

Catalog Competition: Theory and Experiments

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Abstract

This paper studies a *catalog* competition game: two competing firms decide at the same time product characteristics and prices in order to maximize profits. Since Dasgupta and Maskin (1986) it is known that this one-shot Hotelling game admits an equilibrium in mixed strategies but nothing is known about its nature. We consider a discrete space of available product characteristics and continuous pricing and we fully characterize the unique symmetric equilibrium of the catalog competition game for any possible degree of risk aversion of the competing firms. This allows us experimentally test our predictions in both a degenerated and a genuine mixed strategy elicitation mechanism.

Keywords: catalog competition; Hotelling; mixed equilibrium.

JEL classification: D7, H1.

1 Introduction

Hotelling's (1929) first-location-then-price game is the cornerstone of the literature of product differentiation. In this celebrated model the product characteristics space is modeled by the means of a linear city where two competitors set-up their "shops" and where consumers' residences are assumed to be uniformly distributed. Considering a) that each consumer buys one unit of good and b) that the choice from which shop to buy this unit of good is made on the basis of proximity to the consumer's residence (preferences for product characteristic) and the price that the shop charges, this model truly captures the interaction between price and product characteristics choices and, thereafter, the effect of this interaction on the determination of equilibrium outcomes (prices, degree of product differentiation, firms profits and social welfare). Hotelling's (1929) idea to represent the product's characteristic space by a linear segment was indeed ingenious since it captures the basic trade-offs that consumers with different tastes face when they need to choose between differentiated products.

As far as the timing of these choices is concerned, the Hotelling model (1929) - and most of its variants - assumes that competitors *first* decide where to set up their "shops" and *then*, after their location choices become common knowledge, what price to charge for the product that they sell. This timing of choices might fit certain cases of firm competition but it is certainly not relevant for many others. Consider for, example, the common case of two firms which simultaneously have to reveal their new products in a certain exhibition: firms simultaneously announce both product characteristics and prices. In fact, competition in oligopolistic industries such as car or smart-phones industries takes place clearly in this simultaneous form. Hence, Hotelling's (1929) "timing assumptions" seem to exhibit a poor fit with real world oligopolistic competition.

The idea that simultaneous choice of product characteristics and price might better describe real world oligopolistic competition than the first-location-then-price game of Hotelling (1929) is present in the literature for a long time and is by no means ours. In fact it is first encountered in Lerner and Singer (1937) critique of Hotelling's (1929) work. They, specifically argue that player/firm B should "*take both A's location and his price as fixed in choosing his own location*

and price." Dasgupta and Maskin (1986) and Economides (1984, 1987), many years later, discuss and describe some properties of this more intuitive simultaneous model of product differentiation. Following Monteiro and Page (2008) and Fleckinger and Lafay (2010) we use the term *catalog* competition to distinguish this game from the two-stage Hotelling (1929) model.

A catalog consists of a product characteristic and of a price and, hence, in the framework of the linear city model it is just a pair of a shop location and of the price that it charges. It is easy to show that such a catalog game admits no equilibria in pure strategies when each consumer is assumed to always buy exactly one unit of good. Dasgupta and Maskin (1986), however, were the first to provide formal conditions which ensure that this catalog game has an equilibrium in mixed strategies. Monteiro and Page (2008), moreover, proved that a large family of catalog games admits equilibria in mixed strategies and characterized a family of such catalog games which includes the one-shot variation of the Hotelling game that this paper studies.

Despite the fact that we already have these existence proofs, we know nothing regarding the nature of equilibria of catalog competition games. This is because characterizing a mixed equilibrium in a catalog game is not a straightforward task: a mixed strategy in this framework involves a probability distribution with a *multidimensional* support. That is, unlike the price subgames of the two-stage Hotelling game (Osborne and Pitchik, 1987), the all-pay auctions (Baye et al., 1996), Downsian competition with a favored candidate (Aragonès and Palfrey, 2002) or other games which admit equilibria only in mixed strategies whose underlying probability distributions have unidimensional support (the support of these mixed strategies is a subset of \mathbb{R}), the equilibrium of a catalog game involves mixed strategies with two-dimensional support (their support is a subset of \mathbb{R}^2). This additional dimension, obviously, complicates matters in several degrees of magnitude and makes any characterization attempt intractable when considering the model in its general form.

This paper attempts to shed light on the nature of equilibria of catalog games by considering a variation of the model with a discrete set of locations (available product characteristics) and continuous pricing. Discretization of elements of continuous games in order for a mixed equilibrium to be identified is not uncommon in spatial competition literature (see, for example, Aragonès and

Palfrey 2002 and Hummel, 2010). That is, we consider that two firms compete in catalogs (locations and prices at the same time) but, compared to the standard setup in which firms locate at a point in the unit interval we focus on a case in which firms can locate to the western, to the central or to the eastern district of the linear city. We have to stress here, though, that despite the fact that the set of locations in our setup is finite, the set of available prices is not. Therefore, since the strategy space of each firm is the Cartesian product of these two sets, we have that the strategy space of each firm is infinite. This, along with the facts that the game is not a constant-sum game and that firms' payoff functions exhibit discontinuities, implies that existence of a unique symmetric equilibrium and, thereafter, possibility of full characterization are not guaranteed by any known theorem.

We are able to fully characterize an equilibrium and to, moreover, prove that it is the unique symmetric equilibrium of the game. Given that this equilibrium is in mixed strategies it is necessary that we have in mind that its nature depends on the exact level of competitors risk-aversion. We are able to characterize this mixed equilibrium considering that a firm's payoff function is any increasing function of her profits. That is, we can study how equilibrium strategies change when the risk-aversion level of the competitors varies. We find that certain of its qualitative characteristics are robust in variations of the risk attitudes of the two competitors. The location which is more probable for a firm to locate at is the central, the prices that firms charge are never close to zero and the price that a firms which locates at the periphery of the city (that is, at the western or at the eastern district) charges is larger, in expected terms, than the price of a centrally located firm.

The existence and full characterization of a unique equilibrium in mixed strategies for any possible degree of risk aversion of the two competitors allows us conduct an experiment which can credibly test the hypothesis that actual agents behave according to a mixed equilibrium theoretical prediction. In most non-trivial symmetric games with only mixed equilibria (or with only mixed symmetric equilibria) the underlying probability distributions of the mixed strategies are almost always computed under the assumption that agents are risk-neutral. Examples of such experimental studies are Arad and Rubinstein (2012), Collins and Sherstyuk (2000) and Aragonès and Palfrey (2004). In particular, Collins and Sherstyuk (2000) experimentally study a three firm

location model in the unit interval with fixed prices and attribute the divergence between the theoretical predictions¹ - location of a risk neutral firm is a random draw from a uniform distribution with support $[\frac{1}{4}, \frac{3}{4}]$ - and experimental evidence - location choices result in a bimodal distribution - to the fact that agents are risk-averse. They computationally get some approximate equilibria for a case of risk neutral agents and show that these approximate equilibrium predictions fit the data better than the Nash equilibrium of the risk-neutral scenario. Our results provide a unique opportunity of testing whether agents use Nash equilibrium strategies in a game with only mixed equilibria.....

2 The model

We analyze a two-firm competition model in which firms simultaneously decide a product characteristic (location) and a price. Formally, each firm $i \in \{A, B\}$ chooses a catalog $c_i = (l_i, p_i) \in \{W, C, E\} \times [0, 1]$ where $\{W, C, E\} \subset \mathbb{R}$ is our discrete linear city (W stands for the western district, C stands for the central district and E for the eastern district). For analytical tractability, we assume that $W = -E$, $C = 0$ and $E > 1$.

[Insert Figure 1 about here]

We also assume that there exists a unit mass of consumers whose residences are uniformly distributed on the linear city. Formally, we consider that a consumer $j \in [0, 1]$ resides at $h(j) \in \{W, C, E\}$ and, without loss of generality that $h(j) \leq h(j')$ for every $j < j'$. Each consumer buys exactly one unit of good from only one of the two firms. Considering that the utility of a consumer $j \in [0, 1]$ with a residence at $h(j) \in \{W, C, E\}$ from a certain catalog $c = (l, p)$ is given by

$$U_j(c) = -p - |l - h(j)|$$

¹See Shaked (1979).

we assume that this consumer buys the unit of good from firm A if $U_j(c_A) > U_j(c_B)$, from firm B if $U_j(c_A) < U_j(c_B)$ and with probability $\frac{1}{2}$ from each of the two firms if $U_j(c_A) = U_j(c_B)$. We moreover define

$$I_A(c_A, c_B) = \{j \in [0, 1] | U_j(c_A) > U_j(c_B)\}$$

and

$$I_B(c_A, c_B) = \{j \in [0, 1] | U_j(c_A) < U_j(c_B)\}.$$

Then, the profits of the two firms as functions of their catalogs are given by

$$\Pi_A(c_A, c_B) = p_A \times \left[\mu(I_A(c_A, c_B)) + \frac{1 - \mu(I_A(c_A, c_B)) - \mu(I_B(c_A, c_B))}{2} \right]$$

and

$$\Pi_B(c_A, c_B) = p_B \times \left[\mu(I_B(c_A, c_B)) + \frac{1 - \mu(I_A(c_A, c_B)) - \mu(I_B(c_A, c_B))}{2} \right]$$

where $\mu(S)$ is the Lebesgue measure of the set $S \subset \mathbb{R}$. Hence, like Osborne and Pitchik (1987) and many other relevant models we assume zero marginal costs of production.

We consider that each firm $i \in \{A, B\}$ maximizes $v(\Pi_i(c_A, c_B))$ where $v : \mathbb{R} \rightarrow \mathbb{R}$ is any strictly increasing and absolutely continuous function; without loss of generality we normalize $v(0) = 0$. This very general structure of firms preferences allows us to characterize an equilibrium for any kind of a firm's risk preferences.

A mixed strategy profile in this set up is denoted by (σ_A, σ_B) where for each $i \in \{A, B\}$, $\sigma_i = (F_i^W(p), F_i^C(p), F_i^E(p))$. For each $i \in \{A, B\}$ and each $l \in \{W, C, E\}$, $F_i^l(p)$ is the probability that the catalog $c_i = (l_i, p_i)$ of firm $i \in \{A, B\}$ is such that $l_i = l$ and $p_i \leq p$. A Nash equilibrium

in mixed strategies is a mixed strategy profile $(\hat{\sigma}_A, \hat{\sigma}_B)$ such that $\hat{\sigma}_B$ ($\hat{\sigma}_A$) is a best response of firm B (A) to firm A (B) playing $\hat{\sigma}_A$ ($\hat{\sigma}_B$).

Before we advance to the characterization results some comments regarding the employed assumptions are in order. The fact that $p_i \in [0, 1]$ simply captures the fact that a reservation price exists or a government imposed threshold on the product's price in the spirit of Osborne and Pitchik (1987); obviously, the selection of a reservation price equal to one is without loss of generality. On the other hand the assumption $E > 1$ has certain implications on the analysis which is about to follow. This assumption essentially mitigates the intensity of competition between two firms who locate to distinct districts without eliminating it as the assumptions of Economides (1984, 1987) do.² Our assumption implies that if the two firms locate at different districts then each of them will get the consumers of the district she has located in independently of the price choices that they made. But this does not eliminate incentives to compete. If one firm locates at W and the other at E then the consumers who are located at C will buy the product from the firm which offers two lowest price: price competition has a significant intensity. That is, our model may be indeed a rough approximation of a general catalog competition model with a large product characteristics space but, unlike Economides (1984, 1987), it fairly well captures the dynamics that are generated by the simultaneous choice of location and price. Since local-monopolies equilibria are not possible in our setting, the equilibrium outcomes should provide novel insights regarding the nature of oligopolistic competition. If one assumes that $E < 1$ then, indeed, price competition would be even more intense. It very important to note here though that our *equilibrium is robust to considering values of $E < 1$ as long as they are not very small*. But if we let E take any arbitrarily small value then a) the intensity of price competition will lead equilibrium prices near zero, b) derivation of complete analytical results would become intractable and c) the dynamics which form the qualitative features of our equilibrium would not alter in any substantial way. Hence, we work with this assumption acknowledging its limitations but, at the same time, insisting that it generates qualitatively insignificant losses in generality.

²Economides (1984, 1987) considers that consumers have a very low valuation of the good - only the consumers located very near a "shop" will buy the good - and, hence, pure strategy equilibria in the catalog game exist such that each firm is a local monopolist. In our case this can never occur as price competition is intense at a non-degenerate degree.

3 Formal results

The game clearly does not admit any pure strategy equilibrium. If an equilibrium existed in which both firms charged the same positive price, then this should be a minimum differentiation equilibrium (since in all other cases each firm would have incentives to approach the other and marginally undercut the price). But if in a minimum differentiation equilibrium firms charge the same price then each firm has incentives to marginally undercut the other and double her profit. If in equilibrium firms charge different prices then the low price firm has incentives to locate at the same location with the high price firm and take all the market and the high price firm has incentives to go far away from the low price firm - such an equilibrium cannot exist. Finally, it cannot be the case that both firms choose in equilibrium prices equal to zero. In such a case one of these firms could get positive profits by moving away from the other firm and charging a positive price.

In order to improve the way our formal results are presented let us first give a couple of useful definitions.

Definition 1 *We say that $(\hat{\sigma}_A, \hat{\sigma}_B)$ is a symmetric equilibrium if (i) $\hat{\sigma}_A = \hat{\sigma}_B$ and (ii) $\hat{F}_A^W(p) = \hat{F}_A^E(p)$ for every $p \in [0, 1]$.*

That is, symmetry in our analysis means both that the two firms employ the same strategy - the standard game-theoretic meaning of the term - and that each firm uses a mixed strategy which is symmetric about the center of the linear city - this notion of symmetry is used in unidimensional spatial models with mixed equilibria (see, for example, Aragonès and Palfrey, 2002).

Definition 2 *Consider that $(\hat{\sigma}_A, \hat{\sigma}_B)$ is a symmetric equilibrium. Then $q = \hat{F}_A^W(1)$, $G(p) = \frac{\hat{F}_A^W(p)}{\hat{F}_A^W(1)}$ and $Z(p) = \frac{\hat{F}_A^C(p)}{\hat{F}_A^C(1)}$.*

A mixed strategy $\sigma_i = (F_i^W(p), F_i^C(p), F_i^E(p))$ in our setup is bidimensional. The above definition simply defines conditional probability distributions which will help us present our result in

a more intuitive manner. First, q is the probability that a firm locates at the western (eastern) district; obviously, $1 - 2q$ is the probability that a firm locates at the central district. Then $G(p)$ is the probability distribution of the price of a firm conditional that this firm locates in the western (eastern) district. Finally, $Z(p)$ is the probability distribution of the price of a firm conditional that this firm locates in the central district.

We can now state our main result.

Proposition 1 *There exists a unique symmetric equilibrium $(\hat{\sigma}_A, \hat{\sigma}_B)$ which is such that*

$$(i) \hat{F}_A^W(p) = \begin{cases} 0 & \text{if } p \in [0, p'] \\ \frac{v(\frac{1}{3})(1 - \frac{2v(\frac{2}{3})(v(\frac{1}{3}) - v(\frac{p'}{3}))}{v(\frac{1}{3})(-v(\frac{p'}{3}) + v(\frac{2p'}{3}) + v(p)))}{v(\frac{1}{3}) + 2v(\frac{2}{3})}} & \text{if } p \in [p', 1] \\ \frac{v(\frac{1}{3})}{v(\frac{1}{3}) + 2v(\frac{2}{3})} & \text{if } p > 1 \end{cases}$$

$$(ii) \hat{F}_A^C(p) = \begin{cases} 0 & \text{if } p \in [0, p''] \\ \frac{2v(\frac{1}{3}) - v(\frac{2}{3}) + v(\frac{2p}{3}) - (v(\frac{1}{3}) - 2v(\frac{2}{3}))v(p)}{(v(\frac{1}{3}) + 2v(\frac{2}{3}))v(p)} & \text{if } p \in [p'', 1] \\ 1 - 2\frac{v(\frac{1}{3})}{v(\frac{1}{3}) + 2v(\frac{2}{3})} & \text{if } p > 1 \end{cases}$$

and

$$(iii) 0 < p'' < p' < 1$$

where $p' \in (0, 1)$ is the unique solution of $v(\frac{1}{3})(-v(\frac{p'}{3}) + v(\frac{2p'}{3}) + v(p')) = 2v(\frac{2}{3})(v(\frac{1}{3}) - v(\frac{p'}{3}))$ and $p'' \in (0, 1)$ is the unique solution of $2v(\frac{1}{3}) - v(\frac{2}{3}) + v(\frac{2p''}{3}) = (v(\frac{1}{3}) - 2v(\frac{2}{3}))v(p'')$.

Proof. Assume that a symmetric equilibrium $(\hat{\sigma}_A, \hat{\sigma}_B)$ exists and denote by $S \subseteq \{W, C, E\} \times [0, 1]$ the support of its underlying probability distribution. We will first show that a) $(\hat{\sigma}_A, \hat{\sigma}_B)$ is atomless and that b) S has no gaps. By S having no gaps we mean that if for some $l \in \{W, C, E\}$ two distinct catalogs (l, \dot{p}) and (l, \ddot{p}) belong to S then $F_A^l(\dot{p}) \neq F_A^l(\ddot{p})$. The reason why any symmetric equilibrium is atomless is straightforward. Assume the contrary, that is, that there exists a mass

point on catalog $(W, \dot{p}) \in S$: firm B expects that firm A will choose catalog $(W, \dot{p}) \in S$ with probability $F_A^W(\dot{p}) - \lim_{p \rightarrow \dot{p}^-} F_A^W(p) > 0$. First of all, it is trivial to see why this can never be the case for a catalog (W, \dot{p}) with $\dot{p} = 0$. If such a catalog is part of S then, in this symmetric equilibrium, the expected payoff of each firm is zero. But in this game can always secure positive expected payoffs if, for example, she mixes uniformly among $(W, 1)$, $(C, 1)$ and $(E, 1)$. Therefore if there exists a mass point at (W, \dot{p}) it must be such that $\dot{p} > 0$. If firm B chooses $(W, \dot{p}) \in S$ her payoff will be

$$\begin{aligned} & [F_A^W(\dot{p}) - \lim_{p \rightarrow \dot{p}^-} F_A^W(p)]v(\frac{\dot{p}}{2}) + [F_A^W(1) - F_A^W(\dot{p})]v(\dot{p}) + F_A^C(1)v(\frac{\dot{p}}{3}) + \\ & + [F_A^W(\dot{p}) - \lim_{p \rightarrow \dot{p}^-} F_A^W(p)]v(\frac{\dot{p}}{2}) + [F_A^W(1) - F_A^W(\dot{p})]v(\frac{2\dot{p}}{3}) + [\lim_{p \rightarrow \dot{p}^-} F_A^W(p)]v(\frac{\dot{p}}{3}) \end{aligned}$$

and if B chooses $(W, \dot{p} - \varepsilon)$ her payoff will be

$$\begin{aligned} & [F_A^W(\dot{p} - \varepsilon) - \lim_{p \rightarrow \dot{p} - \varepsilon^-} F_A^W(p)]v(\frac{\dot{p} - \varepsilon}{2}) + [F_A^W(1) - F_A^W(\dot{p} - \varepsilon)]v(\dot{p} - \varepsilon) + F_A^C(1)v(\frac{\dot{p} - \varepsilon}{3}) + \\ & + [F_A^W(\dot{p} - \varepsilon) - \lim_{p \rightarrow \dot{p} - \varepsilon^-} F_A^W(p)]v(\frac{\dot{p} - \varepsilon}{2}) + [F_A^W(1) - F_A^W(\dot{p} - \varepsilon)]v(\frac{2(\dot{p} - \varepsilon)}{3}) + [\lim_{p \rightarrow \dot{p} - \varepsilon^-} F_A^W(p)]v(\frac{\dot{p}}{3}). \end{aligned}$$

We can obviously find $\varepsilon > 0$ arbitrary small such that the considered mixed strategy has no mass at $(W, \dot{p} - \varepsilon)$. Therefore, when B chooses $(W, \dot{p} - \varepsilon)$ and $\varepsilon \rightarrow 0$ her expected payoff becomes

$$[F_A^W(1) - \lim_{p \rightarrow \dot{p}^-} F_A^W(p)]v(\dot{p}) + F_A^C(1)v(\frac{\dot{p}}{3}) + [F_A^W(1) - \lim_{p \rightarrow \dot{p}^-} F_A^W(p)]v(\frac{2\dot{p}}{3}) + [\lim_{p \rightarrow \dot{p} - \varepsilon^-} F_A^W(p)]v(\frac{\dot{p}}{3})$$

which is strictly larger than her payoff at (W, \dot{p}) . That is, (W, \dot{p}) cannot belong in the support of a mixed strategy which characterizes a symmetric equilibrium because firms prefer other catalogs to that. This suggests that our assumption - that the symmetric equilibrium might be such that there exists a mass point on a certain catalog (W, \dot{p}) - is wrong. An argument which rules out existence of a mass point at a catalog (E, \dot{p}) is symmetric to the one that we just developed an argument which rules out existence of a mass point at a catalog (C, \dot{p}) is very similar to the present one. This concludes the proof that if a symmetric equilibrium exists then the underlying probability distribution has no atoms.

We now turn attention to our second claim: if a symmetric equilibrium exists then it has no gaps. Again assume the contrary. Consider that we have a gap and, hence, there exist two distinct

catalogs (W, \dot{p}) and (W, \ddot{p}) which belong to S and which are such that $\dot{p} < \ddot{p}$ and $F_A^W(\dot{p}) = F_A^W(\ddot{p})$. Then, given that this symmetric equilibrium is atomless, if B chooses (W, \dot{p}) her payoff will be

$$[F_A^W(1) - F_A^W(\dot{p})]v(\dot{p}) + F_A^C(1)v(\frac{\dot{p}}{3}) + [F_A^W(1) - F_A^W(\dot{p})]v(\frac{2\dot{p}}{3}) + F_A^W(\dot{p})v(\frac{\dot{p}}{3})$$

while if she chooses (W, \ddot{p}) her payoff will be

$$[F_A^W(1) - F_A^W(\ddot{p})]v(\ddot{p}) + F_A^C(1)v(\frac{\ddot{p}}{3}) + [F_A^W(1) - F_A^W(\ddot{p})]v(\frac{2\ddot{p}}{3}) + F_A^W(\ddot{p})v(\frac{\ddot{p}}{3}).$$

Since, $F_A^W(\dot{p}) = F_A^W(\ddot{p})$ it trivially follows that B 's payoff at (W, \dot{p}) is strictly larger than her payoff at (W, \ddot{p}) . Therefore, (W, \dot{p}) cannot be part of the support of a symmetric equilibrium mixed strategy. Arguments which rule out existence of gaps in $F_A^C(p)$ and in $F_A^E(p)$ are very similar. Hence, if a symmetric equilibrium exists it must have no gaps.

Knowing that if a symmetric equilibrium exists it is an atomless equilibrium with no gaps is very useful for our characterization attempt. Notice that the above arguments moreover establish that $(W, 1)$, $(C, 1)$ and $(E, 1)$ are all part of S . Hence, in a symmetric equilibrium the expected payoff, v^* , of each of the firms should coincide with the payoff of firm B when firm A is expected to play the equilibrium mixed strategy and firm B to choose $(W, 1)$ or to choose $(C, 1)$. In the first case (when B plays $(W, 1)$) the equilibrium payoff, v^* , can be shown to be equal to

$$F_A^C(1)v(\frac{1}{3}) + F_A^W(1)v(\frac{1}{3}) = [1 - F_A^W(1)]v(\frac{1}{3})$$

and in the second case (when B plays $(C, 1)$) the equilibrium payoff, v^* , can be shown to be equal to

$$2F_A^W(1)v(\frac{2}{3}).$$

Therefore in a symmetric equilibrium we must have $F_A^W(1) = \frac{v(\frac{1}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})}$, $F_A^C(1) = 1 - 2\frac{v(\frac{1}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})}$ and $v^* = \frac{2v(\frac{1}{3})v(\frac{2}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})}$.

Since S has no gaps and since there are no mass points involved, if firm B chooses $(W, p) \in S$ it must be the case that

$$[\frac{v(\frac{1}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})} - F_A^W(p)]v(p) + [1 - 2\frac{v(\frac{1}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})}]v(\frac{p}{3}) + [\frac{v(\frac{1}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})} - F_A^W(p)]v(\frac{2p}{3}) + F_A^W(p)v(\frac{p}{3}) =$$

$$\frac{2v(\frac{1}{3})v(\frac{2}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})}$$

which is equivalent to

$$F_A^W(p) = \frac{v(\frac{1}{3})(1 - \frac{2v(\frac{2}{3})(v(\frac{1}{3}) - v(\frac{p}{3}))}{v(\frac{1}{3})(-v(\frac{p}{3}) + v(\frac{2p}{3}) + v(p)))}{v(\frac{1}{3}) + 2v(\frac{2}{3})}.$$

We notice that $F_A^W(p) \geq 0$ if and only if $p \geq p'$ where $p' \in (0, 1)$ is the unique solution of $v(\frac{1}{3})(-v(\frac{p'}{3}) + v(\frac{2p'}{3}) + v(p')) = 2v(\frac{2}{3})(v(\frac{1}{3}) - v(\frac{p'}{3}))$.

Moreover, if firm B chooses $(C, p) \in S$ it must be the case that

$$2\frac{v(\frac{1}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})}v(\frac{2p}{3}) + [(1 - 2\frac{v(\frac{1}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})}) - F_A^C(p)]v(\frac{p}{3}) = \frac{2v(\frac{1}{3})v(\frac{2}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})}$$

which is equivalent to

$$F_A^C(p) = \frac{2v(\frac{1}{3}) - v(\frac{2}{3}) + v(\frac{2p}{3}) - (v(\frac{1}{3}) - 2v(\frac{2}{3}))v(p)}{(v(\frac{1}{3}) + 2v(\frac{2}{3}))v(p)}.$$

We notice that $F_A^C(p) \geq 0$ if and only if $p \geq p''$ where $p'' \in (0, 1)$ is the unique solution of $2v(\frac{1}{3}) - v(\frac{2}{3}) + v(\frac{2p''}{3}) = (v(\frac{1}{3}) - 2v(\frac{2}{3}))v(p'')$.

So, there exists a unique candidate for a symmetric equilibrium given by $(\hat{\sigma}_A, \hat{\sigma}_B)$ which is such that

$$(i) \hat{F}_A^W(p) = \begin{cases} 0 & \text{if } p \in [0, p'] \\ \frac{v(\frac{1}{3})(1 - \frac{2v(\frac{2}{3})(v(\frac{1}{3}) - v(\frac{p}{3}))}{v(\frac{1}{3})(-v(\frac{p}{3}) + v(\frac{2p}{3}) + v(p)))}{v(\frac{1}{3}) + 2v(\frac{2}{3})} & \text{if } p \in [p', 1] \\ \frac{v(\frac{1}{3})}{v(\frac{1}{3}) + 2v(\frac{2}{3})} & \text{if } p > 1 \end{cases}$$

$$(ii) \hat{F}_A^C(p) = \begin{cases} 0 & \text{if } p \in [0, p''] \\ \frac{2v(\frac{1}{3}) - v(\frac{2}{3}) + v(\frac{2p}{3}) - (v(\frac{1}{3}) - 2v(\frac{2}{3}))v(p)}{(v(\frac{1}{3}) + 2v(\frac{2}{3}))v(p)} & \text{if } p \in [p'', 1] \\ 1 - 2\frac{v(\frac{1}{3})}{v(\frac{1}{3}) + 2v(\frac{2}{3})} & \text{if } p > 1 \end{cases}$$

and

$$(iii) 0 < p'' < p' < 1$$

where $p' \in (0, 1)$ is the unique solution of $v(\frac{1}{3})(-v(\frac{p'}{3}) + v(\frac{2p'}{3}) + v(p')) = 2v(\frac{2}{3})(v(\frac{1}{3}) - v(\frac{p'}{3}))$

and $p'' \in (0, 1)$ is the unique solution of $2v(\frac{1}{3}) - v(\frac{2}{3}) + v(\frac{2p''}{3}) = (v(\frac{1}{3}) - 2v(\frac{2}{3})) v(p'')$.

To verify that indeed this symmetric strategy profile is a Nash equilibrium we trivially compute the expected payoff of a firm which chooses (l, p) when the other firm is expected to play according to the specified mixed strategy and we get that if (l, p) belongs to the support of the specified mixed strategy the firms expected payoff is $\frac{2v(\frac{1}{3})v(\frac{2}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})}$ and if (l, p) does not belong to the support of the specified mixed strategy the expected payoff of the firm is strictly smaller than $\frac{2v(\frac{1}{3})v(\frac{2}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})}$. Hence, a best response of a firm which expects that her competitor will behave according to the specified mixed strategy is to employ the same mixed strategy. This concludes the existence and uniqueness proof. ■

Notice that in this unique symmetric equilibrium we have $q < \frac{1}{3}$ and, hence, a firm locates to the central district with a probability strictly larger than $\frac{1}{3}$ and it locates to each of the peripheral districts with a probability strictly smaller than $\frac{1}{3}$. This is a very strong result as it holds for any reasonable risk attitude on behalf of the firms.

To better understand the nature of this unique symmetric equilibrium let us use a specific functional form of firms risk preferences and assume that

$$v(x) = \begin{cases} \frac{1-e^{-ax}}{1-e^{-a}} & \text{if } a \neq 0 \\ x & \text{if } a = 0 \end{cases} .$$

That is $v(x)$ exhibits constant absolute risk aversion (CARA), its risk aversion parameter is $a \in \mathbb{R}$ (if $a > 0$ the firm is risk averse, if $a = 0$ the firm is risk neutral and if $a < 0$ the firm is risk lover), it is continuous for every $x \in [0, 1]$ and every $a \in \mathbb{R}$ and $v(0) = 0$ and $v(1) = 1$ for every $a \in \mathbb{R}$.

Given this, our unique symmetric equilibrium $(\hat{\sigma}_A, \hat{\sigma}_B)$ becomes such that

$$\hat{F}_A^W(p) = \begin{cases} 0 & \text{if } p \in [0, p'] \\ -\frac{e^{-a/3} \left(e^{2a/3} - 2e^{ap} - 2e^{a(\frac{1}{3}+p)} - e^{a(\frac{2}{3}+p)} + e^{\frac{2}{3}a(1+p)} + e^{\frac{1}{3}a(2+p)} + 2e^{\frac{1}{3}a(a+2ap)} \right)}{(2+3e^{a/3})(-1+e^{\frac{ap}{3}})(1+e^{\frac{ap}{3}})^2} & \text{if } p \in [p', 1] \\ \frac{1}{3+2e^{-a/3}} & \text{if } p > 1 \end{cases}$$

and

$$\hat{F}_A^C(p) = \begin{cases} 0 & \text{if } p \in [0, p''] \\ \frac{e^{-a/3} \left(-2e^{a/3} - e^{2a/3} + 2e^{ap} + 2e^{a(\frac{1}{3}+p)} + e^{a(\frac{2}{3}+p)} - 2e^{\frac{1}{3}a(2+p)} \right)}{(2+3e^{a/3})(-1+e^{ap})} & \text{if } p \in [p'', 1] \\ 1 - \frac{2}{3+2e^{-a/3}} & \text{if } p > 1 \end{cases}$$

for $0 < p'' < p' < 1$ which satisfy $\hat{F}_A^W(p') = 0$ and $\hat{F}_A^C(p'') = 0$ (their exact closed form solutions are enormous).

In particular for the case of risk neutral firms, that is for $v(x) = x$, our equilibrium significantly simplifies to

$$\hat{F}_A^W(p) = \begin{cases} 0 & \text{if } p \in [0, \frac{1}{2}) \\ \frac{2}{5} - \frac{1}{5p} & \text{if } p \in [\frac{1}{2}, 1] \\ \frac{1}{5} & \text{if } p > 1 \end{cases}$$

and

$$\hat{F}_A^C(p) = \begin{cases} 0 & \text{if } p \in [0, \frac{4}{13}) \\ \frac{13}{15} - \frac{4}{15p} & \text{if } p \in [\frac{4}{13}, 1] \\ \frac{3}{5} & \text{if } p > 1 \end{cases} .$$

In Table 1 we present some brief information for our equilibrium for various values of the risk parameter $a \in \mathbb{R}$.

a	p'	p''	q
-10	0.677	0.338	0.016931
-1	0.54	0.32	0.172675
0	0.5	0.307	0.2
1	0.46	0.29	0.225578
10	0.176	0.136	0.32559

Table 1. Equilibrium features for various levels of risk aversion.

Moreover in Figure 2 we present the evolution of the densities of the conditional price distributions, $G'(p)$ and $Z'(p)$, for the same example values of risk aversion as in Table 1.

[Insert Figure 2 about here]

In the Appendix we characterize symmetric equilibria of a four locations version of our model and we show that it exhibits the same qualitative features with the equilibrium of the three locations case that we presented here. We also present the equilibrium of a trivial modification of our three location game such that prices range from 0 to 1000 (this will be useful for our experiment).

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4 Appendix

4.1 Four Locations

For the case of four locations $\{W, CW, CE, E\}$ we assume that $|W - CW| = |CW - CE| = |CE - E| > 1$ and everything else is identical to the three location model.

In this case we say that $(\hat{\sigma}_A, \hat{\sigma}_B)$ is a symmetric equilibrium if (i) $\hat{\sigma}_A = \hat{\sigma}_B$ and (ii) $\hat{F}_A^W(p) = \hat{F}_A^E(p)$ and $\hat{F}_A^{CW}(p) = \hat{F}_A^{CE}(p)$ for every $p \in [0, 1]$.

When $v(x) = \begin{cases} \frac{1-e^{-ax}}{1-e^{-a}} & \text{if } a \neq 0 \\ x & \text{if } a = 0 \end{cases}$ and $a \geq 0$ the unique symmetric equilibrium takes the form:

$$\hat{F}_A^W(p) = \begin{cases} 0 & \text{if } p \in [0, p'] \\ e^{-\frac{1}{8}a(5+7p)} \operatorname{csch}\left(\frac{a}{8}\right) \left[\frac{e^{a/2} - e^{3a/4} + e^{ap} + e^{\frac{5ap}{4}} + e^{\frac{7ap}{4}} + 2e^{a(\frac{1}{4}+p)}}{41 - 2e^{-a/4} + 3 \coth\left(\frac{a}{8}\right) \left(\sinh\left(\frac{3ap}{8}\right) + \sinh\left(\frac{5ap}{8}\right) + \sinh\left(\frac{7ap}{8}\right) \right)} + \right. \\ \left. + \frac{e^{a(\frac{1}{2}+p)} - e^{a(\frac{3}{4}+p)} + 2e^{\frac{1}{2}a(1+p)} - 2e^{\frac{3}{4}a(1+p)} + e^{\frac{1}{4}a(2+p)} - e^{\frac{1}{4}a(3+p)}}{41 - 2e^{-a/4} + 3 \coth\left(\frac{a}{8}\right) \left(\sinh\left(\frac{3ap}{8}\right) + \sinh\left(\frac{5ap}{8}\right) + \sinh\left(\frac{7ap}{8}\right) \right)} + \right. \\ \left. + \frac{-2e^{\frac{1}{4}a(3+2p)} - 2e^{\frac{1}{4}a(2+3p)} - 2e^{\frac{1}{4}a(3+5p)} + e^{\frac{1}{4}a(2+7p)} + e^{\frac{1}{4}a(3+7p)} - e^{\frac{3}{4}a(a+2ap)}}{41 - 2e^{-a/4} + 3 \coth\left(\frac{a}{8}\right) \left(\sinh\left(\frac{3ap}{8}\right) + \sinh\left(\frac{5ap}{8}\right) + \sinh\left(\frac{7ap}{8}\right) \right)} + \right. \\ \left. + \frac{-2e^{\frac{1}{4}a(a+3ap)} - e^{\frac{1}{2}a(a+3ap)} + e^{\frac{1}{4}a(a+5ap)} - e^{\frac{1}{4}a(a+6ap)} + 3e^{\frac{1}{4}a(a+7ap)}}{41 - 2e^{-a/4} + 3 \coth\left(\frac{a}{8}\right) \left(\sinh\left(\frac{3ap}{8}\right) + \sinh\left(\frac{5ap}{8}\right) + \sinh\left(\frac{7ap}{8}\right) \right)} \right] \\ \frac{1}{1 - 2e^{-a/4} + 3 \coth\left(\frac{a}{8}\right)} & \text{if } p > 1 \end{cases}$$

and

$$\hat{F}_A^{CW}(p) = \begin{cases} 0 & \text{if } p \in [0, p''] \\ e^{-\frac{1}{8}a(4+7p)} \left(\frac{(-e^{a/4} - e^{a/2} - e^{3a/4} + e^{ap} + e^{\frac{3ap}{2}} + e^{\frac{7ap}{4}} + 3e^{a(\frac{1}{4}+p)})}{4 \left(3 \cosh\left(\frac{a}{4}\right) + \sinh\left(\frac{a}{4}\right) \right) \left(\sinh\left(\frac{3ap}{8}\right) + \sinh\left(\frac{5ap}{8}\right) + \sinh\left(\frac{7ap}{8}\right) \right)} + \right. \\ \left. + \frac{+3e^{a(\frac{1}{2}+p)} - e^{a(\frac{3}{4}+p)} - e^{\frac{1}{4}a(1+p)} - 2e^{\frac{3}{4}a(1+p)} - 2e^{\frac{1}{4}a(3+p)} - 4e^{\frac{1}{4}a(3+2p)}}{4 \left(3 \cosh\left(\frac{a}{4}\right) + \sinh\left(\frac{a}{4}\right) \right) \left(\sinh\left(\frac{3ap}{8}\right) + \sinh\left(\frac{5ap}{8}\right) + \sinh\left(\frac{7ap}{8}\right) \right)} + \right. \\ \left. + \frac{-e^{\frac{1}{4}a(2+5p)} - e^{\frac{1}{4}a(3+5p)} + e^{\frac{1}{4}a(2+7p)} + e^{\frac{1}{4}a(3+7p)} - 2e^{\frac{1}{4}a(a+2ap)} + e^{\frac{3}{4}a(a+2ap)}}{4 \left(3 \cosh\left(\frac{a}{4}\right) + \sinh\left(\frac{a}{4}\right) \right) \left(\sinh\left(\frac{3ap}{8}\right) + \sinh\left(\frac{5ap}{8}\right) + \sinh\left(\frac{7ap}{8}\right) \right)} + \right. \\ \left. + \frac{-e^{\frac{1}{4}a(a+3ap)} + e^{\frac{1}{2}a(a+3ap)} - e^{\frac{1}{4}a(a+5ap)} + 3e^{\frac{1}{4}a(a+6ap)} + 3e^{\frac{1}{4}a(a+7ap)}}{4 \left(3 \cosh\left(\frac{a}{4}\right) + \sinh\left(\frac{a}{4}\right) \right) \left(\sinh\left(\frac{3ap}{8}\right) + \sinh\left(\frac{5ap}{8}\right) + \sinh\left(\frac{7ap}{8}\right) \right)} \right) \\ \frac{1}{2} - \frac{1}{1 - 2e^{-a/4} + 3 \coth\left(\frac{a}{8}\right)} & \text{if } p > 1000 \end{cases}$$

for $0 < p' < p'' < 1$ which satisfy $\hat{F}_A^{CW}(p') = 0$ and $\hat{F}_A^C(p'') = 0$.

In particular for the case of risk neutral firms ($a = 0$), that is for $v(x) = x$, our equilibrium significantly simplifies to

$$\hat{F}_A^W(p) = 0 \text{ for every } p \in [0, 1]$$

and

$$\hat{F}_A^{WC}(p) = \begin{cases} 0 & \text{if } p \in [0, \frac{2}{7}) \\ \frac{7}{10} - \frac{1}{5p} & \text{if } p \in [\frac{2}{7}, 1] \\ \frac{1}{2} & \text{if } p > 1 \end{cases} .$$

When $a(0$ the unique symmetric equilibrium takes the form:

$$\hat{F}_A^W(p) = 0 \text{ for every } p \in [0, 1]$$

and

$$\hat{F}_A^{CW}(p) = \begin{cases} 0 & \text{if } p \in [0, p'') \\ \frac{-1+e^{a(-\frac{1}{2}+p)}-e^{\frac{ap}{2}}+e^{ap}}{2(-1+e^{ap})} & \text{if } p \in [p'', 1] \\ \frac{1}{2} & \text{if } p > 1 \end{cases}$$

for p'' such that $\hat{F}_A^{CW}(p'') = 0$.

4.2 Price between 0 and 1000

We now consider that a) in each location there are exactly 10 consumers, b) prices can take values from 0 to 1000, c) distance between districts is properly scaled up to have the same degree of price competition intention as in the case analyzed in the formal part of the paper and d)

$$v(x) = \begin{cases} \frac{1-e^{-a\frac{x}{30000}}}{1-e^{-a}} & \text{if } a \neq 0 \\ \frac{x}{30000} & \text{if } a = 0 \end{cases} . \text{ Therefore our unique symmetric equilibrium takes the form:}$$

$$\hat{F}_A^W(p) = \begin{cases} 0 & \text{if } p \in [0, p') \\ \frac{e^{a/3} \left(-1 - e^{\frac{ap}{3000}} - e^{\frac{ap}{1500}} + e^{\frac{ap}{1000}} + 2 \left(e^{a \left(-\frac{2}{3} + \frac{p}{1000} \right)} + e^{a \left(-\frac{1}{3} + \frac{p}{1000} \right)} - e^{\frac{a(-500+p)}{1500}} \right) \right)}{(2+3e^{a/3}) \left(-1 + e^{\frac{ap}{3000}} \right) \left(1 + e^{\frac{ap}{3000}} \right)^2} & \text{if } p \in [p', 1000] \\ \frac{1}{3+2e^{-a/3}} & \text{if } p > 1000 \end{cases}$$

and

$$\hat{F}_A^C(p) = \begin{cases} 0 & \text{if } p \in [0, p'') \\ \frac{e^{-a/3} \left(-2e^{a/3} - e^{2a/3} + 2e^{a \left(\frac{1}{3} + \frac{p}{1000} \right)} + e^{a \left(\frac{2}{3} + \frac{p}{1000} \right)} + 2e^{\frac{ap}{1000}} - 2e^{\frac{a(2000+p)}{3000}} \right)}{(2+3e^{a/3}) \left(-1 + e^{\frac{ap}{1000}} \right)} & \text{if } p \in [p'', 1000] \\ 1 - \frac{2}{3+2e^{-a/3}} & \text{if } p > 1000 \end{cases}$$

for $0 < p' < p'' < 1000$ which satisfy $\hat{F}_A^W(p') = 0$ and $\hat{F}_A^C(p'') = 0$.

In particular for the case of risk neutral firms, that is for $v(x) = \frac{x}{30000}$, our equilibrium significantly simplifies to

$$\hat{F}_A^W(p) = \begin{cases} 0 & \text{if } p \in [0, 500) \\ \frac{2}{5} - \frac{200}{p} & \text{if } p \in [500, 1] \\ \frac{1}{5} & \text{if } p > 1000 \end{cases}$$

and

$$\hat{F}_A^C(p) = \begin{cases} 0 & \text{if } p \in [0, \frac{4000}{13}) \\ \frac{13}{15} - \frac{800}{3p} & \text{if } p \in [\frac{4000}{13}, 1] \\ \frac{3}{5} & \text{if } p > 1000 \end{cases} .$$

In Table 2 we present some brief information for our equilibrium for various values of the risk parameter a .

a	p'	p''	q
-10	677	338	0.016931
-1	540	320	0.172675
0	500	307	0.2
1	460	290	0.225578
10	176	136	0.32559