient (RCF) models[6], whose coefficients fluctuate randomly about a mean value as follows:

$$\beta_t = \bar{\beta} + \nu_t \tag{9}$$

where ν_t is a random variable following a Gaussian distribution with zero mean and fixed variance.
The third of the four models listed above is the Random Walk (RW) model, first introduced by Rubin (1950) and Rao (1965), that can be written as:

$$\beta_t = \beta_{t-1} + \nu_t \tag{10}$$

Finally, the Mean Reverting (MRV) model was proposed by Bos and Newbold (1984) as:

$$\beta_t = \Phi \beta_{t-1} + (1 - \Phi)\bar{\beta} + \nu_t \tag{11}$$

where ν_t is a gaussian error term with a zero mean and a fixed variance, forcing the parameters return to its mean gradually.
The mean-reverting model represents a general [7] modelisation of the parameters that nests the rest: the OLS model is derived when $var(\nu_t = 0)$; when $\Phi = 1$ we obtain a random walk (RW) model for the varying parameters; and when $\Phi = 0$ we have a random coefficient model (RCF) where the coefficient fluctuates randomly about a mean value. If $\Phi < 1$ the model is convergent (even if convergence is slow).
For convenience, the MRV model can be rewritten as:

$$\left(\beta_t - \bar{\beta}\right) = \Phi\left(\beta_{t-1} - \bar{\beta}\right) + \nu_t \tag{12}$$

that focuses on the transition of the time evolution of the varying component of the parameter.

[5] See Rosenberg (1973), Hsiao (1974), Hsiao (1975), Min and Zellner (1993), Swamy and Mehta (1977), Zellner et al (1991) among others.
[6] Also known as “dispersed coefficient models” (Schaefer et al 1975) or “mean reverting models”.
[7] Ohlson and Rosenberg (1982) formulation of the MRV model that allows for both autocorrelated (predictable) and random (unpredictable) variation within the same model, combining mean reversion to a random mean for parameters, where $\left(\beta_t - \bar{\beta}\right) - \xi_t = \Phi\left[\left(\beta_{t-1} - \bar{\beta}\right) - \xi_t\right] + \nu_t$. In this model, the constant “true” mean of the parameter, $\bar{\beta}$, is perturbed by a random variable, ν_t which has a zero mean and a variance of λ (if $\lambda = 0$, then this model becomes the MRV presented above). $y_t = \left(\bar{\beta} + \xi_t\right)x_t + \left[\beta_t - \left(\bar{\beta} + \xi_t\right)\right]x_t + \omega_t$. This model allows an heteroscedastic variance in the measurement equation, induced by the tendency of the mean of the parameter to vary randomly about its “true” value, being then $u_t = x_t \xi_t + \omega_t$.

3.2 Time-Varying State-Space Models

The time-varying coefficient regression models constitute an interesting application of the state-space representation, as:

$$\underset{(n\times 1)}{y_t} = \underset{(n\times k)}{\mathbf{A}'} \times \underset{(k\times 1)}{x_t} + \underset{(n\times r)}{\mathbf{H}'(x_t)} \times \underset{(r\times 1)}{\xi_t} + \underset{(n\times 1)}{w_t} \tag{13}$$

where A represents a matrix of fixed parameters $\bar{\beta}$. Compared to the general model where the elements of the matrices F, Q, A, H and R are treated as constants, in this model H depends on the observed regressors, as $[H(x_t)]' = x_t$. As stated in Hamilton (1994a), assuming that the eigenvalues of F in (1) are all inside the unit circle, the coefficients can be interpreted as the average or steady-state coefficient vector, and the measurement equation can be written as follows:

$$y_t = x_t^{'}\bar{\beta} + x_t^{'}\xi_t + \omega_t \tag{14}$$

where the vector of unobserved coefficients, $\xi_t = \left(\beta_t - \bar{\beta}\right)$, evolves along time according to the expression:

$$\left(\beta_{t+1} - \bar{\beta}\right) = F\left(\beta_t - \bar{\beta}\right) + \underset{(r\times s)}{B}\underset{(s\times 1)}{Z_t} + \upsilon_{t+1} \tag{15}$$

Equation (15) represents a simple transition equation to be estimated through Kalman Filter, where $\left(\beta_t - \bar{\beta}\right) = \xi_t$ represents the unobserved component of our time-varying parameter, while the fixed component is also included at the measurement equation as $\bar{\beta}$.

3.3 A Panel Time-Varying State-Space extension

In this subsection, we extend the previous time-varying parameters model to a panel setting. Our main goal is to explore the use of the state-space modelisation and the Kalman filter algorithm as an effective method for combining time series in a panel, providing a flexible structure that considers different potential sources of cross-section heterogeneity affecting the model specification. This approach can be a superior alternative to the estimation of the model in unstacked form, commonly employed when there is a small number of cross-sections.
The general model can be presented as follows:

$$y_{i,t} = x_{i,t}^{'}\bar{\beta} + x_{i,t}^{'}\xi_{i,t} + \omega_t \tag{16}$$

or in matrix form:

$$\underset{(n\times 1)}{y_t} = \underset{(n\times n*k)}{\mathbf{A}'} \times \underset{(n*k\times 1)}{x_t} + \underset{(n\times r)}{\mathbf{H}'(x_t)} \times \underset{(r\times 1)}{\xi_t} + \underset{(n\times 1)}{w_t} \tag{17}$$

representing the measurement equation for a $y_t \in \mathbb{R}^n$ vector containing the dependents for a panel of countries. $x_t \in \mathbb{R}^{k\times n}$ is a vector of k exogenous variables, including either (or both) stochastic and deterministic components. The unobserved vector $\xi_t \in \mathbb{R}^r$ influences the dependent through a varying $H'(x_t)$ $(n\times r)$ matrix, whose simplest form is just $H'(x_t) = x_t$. Finally, $w_t \in \mathbb{R}^n$ represents the $(n\times 1)$ vector of N measurement errors.
The specification of the model in Equation (17) relies on a MRV-type modelisation of the measurement equation, which allows for the inclusion of fixed parameters, defined either as a common parameter for all the agents in the panel, $\bar{\beta}$, or country-specific coefficients, $\bar{\beta}_i$. The model also includes time-varying parameters (ξ_t) for some of the regressors, that eventually can be interpreted as deviations from those mean parameters ($\left(\beta_{it} - \bar{\beta}_i\right) = \xi_t$).
The measurement equation for each i-th element for the t-th period $(y_{i,t})$ in the vector of dependents, can be expressed as follows:

$$\begin{aligned} y_{i,t} = & \sum_{ks=ksmin}^{ksmax} \bar{\beta}_{ks,i} x_{ks,i,t} + \sum_{kc=kcmin}^{kcmax} \bar{\beta}_{kc} x_{kc,i,t} \\ & + \sum_{kv=kvmin}^{kvmax} \xi_{kv,it} x_{kv,i,t} + h_i \xi_{r,it} + \underset{(n\times 1)}{w_{it}} \end{aligned} \tag{18}$$

In Equation (18), the measurement equation for each of the individuals (countries) included in the panel allows for the potential inclusion of both fixed (mean) and/or time-varying parameters for the regressors included in the

model to be estimated. For the fixed parameters case different partitions can be considered. First, one can choose a subset of $ksnum = ksmax - ksmin + 1$ regressors $(x_{ksmin}, ..., x_{ks}, ..., x_{ksmax})$ in the interval $[0,k]$, whose coefficients will be modelled as country-specific or idiosyncratic. A second subset of $kcnum = kcmax - kcmin + 1$ regressors $(x_{kcmin}, ..., x_{kc}, ..., x_{kcmax})$, also defined in the interval $[0,k]$, is also possible, but related in this case to the dependent vector through a common coefficient for all the countries included in the panel. Last, the varying parameters are associated to a third subset of $kvnum = kvmax - kvmin + 1$ regressors $(x_{kvmin}, ..., x_{kv}, ..., x_{kvmax})$, also belonging to the interval $[0,k]$,
Moreover, we can also consider the possibility of the fixed parameters being affected by a deterministic time trend:

$$y_{i,t} = \sum_{ks=ksmin}^{ksmax} (\bar{\beta}_{ks,i} + \bar{\delta}_{ks,i} \cdot t) x_{ks,i,t} + \sum_{kc=kcmin}^{kcmax} (\bar{\beta}_{kc} + \bar{\delta}_{kc,i} \cdot t) x_{kc,i,t} + \sum_{kv=kvmin}^{kvmax} \xi_{kv,it} x_{kv,i,t} + h_i \xi_{r,it} + \underset{(n\times 1)}{w_{it}} \tag{19}$$

Equation (19) presents a more general panel representation of the measurement equation, that also incorporates a deterministic time trend accompanying (or not) the fixed parameters explained above, and modelled either idiosyncratic $(\delta_{ks,i} \cdot t)$ or common for all the countries $(\delta_{ks,i} \cdot t)$.
The panel specification presented in Equations (18) and (19) can also be enriched to allow for the potential inclusion, as in Broto and Perez-Quiros (2015), of a dynamic common-factor in the measurement equation driving the dependent vector, $y_{i,t}$. This common factor is modelled simply as an additional unobserved state in the state-vector (whose number of rows ups now until $n * kv + 1$), that enters each country's measurement equation with a country-specific loading parameter, h_i[8].
Regarding the State-Transition Equation, the $\xi_t \in \mathbb{R}^r$ vector of unobserved coefficients evolves according to the following expression,

$$\underset{(r\times 1)}{\xi_{t+1}} = \underset{(r\times r)}{F} \underset{(r\times 1)}{\xi_t} + \underset{(r\times s)}{B} \underset{(s\times 1)}{Z_t} + \underset{(r\times 1)}{v_t} \tag{20}$$

In equation (20), the vector of unobserved ξ_t follows an autoregressive process where F denotes an $(r \times r)$ state-transition matrix, while $Z_{i,t}$ is the vector containing any control inputs affecting the state through the control input matrix B, which applies the effect of each control input parameter in the vector on the state vector. These control variables are frequently employed in engineering but are not so commonly applied to state-space models in Economics. Their use could be interpreted as the "coefficient-drivers" of the second-generation TVP models, described in Swamy and Tavlas (2003) and related work. As stated in Gourieroux and Monfort (1997), with the introduction of an input in the "transition equation" or in the "measurement equation", all the formulae of the filter remain valid with the exception of the introduction of the variable in the update equation. Finally, v_t represents the $(r \times 1)$ vector containing the process noise terms for each parameter in the state vector and is assumed to be i.i.d. $N(0,Q)$.
Each one of the $(kv \times n)$ first components of ξ_{t+1} evolve along time driven by the following expression:

$$\xi_{kv,i,t+1} = \phi_{kv,i} \cdot \xi_{kv,i,t} + \sum_{js=jsmin}^{jsmax} \bar{\mu}_{js,i} \cdot z_{j1,i,t} + \sum_{jc=jcmin}^{jcmax} \bar{\mu}_{jc} \cdot z_{jc,i,t} + v_{kv,i,t+1} \tag{21}$$

where every unobserved component follows an autoregressive process with coefficients ϕ to be estimated. The unobserved is also influenced by the evolution of a vector containing s observed variables, z_t. Input controls in the state transition equation can be included as common/specific for all countries in the panel and/or common/specific for any control variables considered. Thus, a subset of $jsnum = jsmax - jsmin + 1$ control inputs $(z_{jsmin}, ..., z_{js}, ..., z_{jsmax})$ in the interval $[0,s]$, whose coefficients $\bar{\mu}_{js,i}$ are modelled as country-specific or idiosyncratic. Similarly, for a second subset of $jcnum = jcmax - jcmin + 1$ regressors $(z_{jcmin}, ..., z_{jc}, ..., z_{jcmax})$, also defined in the interval $[0,s]$, the coefficient $\bar{\mu}_{jc}$ is estimated as common for all the countries included in the panel. This specification is particularly helpful when economic theory suggests asymmetries among countries for the impact of any particular variable (i.e. while country size should exhibit a common parameter for all countries, global risk aversion should affect differently to the attractiveness of a country for investments). If a dynamic common factor is introduced in the model, its transition equation can be eventually restricted, so it does not require the inclusion of potentially idiosyncratic nature of control variables.

[8]Note that our Gauss code allows for multiple common-factors as well as the inclusion of potential restrictions on them

As in Hamilton (1994a), the autoregressive parameter in the state-transition equation is not restricted to a random walk model, but allowed for its estimation. Following this author, if the eigenvalues of F remain inside the circle unit, then the system is stable and the vector process defined by (20) is stationary. In this case, the inclusion of both fixed and varying parameters for the regressors can be interpreted in terms of the estimation of a mean-reverting model. the fixed parameter, either idiosyncratic ($\bar{\beta}_i$) or common ($\bar{\beta}$) has the interpretation as the average or steady-state coefficient vector, being the varying parameter the deviation from this mean in a mean-reverting framework, where:

$$\xi_{kv,i,t} = \left(\beta_{kv,i,t} - \bar{\beta}_{kv,i}\right) \tag{22}$$

or

$$\xi_{kv,i,t} = \left(\beta_{kv,i,t} - \bar{\beta}_{kv}\right) \tag{23}$$

Finally, the previous specification can be adapted to introduce multiple restrictions on the variances on measurement and transition equation, i.e. by assuming measurement error to be null or of identical variance size for all countries, or defining the signal-to-noise ratio (ratio between the two variances)[9]. Further restrictions can also be introduced on any of the components of the hyper-parameter vector.

4 Empirical Application to the Feldstein Horioka puzzle

4.1 A brief review of the literature on FH puzzle

Over the recent decades, global capital markets have witnessed an unprecedented expansion of international financial transactions. This process has been boosted by advances in information and communication technologies (ICT) and has been accompanied by a substantial financial market deregulation and a generalized abolishment of capital controls in virtually all countries. Barriers to capital mobility started to be removed by the seventies in the US and the UK, followed by other developed countries during the eighties[10]. In the European Union (EU), this liberalization process started in the mid-eighties due to the Single Market initiative, that implied full capital mobility before 1992 [11]. Under theoretically-perfect international capital mobility[12], domestic savings should flow to finance the most attractive projects, either domestic or internationally, around the globe. Similarly, local investment should be indifferently financed with domestic or foreign saving. As a result, if capital is mobile domestic investment will not be restricted by domestic saving, so there will be a low correlation between investment and saving.

On the contrary, since the seminal papers by Feldstein and Horioka (1980) and Feldstein (1983), the evidence points toward domestic investment tracking domestic saving. Feldstein and Horioka (1980), proposed a test to measure capital mobility through the regression of gross domestic investment on gross domestic savings, both expressed in terms of GDP, for the period 1960-74. They estimated an OLS regressions of the form:

$$\left(\frac{I}{Y}\right)_{it} = \alpha + \beta\left(\frac{S}{Y}\right)_{it} \tag{24}$$

where S is gross domestic savings (GDP minus private and government consumption), I is gross domestic investment, both in terms of gross domestic product Y, and nominal terms. In this framework, the constant term, α, captures the impact of the common shocks that affect average savings and investment rates all over the world. The coefficient β (also known as the “saving-retention coefficient”) measures the degree to which domestic saving and investment ratios to GDP are correlated, for a panel of countries.

Surprisingly, Feldstein and Horioka (1980) found a very high saving-investment coefficient when they tested this relationship for 16 OECD countries for the period 1960–74. To handle the cyclical endogeneity of savings and investment rates, their research took the long-period averages of these rates, estimating the coefficients for three subperiods (1960-1964, 1965-1969 and 1970-1974). These findings were confirmed by Feldstein (1983) and Feldstein and Bacchetta (1991), who extended the sample period up to 1986 and concluded that capital mobility was low at that time.

In order to justify these findings Feldstein and Horioka (1980) argue that a number of barriers, such as exchange rate and sovereignty risks, legal difficulties, transactions costs, and information limitations remain. Therefore,

[9] Increasing signal-to-noise ratio would weigh the observation heavier in the correction equations of the Kalman filter

[10] See Wyplosz (1999) and Pagoulatos (1999) for a detailed description of this process.

[11] See Camarero et al (2002) for an empirical assessment of the convergence process

[12] I.e. free from intermediation costs, asymmetric information, and other frictions.

capital mobility would be impaired and domestic saving would still be a very importante source of financing for domestic investment. In the end, these early papers support the lack of full capital mobility and the existence of constraints to international capital mobility, as they interpret the saving-retention coefficient as an indicator of the degree of international capital mobility.

There have been numerous attempts to address or explain what became the Feldstein-Horioka (FH hereafter) puzzle, looking for evidence of a falling coefficient. As stated in Frankel (1992), "if the saving-investment regressions were a good test for barriers to financial-market integration, one would expect to see the coefficient falling over time". As empirical evidence does not seem to support clearly this hypothesis, Obstfeld and Rogoff (2000) regarded the FH puzzle as "the mother of all puzzles "in international monetary economics.

The presence of this high correlation between domestic savings and investment remained stubbornly high in the decades following the publication of FH's paper (using different samples, country coverage and estimation methods), even though interest parity studies and casual empiricism have revealed a very high degree of international capital mobility ((Camarero et al 2009, 2010)). Therefore, this empirical finding constituted a puzzle and has produced an extensive research attempting to solve it either looking for alternative interpretations of the saving-retention coefficient away from a financial integration indicator or challenging the result on methodological grounds[13]. According to the aforementioned criticisms, the literature on the FH puzzle can be grouped into three main categories.

A first group of papers has attempted to reconcile the high saving-investment correlation with high capital mobility. Coakley et al (1996); Jansen (1998, 1997, 2000); Obstfeld and Rogoff (1995); Shibata and Shintani (1998); Sinn (1992), explain the high saving-investment correlation by the inter-temporal budget constraint. Even under perfect capital mobility rather than low capital mobility, the long-run solvency constraint would imply the stationarity of the current account balance and thus the cointegration between saving and investment with a cointegrating vector $(1,-1)'$. Therefore, government policy or market discipline might constrain the dynamics of the current account, implying a correlation between saving and investment which does not reveal anything about capital mobility per se.

Some other paper papers associate the savings-investment correlation to government interventions (i.e. temporary capital controls, or more generic forms of financial repression) to offset private net capital flows (Fieleke (1982); Summers (1988); Tobin (1983); Westphal (1983)) while others introduce financial frictions ((Bai and Zhang 2010)), currency and political risks or long-run risk component in global shock processes (Chang and Smith (2014)). All these elements create a sort of 'home country bias' phenomenon.

Finally, Feldstein and Horioka (1980) claimed that, as a country gets larger, it tends to behave like a closed economy, attributing the high saving-correlation to a country size effect. One of the most important criticisms has to do with the choice of the countries in their paper: FH only include large industrialized countries that may cause an upward bias on the saving-retention coefficient, as savings shocks hitting the large economies are expected to affect the world interest rate and hence their domestic investment. More recently, Ho (2003) supports this hypothesis, as he finds, using a threshold model, that the saving-retention coefficient increases as country-size augments in a panel of OECD countries.

A second line of research includes papers that focused on the fact that saving is endogenous, so the FH-type regressions would be plagued by simultaneity bias (Tobin (1983)). This problem may arise if both variables react to population or productivity shocks or to a government response to correct a current account imbalance. It has also been noted that it is not clear whether the FH finding, particularly for panel regressions, captures permanent or transitory effects, the latter being less relevant to an assessment of capital mobility. An attempt to solve this problem has been made accounting for the possible non-stationary properties of the variables (Coiteux and Olivier 2000; Jansen and Schulze 1996). More recently, Chu (2012) showed that a common deflator might cause the spurious saving-investment correlation. Relying on the previous objections, the empirical literature has tried to solve the puzzle by using improved econometric techniques, mainly time series and panel data methodologies. However, the evidence using both time series and panels tend to be non-conclusive. Westerlund (2006) shows that once we account for structural breaks in the panel, we find more evidence of cointegration between saving and investment. However, there is not much literature on the FH puzzle that takes into account structural breaks. Ho (2002) allows for structural changes in the cointegrated relationship arguing that the saving-investment relationship may be subject to regime changes. Also Telatar et al (2007), Mastroyiannis (2007), Özmen and Parmaksiz (2003) allow for structural breaks. The last two papers found that when a regime shift is allowed there is no evidence in favor of the puzzle. Kejriwal (2008) revisits the FH puzzle for 21 OECD countries with data ending in 2000. He finds evidence from 1 to 3 regime changes and considers that part of the evidence in favor of the puzzle may be related to the non-stationarity of the variables and de existence of structural breaks, both unaccounted for. The same methodology is also used by Ketenci (2012), who incorporates the breaks found to correct the β coefficient.

[13] Exhaustive reviews can be found at Coakley et al (1998) or Apergis and Tsoumas (2009).

His findings show a decreasing saving-retention coefficient for the EU countries. This approach assumed abrupt structural breaks. However, it may seem unlikely that a structural break could be immediate, and it might seem more reasonable to allow a structural change to take a period of time to take effect. Indeed, given the menu cost, the effect of the technological progress and policy switch might have time lags. Accordingly, we think that to capture this effect it is more realistic to let the parameter in the FH regression vary overtime. Indeed, there is a recent and growing literature that focus on this kind of statistical approach that is the one we follow in the present research. Therefore, in the next section we assume that the saving retention coefficient evolves smoothly, which implies a gradual variation over time.

4.2 A time-varying model for the Feldstein-Horioka puzzle

To estimate the relationship between saving and investment in a dynamic panel time series framework, we have obtained annual savings and investment data (relative to GDP) from 1970 to 2016 for 17 countries, 12 from the original Eurozone (Luxembourg, Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal and Spain) and 5 other large industrialized or emerging economies (Canada, China, Japan, United Kingdom and USA). The data has been obtained from the World Development Indicators (WDI) database[14]. The graphs for the two variables for each of the countries analyzed is presented in Figure 1.

INSERT FIGURE 1 HERE

We claim that the approach adopted in this paper permits to account for several factors that have been considered crucial in the theoretical and empirical literature to explain the FH puzzle. Thus, in this section, we provide a detailed justification of the role that not only the additional variables we include in the state equation (a mesure of country-size and a mesure of risk) but also the flexibility provided by the econometric framework, play in our empirical application.

A strand of the literature stresses the fact that country size is one of the potential determinants of the saving retention coefficient. On one side, Harberger (1980) shows that FH conclusions may be influenced by size-of-country considerations [15]. In the same vein Murphy (1984), argues that the estimates of an equation where all the countries are treated equally may yield biased coefficients if the ability of a country to influence conditions in the world capital market depends on its size. Thus, if we think that financial conditions are an important factor to determine the relationship between saving and investment rates, size should be taken into account. Ho (2003) has also found that the saving-retention coefficient increased with the size of the country using threshold models. Several measures of size have been used in the empirical literature, such as average saving, investment, and GDP shares in the respective totals. In our case we have used the share of each of the countries analyzed in world GDP as a proxy for country-size.

More recently, Giannone and Lenza (2010) and Lenza (2018) have put forward the importance of using general equilibrium explanations of the FH findings. The predictions of the partial equilibrium inter-temporal theory of the current account refers to idiosyncratic components (country-specific or regional shocks) of saving and investment rates that, not affecting all the countries, are unlikely to generate imbalance in the world capital market. Since global shocks are acknowledged to be an important driving force of the world business cycle, general equilibrium effects should be the ones able to reconcile theory and evidence. However, Giannone and Lenza (2010) claim that some previous attempts to find a decreasing saving-investment correlation when controlling for global shocks failed because they assumed homogeneity across countries [16]. It is important to bear in mind that if global shocks have heterogeneous (asymmetric) effects, they can create imbalances on the world capital market. Giannone and Lenza (2010) propose the use of factor-augmented panel regressions, a new methodology, to isolate idiosyncratic sources of fluctuations. They improve on existing studies since countries are allowed to react with specific sign and magnitude to global shocks. Unlike previous studies that reached biased estimations of the saving-retention coefficient, they find that allowing for a heterogeneous propagation mechanism of global shocks yields dropping saving-retention coefficients from the 1980s.

In the present paper, as in Paniagua et al (2017), we also account for both idiosyncratic and global shocks, allowing the latter to have asymmetric effects. Indeed, after the global financial crisis, the literature on international capital markets shifted the attention from idiosyncratic factors towards a common factor, which would reflect a change in

[14]The data has been obtained from https://data.worldbank.org/. Gross fixed capital formation (formerly gross domestic fixed investment) includes land improvements (fences, ditches, drains, and so on); plant, machinery, and equipment purchases; and the construction of roads, railways, and the like, including schools, offices, hospitals, private residential dwellings, and commercial and industrial buildings. According to the 1993 SNA, net acquisitions of valuables are also considered capital formation. Gross domestic savings are calculated as GDP less final consumption expenditure (total consumption). As in the literature, both variables have been expressed as a percentage of GDP.

[15]Harberger notes that the difference between gross domestic saving and investment to GDP has greater variability and larger absolute value for small countries than for large ones.

[16]Examples of this are Glick and Rogoff (1995) and Ventura (2003).

investors' risk aversion. Risk aversion is associated with the willingness of investors to take risks (the so-called "risk appetite"). As investors continuously adjust their risk-return preference function, even if the "amount of risk" embedded in a security remains unchanged, the risk premium may vary depending on the "price of risk". From that perspective, variations in global risk-aversion might have also contributed to increase distrust and perceived risk (Litterman and Scheinkman 1991), changing the attitude of investors towards risk. In times of high uncertainty, investors are supposed to be more risk-averse. In practice, shifts in investors' "risk appetite" are not directly observable and the impact on global risk repricing should affect through its interaction with the risk-content of a particular asset, which could also be time-varying.

As in Bernoth and Erdogan (2012), we use the yield spread between low grade US corporate bonds (BAA) and the 10-year Treasury bonds as an empirical proxy for this overall investors' risk attitude ("BAAS"). This variable has been obtained from the Federal Reserve Economic Database (FRED), provided by the Federal Reserve Bank of St. Louis.

An alternative for global risk aversion, CBOE Volatility Index, known by its ticker symbol VIX, was not available for the whole period[17]. Moreover, although the literature finds a relevant role for both proxies, while BAA spread measures risk appetite, a variation in implied volatility on a market may stem from a change in the quantity of risk on this market and not necessarily from a change in the investor's risk aversion.

After these qualifications, we present the equations that we effectively estimate in this paper. Our time-varying parameters specification of the investment ratio to gdp (I/Y) on savings ratio to gdp (S/Y), is represented by the equation:

$$inv_{i,t} = \beta_{0i} + (\beta_{1i} + \delta_i \cdot t)\, sav_{i,t} + \xi_{i,t} sav_{i,t} + \omega_{i,t} \tag{25}$$

where $inv_{i,t}$ is the investment to GDP ratio, $sav_{i,t}$ is the saving to GDP ratio and, in addition to the fixed (country-specific) savings retention coefficient, β_{1i}, $\xi_{i,t}$ stands for the varying component of the $sav_{i,t}$ parameter. In a mean-reverting framework, this varying component can be interpreted as the deviation from the saving-retention mean parameter, $\left(\beta_{1i,t} - \bar{\beta_{1i}}\right)$. The model includes also a deterministic trend in the parameter variation $\delta_i \cdot t$, so that the varying parameter $\beta_{1i,t} = \bar{\beta_{1i}} + \delta_i \cdot t + \xi_{i,t}$ is reverting towards a mean parameter potentially including a deterministic trend as well, $\bar{\beta_{1i}} + \delta_i \cdot t$. Hence, our state-space model incorporates heterogeneity by allowing for both country-specific intercepts and slopes.

Then, the model is estimated by maximum likelihood using the Kalman Filter algorithm, where each of the elements of the unobserved vector, $\xi_{i,t}$, follows a stochastic process such as:

$$\xi_{i,t+1} = \phi \xi_{i,t} + \mu_1 size_{i,t} + \mu_{2i} baas_{i,t} + \upsilon_{i,t+1} \tag{26}$$

where $\upsilon_t \sim N(0,Q)$ and the transition of the unobserved vector includes, in addition of the autoregressive component, a (common parameter, μ_1) that corresponds to the country-size indicator, with $size = \frac{gdp}{worldgdp}$, that is, the relative size of each country in terms of world's gdp. The second control instrument, a global risk aversion measure, in our case the BAA spread, is denoted $baas_{i,t}$ and assumed to affect the varying saving-retention through a country-specific parameter, μ_{2i}.

5 Results and discussion

In this section we estimate the empirical specification described in section 4 using monthly data for the period 1970 to 2016 in a panel of 17 countries. Prior to the estimation of our model using TVP, we analyze the univariate properties of the series using panel unit root tests, allowing for both cross-country dependence and structural breaks.

5.1 Univariate properties of the data.

Regarding the analysis of the order of integration of the variables included in the estimated model, we have considered the existence of potential and unknown structural changes. This issue is non-trivial given that unit root tests can lead to misleading conclusions if the presence of structural breaks is not accounted for, as Perron (1989) argued in his seminal paper.

As most of the variables (with the exception of the BAAS spread $baas_t$) have been defined at country-level, we can construct a panel consisting of the different individuals and test for unit roots using Bai and Carrion-i Silvestre

[17] CBOE VIX measures the stock market's expectation of volatility implied by S&P 500 index options, calculated and published by the Chicago Board Options Exchange.

(2009) panel unit root test. In the case of the BAA spread, we apply the univariate unit root tests proposed by Carrion-i Silvestre et al (2009). Both, the panel and the univariate tests, allow for multiple and unknown structural breaks.
Bai and Carrion-i Silvestre (2009) propose a set of panel unit root statistics that pool the modified Sargan-Bhargava (hereafter MSB) tests (Sargan and Bhargava 1983) for individual series, taking into account the possible existence of multiple structural breaks[18] and cross-section dependence modelled as a common factors model[19]. The common factors may be non-stationary, stationary or a combination of both. The number of common factors is estimated using the panel Bayesian information criterion proposed by Bai and Ng (2002).
The possible presence of non-stationary variables, as the potential structural changes, are non-trivial features. A non-stationary dependent variable, without further restrictions, could be most likely driven by a non-stationary state, such as one following a random walk[20]. In our case we do not assume that the transition equation follows a random walk, but let it be estimated as a component of the hyper-parameter vector. On the other hand, the presence of the structural breaks could, in fact, be an indication of a time-varying relationship for the model.
Concerning the panel unit root tests, we have allowed for a maximum number of 4 breaks, determined through the Bai and Perron (1998) procedure[21]. In Table 1 we present the unit root results for the variables inv_it, sav_it and $size_it$, estimated for our group of seventeen countries. The panel unit root tests have a constant and a trend and allow for structural changes in both (Model 2, trend break model). In the case of inv_{it} the evidence points to a non-rejection of the unit root hypothesis and a similar outcome is found for $size_{it}$, with the exception of the Z^* test. As Bai and Perron (1998) suggest that the Z and P statistics have better small sample properties, we conclude that the latter variable is non-stationary. Regarding sav_{it}, all the P-tests point toward stationarity at different levels of significance, but the null hypothesis of a unit root cannot be rejected using the Z-tests. Thus, the evidence is inconclusive in this case.

INSERT TABLE 1 HERE

The position of the structural breaks found for each of the panel-dimension variables is reported in Table 2. Due to the length of the sample, the structural breaks are distributed along the whole period, although we can find some patterns. Spain, Luxembourg, the Netherlands and Belgium present instabilities at the end of the seventies in some of the variables, whereas the beginning of the nineties also concentrates some breaks in Luxembourg, the Netherlands and China. Concerning the financial crisis, Greece, Ireland, the Netherlands, Portugal, Spain and China seem hit by changes between 2006 and 2009. The applied algorithm does not detect any structural break in another group countries, such as Austria, Finland, France, Italy, the UK and the US.

INSERT TABLE 2 HERE

Finally, for the BAA yield spread to long-term federal bonds, denoted $baas_{it}$ we have used the GLS-based unit root test statistics proposed in Kim and Perron (2009) and extended in Carrion-i Silvestre et al (2009). The advantage of this proposal is that it solves many of the problems of previous standard unit root tests with a structural breaks [22], and allows for multiple breaks at an unknown time under both the null and alternative hypothesis.
The unit root test results of $baas_{it}$ are mixed, as some of them reject the null hypothesis of a unit root whereas others do not, as shown in Table 3.

INSERT TABLE 3 HERE

Therefore, although with some mixed results, the null hypothesis of a unit root (with structural breaks) cannot be rejected for all the series at the 5% level of significance. Accordingly, we can conclude that the variables in Tables 2 and 3 are I(1) with structural breaks.

5.2 Panel TVP model estimation for the FH savings-retention coefficient

The estimation of the mean-reverting time-varying savings-retention model with deterministic trend presented above has been obtained using a GAUSS code written by the authors that extends the traditional approach by Hamilton (1994a) and includes all the elements of the model presented in Subsection 3.3. The results for the maximum likelihood estimation of the elements of the hyper-parameter vector, ϕ, are reported in Table 4. The first three columns display the "measurement equation" results (see equation (25) above) , while the "state-update equation" is included in columns 4 to 6 (see equation (26)). Also a horizontal line separates the Eurozone (EZ) countries from non-members. We are interested in the two equations, as the measurement equation presented in the

[18] Adapting Bai and Perron (2003) methodology to a panel data framework.
[19] Following Bai and Ng (2004) and Moon and Perron (2004).
[20] An important part of the literature on time-varying parameter models would make this choice.
[21] See Bai and Carrion-i Silvestre (2009) for details.
[22] See Zivot and Andrews (2002), Perron (1997), Vogelsang and Perron (1998), Perron and Vogelsang (1992a,b), among others.

table would be equivalent to many OLS models reported in the empirical literature, whereas our first contribution is the analysis of the unobserved component obtained in the state-update equation. To ease the interpretation of the results, we should complement the estimated coefficients presented in the Table with the graphs of the time-varying parameters shown in Figure 2.

According to Hamilton (1994a), as it happens in our case, if ϕ in the state-update equation is lower than one, the varying coefficient can be considered a deviation from the mean (the parameter obtained in the measurement equation). The estimated value for the parameter is 0.843, which shows its highly persistent nature but lower than zero. This means that a random walk would have been an unsatisfactory modelization of the process. Our state-space equation also includes control variables, driving the evolution of the varying component of the parameter: first, $size_{it}$, that represents the relative size of the country (defined as a common-parameter control instrument) and captures the fact that larger countries tend to be less open than small ones, and second, $baas_t$, the BAA spread, a frequently employed measure of the global risk aversion, defined as a country-specific parameter control instrument.

The estimated parameters of the model are shown in Table 4, for both the measurement equation and the state-transition equations. In the case of the first one, we report the single-country estimated fixed-parameters, where β_{0i} is the intercept, and β_{1i} and δ_i are the mean and time-trend parameters, respectively, for the savings-retention coefficient. In the state-transition equation, the autoregressive parameter ϕ is in the fourth column, while columns 5 and 6 show the output of the common coefficient for $size_{it}$, denoted μ_1 and an idiosyncratic coefficient for the global risk variation μ_{2i} . In Figure 2 we plot the time-varying coefficients for the savings-retention coefficient estimated for each country as a deviation from the mean-parameter, that includes a deterministic trend in some cases.

In addition to the persistent nature of the autoregressive coefficient, a second noteworthy result is the heterogeneity of the saving-retention coefficients across countries (see the second column of Table 4, under the heading β_{1i}). Luxembourg is an obvious outlier in the sample (as the parameter is negative), which may be explained by its special economic status as an attractive financial center. The remaining countries have positive saving-retention parameters ranging from 0.213 in Italy to 0.804 for Greece and 0.834 for Spain. Regarding the deterministic time-trend included δ_i, it happens to be significant and negative with a downward slope for Austria, Finland, Germany, Italy and Portugal. For Greece, the slope in the mean parameter is found to be significant but positive. This pattern can be clearly observed in Figure 2.5

When focusing on the temporal evolution of the coefficients, we find in the state-transition equation the positive effect of $size_i$ in the evolution of the unobserved state, that represents the varying component ξ_{it} of the FH coefficient. This finding confirms previous results by Harberger (1980) and Ho (2003), among others.

Concerning the global risk indicator, also in this case the results are heterogeneous. For the majority of countries, the coefficient on $baas_{it}$ is significant and negative (in Austria, Belgium, France, Germany, Greece, Italy, Netherlands, Portugal, Spain, Japan and USA). The exception is China, with a positive coefficient. For the remaining countries, this variable is nonsignificant (Luxembourg, Finland, Ireland, Canada or the United Kingdom). These results can be capturing an increase of international capital mobility in the post-crisis period and this may be primarily due to investor's preference for safe haven assets. A similar argument may apply for the China's positive coefficient, representing a redirection of Chinese capital to internal financial demands, once international markets were riskier after the crisis and foreign investors more difficult to find. The negative coefficient for $baas_{it}$ in the case of EU peripheral countries such as Portugal seem to contradict the negative effect on short-term international capital mobility due to the global financial crisis. This surprising finding seems to be the result of declining investment rates in front of high and stable savings rates accompanied by current account surpluses which turn into lowering temporally saving-retention coefficients [23]. These results are consistent with previous research, such as Bautista and Maveyraud-Tricoire (2007) for the Asian financial crisis, or Sachs et al (1981) who found a negative relation between investment and the current account balance for 14 developed economies. Hence, those countries most heavily affected by the global financial crisis, will have a decline in output and in economic activity, but this would produce current account surpluses due to the decline in imports.

Up to now, our findings show that the Feldstein-Horioka puzzle vanishes when we consider an extended time period and allow for time-varying coefficients. Let us focus on specific countries in the sample. In the case of Austria, a small EZ country, the saving-retention parameter is small (0.300) but positive and significant. Thus, for Austria, there would be (apparently) no puzzle. Moreover, the time-varying coefficient evolves around a decreasing trend that is crossed twice: at the beginning of the eighties and later, at the time of the financial crisis. During the majority of the sample the time-varying parameter has been positive but with a decreasing slope, meaning that domestic saving contributed more to domestic investment, whereas during the first half of the eighties it was

[23]The conventional wisdom is for capital to flow in the opposite direction: insufficient domestic saving is augmented by foreign saving to match investment demand, i.e., capital flows in, and this should be reflected by a current account deficit.

negative. The negative trend of this parameter points to an increase of capital mobility during the sample and, therefore, a reduction in the total saving-retention parameter.

Germany's saving-retention parameter β_{1i} is higher (0.492) than the Austrian one, but it has follow a similar (decreasing) trend. The time-varying parameter, however, has remained below the mean during the majority of the period, what implies that also in Germany capital mobility has increased with the monetary union. The Netherlands can be also included in this group, as the parameter in the measurement equation is 0.340 and the time-varying one is also below the mean during the whole period. One of the most remarkable features of these three countries is that the financial crisis did not reduce their degree of capital openness.

Belgium and France have also common characteristics. Both have a relatively high saving-retention parameters (0.568 in Belgium and 0.396 in France). Also in both cases the control variables (risk and size) are significant. During the sample period, the time-varying parameter has fluctuated around the mean, but most of the time has remained below it. Their inclusion in the EZ did not have the same type of effect as in the previous group of countries: their degree of capital mobility remained unaltered at the end of the period and the time-varying parameter increased with the financial crisis. Spain could be also included in this group, although the value of the savings parameter (0.834) is one of the highest of the EZ countries. A common feature with France and Belgium is that also in Spain the time-varying parameter increased in the years previous to the crisis, when the financial bubble was growing, an effect that was corrected when the bubble burst.

We should pay special attention to the countries that were intervened or assisted during the crisis. In the case of Greece, the saving-retention parameter is one of the highest among EZ members (0.804). Looking at the graph, during the whole period domestic investment has been mostly possible using domestic savings, although the time-varying parameter has remained below the mean during the whole sample. The graph shows that, although the mean is 0.804, the trend has systematically added to the value of the parameter, that has remained around 1 since 2012, after the bail-out programme. As for Ireland, the value of the saving-retention parameter in the measurement equation is not significant. The most plausible explanation for this finding is that Ireland is a very small country compared with the size of its financial system. An attractor of FDI due to its low taxes, the country has very high capital mobility.

Concerning non-EZ countries, the majority of them are large, so that the measurement equation coefficient is also relatively large (0.48 in Japan, 0.476 in China, 0.621 for the US, or 0.59 in the UK). Canada is the exception, with a small mean parameter (0.226) and an increase in capital openness since the beginning of the nineties. However, after the international financial crisis the time-varying parameter stays above the mean, remaining more conservative and, therefore, more dependent on domestic savings. The larger countries show a very stable value of the saving-retention parameter, around 0.6-0.7, with the time-varying coefficient fluctuating around the mean and no clear pattern. Finally, although the β_{1i} parameter (that is, the mean of the saving-retention) is also near 0.5 in Japan, the time-varying parameter has had a very clear negative coefficient during the whole sample, showing an increase in capital openness in this country. The behavior of some non-EZ countries, such as the UK and the US, is in sharp contrast to the larger countries in the EZ (Germany and France, and even Italy) that are significantly more open to capital mobility, especially after the launching of EMU and despite the world financial crisis. Our main conclusion is that we can hardly talk about the FH puzzle in the EZ if we allow for structural breaks and time-varying parameters. The same applies to other small open countries, such as Canada or even Japan, where since the end of the nineties the time-varying parameter has remained below the mean as well.

INSERT TABLE 4 HERE

INSERT FIGURE 2 HERE

Summing up, our findings suggest that there is substantial cross-country variation in the degree of capital mobility, thereby validating the random coefficients approach. Nevertheless, the results also show that, for most countries, the FH coefficients exhibit a noticeable decline over time and an increase in capital market mobility and international openness. Moreover, the role of global risk aversion is also very different depending on the country, something that should be taken into account to explain the pattern of global capital reallocation. Finally, the effect of the financial crisis on capital openness has been lower or negligible in the large EZ countries in sharp contrast to non-members, as the US or the UK, possibly due to the effects of bailouts.

6 Conclusions

In this paper we apply an extended state-space framework for panel time series modelling to the Feldstein-Horioka puzzle. Our research contributes to the literature by extending the simplest specification generally employed by the practitioners in the fields of economics and finance, into a panel-data time-varying framework for the measurement equation, that combines fixed and time-varying parameters. Under specific circumstances (see, for example, Hamilton 1994a), this setting can be understood as a mean-reverting panel time-series model, or even a mean-reverting

model around a deterministic trend. Moreover, the panel modellization has been enriched by the inclusion of either common or country-specific fixed parameters, as well as a common unobservable factor with idiosyncratic fixed parameters in the measurement equation.

Regarding the transition equation, our panel structure allows for the estimation of different specifications for the autoregressive component. Control instruments are also possible, that can also be either common or idiosyncratic. All these specifications have been written in a GAUSS code, which has been adapted from James Hamilton's univariate code. The new code also allows for the introduction of restrictions in the variances of both the transition and the measurement equations.

Finally, this panel-TVP state-space framework is used to re-examine the so-called Feldstein-Horioka puzzle. We analyze the existence of a correlation between domestic saving and investment ratios to GDP, that should not exist if capital markets were highly integrated. In our case we test for this relation during the period 1970-2016 in a sample of 17 countries, 12 Eurozone members (Luxembourg, Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal and Spain) and 5 large non-member countries (Canada, China, Japan, United Kingdom and USA).

The time-varying estimate of the FH equation allows to disentangle the variations in the degree of responsiveness along time. Our results reveal certain heterogeneity not only in the fixed but also in the varying components of the saving-retention coefficient. Moreover, many of the relationships evolve around a deterministic trend, either positive or negative. We find that both country size and global risk aversion have a significant role to explain international capital allocation. In addition, the large and stable EZ countries have been more isolated from the international financial crisis than other large non-member economies. Our results indicate that the effects of bailouts have been non-negligible for bailed-out countries.

This article adds to the literature by showing that time-varying models can be useful for the analysis of both non-linearities and country heterogeneity in panel data time series, as is the case of the relationship represented by the Feldstein-Horioka puzzle.

Acknowledgements The authors are indebted to James D. Hamilton and J. LL. Carrion-i-Silvestre for providing them with the Gauss codes to implement some of the tests used in the paper. They also thank comments and suggestions from participants in the 5th ISCEF Symposium 2018 (Paris).

References

Ansley CF, Kohn R (1985) Estimation, filtering, and smoothing in state space models with incompletely specified initial conditions. The Annals of Statistics pp 1286–1316

Apergis N, Tsoumas C (2009) A survey of the feldstein–horioka puzzle: what has been done and where we stand. Research in Economics 63(2):64–76

Bai J, Ng S (2002) Determining the number of factors in approximate factor models. Econometrica 70(1):191–221

Bai J, Ng S (2004) A PANIC Attack on Unit Roots and Cointegration. Econometrica 72(4):1127–1177

Bai J, Perron P (1998) Estimating and Testing Linear Models with Multiple Structural Changes. Econometrica 66(1):47–78

Bai J, Perron P (2003) Computation and analysis of multiple structural change models. Journal of Applied Econometrics 18(1):1–22, DOI 10.1002/jae.659

Bai J, Carrion-i Silvestre JL (2009) Structural Changes, Common Stochastic Trends, and Unit Roots in Panel Data. Review of Economic Studies 76(2):471–501

Bai Y, Zhang J (2010) Solving the feldstein–horioka puzzle with financial frictions. Econometrica 78(2):603–632

Bautista C, Maveyraud-Tricoire S (2007) Saving-investment relationship, financial crisis and structural changes in east asian countries. Économie internationale (3):81–99

Bernoth K, Erdogan B (2012) Sovereign bond yield spreads: A time-varying coefficient approach. Journal of International Money and Finance 31(3):639–656

Bos T, Newbold P (1984) An Empirical Investigation of the Possibility of Stochastic Systematic Risk in the Market Model. Journal of Business 57(1):35–41

Broto C, Perez-Quiros G (2015) Disentangling contagion among sovereign cds spreads during the european debt crisis. Journal of Empirical Finance 32:165–179

Camarero M, Ordóñez J, Tamarit C (2002) Tests for interest rate convergence and structural breaks in the ems: further analysis. Applied Financial Economics 12(6):447–456

Camarero M, Carrion-i Silvestre JL, Tamarit C (2009) Testing for real interest rate parity using panel stationarity tests with dependence: A note. The Manchester School 77(1):112–126

Camarero M, Carrion-i Silvestre JL, Tamarit C (2010) Does real interest rate parity hold for oecd countries? new evidence using panel stationarity tests with cross-section dependence and structural breaks. Scottish Journal of Political Economy 57(5):568–590

Chang Y, Smith RT (2014) Feldstein–horioka puzzles. European Economic Review 72:98–112

Chu KH (2012) The feldstein-horioka puzzle and spurious ratio correlation. Journal of International Money and Finance 31(2):292–309

Coakley J, Kulasi F, Smith R (1996) Current account solvency and the feldstein–horioka puzzle. The Economic Journal pp 620–627

Coakley J, Kulasi F, Smith R, et al (1998) The feldstein–horioka puzzle and capital mobility: a review. International Journal of Finance & Economics 3(2):169

Coiteux M, Olivier S (2000) The saving retention coefficient in the long run and in the short run: evidence from panel data. Journal of International Money and Finance 19(4):535–548

Faragher R (2012) Understanding the basis of the kalman filter via a simple and intuitive derivation [lecture notes]. IEEE Signal processing magazine 29(5):128–132

Feldstein M (1983) Domestic saving and international capital movements in the long run and the short run. European economic review 21(1-2):129–151

Feldstein M, Bacchetta P (1991) National saving and international investment. In: National saving and economic performance, University of Chicago press, pp 201–226

Feldstein M, Horioka C (1980) Domestic Saving and International Capital Flows. The Economic Journal 90(358):314–329, DOI 10.2307/2231790

Fieleke N (1982) National saving and international investment. Saving and government policy pp 138–58

Frankel JA (1992) Measuring international capital mobility: a review. The American Economic Review 82(2):197–202

Giannone D, Lenza M (2010) The feldstein-horioka fact. In: NBER International Seminar on Macroeconomics 2009, University of Chicago Press, pp 103–117

Glick R, Rogoff K (1995) Global versus country-specific productivity shocks and the current account. Journal of Monetary Economics 1(35):159–192

Gourieroux C, Monfort A (1997) Time series and dynamic models. Cambridge University Press

Granger CWJ (2008) Non-Linear Models: Where Do We Go Next - Time Varying Parameter Models? Studies in Nonlinear Dynamics & Econometrics 12(3):1–9

Hamilton JD (1994a) State-space models. In: Robert F Engle and Daniel L McFadden (ed) Handbook of Econometrics, vol Volume 4, Elsevier, chap Chapter 50, pp 3039–3080

Hamilton JD (1994b) Time series analysis, vol 2. Princeton university press Princeton

Harberger AC (1980) Vignettes on the world capital market. The American Economic Review 70(2):331–337

Harvey A (1989) Forecasting, structural time series models and the Kalman filter. Cambridge University Press, Cambridge [etc.]

Harvey AC (1993) Time series models, 2nd edn. Harvester Wheatsheaf

Harvey AC, Phillips GD (1979) Maximum likelihood estimation of regression models with autoregressive-moving average disturbances. Biometrika 66(1):49–58

Hildreth C, Houck JP (1968) Some Estimators for a Linear Model with Random Coefficients. Journal of the American Statistical Association 63(322):584–595, DOI 10.2307/2284029

Ho Tw (2002) A panel cointegration approach to the investment-saving correlation. Empirical Economics 27(1):91–100

Ho TW (2003) The saving-retention coefficient and country-size: the feldstein–horioka puzzle reconsidered. Journal of Macroeconomics 25(3):387–396

Hsiao C (1974) Statistical Inference for a Model with Both Random Cross-Sectional and Time Effects. International Economic Review 15(1):12, DOI 10.2307/2526085

Hsiao C (1975) Some Estimation Methods for a Random Coefficient Model. Econometrica 43(2):305, DOI 10.2307/1913588

Jansen WJ (1997) Can the intertemporal budget constraint explain the feldstein-horioka puzzle? Economics Letters 56(1):77–83

Jansen WJ (1998) Interpreting saving-investment correlations. Open Economies Review 9(3):207–219

Jansen WJ (2000) International capital mobility: evidence from panel data. Journal of International Money and Finance 19(4):507–511

Jansen WJ, Schulze GG (1996) Theory-based measurement of the saving-investment correlation with an application to norway. Economic Inquiry 34(1):116–132

Kalman RE (1960) A New Approach to Linear Filtering and Prediction Problems. Journal of Fluids Engineering 82(1):35–45, DOI 10.1115/1.3662552

Kalman RE (1963) New methods in wiener filtering theory. In: Proceedings of the First Symposium on Engineering Applications of Random Function Theory and Probability, edited by JL Bogdanoff and F. Kozin, John Wiley & Sons, New York

Kalman RE, Bucy RS (1961) New results in linear filtering and prediction theory. Journal of basic engineering 83(3):95–108

Kejriwal M (2008) Cointegration with structural breaks: An application to the feldstein-horioka puzzle. Studies in nonlinear dynamics & econometrics 12(1)

Ketenci N (2012) The feldstein–horioka puzzle and structural breaks: evidence from eu members. Economic Modelling 29(2):262–270

Kim CJ, Nelson CR (1999) State-space models with regime switching: classical and gibbs-sampling approaches with applications. MIT Press Books 1

Kim D, Perron P (2009) Unit root tests allowing for a break in the trend function at an unknown time under both the null and alternative hypotheses. Journal of Econometrics 148(1):1–13, DOI 10.1016/j.jeconom.2008.08.019

Koopman SJ (1997) Exact initial kalman filtering and smoothing for nonstationary time series models. Journal of the American Statistical Association 92(440):1630–1638

Koopman SJ, Durbin J (2003) Filtering and smoothing of state vector for diffuse state-space models. Journal of Time Series Analysis 24(1):85–98

Lenza M (2018) Special feature c: An empirical assessment of the feldstein-horioka's saving-retention coefficient as a measure of financial integration in the euro area. In: Financial integration in Europe, May 2018, ECB, pp 119–126

Litterman RB, Scheinkman J (1991) Common factors affecting bond returns. The Journal of Fixed Income 1(1):54–61

Mastroyiannis A (2007) Current account dynamics and the feldstein and horioka puzzle: The case of greece. The European Journal of Comparative Economics 4(1):91

Min Ck, Zellner A (1993) Bayesian and non-Bayesian methods for combining models and forecasts with applications to forecasting international growth rates. Journal of Econometrics 56(1–2):89–118, DOI 10.1016/0304-4076(93)90102-B

Moon HR, Perron B (2004) Testing for a unit root in panels with dynamic factors. Journal of Econometrics 122(1):81–126, DOI 10.1016/j.jeconom.2003.10.020

Murphy RG (1984) Capital mobility and the relationship between saving and investment rates in oecd countries. Journal of international Money and Finance 3(3):327–342

Obstfeld M, Rogoff K (1995) The intertemporal approach to the current account. Handbook of international economics 3:1731–1799

Obstfeld M, Rogoff K (2000) The six major puzzles in international macroeconomics: is there a common cause? NBER macroeconomics annual 15:339–390

Ohlson J, Rosenberg B (1982) Systematic Risk of the CRSP Equal-Weighted Common Stock Index: A History Estimated by Stochastic-Parameter Regression. The Journal of Business 55(1):121–145

Özmen E, Parmaksiz K (2003) Exchange rate regimes and the feldstein-horioka puzzle: The french evidence. Applied Economics 35(2):217–222

Pagoulatos G (1999) Financial repression and liberalization in europe's southern periphery: From "growth state" to "stabilization state". Tech. rep., European Community Studies Association (ECSA) Sixth Biennial International Conference, Pittsburg

Paniagua J, Sapena J, Tamarit C (2017) Sovereign debt spreads in emu: The time-varying role of fundamentals and market distrust. Journal of Financial Stability 33(Supplement C):187 – 206, DOI https://doi.org/10.1016/j.jfs.2016.06.004

Perron P (1989) The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis. Econometrica 57(6):1361–1401

Perron P (1997) Further evidence on breaking trend functions in macroeconomic variables. Journal of Econometrics 80(2):355–385, DOI 10.1016/S0304-4076(97)00049-3

Perron P, Vogelsang TJ (1992a) Nonstationarity and Level Shifts With an Application to Purchasing Power Parity. Journal of Business & Economic Statistics 10(3):301–320, DOI 10.1080/07350015.1992.10509907

Perron P, Vogelsang TJ (1992b) Testing for a Unit Root in a Time Series with a Changing Mean: Corrections and Extensions. Journal of Business & Economic Statistics 10(4):467–70

Rao CR (1965) The Theory of Least Squares When the Parameters are Stochastic and Its Application to the Analysis of Growth Curves. Biometrika 52(3/4):447, DOI 10.2307/2333697

Rosenberg B (1973) Random Coefficients Models: The Analysis Of A Cross Section Of: Time Series By. NBER Annals of Economic and Social Measurement 2(4):397–428

Rubin H (1950) Note on random coefficients. Statistical inference in dynamic economic models 10:419–421

Sachs JD, Cooper RN, Fischer S (1981) The current account and macroeconomic adjustment in the 1970s. Brookings Papers on Economic Activity 1981(1):201–282

Sargan JD, Bhargava A (1983) Testing Residuals from Least Squares Regression for Being Generated by the Gaussian Random Walk. Econometrica 51(1):153–174, DOI 10.2307/1912252

Schaefer S, Brealey R, Hodges S, Thomas H (1975) Alternative models of systematic risk. International capital markets, North Holland, Amsterdam pp 150–161

Shibata A, Shintani M (1998) Capital mobility in the world economy: an alternative test. Journal of International Money and Finance 17(5):741–756

Carrion-i Silvestre JL, Kim D, Perron P (2009) GLS-based Unit Root tests with multiple structural breaks under both the null and the alternative hypotheses. Econometric Theory 25(Special Issue 06):1754–1792, DOI 10.1017/S0266466609990326

Sinn S (1992) Saving-investment correlations and capital mobility: On the evidence from annual data. The Economic Journal 102(414):1162–1170

Summers LH (1988) Tax policy and international competitiveness. In: International aspects of fiscal policies, University of Chicago Press, pp 349–386

Swamy P, Tavlas GS (2003) Random Coefficient Models. In: Baltagi BH (ed) A Companion to Theoretical Econometrics, Blackwell Publishing Ltd, pp 410–428

Swamy PAVB (1970) Efficient Inference in a Random Coefficient Regression Model. Econometrica 38(2):311–323, DOI 10.2307/1913012

Swamy PAVB, Mehta JS (1977) Estimation of Linear Models with Time and Cross-Sectionally Varying Coefficients. Journal of the American Statistical Association 72(360a):890–898, DOI 10.1080/01621459.1977.10479978

Swerling P (1959) First-order error propagation in a stagewise smoothing procedure for satellite observations. The Journal of the Astronautical Sciences 6:45–52

Telatar E, Telatar F, Bolatoglu N (2007) A regime switching approach to the feldstein–horioka puzzle: Evidence from some european countries. Journal of Policy Modeling 29(3):523–533

Thiele TN (1880) Om anvendelse af mindste kvadraters methode i nogle tilfælde, hvor en komplikation af visse slags uensartede tilfældige fejlkilder giver fejlene en 'systematisk'karakter. Det Kongelige Danske Videnskabernes Selskabs Skrifter-Naturvidenskabelig og Mathematisk Afdeling pp 381–408

Tobin J (1983) "domestic saving and international capital movements in the long run and the short run" by m. feldstein. European Economic Review 21(1-2):153–156

Ventura J (2003) Towards a theory of current accounts. The world economy 26(4):483–512

Vogelsang TJ, Perron P (1998) Additional Tests for a Unit Root Allowing for a Break in the Trend Function at an Unknown Time. International Economic Review 39(4):1073–1100, DOI 10.2307/2527353

Wells C (1996) The Kalman Filter in Finance. Springer Science & Business Media

West M, Harrison J (1997) Bayesian Forecasting and Dynamic Models (2nd Edition). Springer-Verlag New York, Inc., New York, NY, USA

Westerlund J (2006) Testing for panel cointegration with multiple structural breaks. Oxford Bulletin of Economics and Statistics 68(1):101–132

Westphal U (1983) "domestic saving and international capital movements in the long run and the short run" by m. feldstein. European Economic Review 21(1-2):157–159

Wyplosz C (1999) International financial instability. Global Public Goods: International Cooperation in the 21st Century pp 152–89

Zadeh LA, Desoer CA (1963) Linear system theory: the state space approach. McGraw-Hill, New York

Zellner A, Hong C, Min Ck (1991) Forecasting turning points in international output growth rates using Bayesian exponentially weighted autoregression, time-varying parameter, and pooling techniques. Journal of Econometrics 49(1–2):275–304, DOI 10.1016/0304-4076(91)90016-7

Zivot E, Andrews DWK (2002) Further Evidence on the Great Crash, the Oil-Price Shock, and the Unit-Root Hypothesis. Journal of Business & Economic Statistics 20(1):25–44, DOI 10.1198/073500102753410372

Appendix A. The Kalman Filter

The objective of the state-space formulation is to define the state vector ξ_t in a way that guarantees the minimization of the number of elements and the comprehension of all the available information at time t. To estimate the model we use the Maximum Likelihood technique.

Essentially, the Kalman[24] filter (hereafter KF) is, in fact, an algorithm composed by a set of equations, which, performed sequentially, allows to obtain in a forward prediction procedure (called "filtering"), the sequence of linear least squares forecasts of the state vector, $\hat{\xi}_{t|t-1} = \mathrm{E}[\xi_t|\zeta_{t-1}]$ (and hence for the vector of dependents, $\hat{y}_{t|t-1}$), on the basis of the information available at $t-1$, summarized by the vector $\zeta_{t-1} \equiv (y_{t-1}, y_{t-2}, ..., y_1, x_{t-1}, x_{t-2}, ..., x_1)$: In general, the KF, described in Zadeh and Desoer (1963), Harvey (1989), or Hamilton (1994a)[25], is particular suited for Bayesian analysis where, at each time t, there are prior distributions for state variables ξ_t and parameters produced at time $t-1$, from which posterior distributions at time t are calculated as observation y_t becomes available.

The iteration is started by assuming that the initial value of the state vector ξ_1 is drawn from a normal distribution with mean denoted $\hat{\xi}_{1|0}$ and unconditional variance $P_{1|0}$,

$$\xi_1 \sim N\left(\hat{\xi}_{1|0}, P_{1|0}\right) \tag{27}$$

In this case, the distribution of ξ_t conditional on ζ_{t-1} turns out to be normal for $t = 2, 3, , .., T$. The mean of this conditional distribution is represented by the $(r \times 1)$ vector $\hat{\xi}_{t|t-1}$ and the variance of this conditional distribution is represented by the $(r \times r)$ matrix $P_{t|t-1} = \mathrm{E}\left(\xi_t - \hat{\xi}_{t|t-1}\right)\left(\xi_t - \hat{\xi}_{t|t-1}\right)'$.

When all elements of the state vector ξ_t defined by 1 are stationary, i.e. if the eigenvalues of F are all inside the unit circle, the initial means, variances and covariances of these initial state elements can be derived from the model parameters, the system is stable, and $\hat{\xi}_{1|0}$ would be the unconditional mean[26] of this stationary vector, while $P_{1|0}$ would be the unconditional variance, that can be calculated from:

$$vec\left(P_{1|0}\right) = \left[I_{r^2} - (F \otimes F)\right]^{-1}.vec\left(Q\right) \tag{28}$$

where $vec\left(P_{1|0}\right)$ is the $(r_2 \times 1)$ vector formed by stacking the columns of $P_{1|0}$, one on top of the other, ordered from left to right, I_{r^2} represents a r^2 dimension identity matrix, and the $\otimes$ operator represents the Kronecker product.

On the contrary, if (at least some of) the elements of the state vector ξ_t are non-stationary, the process of starting up the series is said to be diffuse initialization of the filter, as at least some of the elements of $\hat{\xi}_{1|0}$ and $P_{1|0}$ are unknown. Hence, for time-variant or non-stationary systems, $\hat{\xi}_{1|0}$ represents a guess as to the value of ξ_1 based on prior information, while $P_{1|0}$ measures the uncertainty associated with this guess [27]. This prior cannot be based on the data, since it is assumed in the derivations to follow that v_{t+1} and w_t, are independent of ξ_1 for $t = 1, 2, ..., T$. Harvey and Phillips (1979) propose a simple approximate technique for the diffuse initialization of the filter, that consists in to initialize nonstationary components of the state vector by any value (say ,$\hat{\xi}_{1|0} = 0$), and an arbitrary large variance, $P_{1|0}$, relative to the magnitude of the series, and then use the standard Kalman filter. The larger the variance, the lesser informative the initialization is for the filter. Koopman (1997) and Koopman and Durbin (2003) propose a more transparent treatment of diffuse filtering and smoothing based on Ansley and Kohn (1985).

Departing from the initial conditions, the algorithm works sequentially in a two-step process. Having described the values of $\hat{\xi}_{t|t-1}$ and $P_{t|t-1}$ for $t = 1$, once the outcome of the next measurement (possibly corrupted with some amount of error, including random noise) is observed, these estimates are updated into the *a posteriori* estimate, as in (29) and (30):

$$\hat{\xi}_{t|t} = \hat{\xi}_{t|t-1} + \cdot P_{t|t-1} H (H' P_{t|t-1} H + R)^{-1} (y_t - A' x_t - H' \hat{\xi}_{t|t-1}) \tag{29}$$

$$P_{t|t} = \cdot P_{t|t-1} - \cdot P_{t|t-1} H (H' P_{t|t-1} H + R)^{-1} H' P_{t|t-1} \tag{30}$$

[24] The Kalman Filter owes its name to the Hungarian Rudolf E. Kalman and his contributions in Kalman (1960), Kalman and Bucy (1961), and Kalman (1963), although similar algorithms had been developed earlier by Thiele (1880) and Swerling (1959), and also at Zadeh and Desoer (1963), constituting an efficient way to formulate the likelihood (usually Gaussian) for many complex econometric models for estimation and prediction purposes.

[25] This section, and the notation employed, draws heavily on the exposition in chapter 13 of Hamilton (1994).

[26] When $\hat{\xi}_{1|0}$ is covariance stationary, a candidate value for $\hat{\xi}_{1|0}$ is zero so that all state variables are initially in steady state.

[27] The greater our prior uncertainty, the larger the diagonal elements of $P_{1|0}$.

The predictive phase uses the state estimate from the previous timestep to produce an estimate of the state at the current timestep. This predicted state estimate is also known as the *a priori* state estimate, along with their uncertainties:
As $\hat{\xi}_{t+1|t} = F\hat{\xi}_{t|t}$ and $P_{t+1|t} = F \cdot P_{t|t}F' + Q$, to calculate the sequence $\left\{\hat{\xi}_{t+1|t}\right\}_{t=1}^{T}$ and $\left\{P_{t+1|t}\right\}_{t=1}^{T}$, one has simply to iterate on equations (31) and (32) for $t = 1, 2, ..., T$.

$$\hat{\xi}_{t+1|t} = F\hat{\xi}_{t|t-1} + F \cdot P_{t|t-1}H(H'P_{t|t-1}H + R)^{-1}(y_t - A'x_t - H'\hat{\xi}_{t|t-1}) \tag{31}$$

$$P_{t+1|t} = F \cdot P_{t|t-1}F' - F \cdot P_{t|t-1}H(H'P_{t|t-1}H + R)^{-1}H'P_{t|t-1}F' + Q \tag{32}$$

The output obtained for step t is used sequentially as the input for the step t + 1.
When the values of the matrices F, Q, B, A, H and R are unknown, we collect the unknown elements of these matrices in a hyperparameter vector, θ, and obtain their maximum likelihood estimates. Casual starting values are assigned to the ξ_t vector and to the unknown elements of matrices included in the vector of parameters θ and the estimation procedure maximizes the likelihood function. In general, Q and R are assumed to be positive semi-definite (which includes the possibility that some of the error terms may be zero).
In some cases it is desirable to use information through the end of the sample (date T) to help improving the inference about the historical value that the state vector took on at any particular date t in the middle of the sample. Such an inference is known as a smoothed estimate, $\hat{\xi}_{t|T} = \mathrm{E}[\xi_t|\zeta_T]$. The mean squared error of this estimate is denoted $P_{t|T} = \mathrm{E}\left(\xi_t - \hat{\xi}_{t|T}\right)\left(\xi_t - \hat{\xi}_{t|T}\right)'$. The estimation of the sequence $\left\{\hat{\xi}_{t|T}\right\}_{t=1}^{T}$ can be calculated by iterating in reverse order for for $t = T-1, T-2, ..., 1$ on:

$$\hat{\xi}_{t|T} = \hat{\xi}_{t|t} + J_t(\hat{\xi}_{t+1|T} - \hat{\xi}_{t+1|t}) \tag{33}$$

$$P_{t|T} = P_{t|t} + J_t(P_{t+1|T} - P_{t+1|t})J_t' \tag{34}$$

where

$$J_t = P_{t|t}F'P_{t+1|t}^{-1} \tag{35}$$

Appendix B. Tables and Figures

Table 1 Bai & Carrion-i-Silvestre (2009) Panel Unit Root Test with common factors and structural breaks. Period 1970-2016.

Variable	Model 2. Trend Break Model									
	Z	P_m	P	Z^*	P_m^*	P^*	T	N	m	fr
inv_{it}	0.023	-0.695	28.264	0.610	-0.059	26.093	47	18	4	72
sav_{it}	-1.121	1.790**	48.764***	-1.217	1.373*	45.326**	47	18	4	72
$size_{it}$	0.690	-1.588	20.902	4.785***	-2.127	16.460	47	18	4	72

Notes: Z, P and Pm denote the test statistics proposed by Bai and Carrion-i-Silvestre (2009). Z and P_m follow a standard normal distribution and their 1%, 5% and 10% critical values are 2.326, 1.645 and 1.282; whereas P follows a Chi-squared distribution with n×(breaks+1) degrees of freedom with critical values *46.459, 43.188 and 37.485*, at 1%, 5% and 10% respectively). The number of common factors are estimated using the panel Bayesian information criterion proposed by Bai and Ng (2002). Z^*, P^* and P_m^* refer to the corresponding statistics obtained using the p-values of the simplified MSB statistics. The null hypothesis of a unit root is rejected at * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$ significance level, respectively, if the statistic is greater than the upper level.

Table 2 Bai and Carrion-i-Silvestre (2009) estimated structural breaks ($m = 4$). Position of the breaks. Period 1970-2016.

	inv_{it}	sav_{it}	$size_{it}$	**Observations**
LUX		1984 1991	1982 2997	47
AUT				47
BEL		1983	1977 1984	47
FIN				47
FRA				47
DEU		1981		47
GRC			2009	47
IRL			1998 2007	47
ITA				47
NETH		1979 1992 2002	1977 1992 2008	47
POR		2009		47
ESP	1979 2006			47
CAN				47
CHN		2009	1993 2006	47
JPN			1995	47
UK				47
USA				47
WORLD				47

Table 3 $baas_t$. Kim and Perron (2009) and Carrion-i-Silvestre et al. (2009) GLS unit roots tests with multiple structural breaks

	Test Statistics	5% Critical Values
P_T^{GLS}	13.766**	7.675
MP_T^{GLS}	14.026**	7.675
ADF	-4.0760**	-4.043
Z_α	-24.607**	-32.846
MZ_α^{GLS}	-18.007**	-32.846
MSB^{GLS}	0.165**	0.122
MZ_t^{GLS}	-2.978**	-4.043
Break dates	1974 / 1979 /2007	

Notes: * denotes significance at $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$ significance level, respectively. The critical values were obtained from simulations using 1,000 steps to approximate the Wiener process and 10,000 replications. Note that for the MSB and MP_T tests the null hypothesis of a unit root is rejected in favour of stationarity when the estimated value of the statistic is smaller than the critical value.

Table 4 Savings Retention Function 1970-2016. European and Non-European Countries.

	Measurement equation			State Update equation		
	β_{0i}	β_{1i}	δ_i	ϕ	μ_1	μ_{2i}
LUX	29.280***	-0.143	0.001	0.843***	0.0004***	-0.008
	8.457	-1.356	0.472	40.453	2.430	-1.029
AUT	19.167***	0.300**	-0.005*			0.003
	5.816	2.543	-1.895			0.406
BEL	10.465***	0.568***	0.003			-0.013**
	4.404	6.017	1.135			-2.012
FIN	14.325***	0.449***	-0.007***			0.008
	5.954	5.175	-2.643			1.173
FRA	14.388***	0.396***	0.001			-0.006
	4.450	3.219	0.225			-0.872
DEU	14.848***	0.492***	-0.005*			-0.005
	5.679	4.729	-1.924			-0.796
GRC	5.685*	0.804***	0.012*			-0.008
	1.736	6.330	1.658			-0.574
IRL	24.942***	0.082	0.012*			-0.037***
	6.473	0.333	1.934			-2.806
ITA	20.756***	0.213*	-0.005**			-0.008
	6.627	1.667	-2.012			-1.236
NETH	18.722***	0.340**	0.001			-0.020***
	4.582	2.300	0.354			-3.194
POR	22.500***	0.321**	-0.017***			0.013
	9.302	2.436	-4.041			1.094
SP	6.500	0.834***	0.001			-0.010
	1.450	4.706	0.340			-1.305
CAN	16.096***	0.226***	0.001			0.000
	9.253	2.659	0.496			-0.050
CHN	7.616	0.476***	0.001			0.011*
	1.375	2.672	0.401			1.678
JPN	18.701***	0.480***	-0.004			-0.009
	5.665	5.326	-1.386			-1.460
GBR	9.845***	0.590***	-0.003			-0.002
	6.452	7.567	-0.963			-0.404
USA	8.274***	0.621***	0.004			-0.010*
	3.770	5.717	1.636			-1.650
WORLD	15.481***	0.304	-0.001			-0.014*
	3.192	1.614	-0.517			-1.762
Observations: 47						

Notes: t-tests in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Fig. 1 Gross Domestic Investment and Savings Ratio to GDP.

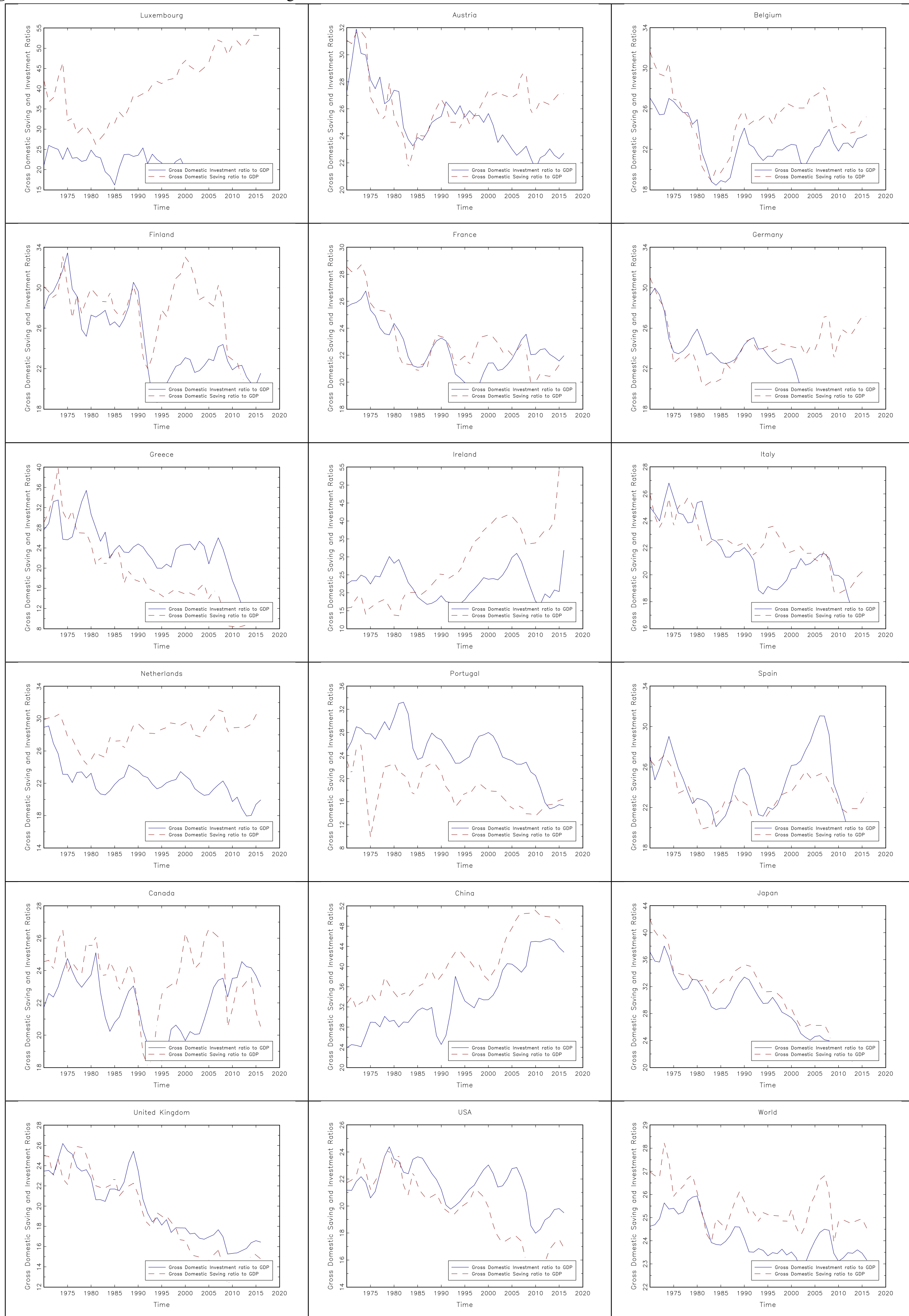

Fig. 2 Fixed and Time-Varying component of Domestic Savings Retention Ratio.