

# The effect of herding in financial markets



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DEGREE IN ECONOMICS

EC1049 – FINAL PROJECT OF DEGREE

Academic year: 2016 – 2017

November 2016

## **ABSTRACT**

In this research we present a stylized financial agent-based model with heterogeneous noise traders that imitate each other on a dynamic network structure. Following Tedeschi et al. (2009, 2012), we show how an expectation feedback system can reproduce synchronization effects generating large fluctuations in returns. Moreover, we assess how ‘herding’ can give rise to some stylized facts such as volatility clustering and fat tailed distributions in some investigated variables such as indegree or returns, (see Cont, 2001). We demonstrate how the transition from periods of network centralization, corresponding to high synchronization in agents’ expectations, to periods of decentralization, when traders play randomly, is the key ingredient to reproduce these statistical properties above-mentioned. The model is an evolution of Tedeschi’s (2009), since we introduce an endogenous evolution mechanism of the intensity of choice,  $\beta$ . Here this parameter is updated daily according to the guru’s surviving period. Our findings show that there exists a strong correlation between the “guru evolution” and the returns time series.

Keywords: volatility clustering, fat-tail distributions, noise traders, intensity of choice, guru

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# 1. THEORETICAL FRAMEWORK.

## 1.1. Agent Based Model.

Agent Based Models (from now onwards AMBs) are computational systems used to simulate and observe the actions and interactions of individuals within artificial markets. Basically, they help us to analyze more complex environments via the interaction of many heterogeneous agents. According to this view, a market economy can be analyzed as a self-organizing entity. In many complex systems in nature, there are global phenomena that are the irreducible result of local interactions between components whose individual study would not allow us to see the global properties of the whole combined system. Thus, a growing number of researchers show that many macro properties of the economic system are not directly encoded by any of the single components involved, but are the self-organization outcomes of the interactions of the components. Thus, given the presence of imperfections in the market organization, economic dynamics is the result of the communication and interaction of a myriad of heterogeneous agents and not the fruit of some invisible hand optimal process.

In a recent paper, Iori & Porter (2012) show strengths and weaknesses of this approach. On the one hand, the usage of these simulations are supported in order to better understand the economy and the financial markets as an evolving system to prevent the financial crisis. On the other, some skeptical economists hardly criticize the usage of these simulations by claiming that these models pay too much attention to the well-known “stylized facts”<sup>1</sup> from the temporary financial series, which are inconsistent with the standard asset pricing models. However, Farmer & Foley (2009) highlight the lack of clarity about asset pricing models and recognize the necessity of going beyond as well as applying the ABM methodology to create wider models able to incorporate multiple markets. Indeed, the CRISIS Project and FuturICT knowledge Accelerator are the most relevant undertaken projects that have made an attempt to create a large-scale market model. However, as Iori & Porter (2012) suggest

it is generally accepted that there are many empirical financial phenomena which are difficult to explain using traditional models. As many authors have noted the empirical distributions of returns of many market indices and currencies, over different but relatively short time intervals, shows an asymptotic power law decay (Mandelbrot (1963); Pagan (1996); Guillaume et al. (1997); Gopikrishnan et al. (1999).

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<sup>1</sup> It has been detected a set of empirical common properties in asset returns in most of research. This “stylized facts” are analyzed by Cont (2001).

## 1.2. Modelling approaches.

The ABM that we introduce is a financial market model where the market mechanism is the major area of interest. In order to build a model with these characteristics, it is important to understand both the structure of the market and the modeling of behavior. First, regarding the market structure, as we will see in the following sections, our model is based on a decentralized market where it does not exist a market maker such as risk neutral and endowed with unbounded liquidity, whose function is absorbing surpluses and making trading always viable. Second, regarding the modeling of agents' behaviors, we refer to the herding and social learning phenomena further developed by Tedeschi (2016) and Tedeschi, Iori & Gallegati (2009, 2012) where the agents known as "noise traders" or "zero intelligence agents" base their expectations on the expectations of others investors to whom they are connected, which are "gurus" or links that will be updated through a fitness mechanism.

In order to comprehend the proposed modeling approach, we have to highlight its three key ingredients: i) zero intelligence agents, ii) which interact through a market mechanism in order trade stocks and iii) interacting directly in order to form their expectations on prices. In the next subsections, we present how agent-based literature deals with these three classes of models.

### 1.2.1. Zero Intelligence Agents

ZI Agents Models are mainly based on an investigation or research with noise traders, who are able to significantly vary from a completely random behavior to the fact of having a budgetary limitation or some sort of specific strategy. In general, we can pinpoint two main characteristics in the vast majority of these models: lack of (explicit) learning and a minimalist approach to agent behavior.

Even though there exists some previous work elaborated by Becker (1962), the first investigation on ZI trading for financial market was conducted by Gode & Sunder (1993). While they were trying to describe what a ZI agent is about, they noted that "it has no intelligence, does not seek or maximize profits, and does not observe, remember, or learn. It seems appropriate to label it as a zero-intelligence trader" (p.4, 1993). In fact, the researchers also compared the behavior of profit-motivated human traders to ZI agents and then divided their investigation into three main stages. Firstly, they use the mechanism of a double auction where buyers and sellers submit limit orders bids or asks and can accept these bids or asks, all of them subject to budget constraints. In the second double auction, the same subjects are subject to no budget constraint. In the

third stage, they deeply analyze the results and draw on the main conclusions. The key result is that for their model, in terms of the aggregate property of allocative efficiency<sup>2</sup>, ZI traders perform comparably well to human traders.

In the same line of research, Duffy & Ünver (2006) conducted an experiment in order to examine whether a simple ABM can generate bubbles and similar crashes to the ones that have already been observed in previous experiments. This study focuses on better understand the behavior of ZI agents in order to deal with the characteristics of asset bubble environments, making it possible through near-zero-intelligence traders in a double auction context as in the previous model.

### 1.2.2. *Heterogeneous Agents with Market Mediated interactions.*

This class of models try to emphasize some important characteristics of the different mechanisms of price formation, that is the detail of how exchanges occur in financial markets. In this regard, stock markets can be organized in different ways. There are, for example, supply demand in-balance markets which require the presence of a market maker, double action and order-driven markets. Among the several financial agent-based models, some works have paid more attention to the fact of reproducing the behavioral rules of traders, others, instead, have focused the analysis on the market microstructures. Very few models, however, have jointly combined agents' microfoundation with markets' microstructures.

The model of Caldarelli et al (1997) was one of the former ones which introduced this sort of models. Thus, a prototypical stock market model is proposed, in which only the interactions among traders without external influences are taken into account. Each agent trades according to his/her own strategy by speculating about the prices fluctuations of the artificial market. Again, this model reproduces a record of prices very close to the ones observed in the real financial markets.

Moreover, Lux & Marchesi (1999) described a financial market model with chartists and fundamentalists, which gave rise to scaling laws. Here there is a market maker reacting to imbalances between demand and supply. The fundamentalist agents purchase or sell if the value of the market is beyond or below the fundamental value while the chartists form a more heterogeneous group due to the fact that they are constantly changing from positive to negative expectations and the other way round, which will clearly define their role as buyers or sellers. A burst of volatility is produced if

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<sup>2</sup> Allocative efficiency is total profit divided by maximum total profit, or sum of consumer and producer surplus.

the proportion of agents, which use chartist strategies, overcomes a given threshold. These stages of instability are rapidly over and it is known as “on-off-intermittency”<sup>3</sup>.

The above models use some kind of switching, either between classes or strategies. For instance, Chiarella & Iori (2002) built a market model with agents that used mixed strategies with different weight or components. Introducing an order-driven market model with heterogeneous agents trading via a central order matching mechanism, the researchers examined how different trading strategies may affect the dynamics of price, bid-ask, trading volume and volatility. Attention is also given to how some features of market design (tick size and order lifetime) affect market liquidity.

In the same vein, Chiarella & Iori (2009) show an order driven market similar to the one designed in 2002 where agents based their expectations on future asset returns on weighted average of three different components: fundamentalist, chartist and noise trader. Furthermore, agents differ in the characteristics describing these components, such as time horizon, risk aversion and the weights given to the various components. The key result from this investigation is that the chartist strategies are responsible of generating fat tails and clustering in the artificial price data generated by the model. Also, it shows that the increase of volatility is probably due to the presence of large gaps in the book because of the expectations from both chartist and fundamentalist components.

### 1.2.3. Heterogeneous Agents with Direct Interactions.

In the models with direct interactions, that is to say, without any mechanism that mediates among agents, we can observe that these include sophisticated learning behaviors and explicit modeling of the direct interactions of agents. Thus, there exist three different levels of direct interactions: (a) global interactions, where an agent uniformly randomly interacts with another agent, (b) local interactions on a lattice, where interactions are constrained to a set of neighbors but in a regular way and (c) local interactions on a network which can evolve from early models where the network is assumed to later models where the network structure may arise endogenously.

A canonical investigation on this type of models was developed by Kirman (1993). This investigation included the idea of herding by using ants as the object of their study. The ants rely on two random food sources from which they can change through an individual process of recruitment: two ants meet and one switches to another’s source with probability  $(1-\delta)$  even though there exists an additional probability  $\epsilon$  that an ant

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<sup>3</sup> This concept of market behavior is explain more extensively by Lux and Marchesi (2000).



changes from one source to another with no interaction. The experiment shows that initially the ants are more focused on one of the sources and that later on they change their attention through herding, phenomenon that is compared to human when choosing a certain restaurant.

Based on the experiments carried out by Kirman (1993) and Lux & Marchesi (2000), Westerhoff (2009) develops an agent-based financial market model where the agents may follow technical and fundamental rules to determine their speculative investment position. The mechanism of opinion changes here is more sophisticated than Kirman's (1993). The probability that agents adopt a new speculation strategy will depend on the past profits of that strategy. This is achieved by means of fitness variables  $A^C$  y  $A^F$  for chartist strategies and fundamental ones respectively, each a discounted sum of the past returns. This switching leads to periods where fundamental strategies dominates the markets and consequently prices fluctuate around its fundamental value and there will be other periods dominated by technical strategies, which result in an increase in volatility, spectacular bubbles and crashes.

Challet, Marsili & Zhang (2000) introduced the concept known as Minority Game<sup>4</sup> in their model. One of the most relevant parts of this investigation is based on observing what happens when an agent knows ahead of time the actions of a subset of other agents. This can vary its strategies in relation to the tendencies that it observes and consequently, it always wins at least the average of other agents.

A simple model of stock market with a random communication structure is also provided in Cont & Bouchaud (2000) where the interactions among agents give raise to heavy tails in the distribution of stock price variations in the form of an exponentially truncated power law. The key element of the model is communication between agents, which is modeled here by a set of clusters that coordinate individual demand. This model thus indicates a relation between the excess kurtosis observed in asset returns, the market order flow, and the tendency of market participants to imitate each other.

Last but not least, Tedeschi, Iori & Gallegati (2012) suggest a very similar model to the one suggested in our experiment. The researchers introduce an order-driven market with heterogeneous traders where the concept of herding becomes completely relevant. The agents, mainly "noise traders" have the incentive of imitating themselves

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<sup>4</sup> The basic Minority Game was formulated by physicists Damien Challet and Yi-Cheng Zhang in 1997 to avoid the main obstacle of the El Farol problem, the definition of agents' strategies.

among them, which causes the appearance of the so-called “gurus”<sup>5</sup>. Thanks to the fitness mechanism that measures the relative wealth from the previous period of an agent  $i$ , agents ZI are able to change their willingness to follow an agent  $k$  by another new agent (guru)  $j$ . Thus, thanks to this mechanism, gurus can rise and fall in popularity over time. One of the main results of this experiment is that the suggestions that noise traders quickly go bankrupt and are eliminated from the market is unrealistic in presence of herding and the expectations feedback system<sup>6</sup> and positive intelligence agents cannot invade a market populated by noise trader when herding is high.

### **1.3. What we propose: and introduction to our model.**

In this section we present a financial market model able to reproduce important regularities emerging in financial time series. The main goal of this research is to identify the main conditions under which imitation leads to fat tails and volatility clustering and how herding effects may be responsible for the persistence of asset price. Even though stocks returns are uncorrelated, the absolute values are auto correlated. For this reason, they reflect an inclination to move from quiet periods to more turbulent ones. Regarding the surviving period of significant positive autocorrelation of absolute stock returns, they last for a year or more, and they decay at a rate which is slower than exponential.

We develop an expectation feedback system populated by noise traders. Initially, these agents start with the same amount of wealth. As time goes by, some of them will become richer than others and consequently, heterogeneous, according with the empirical evidence that the market participants are very heterogeneous in size. Moreover, in contrast with the prevailing economic view that informed agents need to hide their private informations in order to profit it, (see Benabouy and Laroque, 1992; Caldentey and Stacchetti, 2007), our uninformed gurus gain the highest profits when they reveal their expectations to the highest number of followers. The generation of “herding” that provokes excess demand, bubbles and volatility clustering is conducted by the introduction of an endogenous mechanism of imitation. By using a preferential attachment rule (Barabasi and Alberti, 1999), each trader is followed by another one with a probability proportional to its profit.

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<sup>5</sup> In our artificial financial market model each agent has an outgoing link and can have various incoming links. The agent with most incomings links is known as the “guru”.

<sup>6</sup> “Positive feedback in a stock market refers to the situation where positive (negative) expectations about the price do lead to a price increase (decrease)” (Tedeschi, Iori and Gallegati, 2012).

Besides, this mechanism generates an evolving network structure ranging from the random graph to the power-law one<sup>7</sup>. This allows us to analyze how the imitation among traders affects the evolution of the network topology and how it has influence on market returns. Most of the studies on herding effects have examined how herding can go along with large price fluctuation but only a few papers have investigated its role on the communication network structure and on traders' wealth, which is one of the main aims of the present study.

It is crucial to understand how expectations feedback system with 'zero intelligence agents' works. If a guru shares pessimistic expectations about an asset, he/she decides to sell. His/her followers imitate his/her strategy driving to a higher fall in the prices. Thus, the guru has cash to buy a higher amount of assets than he/she had at the beginning. Conversely, if a guru shares an optimistic expectation about an asset, he/she decides to buy. The neighbors that follow his/her strategies imitate the guru's strategy driving prices up. In this moment, the guru has assets that he/she bought cheaper.

The agents in our market are zero intelligence agents which present random expectations about future returns and a random demand function. This differs to other models where sophisticated strategies are implemented (Brock and Hommes 1998; Chiarella et al. 2009; Lux and Marchesi 2000). This is due to several reasons. Firstly, they help us to deeply examine the impact of imitation on prices dynamics. Secondly, the analysis of the impact of noise traders on prices' movement is thorough in the literature. For instance, some authors (Figlewski et al., 1979; Shiller, 1984; De Long et al., 1990) show that "irrational" aggressive noise traders can destabilize prices driving them away from the fundamental value and earn larger returns.

The main reason for using this AMB to develop our model is that the mainstream economy literature does not provide us with satisfactory and adequate information to conduct a deep analysis of the situations of crisis. As Iori and Porter claim

The highly stylized, analytically tractable traditional models in economics and finance are not well-suited to study crisis situations (Bouchaud [2008], Farmer and Foley [2009], Kirman [2010]); in fact there is no framework in classical economics for the understanding crises. ABMs on the contrary can

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<sup>7</sup> A power-law distribution is observed when, on the one hand, there are many individual elements with almost no popularity, and on the other, few elements with high popularity.

represent unstable systems with crashes and booms that develop out of non-linear responses to proportionally small change (2012, p.2)

Our findings show that the phenomenon of herding leads to a market populated by zero intelligence agents to present fat tail distributions and volatility clustering. In this context, we can observe two possible scenarios: (a) coordinated periods of strategies that produce bubble prices and volatility in returns, and (b) non-coordinated periods of strategies where returns remain stable and some agents compete to concentrate the majority of incoming links.

The rest of the research is organized as follows. In section two we explain how the market works and which are its characteristics. In the next section we comment the main results obtained in the investigation. In section four we develop the main conclusions and to conclude we list the references used to conduct the experiment.

## **2. THE MODEL.**

In this section we present how the communication and imitation among agents affect the expected returns and how our artificial market works. We implement a realistic mechanism of price formation based on the Euronext and the London Stock Exchange mechanism. In this financial market the price dynamics is determined by the structure of the exchange process without using any ad hoc mechanism. Agents of the artificial market may execute two types of orders: buy or sell. On the one hand, a trader can execute an order known as “market order to sell (buy)”. This instruction is automatically executed when the selling price (buying price), called ask (called bid) is lower (higher) than the market price. On the other hand, when a sell order (buy order) is higher (lower) than the market price, there is a “limit order to sell (buy)”. Limit orders are thus stored in the exchanges books expecting to be executed due to the market’s prices fluctuations. Thus, limit orders still unmatched after a predetermined time horizon are removed from the book.

### **2.1. The market microstructure.**

We introduce an order driven market with all buyers and sellers displaying the prices at which they are willing to buy or sell securities, and the quantity of securities desired to be bought or sold. Our artificial model is populated by N traders that can place market and limit orders. Market orders are placed when a purchase or sale offer reaches a quote in the opposite side of the market and are immediately executed at the current best price on the list. On the contrary, limit orders are placed when a purchase or sale offer does not reach a quote on the opposite side of the market and are kept and executed in the exchange’s book using time and price priority.

It is through a number of periods (days)  $t_k$  with  $k = 1, \dots, T$  that trading takes places. In this way, there are  $\tau$  intraday operations each day of trading. When the period starts, traders make expectations about the price at the end of a given time horizon  $T$ . Thus, the future price expected by agent  $i$  at time  $t_k + \tau$  is given by

$$\hat{p}_{t_k, t_k + \tau}^i = p_{t_k} e^{\hat{r}_{t_k, t_k + \tau}^i \sqrt{\tau}} \quad (1)$$

Where  $\hat{r}_{t_k, t_k + \tau}^i$  is the agent's expectation on the spot return which may be affected by the expectations of other agents due to the communication network among traders and is the price that  $p_{t_k}$  all agents can see at the beginning of each period.

Once agents have determined their expectations about the future price, they randomly enter the market to establish a limit price and an order size to submit their orders. Those traders with bullish expectations decide to purchase at price  $b_t^i$ , which will obviously be lower than the expected future price  $\hat{p}_{t_k, t_k + \tau}^i$ . Those traders with bearish expectations decide to sell at price  $a_t^i$ , which will be higher than in expected price. The purchase and sale price, namely bid and ask, are uniformly distributed around the current price and calculated according the following rule:

$$\begin{aligned} b_t^i &\sim U(p_{\min}^b, \hat{p}_{t_k, t_k + \tau}^i), \quad p_{\min}^b = p_t(1 - \gamma_t^1), \\ a_t^i &\sim U(\hat{p}_{t_k, t_k + \tau}^i, p_{\max}^a), \quad p_{\max}^a = p_t(1 + \gamma_t^2) \end{aligned} \quad (2)$$

Where  $\gamma_t^{1,2}$  are random variables uniformly distributed in the interval (0,1) and  $p_t$  is the price at the time the order is submitted. In formula (2) it can be observed that the price that buyers are willing to pay is between  $p_{\min}^b$ , which is the worst buy order, and the expected future price previously calculated  $\hat{p}_{t_k, t_k + \tau}^i$ . Conversely, the price that sellers are willing to sell is between their future price expectation  $\hat{p}_{t_k, t_k + \tau}^i$  and  $p_{\max}^a$  which corresponds to the best sell order in this moment.

When transactions occur, price is recalculated as follows:  $p_t$  is calculated by the price at which a transaction occurs. If there are not occurring transactions, a proxy for the price is given by the average of the quoted ask  $a_t^q$  (the lowest ask listed in the book) and the quoted bid (the highest bid listed in the book):  $p_t = \frac{(a_t^q + b_t^q)}{2}$

For this reason, the updated price with this proxy variable will be between the lowest selling price and the highest purchase price included in the book  $a_t^q < p_t < b_t^q$ .

The order book is the list of all buy and sell limit orders at a given instant of time, with their corresponding price and volumes. A limit order indicates the maximum (or minimum) price at which a trader is willing to buy (or sell) a certain quantity of shares. At a given point in time, all limit buy orders are below the best buy order called the 'bid price'. On the other hand all sell orders are above the best sell order called the 'ask price'. With the appearance of a new order, such as buy order, it either adds to the book if it is below the ask price, or generates a trade at the ask price if it is above (or equal to) it, (we call all these 'market orders'). More in detail, if the maximum price that a buyer is willing to offer is higher than the price a seller is willing to sell, that is  $b_t^i > a_q^t$ , agents place market orders to buy stocks at a current quoted ask  $a_q^t$ . If it appears that the buyer wants to buy more stocks than available at this price, he/she will continue buying the next offer available until it satisfied their demand or when there is no more sell orders in the book at a price smaller than  $b_t^i$ . If  $b_t^i > a_q^t$  traders submit limit orders.

Symmetrically, if the price at which a seller is willing to sell is lower than the current quoted bid, that is  $a_t^i < b_q^t$ , then, investors place market orders to sell stock at a price  $b_q^t$ . If the demand available on the book at this price is not sufficiently large, he fills the available demand at the bid and then moves on to check the second best bid price, iterating the process until the agent has no more stock to sell or there are no more buy orders in the book at a price greater than  $a_t^i$ .

In our financial market, agents have a random demand function and the size of their order is only bounded by budget constraints. Agents hold a finite amount of cash  $C_t^i$  and stocks  $S_t^i$  in their portfolio. The size  $s_t^i$  of agents' orders is determined as follows:

- If the agents expect a price decrease, they sell a random fraction of their assets  $s_t^i = \xi_t S_t^i$
- If the agents expect a price increase, they invest a random fraction of their cash in the assets equal to
  - $s_t^i = \xi_t C_t^i / b_t^i$  if they execute/submit a limit order
  - $s_t^i = \xi_t C_t^i / a_q^t$  if they execute/submit a market order (agents buy at the current ask)

with  $\xi_t$  to be a random variable uniformly distributed on the interval (0,1).

The essential details of the trading mechanism can be explained easily. We can find four different situations:

- When an agent is willing to sell a random fraction of his/her stock at a lower price (price per asset) than the buying quoted price, place a market order to sell that will be executed at the best buying price in that specific moment.
- When this agent is willing to sell a random fraction of his/her stock a higher price (price per asset) than the buying quoted price, place a limit order to be kept in the order book awaiting for the selling quoted price to decrease in order to be executed.
- When an agent is willing to buy a random amount of stock at a higher price (price per asset) than the selling quoted price, place a market order to buy that will be executed at the lowest selling price in that specific moment.
- When this agent is willing to buy an amount of stock at a lower price (price per asset) than the buying quoted price, place a limit order to buy to be kept in the order book awaiting for the selling quoted prices to decrease in that specific moment.

When agents execute a market order, their cash and stock amounts proportionally vary to such order. When agents execute a limit order, even though there is no change in the composition of the portfolio in that moment, the cash they commit to buy and the stocks they commit to sell are also temporarily removed from their portfolios. In this way, agents cannot spend money or sell stocks that have already been committed in the book. If the order is cancelled, the stocks and cash that were retained are returned in the portfolios of the corresponding agents.

## 2.2. The agents' microfoundation.

The market we present consists of zero intelligent agents, also known as “noise traders”. These agents are heterogeneous. Therefore, at the beginning of each day they have different expectations about the spot return  $r_{t_k, t_k + \tau}^i$  in the interval  $(t_k, t_k + \tau)$  and have different forecasts of the returns' volatility,  $\sigma_{t_k}^i$ . Expected returns are thus given by

$$r_{t_k, t_k + \tau}^i = \sigma_{t_k}^i \epsilon_{t_k} \quad (3)$$

where  $\sigma_{t_k}^i$  is a positive, agent specific, constant and  $\epsilon_{t_k} \sim N(0,1)$  is a normal noise. In order to determine how the interaction among agents affects asset price and volatility returns, we have introduced a communication structure in which nodes represent agents and the hedges are the connective links between them.

Thanks to this communication network, traders ask opinion on other traders, more precisely on their neighborhood, so they can revise and change their expectations and consequently their future price (see Eq.1). Specifically, we model an imitation mechanism, by implementing a preferential attachment rule (Barabasi and Albert, 1999) such that each trader is imitated by others with a probability proportional to its wealth. The noise trader is the agent  $i$ , who imitates agent  $j$ , known as the guru. Therefore, the agent  $i$  updates his/her expected returns according to the trader  $j$ , this is

$$\hat{r}_{t_k, t_k + \tau}^i = r_{t_k, t_k + \tau}^j$$

All agents depart from the same level of economic wealth  $W_{t=0} = C_{t=0} + p_{t=0}S_{t=0}$ . Clearly, as times passes by and operations are performed, some of them become richer than others, what strongly determines who imitates and who is being imitated. In order to quantify the agents' success, we use a measure that indicates us the fitness from every agent at time  $t$  as well as their wealth relative to the wealth  $W_t^{\max}$  of the richest agent  $i_{\max}$ :

$$f_t^i = \frac{W_t^i}{W_t^{\max}} \quad (4)$$

Each agent  $i$  starts with one outgoing with a random agent  $j$ , that is to say, each agent can only imitate one guru's strategy and, instead, can have several incoming links from other agents. At the beginning of each period, links are rewinded as it follows: each agent  $i$  cuts his outgoing link, with agent  $k$ , and forms a new link with a randomly chosen agent  $j$ , with a probability:

$$p_t^i = \frac{1}{1 + e^{-\beta_t(f_t^j - f_t^k)}}, \quad (5)$$

and with a probability equal to  $1 - p_t^i$  of keeping his/her actual link. Gurus are the most successful ones, which will make them obtain higher fitness, and as consequence they will have more probabilities to obtain more incoming links. In order for the links not to be directed and exclusively to the gurus with more fitness, the algorithm introduces a certain amount of randomness modeling imperfect information and bounded rationality of agents so as the links with more successful agent have a finite probability to be cut in favor of links with less successful ones. The parameter  $\beta$  in Eq. (5) represents the *intensity* of



*choice* and indicates the trust that traders have in the information (expectation) about other agents' performance. If  $\beta$  is zero, agents act in an independent way from one another. However, when  $\beta$  increases, the agents have more similar behaviors and their rationality increases due to the fact that they have more confidence on the guru's expectations and there are more agents that actually copy their strategies. Somewhat,  $\beta$  measures the "imitative behavior", since the more beneficial strategy will attract more confidence from the noise traders and, thus, it will have more agents imitating it. In our model,  $\beta$  is related to the guru's life span: the longer the period the guru survives, the higher his signal credibility. We set  $\beta$  equal to 1 and increase it by more than 1 every day the guru survives. When the guru is replaced, the new  $\beta$  is set again to the value of 1.

### 3. SIMULATIONS AND RESULTS.

#### 3.1. Implementation of the model.

In order to proceed with the market simulation, we have worked with data in C language. By using the software "Code Blocks", the numerical orders have been transformed into financial market simulations. For this reason, before advancing the results obtained in this research, some fundamental aspects of the programming process have to be mentioned. Specifically, we present five essential elements that cannot be omitted in order to compile the C source code and the two principal functions of our model: the Matrix and the Trade functions. Once we explain its main characteristics, the basic syntax of the C language can be further understood. Thus, the programming process includes the following steps: libraries, defines, global variables, functions declaration and main function.

##### 3.1.1. Libraries

This is the first stage of the programming process. As shown in Figure 1, libraries include useful functions for different types of tasks. For this reason, depending on the experiment conducted we decide to execute the libraries that contain the specific functions in order to achieve it. This operation uses the directive `#include`.

```
1 | #include <stdio.h>
2 | #include <stdlib.h>
3 | #include <math.h>
```

Figure 1. Examples of libraries to be executed.

### 3.1.2. Defines

Once we have established the library to be employed, the second stage is to define the set of basic parameters. Besides, we assign the initial values to these parameters by establishing the starting conditions of the system. This operation uses the directive #define.

```
1 | #define VOL 100
2 | #define DAY 300
3 | #define TOT_SIM 100
4 | #define GDAY 10
5 | #define NDAY 2000
6 | #define NTIMES 200000
```

Figure 2. Examples of parameters employed in this model.

Each line of Figure 2 shows a variable along with its respective initial value. For instance, the variable “VOL” indicates that one hundred traders compose our market. In the same way, the variable “DAY” shows the different intra-day negotiation periods so we can deduce that there are three hundred negotiation periods every single day.

### 3.1.3. Global Variables

In this third stage, two types of variables using data in language C can be defined: global and local. After the definition of #define, the global variable is immediately called in the main body of the source code apart from the rest of functions. Besides, these variables exist in any part of the source code in contrast to the local, which only exists inside a function that performs a specific task. In this way, local variables recur each time that the function is called or executed. On the contrary, global variables do not need to be created each time that a function is called as we can see in Figure 3.

```
1 | long idum;
2 | ...
3 | int dMarketMatrix[VOL][VOL];
4 | ...
5 | float wealth[VOL];
```

Figure 3. An example of a global variable.

Both global and local variables may differ depending on the type. The language C provides several, but we have only employed five types of available variables: char (characters), int (basic integer type), long (long integer type), float (single precision floating point) and double (double precision floating point).

### 3.1.4. Functions declaration

In this stage, the programmer must declare the functions that he/she is going to use in the source code. Moreover, with the declaration function we determine the set of instructions that implement the model’s behaviors into the source code.

There are two different options to declare a function, but it is the void function the one we have employed in this model due to its flexibility and simplicity (see Figure 4).

```
1 void FunctionName(void)
2 ...
```

Figure 4. The typical function declaration in our Financial Model.

### 3.1.5. Main Function

All the elements used in our model must be included into the main function in order to be compiled and executed by the machine. The compiler only executes what lies within its curly braces { }. For this reason, we claim that the main function is the core of every program. Figure 5 shows a stylized example of how all elements are included into the main function.

```
1 int main()
2 {
3     int i;
4     idum = -123456789;
5     ...
6     //File printing
7     sprintf(file_name, "df_vol_NG_%d_p%.2f_w%.2f", NGURU, prob
8         ,w);
9     ...
10    //File writing
11    out=fopen(file_name, "w");
12    ...
13    // Monte Carlo Simulations Loop.
14
15    for (n_sim = 0; n_sim < TOT_SIM; n_sim++)
16    {
17        /*List of function to execute each run*/
18        Init_all();
19
20        InitializeMatrix();
21
22        Trade();
23    }
24
25    return 0;
26 }
```

Figure 5. A stylized example of included elements into the main function in our Financial Model.

In the previous section, we have just emphasized the importance of including all the elements into the main function to be executed by the machine. In order to conclude with the description of the programming process, the most important functions of the main:

### 3.1.5.1. The Matrix Functions.

There are two functions that describe the network topology that we use: Matrix1 and Matrix3.

**Matrix1:** This function implements a random attachment among traders. The most common situation is to assume a random interaction as the initial configuration of an evolving network at the beginning of the simulation period and when we have no connections between nodes. However, we have slightly modified the mechanism of attachment in order to allow a set of topologies, ranging from a pure random network to a star one<sup>8</sup>.

**Matrix3:** The objective of this function is to create a fitness network. By using this function, three actions in the market are executed on a daily basis:

- Traders randomly select a new possible neighbor,  $j$ . As previously explained, each period links are rewinding according to the probabilistic rule express by Eq. 5.
- Traders calculate the probability of switching from the old neighbor,  $k$ , to the new selected,  $j$ .
- Each trader chooses his/her neighbors.

By using Equation 4, we are able to detect the trader with the highest fitness and consequently the highest profit. Thus, the agent with most fitness- namely the guru- will have the maximum probability to acquire the highest amount of incoming links.

Also, we have to keep in mind that both Matrix1 and Matrix 3 are dichotomous since they perform the same task in different ways. We run Matrix1 in the first steps of the simulation and then only Matrix3 is executed.

### 3.1.5.2. The Trade Function.

This is the function that represents the real core of the model's implementation. In the first part, we find a set of definitions of local variables, which set up the initial condition of both traders and book's variables. The next step is to write down the loops that govern the passage of time. We have to do it carefully though since the time's loops have daily and intra-day operations.

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<sup>8</sup> A star network is a topology of centralized network where there is a central node which connect all nodes. In this study it is a guru that will concentrates the vast majority of incoming links as time passes by.

In the first days of simulation, we use the Matrix1 function to initialize our interaction structure. Consecutively, we execute the Matrix3 to load the fitness network. Once the **dMarketMatrix** is filled, the total amount of incoming and outgoing links can be counted to establish the id of the neighbor that each node is going to imitate. This information will be helpful to better understand how imitation spreads among traders within the network and to detect the agent that will be the guru that specific day.

When the guru of that day is already identified, the most important parameter of our study, the intensity of choice,  $\beta^9$ , comes into play. In order to set the value of this parameter, we need to count the consecutive days in which a certain agent survives as a guru.

Finally, the last part of the Trade Functions is where traders form expectations; submit their orders of buying and selling and where all the operations are registered in the check\_book function.

### **3.2. Parameters of the model.**

In the artificial market there are a number of traders equal to  $N = 100$ . All agents start with the same amount of wealth. Precisely, each agent is initially given an amount of stock equal to  $S_0 = 100$  and an amount of cash equal to  $C_0 = 1000$ . The initial stock price is chosen at  $p_0 = 1000$ . We also fix  $T = 300$ . Also, we conduct  $T = 2000$  periods of simulation. There are  $N_t = 300$  trades per period. Simulations are repeated  $M = 10$  times with a different random seed.

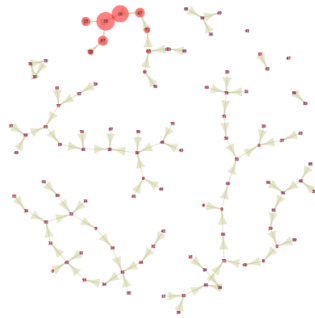
After describing all the variables and parameters, the following section presents how these simulations reproduce some stylized facts observed emerging in financial markets (see Cont 2001). We begin by showing that the market evolves endogenously to periods of decentralized network to periods of centralized network. Then we study some statistical properties of the network topology to demonstrate that our network is not a random graph and we show how the wealth of agents keeps evolving. Then we show the correlation among the guru life, the in-degree and the fitness measure and how it is correlated with returns times series and the intensity of choice. To conclude the discussion of results, we introduce how the autocorrelation of absolute returns are distributed as a power law.

---

<sup>9</sup> As we have already explained, the longer is the guru life, the higher  $\beta$ . In other words, the more consolidated an agent is while being the most imitated trader, the more confident the other traders will be in their strategies and thus, the probability of being imitated will also increase

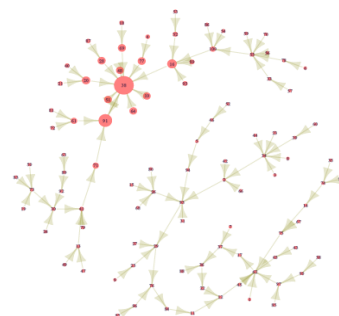
### 3.3. Discussion: the network analysis.

At this point, the reader may already acknowledge that the network of our model is dynamic, what means that its architecture evolves as times passes. Figure 6 shows us the step 939 of our research, where traders 38 and 10 are competing to become a guru. This is an unstable period in which few gurus co-exist and compete for popularity. In the absence of a guru dominating the market, the strategies that noise traders imitate are diverse and there exists no coordination among their strategies.



*Figure 6. Decentralized model.*

Figure 7 illustrates that after a certain period of time, concretely in the step 973, one agent has achieved to be the guru even though the network still is decentralized. The reason is that the parameter  $\beta$  (the intensity of choice) is not really high due to the fact that this agent is guru for a brief period of time and that there exist some agents that do not trust his/her investment strategies.



*Figure 7. Model with a guru but not stable.*

The last possible modality of our model is a centralized network, where there exists coordination in the decision-making process. Now, the guru has a higher  $\beta$ . For this reason, the guru has more confidence on behalf of the agents and the vast majority of

them tend to imitate the strategies of the new guru. At this moment, price bubbles and volatility clustering are generated due to the strategic coordination.

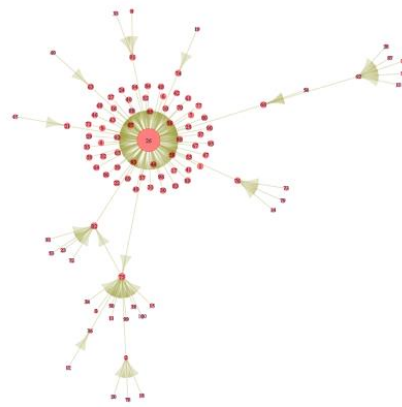


Figure 8. Centralized network and stable time of

Once explained how our network evolves, we investigate some important statistical properties identifying the network topology (see Figure 9). Specifically, we study the degree distribution (left panel), which shows the connectivity degree among traders within the network. We also check how the network architecture crucially depends on the intensity of choice  $\beta$ , (central panel), and finally, we analyze the average degree network centrality (right panel). This last concept indicates us how concentrated or centralized the direction of the links are.

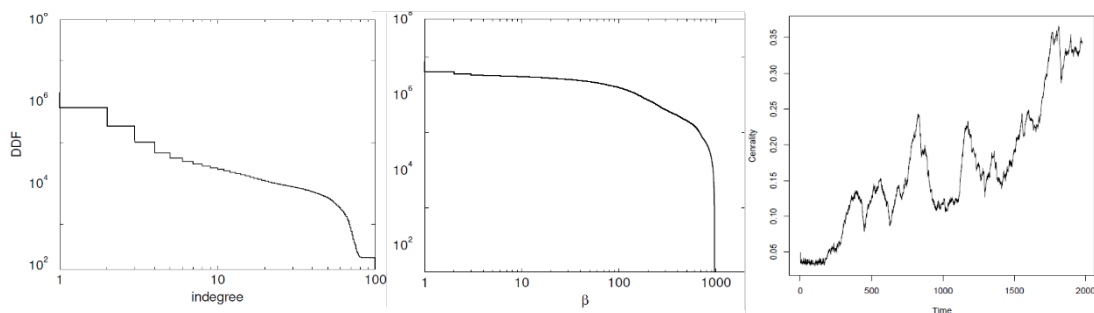


Figure 9. The decumulative distribution (DDF) of the in-degree (left panel), the intensity of choice  $\beta_t$  (central panel) and the average network centrality over simulation (right panel).

The left panel of Figure 9 shows that our network topology is not a random graph but that some agents will concentrate the majority of the network links. This fact let us classify the imitation network as a scale free<sup>10</sup> whose degree distribution follows a power law.

<sup>10</sup> A scale-free network in a complex type of network where some nodes poses a high amount of links to others nodes although most of their degree of connection is quite low.

The central panel displays the distribution of the intensity of choice. In our model  $\beta$  plays a crucial role in the network architecture because, unlike previous studies, we introduce an endogenous evolution mechanism of this. This parameter positively depends on the guru's live, which is in turn function of the guru fitness, of Eq.4. We briefly explain how  $\beta$  works: lower values of  $\beta$  make the network distributes randomly since there is no agent that has a higher confidence level. When the parameter  $\beta$  increases substantially, our market self-organizes into a pseudo-star where the guru concentrates almost all links. This is due to the fact that the market agents trust him/her. Central panel shows that the guru's time distribution is heterogeneous. On the one hand, there are stable periods where there is a stable guru and the network is centralized. On the other hand, there are unstable periods where traders compete in order to become the guru and the network is decentralized.

In the last panel (right panel), we observe the average degree network centrality<sup>11</sup>. In the graph we can clearly observe that the degree of centrality is low in the researching period, which are periods where the network is decentralized. Conversely, as time passes by, the degree of centrality increases, what means that some agent achieves a high level of intensity of choice and concentrates the majority of links.

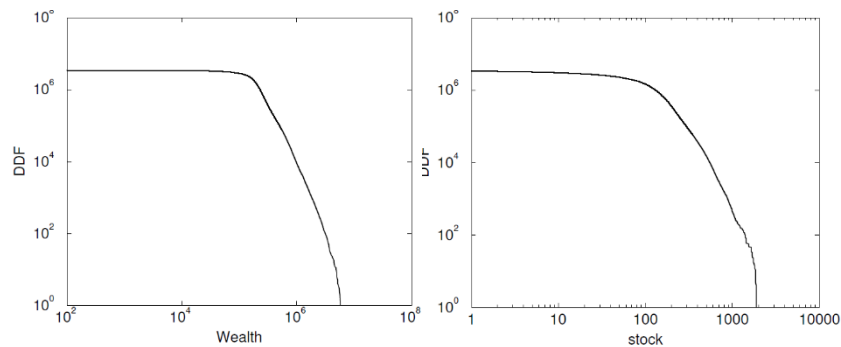


Figure 10. The decumulative distribution (DDF) of the wealth (left side) and the decumulative distribution (DDF) of the stocks (right side)

In our model, traders initially start with the same amount of stock and cash. However, as time goes by, herding generates a fat tail distribution in individuals' wealth and stock (see Figure 10), in accordance with the initial hypothesis of our model that market participants are very heterogeneous in size. Now, we explain the mechanism driving the evolution of the imitation network and its impact on prices. As already stressed, traders start with the same amount of wealth, but thanks to the fitness measures (Eq. 4), some investors extend their numbers of followers and as a

<sup>11</sup> We are using the Degree centrality but there exist many other such as closeness, betweenness, eigenvector and percolation.



consequence, their wealth. We can easily understand the process with the following example: Let's imagine that the guru has a bullish expectation on an asset and he/she decides to buy a certain amount at a price equal to 10. If he/she is able to execute a market order, all of his/her followers will imitate the strategy and the price of the asset will increase. For this reason, the guru obtains benefits since he/she possesses an asset which cost 10 and whose value is now, for instance, 14. Furthermore, since it has a higher wealth, his/her fitness and the number of followers also increase. With a major fitness, the likelihood that he/she remains the guru in the aftermath increases. In the same vein, the intensity of choice,  $\beta$ , is also increased, which further strengthens the guru fitness and his/her chances to persist over time. With this example, we can check how, in contrast to the prevailing economic view where informed agents need to hide their private information to obtain benefits (see Behanou and Laroque, 1992; Caldenteu and Stacchetti, 2007; Chakraborty and Yilmaz, 2008), in our market the guru gains in the highest profits when they reveal their expectations to the highest number of followers. Specifically, the average wealth over time and simulation of guru, followers and non-followers are 598280 (st.desv 5629), 36637 (st. dev 2543) and 2563 (st. dev 1067) respectively.

### 3.4. The impact of herding on the returns.

In order to analyze the competition and replacement dynamics driving “guru life cycle”, we run one Monte Carlo simulation of  $T = 10000$  periods<sup>12</sup>.

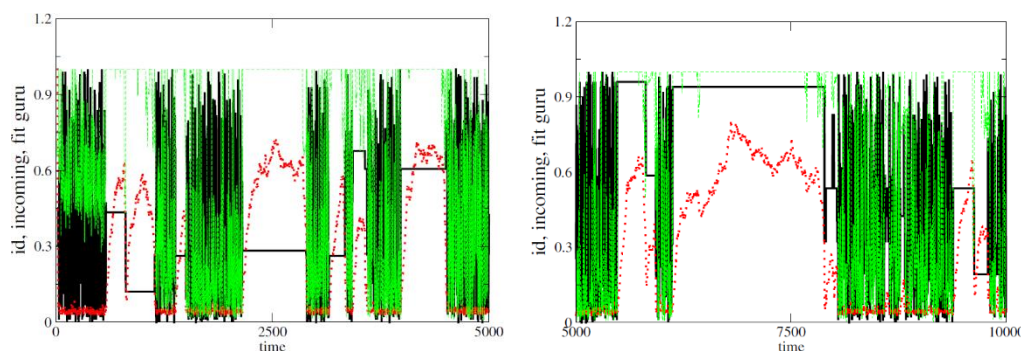


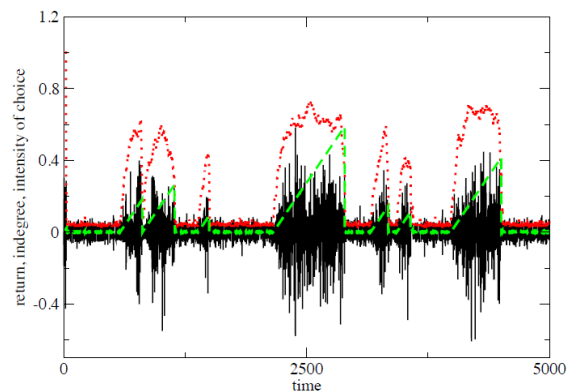
Figure 11. The index of the current guru (black solid line), the percentage of incoming link to current guru (red dotted line) and fitness of current guru (green dashed line).

Figure 11 shows the transition from unstable periods of decentralized networks in which agents compete to become the guru to stable periods in which the net is centralized and there is a guru that concentrates the majority of links. The black solid line is fragmented and very volatile in the periods that there is no guru. Clearly, these

<sup>12</sup> The figure has been divided in two sub-graph in order to be clearer. In order to understand how the market works, it is enough with  $T = 5000$ . However, it is important to take into account that the cycle does not finish in this moment the network keeps evolving.

periods will present a low percentage of in-degree (red dotted line low values) and, also, with high fitness values of volatility since there is no agent that stands among the others (green dashed line unstable). Conversely, the black solid line is continuous when there exists a guru in the market. Generally, these moments coincide with a high percentage of in-degree (red dotted line high values) and with a much more stable and close fitness to one.

During the periods of the guru's stability, one or more of his/her followers may become richer than the guru himself/herself, as signaled by the fact that the fitness (green dashes line) of the guru becomes, at times, smaller than 1. As other agents become richer, they present a higher fitness and start to be imitated until they become the new guru. The mechanism of the guru replacement is very simple. A possibility of replacement may occur when the guru places a limit order, which is a buy/sell order not immediately executed. At this moment, one of his/her followers may take advantage of the guru's information in order to place a market order, thus increasing his/her wealth at the expense of the guru. In doing so, the guru fitness will decrease and it may also determine his/her replacement. Another possibility is when, despite the guru placing a market order, the size of this order is less than the size of imitators. In this occasion, the guru's wealth will increase but less than his/her imitator's, what generates a replacement dynamics similar to that described above.



*Figure 12. Return time series (black solid line), the percentage of incoming link to current guru (red dotted line) and the intensity of choice, (green dashed line).*

As can be observed in Figure 12, there is a strong correlation between volatility clustering (black solid line) and in-degree (red dotted line). This is due to the creation and destruction mechanism as well as the replacement of the guru in order to generate stock return volatility. It is not by chance that the highest returns moments coincide with the highest levels of in-degree percentages and intensity of choice (green dashed line).

By having a high  $\beta$ , the guru has a higher in-degree and bubbles are produced up to the moment he/she is replaced. Moreover, the figure also shows the correlation between the life of the guru, the in-degree and the intensity of choice: the longest the life of the guru is, the greater the number of his/her imitators, the higher the parameter  $\beta$ . By comparing Figure 12 to the previous (Figure 11), we can examine the existing correlation between the “guru evolution” and the returns’ time series. Specifically, we observe that the moments of the return time series associated to coordination periods are far away from those observed in the Normal distribution (distribution will have a leptokurtic and asymmetric distribution and display fat-tails), while those associated to non-coordination periods better approach the Gaussian distribution.

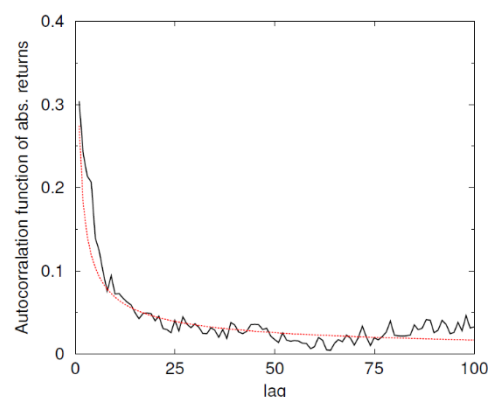


Figure13. Autocorrelation of absolute returns and the power law best fit (red line).

The last figure of this section (Figure 13) confirms the presence of volatility clustering. Figure 13, in fact, shows a positive and slowly decaying autocorrelation of absolute returns. Moreover, in line with the empirical evidence (see Cont 2001), the autocorrelation function of absolute returns is well fitted by a power law. If we presented the distribution of the returns’ autocorrelation, we could observe that it would show a fast decaying to zero, presenting thus no autocorrelation. For this reason, if there existed autocorrelation, the agents could conduct arbitrary strategies to obtain higher returns.

## 4. CONCLUDING REMARKS.

In this work, we have presented a stylized agent-based model able to reproduce the standard stylized facts emerging in financial market. Through this toy model that relies on a number of ad-hoc exogenously imposed rules, we show that the model reproduces some of the most important properties emerging in the returns' time series: returns have a leptokurtic and asymmetric distribution and display fat-tails.

We have demonstrated that in an order driven market populated by zero intelligent agents emerge the well-known stylized facts such as fat tails and volatility clustering. By introducing an endogenous mechanism of imitation, it allows a guru to emerge spontaneously in the system, rise and fall in popularity over time, and possibly be replaced by a new guru. Moreover, this artificial market shows that, conversely to real markets where the highest benefits are obtained by hiding information regarding their investment strategies, our gurus obtain more benefits as more investors follow their strategies. This imitation of beliefs can generate coordination of trading and large returns fluctuations when the popularity of the guru slowly changes over time. In fact, it is the transition from coordination periods (centralized network with a guru absorbing the majority of links) to no-coordination periods (decentralized network with a few agents competing to become the guru), emerging thanks to a fitness mechanism based on agents wealth, which generates interesting prices dynamics.

Furthermore, the endogenous communication structure has allowed us to analyze important properties of the network. Firstly, we have shown that herding generates fat tail distributions and leptokurtosis, since there is a guru in the model with a big probability of indegree (leptokurtic) and almost all agents follow him/her (heavy tail). Secondly, we have introduced an endogenous modification in the guru's intensity of choice that let the model self-organize themselves into some different network topologies, ranging from random graph to scale-free topologies.

To conclude this study, we have shown some important properties of the market. Specifically, herding is able to reproduce bubbles and consequently crashes and stages of aggragation of volatility (volatility clustering). Moreover, the model is able to reproduce correlation between volatility clustering and the percentages of in-degree, and between the percentage of incoming links and the fitness of the current guru.

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