



Título artículo / Títol article:

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Revista:

Journal of Futures Markets, 2014, 34.4: 374-398

Versión / Versió:

PDF Preprint Autors

Cita bibliográfica / Cita bibliogràfica (ISO 690):

SALVADOR, Enrique; ARAGÓ, Vicent. Measuring Hedging Effectiveness of Index Futures Contracts: Do Dynamic Models Outperform Static Models? A Regime-Switching Approach. *Journal of Futures Markets*, 2014, 34.4: 374-398.

url Repositori UJI:

<http://hdl.handle.net/10234/126305>

Measuring the hedging effectiveness of index futures contracts: Do dynamic models outperform static models? A regime-switching approach

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Abstract

This paper estimates linear and non-linear GARCH models to find optimal hedge ratios with futures contracts for some of the main European stock indexes. By introducing non-linearities through a regime-switching model, we can obtain more efficient hedge ratios and superior hedging performance in both in-sample and out-sample analysis compared with other methodologies (constant hedge ratios and linear GARCH). Moreover, non-linear models also reflect different patterns followed by the dynamic relationship between the volatility of spot and futures returns during low and high volatility periods.

Keywords: Futures indices, Dynamic hedging, Hedging Effectiveness, Markov Regime Switching, Asymmetric volatility, Non-linear GARCH

JEL codes: C13, G11.

¹ The authors are grateful for financial support from the Ministerio de Educación y Ciencia project ECO2011-27227, Programa de Fomento de Proyectos I+D Universitat Jaume I P1 1B2012-07 and Fundació Caixa Castelló Bancaixa project P1 1B2009-54. They also appreciate the insightful comments from one anonymous referee throughout the refereeing process

1. Introduction

Over the past two decades with the development of derivatives markets, plenty of literature has focused on techniques to reduce investment risk. One simple technique for this purpose is hedging with futures contracts, which despite its simplicity has received extensive research attention (Johnson, 1960, Ederington, 1979, Myers and Thompson, 1989; Cheung et al., 1990; Chen et al., 2003)

There is a great controversy in the literature whether dynamic hedging (hedge ratios are updated with the arrival of new information into the market) overcomes significantly the effectiveness reached with static strategies. Several authors (Myers, 1991; Kroner and Sultan, 1993; Park and Switzer, 1995) show that dynamic hedge ratios outperform constant hedge ratios in terms of reducing portfolio risk. However, there are some papers where the main conclusion is just the opposite (Lien and Tse, 2002; Cotter and Hanly, 2006; Park and Jei, 2010).

One of the motivations behind this paper is to provide empirical evidence on these contradictory results and analyze whether more complex models better fit financial series patterns and provide superior hedging effectiveness or not. The results of the paper show that considering non-linearities in the volatility specification leads to differences in the estimations and forecasts of volatility. These differences have an impact on the hedge ratios obtained and the effectiveness reached, causing non-linear models to achieve better effectiveness. Results and conclusions reached are robust across countries, independent of the effectiveness measure considered and the consideration of ex-ante or ex-post analysis.

The main contribution of our work consists in considering jointly most of the financial series characteristics analyzed in previous works which may have an effect on the optimal hedge ratio and its effectiveness: a) cointegration relationships between spot and future markets (Alizadeh et al, 2008; Ghosh, 1993)); b) asymmetric response of volatility against positive and negative shocks (Brooks et al.,2002); c) non-linearities originated by regime-switches in the spot-future relationship (Lee, 2010). The no consideration of one of these aspects may have influence in the estimation and the forecast of the covariance matrix and, therefore, in the hedge effectiveness obtained.

No previous paper has jointly considered these three characteristics. On one side, Alizadeh et. al (2008) do not include the asymmetric volatility term in their model. Brooks et al. (2002) highlight the importance of allowing optimal hedge ratios to be both time varying and asymmetric. On the other side, Lee and Yoder (2007a, b); Lee (2009a, b); and Lee (2010) do not reflect potential cointegration relationships between spot and future markets. However, the empirical evidence suggests that not considering this aspect lead to misspecification models and underestimations of the optimal hedge ratio (Lien, 1996; Brooks et al 2002). In this paper we take a step further and we consider regime-switching parameters in the error correction term which let us analyse whether the speed of adjustment to the long-run equilibrium varies or not among regimes (Alizadeh et al, 2008).

In our empirical study, we use several multivariate GARCH models. More specifically, the traditional BEKK model (Baba, Engle, Kraft and Kroner, 1990; Engle and Kroner, 1995) and we estimate asymmetric BEKK models (Brooks et al., 2002) to include the well-known ‘leveraged effect’² of volatility. The existence of cointegration relationships between spot and futures markets leads us to the incorporation of an ECT (error correction term) in the mean equation (Ghosh, 1993; Lien, 1996; Alizadeh et al., 2008). Finally, we also propose more complex models that consider non-linear relationships by using a regime-switching GARCH specification (Lien, 2011). This approach let us compare the effectiveness of linear GARCH models with that of non-linear GARCH models. The effectiveness of the hedging strategy is measured through several approaches. Firstly, we compute the variance reduction of the different hedging strategies over the unhedged portfolio (Ederington, 1979). Secondly, we analyze the economic significance of the risk reduction in terms of investor utility (Kroner and Sultan, 1993). Variance reduction is a good risk measure of a hedge strategy if the returns follow a normal distribution but this assumption is not always satisfied (Park and Jei, 2010). To avoid this problem, we also estimate alternative effectiveness measures based on loss distribution tails such as Value at Risk (VaR) (Jorion, 2000) and Expected Shortfall (ES) (Artzner et al., 1999).

The study is performed for several European markets using the main stock index in each case (namely FTSE for the UK, DAX for Germany and Eurostoxx50 for Europe) and their future contracts considering an ex post and ex ante analysis, with the last approach closer to the decision process followed by an investor/hedger. The out-sample analysis also includes the last financial crisis to show the best hedging models in periods of market jitters.

The outline of the paper is as follows. Section 2 reviews the controversy whether static models provide more effective hedging strategies than dynamic models. Section 3 presents the database used in the study. Section 4 introduces the empirical methodology. Section 5 shows the main empirical results of the study analyzing the optimal hedge ratio estimations and the effectiveness measures proposed. Finally, we present the main conclusions of the study.

2. Static versus dynamic models

The pioneering work using constant hedge ratios was performed by Ederington (1979).

In this approach, the hedge ratio is $\left(HR = \frac{\sigma_{sf}}{\sigma_f^2} \right)$. This hedge ratio is estimated through

the slope of the ordinary least squares (OLS) regression between the spot and futures returns.

However, this approach exhibits several problems. One of them is that it does not account for the long-run disequilibrium between spot and futures markets (Ghosh, 1993; Lien, 1996). Another problem is that it assumes constant conditional second-order

² The ‘leverage effect’ is the different response of volatility to shocks of different sign (Nelson, 1991; Glosten et al., 1993).

moments and, therefore, static hedging not conditional on the arrival of information into the market. There are essentially two approaches to obtain dynamic hedge ratios. The first one consists of allowing hedge ratios to be time-varying coefficients and estimating these coefficients directly (Alizadeh and Nomikos, 2004; Lee et al., 2006). The second approach (Kroner and Sultan, 1991; Brooks et al., 2002) uses conditional second-order moments of the spot and futures returns from multivariate GARCH models, which allow for the estimation of hedge ratios at time t adjusted to the information set available to the investor at $t-1$:

$$HR_t = \frac{\sigma_{s,f}}{\sigma_f^2} | \Omega_{t-1}$$

Most of the literature has focused on this second approach, proposing increasingly more complete models that more accurately capture the characteristics of the financial data and thereby overcome the limitations of the simpler GARCH models. One of the limitations of GARCH models is that they are incapable of reliably capturing the patterns of financial data series, specifically the asymmetric impact of news (Glosten et al., 1993; Engle and Ng, 1993; Kroner and Ng, 1998). Negative shocks are widely known to have a greater impact on financial series than do positive shocks. This fact should be taken into account when hedge ratios are estimated. Brooks et al. (2002) conclude that hedging effectiveness is greater when this asymmetric behavior is considered. A further limitation of GARCH models is that they consider high volatility persistence. This high persistence level suggests the presence of several regimes in the volatility process (Marcucci, 2005). Ignoring these regime shifts could lead to inefficient volatility estimations. Therefore, the consideration of several regimes in the volatility process could lead to more accurate estimations of volatility and thus a better performance of hedging strategies. This approach is described in Hamilton and Susmel (1994), who use a switching ARCH (SWARCH) model to introduce regime switches. Susmel (2000) also analyzes the possibility of regime switches, but uses an E-SWARCH specification that let him consider asymmetry, and concludes that both ARCH and asymmetric effects are reduced when regime switches are introduced.

In recent years, regime-switching models have taken on a new dimension with the development of Markov regime switching (MRS) models. Sarno and Valente (2000) propose a multivariate version of Hamilton's (1989) MRS model. Alizadeh and Nomikos (2004) were the first to use this methodology to estimate time-varying hedge ratios. These authors consider the slope of the OLS regression between spot and futures returns (minimum variance hedge ratio) to be regime-dependent. Chen and Tsay (2011) use the methodology proposed in the previous paper including the state-dependent autoregressive terms of the spot and future returns. In all these papers the regime-switching is considered in the mean equation assuming the variance to be constant over time but dependent on the state.

Another way to consider the regime-switching influence on the optimal hedge ratio estimation is through regime-switching-GARCH models (RS-GARCH) (Lee and Yoder 2007a, b; Alizadeh et al. 2008; Lee 2009 a,b; Lee 2010). Lee and Yoder (2007a) first consider a regime-switching time varying correlation model. Then, Lee and Yoder

(2007b) develop a new MRS-BEKK model in which they extend the work of Gray (1996) to the bivariate case. These studies propose a recombining method for conditional covariance matrices that allow the models to be tractable. They focus on modeling the variance and disregard the behavior of the mean. Alizadeh et al. (2008) incorporate an error correction term (ECT) that allows series characteristics to be related in the short- and long-run. Regarding the previous studies, these works allow for both time-varying and state-dependent conditional variances.

Besides these studies, Lee (2009a) develops a regime-switching Gumbel-Clayton copula GARCH model where the return series are modeled using a switching copula instead of assuming bivariate normality. An independent switching GARCH process for every state is considered to avoid the path dependency problem. Lee (2009b) also develops a Markov regime switching Orthogonal GARCH with conditional jumps dynamics. In the paper of Lee (2010), it is considered an independent switching dynamic conditional correlation GARCH with more than 2 regimes (2 is the number of regimes commonly used in previous studies).

The evidence from studies including regime switches shows that more robust estimates are generated if volatility is allowed to follow different regimes depending on the market conditions, with the result that the hedge effectiveness will be greater. Lien (2009) analyses and demonstrates why static models (OLS) may outperform GARCH models. Recently, Lien (2011) derives the form of the hedge ratio when Regime switching GARCH models are considered³ and compares it with the static and GARCH models. Lien (2009, 2011), points out that the existence of structural breaks or different regimes of volatility in financial series may improve the performances of dynamic models (RS-GARCH models) or at least that the consideration of these in estimated models improves effectiveness. More specifically, this author determines a theoretical superiority in terms of risk reduction of the RS-GARCH model over the OLS and GARCH hedging strategies.

3. Description of the data and preliminary analyses

The data used in this study include weekly closing prices⁴ (Alizadeh and Nomikos, 2004; Alizadeh et al., 2008; Chen and Tsay, 2011) for some of the main European stock indexes and their futures contracts. Specifically, we use information on the UK (FTSE100), Germany (DAX30), and Europe (Eurostoxx50). The time horizon includes observations from 1 July 1998 to 30 September 2010. We divide this data into two sub-samples: observations from 1 July 1998 to 31 December 2008 (548 observations) are used for the in-sample analysis and observations from 1 January 2009 to 30 September 2010 (92 observations) are used for the out-sample study. We obtained the indexes data from Thomson DataStream and the futures information from the Institute of Financial Markets Data Center.

³ Lee (2009a) also derives a formula for the two-state regime switching hedge ratio.

⁴ Wednesday closing prices are used as weekly observations. If a Wednesday is not available in a week is replaced by the Tuesday in that week.

We construct the continuous futures series using the contract closest to maturity⁵. Weekly return series are computed as the logarithmic differences multiplied by 100.

$$r_{i,t} = 100 \left(\log \frac{P_{i,t}}{P_{i,t-1}} \right) \quad \text{for } i = \{s, f\} \quad (1)$$

Descriptive statistics are presented in Table 1. Panel A shows the main summary statistics for the spot and futures indexes. Certain results are noteworthy. For the returns, negative values are present in the third-order moments. There is also excess kurtosis in the returns (fat tails). Finally, note that the Jarque–Bera normality test (1980) is rejected because of the asymmetric and leptokurtic characteristics of the series. Results for the out-sample period differ only slightly from those of the in-sample period⁶. Panel B displays the serial autocorrelation tests for the series in levels and squares. The Ljung–Box statistics for the squared series suggest evidence of conditional heteroskedasticity for both series. There is also evidence of serial correlation for returns in levels so it is necessary include structure (lags) in the mean equation⁷. Panel C reflects the stationarity tests performed over the price series and reveals that the price series are I(1), so we have to work with the returns series for stationarity reasons. Finally, panel D presents the results of the cointegration tests for the series studied. The results also show that both series are cointegrated. Therefore, these relationships should be introduced in the specification of the model used to calculate the hedge ratios, since otherwise we would obtain inefficient hedges (Lien, 1996).

[INSERT TABLE 1]

4. Methodology

This section explains and develops the empirical models used to estimate time varying volatilities and hedge ratios. We start with the symmetric and asymmetric linear specifications (BEKK and GJR-BEKK) to model the dynamic relationship between spot and futures returns. After that, we assume non-linear dynamics through a regime-switching process, thereby allowing hedge ratios to be dependent on the state of the market.

4.1. Linear bivariate GARCH models

Linear bivariate GARCH models have been widely used in the analysis of dynamic hedge ratios (Baillie and Myers, 1991; Park and Switzer, 1995). One of the most frequently used is the BEKK model (Baba et al., 1990) since it incorporates certain characteristics⁸ that make it particularly attractive for this type of study. In this specific

⁵ Carchano and Pardo (2008) show that rolling over the futures series has no significant impact on the resultant series. Therefore, the least complex method can be used for series construction to reach the same conclusions.

⁶ The out-sample data run from 1 January 2009 to 30 September 2010 (observations). The descriptive statistics, not presented in the paper, are available from the authors upon request.

⁷ The existence of serial correlation in returns may bias the estimation of conditional second moments. To remove this undesirable feature we add an AR(1) term in the mean equation.

⁸ The main advantage of this model is that it guarantees that the covariance matrix will be a positive definite by construction (quadratic form).

case, we incorporate an ECT in the mean equation⁹ because both series are cointegrated. Let $r_{s,t}$ and $r_{f,t}$ be the spot and futures returns at period t respectively; thus, we define the mean equation as:

$$r_{s,t} = a_0 + a_1 r_{s,t-1} + a_2 r_{f,t-1} + a_3 ECT_{t-1} + e_{s,t} \quad (2)$$

$$r_{f,t} = b_0 + b_1 r_{f,t-1} + b_2 r_{s,t-1} + b_3 ECT_{t-1} + e_{f,t} \quad (3)$$

$$e_t | \Omega_{t-1} = \begin{pmatrix} e_{s,t} \\ e_{f,t} \end{pmatrix} | \Omega_{t-1} \sim BN(0, H_t) \quad (4)$$

where a_i, b_i for $i = \{0, 1, 2, 3\}$ are the parameters to be estimated. The subscripts s and f indicate spot or futures respectively, $e_{s,t}$ and $e_{f,t}$ indicate innovations, Ω_{t-1} denotes the information set available up to $t-1$, BN refers to the bivariate normal distribution and H_t is a positive definite time-varying 2×2 matrix defined as follows:

$$H_t = \begin{pmatrix} h_{s,t}^2 & h_{sf,t} \\ h_{sf,t} & h_{f,t}^2 \end{pmatrix} = C' C + A' e_{t-1} e_{t-1}' A + B' H_{t-1} B \quad \text{where } C = \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix} \quad A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \quad (5)$$

where C is a triangular matrix of constants and A, B are 2×2 square matrices of coefficients to be estimated.

Assuming that the innovations follow a bivariate normal distribution, the unknown parameters $\theta = (a_i, b_i, C_{2 \times 2}^l, A_{2 \times 2}, B_{2 \times 2})$ for $i = \{0, 1, 2, 3\}$ are estimated by maximizing the following likelihood function with respect to θ :

$$f(r_t; \theta) = (2\pi)^{-1} |H_t(\theta)|^{-\frac{1}{2}} \exp\left(-\frac{1}{2} e_t(\theta)' H_t^{-1} e_t(\theta)\right) \quad (6)$$

$$L(\theta) = \sum_{i=1}^T \log f(r_i; \theta) \quad (7)$$

where T is the number of observations.

GARCH models allow us to obtain an estimation of the variance–covariance matrix for each period. We obtain the dynamic hedge ratio (HR_t) estimations, according to the expression (8):

$$HR_t = \frac{\hat{h}_{sf,t}}{\hat{h}_{f,t}^2} \quad (8)$$

⁹ We also include an AR(1) term in the mean equation following previous studies (Lo and Makinley, 1990) which claim that the omission of this autoregressive component has serious implications for the variances, autocorrelations, and cross-autocorrelations of individual stocks as well as portfolios due to the asynchronous trading of some components in the stock indexes. Besides, the model selection criteria (likelihood-ratio test) also recommends the inclusion of the AR(1) term. These results are available upon request.

This simplest variance specification (shown in 5) can be used to incorporate other financial series characteristics such as asymmetries in volatility. One of the most popular approaches in the literature is the GJR model of Glosten et al. (1993), which uses specific variables to incorporate this asymmetric behavior.

$$H_t = \begin{pmatrix} h_{s,t}^2 & h_{fs,t} \\ h_{sf,t} & h_{f,t}^2 \end{pmatrix} = C' C + A' e_{t-1} e_{t-1}' A + B' H_{t-1} B + D' \eta_{t-1} \eta_{t-1}' D \quad (9)$$

where D is a diagonal 2×2 matrix of parameters (similar than matrices A and B) to be estimated and $\eta_t = \min(e_t, 0)$. The remaining parameters and variables are the same as those in equations 2–4 and the estimation procedure is similar to that above.

4.2. Non-linear bivariate GARCH models

In contrast to previous models, in which the dynamic relationship between spot and futures returns is characterized by linear patterns, the model presented by Lee and Yoder (2007a) allows regime shifts, which suggests that one can obtain more efficient hedge ratios and superior hedging performance compared with other methods. These types of non-linear models open up a new line for dynamic hedging in which the returns process is state-dependent. Let $r_{s,t,st}$ and $r_{f,t,st}$ be the state-dependent spot and futures returns at t respectively; we define the state-dependent mean equations as:

$$r_{s,t,st} = a_0 + a_1 r_{s,t-1} + a_2 r_{f,t-1} + a_{3,st} ECT_{t-1} + e_{s,t,st} \quad (11)$$

$$r_{f,t,st} = b_0 + b_1 r_{f,t-1} + b_2 r_{s,t-1} + b_{3,st} ECT_{t-1} + e_{f,t,st} \quad (12)$$

$$e_{t,st} | \Omega_{t-1} = \begin{pmatrix} e_{s,t,st} \\ e_{f,t,st} \end{pmatrix} | \Omega_{t-1} \sim BN(0, H_{t,st}) \quad (13)$$

where a_i, b_i for $i = \{0, 1, 2, 3\}$ are the parameters to be estimated. Based on model selection criteria¹⁰, they are not considered to be state-dependent. However, following Alizadeh et al. (2008), the parameters accompanying the ECT depend on regime $s_t = \{1, 2\}$.

The state-dependent innovations $e_{t,st}$ follow a bivariate normal distribution that depends on state $s_t = \{1, 2\}$. This state variable follows a two-state first-order Markov process with transition probabilities:

¹⁰ Following the insightful suggestions from an anonymous referee we perform the whole in-sample analysis for the non-linear models using different mean equation specifications, a) allowing all parameters to switch between regimes, b) not allowing any parameter to switch between regimes; c) not including the ECT. However, based on the hedging effectiveness results and the selection criteria between models (likelihood-ratio test), none of the alternative models improved the model presented in the text. Results are available from authors upon request. So, these results in line with studies such as Alizadeh et al (2008) lead us to only consider as switching parameters the parameters measuring the speed of adjustment to the long-run equilibrium between spot and future prices. .

$$P = \begin{pmatrix} \Pr(s_t = 1 | s_{t-1} = 1) = p & \Pr(s_t = 1 | s_{t-1} = 2) = (1-q) \\ \Pr(s_t = 2 | s_{t-1} = 1) = (1-p) & \Pr(s_t = 2 | s_{t-1} = 2) = q \end{pmatrix} \quad (14)$$

where p represents the probability of continuing in state 1 if it was previously in state 1 and q represents the probability of continuing in state 2 if it was previously in state 2.

The state-dependent conditional second-order moments $H_{t,st}$ follow an asymmetric BEKK¹¹ specification model that takes different values depending on the value of $s_t = \{1, 2\}$. Because of this state dependence, the model will become intractable as the number of observations increases. In order to resolve this problem we apply the recombining method used in Gray (1996) where the path dependency problem is solved for univariate models. Lee and Yoder (2007a) extend this recombining method for the bivariate case. Thus, the variance specification in each state is defined as follows:

$$H_{t,st} = \begin{pmatrix} h_{s,t,st}^2 & h_{fs,t,st} \\ h_{sf,t,st} & h_{f,t,st}^2 \end{pmatrix} = C_{st}' C_{st} + A_{st}' e_{t-1} e_{t-1}' A_{st} + B_{st}' H_{t-1} B_{st} + D_{st}' \eta_{t-1} \eta_{t-1}' D_{st} \quad (15)$$

where $h_{s,t,st}^2$ and $h_{f,t,st}^2$ are the conditional variances of the spot and futures in period t for each state s_t and $h_{sf,t,st}$ is the conditional covariance in t for each s_t . C_{st} , A_{st} , B_{st} and D_{st} are the matrices of parameters to be estimated as in previous models.

The consideration of several states leads to a noteworthy rise in the number of parameters to estimate. In order to reduce this over-parameterization the difference among states is defined by four new parameters sa , sb , sc and sd that properly weight the estimations obtained in one state for the other state¹² (Capiello and Fearnley, 2000). Therefore, the state-dependent covariance matrices in our model are:

$$H_{t,s_t=1} = \begin{pmatrix} h_{s,t,1}^2 & h_{fs,t,1} \\ h_{sf,t,1} & h_{f,t,1}^2 \end{pmatrix} = C_1' C_1 + A_1' e_{t-1} e_{t-1}' A_1 + B_1' H_{t-1} B_1 + D_1' \eta_{t-1} \eta_{t-1}' D_1 \quad (16.1)$$

$$H_{t,s_t=2} = \begin{pmatrix} h_{s,t,2}^2 & h_{fs,t,2} \\ h_{sf,t,2} & h_{f,t,2}^2 \end{pmatrix} = C_2' C_2 + A_2' e_{t-1} e_{t-1}' A_2 + B_2' H_{t-1} B_2 + D_2' \eta_{t-1} \eta_{t-1}' D_2 \quad (16.2)$$

where $C_2 = sc \cdot C_1$, $A_2 = sa \cdot A_1$, $B_2 = sb \cdot B_1$, $D_2 = sd \cdot D_1$, A_1 and B_1 are 2×2 matrices of parameters, C_1 is a 2×2 lower triangular matrix of constants and D_1 is a diagonal 2×2 matrix of parameters.

¹¹ We also present the results for the symmetric MRS-BEKK model. This model is similar to that presented in the paper except for the variance equation where the last summation $D_{st}' \eta_{t-1} \eta_{t-1}' D_{st}$ is not considered.

¹² The economic interpretation of the parameters sc , sa , sb and sd is how much the constant term, the weight of the shocks, the weight of the past variance and the impact of negative shocks on the volatility formation differ between each state respectively.

The basic equations of the recombining method¹³ used to collapse the variances and covariances of the spot and futures errors and to ensure the model is tractable are described below:

$$h_{i,t}^2 = \pi_{1,t} (r_{i,t,1}^2 + h_{i,t,1}^2) + (1 - \pi_{1,t}) (r_{i,t,2}^2 + h_{i,t,2}^2) - (\pi_{1,t} r_{i,t,1} + (1 - \pi_{1,t}) r_{i,t,2})^2 \quad (17)$$

for $i = \{s, f\}$

$$e_{i,t} = \Delta S_t - (\pi_{1,t} r_{i,t,1} + (1 - \pi_{1,t}) r_{i,t,2}) \quad (18)$$

$$h_{sf,t} = \pi_{1,t} (r_{s,t,1} r_{f,t,1} + h_{sf,1,t}) + (1 - \pi_{1,t}) (r_{s,t,2} r_{f,t,2} + h_{sf,2,t}) - (\pi_{1,t} r_{s,t,1} + (1 - \pi_{1,t}) r_{s,t,2}) (\pi_{1,t} r_{f,t,1} + (1 - \pi_{1,t}) r_{f,t,2}) \quad (19)$$

where $h_{i,t}^2$, $h_{sf,t}$ are the state-independent variances and covariances aggregated by the recombining method and $h_{i,st,t}^2$, $h_{sf,st,t}$ are the state-dependent variances and covariances for $s_t = \{1, 2\}$.

The terms $r_{i,t,st}$ represent the state-dependent mean equations and $\pi_{1,t}$ is the probability of being in state 1 at time t obtained by the expression:

$$\pi_{1,t} = p \left(\frac{g_{1,t-1} \pi_{1,t-1}}{g_{1,t-1} \pi_{1,t-1} + g_{2,t-1} (1 - \pi_{1,t-1})} \right) + (1 - q) \left(\frac{g_{2,t-1} (1 - \pi_{1,t-1})}{g_{1,t-1} \pi_{1,t-1} + g_{2,t-1} (1 - \pi_{1,t-1})} \right) \quad (20)$$

where

$$g_{i,t} = f(r_t | s_t = i, \Omega_{t-1}) = (2\pi)^{-1} |H_{t,i}|^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2} e_{t,i}' H_{t,i}^{-1} e_{t,i} \right\} \quad \text{for } i = \{1, 2\} \quad (21)$$

and p and q are as described in equation 14.

Thus, the parameters of the model can be estimated using the following maximum likelihood function where each state-dependent likelihood function is properly weighted by the filtered probability of being in state 1 at time t ($\pi_{1,t}$) and the filtered probability of being in state 2 ($\pi_{2,t}$).

$$f(r_t | \theta) = \pi_{1,t} \left[(2\pi)^{-1} |H_{t,1}|^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2} e_{t,1}' H_{t,1}^{-1} e_{t,1} \right\} \right] + \pi_{2,t} \left[(2\pi)^{-1} |H_{t,2}|^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2} e_{t,2}' H_{t,2}^{-1} e_{t,2} \right\} \right] \quad (22)$$

$$L(\theta) = \sum_{t=1}^T \log f(r_t; \theta) \quad (23)$$

¹³ For further details on the recombining method, see Gray (1996) and Lee and Yoder (2007a).

Based on the estimations obtained, we calculate the optimal hedge ratio from the results of the state-independent covariance matrix given by the recombining method, substituting the resulting second-order moments in expression (8).

5. Empirical results

This section presents the main empirical results of the study. Section 4.1 shows the parameter estimation results for all the models proposed. Section 4.2 describes the volatility evolution and the hedge ratios estimated using each model. Section 4.3 proposes several effectiveness measures to analyze the performances of the different hedging policies. Finally, section 4.4 performs specification tests over the estimation residuals to detect any problems related with a potential misspecification of the empirical model.

5.1 Model estimation

In this section, we show the evolution of the patterns followed by the volatility¹⁴ in the linear and non-linear frameworks proposed in the study. The estimations of the models are presented in Table 2 for all markets considered. A two-state specification is used for the MRS models. This specification allows the states to be associated with high and low volatility regimes¹⁵ using the median of the estimated state-dependent volatilities for the stock indexes¹⁶, which present a value of 6,766 (6,510) for state 1 and 8,188 (7,021) for state 2 in Europe, 7,192 (2,426) for state 1 and 2,588 (5,693) for state 2 in the UK and 8,882 (7,660) for state 1 and 9,359 (8,153) for state 2 in Germany¹⁷. Therefore, the state with the highest value of estimated conditional variance in each model corresponds to the high volatility state.

[INSERT FIGURE 1]

Moreover, Figure 1 shows the smooth probability of being in the low volatility state in each data series used¹⁸. The figure corresponding to Eurostoxx is governed essentially by this state, which corresponds with a calm period in financial markets (2003–2007). When the state governing the process is state 2, this corresponds to periods of market jitters such as the dot-com bubble (2002–2003) and the last financial crisis (2008). The probabilities for the rest of markets share these periods of high volatility states and, moreover, present other high volatility periods probably related with their own country-idiosyncratic market evolution.

[INSERT TABLE 2]

¹⁴ We focus mainly on the interpretation of the variance equation parameters since this determines the estimated covariance matrix and, therefore, the optimal hedge ratio.

¹⁵ Sarno and Valente (2000) use a three-state process, but the third state seems to capture spurious state changes that are not related to market regime switches. But the selection of the two-state process is mainly due to economic interpretations.

¹⁶ The estimated volatility for the futures indexes follow the same order and they are not displayed to save space. The results are available from the authors upon request.

¹⁷ Values in parentheses refer to medians in the asymmetric models.

¹⁸ The estimation process itself determines whether state 1 corresponds to high or low volatility states. Depending on the country, state 1 could refer to a high volatility state in one market and to a low volatility state in another market. The figure represents the probability of low volatility states.

For each market in Table 2, the first two columns in each market show the parameter estimations for the linear models (BEKK and ASYM-BEKK). We can observe that linear models reflect in most cases a weak significance of the parameters representing the persistence of the impact of shocks in volatility (a11, a22). Furthermore, the impact of one market's shocks on the other markets' volatility is generally not significant (a12, a21).

The evidence for a significant influence of past volatility on volatility formation is more evident both for spot (b11) and futures markets (b22) but this is not observed for the cross parameters (b12, b21). Generally, there is also an asymmetric response of volatility against negative shocks, although in markets such as Eurostoxx and the UK this evidence is only observed in the futures markets (d22).

Finally, there is another remarkable result about volatility dynamics; the persistence level in linear models is relatively high. This result suggests the presence of several regimes in the volatility process and, therefore, potential non-linearities and the adequacy of using MRS-GARCH models.

It is also interesting to analyze the differences in the volatility parameters between states of the market in non-linear models. For example, the constant term is usually lower in low volatility states than it is in high volatility states¹⁹. That is understandable because the constant term in our model reflects the unconditional volatility, and this is supposed to be higher in high volatility states. Second, the presence of shocks on volatility formation is higher in high volatility states than it is in low volatility states.²⁰ However, the impact of past variance on the formation of volatility is lower in high volatility states than it is in low volatility states²¹. There seems to be a trade-off between the impact of shocks and past variance on the formation of volatility between states. In low volatility states, there is a greater past variance persistence and a lower presence of shocks in volatility. In high volatility states, there is a higher presence of shocks but a lower impact of past variance. These results are similar to those of Marcucci (2005), who explain these differences in volatility dynamics between low and high volatility periods by arguing that there is a greater amount of news during high volatility periods. Therefore, the continuous arrival of new information into the market causes volatility formation to occur largely because of the impact of these shocks rather than the past variance observed in the market, as occurs in low volatility periods when less news affects the markets. Finally, we find that the asymmetric response of volatility is significant in spot and futures markets in the non-linear specification. We also find that there is a different asymmetric response of volatility in low and high volatility periods. However, there is no common result on how the asymmetric response changes with

¹⁹ In the models where state 1 corresponds to low volatility periods this is observed because the scale sc for state 2 (high volatility) is higher than 1; in the cases where state 1 corresponds to high volatility periods, the scalar sc is lower than 1.

²⁰ Similar to footnote 16 but using scalar sa instead of sc .

²¹ In this case, when state 1 corresponds to low volatility periods the scale sb for state 2 (high volatility) is lower than unity; in the cases where state 1 corresponds to high volatility periods, the scalar sb is higher than 1.

volatility regime. In Europe and Germany, this asymmetric response is higher in low volatility periods, while it is less acute in high volatility periods but in the UK, the opposite occurs.

We also consider interesting to determine the average durations of the different states in the economy. This duration value can be obtained according to the transition probability estimates p and q in equation 14. For example, Europe presents a value of $p=0.966$ and $q=0.962$; this means that once in state 1, the probability of remaining in that state is 96.6%, while the probability of remaining in state 2 is 96.2%. Therefore, the average duration of being in state 1 when the volatility process is governed by this state will be approximately 29 weeks ($1/(1-0.966)$). A similar duration can be calculated in the high volatility regime state ($1/(1-0.962)$). This indicates that the regime switches present a smooth evolution, keeping the process in each state during relatively long periods. For the remainder of markets these values are very similar.

5.2.- Volatility and hedge ratios

At this point, it is time to analyze the evolution and differences in the estimated variances obtained in each model, which will then lead us to the differences in the estimated hedge ratios. The estimated covariance matrix for the linear models is obtained using equation 5 for the symmetric and equation 9 for the asymmetric cases. For a proper comparison between models, we use the estimation for the independent covariance matrix (equations 17 and 19) for the non-linear models. Figure 2 shows the estimated variance for the spot market²² for all markets considered.

[INSERT FIGURE 2]

All figures seem to exhibit similar patterns, although there are obvious differences between them. Common to all the estimations, there are two periods corresponding to 2001–2003 and 2008 that present higher estimations of volatility. These periods of high volatility coincide with the dot-com bubble and the last financial crisis, which are periods of market jitters. Figure 1 shows that the mentioned periods correspond with periods governed by high volatility states and the rest of the sample is often governed by low volatility states. The volatility estimations in high volatility periods using non-linear models are higher than are those obtained with linear models, but in the rest of the sample coinciding with calm periods the volatility estimations using linear models are higher than are those obtained with non-linear models²³. If we do not distinguish between states, one state would define the volatility process, and this may not properly reflect the patterns during turbulent periods, which exhibit different dynamics than do those present during calm periods. Therefore, the volatility estimations tend to be

²² For brevity, only the spot market volatility is shown. The estimated volatilities for futures markets and the covariance between spot and futures markets are similar. The results are available from the authors on request.

²³ Using the filtered probability in each market, we find that the average for high volatility states using symmetric linear models in Europe, the UK and Germany are 8.83, 21.14 and 18.51 respectively, while for the non-linear case they are 9.10, 22.33 and 22.93 respectively. For low volatility states, the average estimated volatility is 4.51, 9.12 and 11.92 for linear models against 3.14, 8.55 and 10.98 for non-linear models.

underestimated using linear GARCH models in the periods corresponding to high volatility states and overestimated in low volatility periods, and this may influence the effectiveness of the hedge policy.

Finding the optimal hedge ratios for the in-sample analysis is simple. For linear GARCH models, we use equation 8 and the covariance matrix estimates at each moment t (Kroner and Sultan, 1993). For non-linear models, we also use equation 8 and the state-independent estimations of the covariance matrix.

Finding hedge ratios for the out-sample period is more complex and differs by model. Common to all models is the construction of a rolling window in which the model is re-estimated for each window period, removing the first observations and adding new ones as the window advances. The parameter values are found for each estimation period, which allows us to make one period ahead forecasts of the covariance matrix. Note that this procedure is performed for the linear BEKK models both with and without asymmetries.

The process of forecasting the covariance matrix for the non-linear BEKK models with (and without) asymmetries is more complex because of the existence of two possible states. This forecast is performed in a three-stage process (Alizadeh et al., 2008). In the first stage, we use the estimations of the transition matrix in t (equation 14) and the smoothed probabilities in t to obtain the prediction of the probability of being in each one of the two states $s_t = 1, 2$ in the period $t+1$.

$$E \begin{bmatrix} \pi_{1,t+1} \\ \pi_{2,t+1} \end{bmatrix} = \begin{pmatrix} \hat{\pi}_{1,t} \\ \hat{\pi}_{2,t} \end{pmatrix} \begin{pmatrix} \hat{p} & (1-\hat{q}) \\ (1-\hat{p}) & \hat{q} \end{pmatrix} \quad (24)$$

In the second stage, we make a prediction one period ahead of the state-dependent mean and variance equations (equations 11–13 and 15) using the parameters estimated. In the third stage, the recombining method is used as in equations 17–19 to obtain the predictions of the state-independent covariance matrix. Once we have the one period ahead prediction of the covariance matrix for each model, we obtain the predicted hedge ratio using the equation 8 for $t+1$.

Figure 3 presents the hedge ratios obtained for both the in-sample and out-sample period, together with their evolution²⁴. The top figures show the evolution for the in-sample analysis and the bottom graphs reflect the forecasts performed for each model. We compare symmetric (MRS-BEKK) against asymmetric (MRS-ASYM-BEKK) non-linear models on the left-hand figures and linear (BEKK) against non-linear (MRS-BEKK) on the right-hand side, with the continuous line, the MRS-BEKK model and the alternatives in each case plotted with dashed lines.

[INSERT FIGURE 3]

²⁴ The estimated hedge ratios of the remaining models are not presented here for brevity, but are available from the authors upon request.

The differences among models are evident both between linear (dashed line) and non-linear (continuous line) specifications and between symmetric (continuous line) and asymmetric (dashed line) specifications.

[INSERT TABLE 3]

There also exist differences in the averages and in the variability of the estimated and forecasted ratios (Table 3). Therefore, it seems the omission or inclusion of one of these characteristics could lead to significant differences in the estimated hedge ratios and, therefore, in the effectiveness reached. Therefore, concerning the evident differences between the estimated and forecast hedge ratios obtained in each strategy, we try to explain in the next section which hedge strategy allows us to achieve a more effective hedge policy. The study in the next section is especially attractive because the out-sample analysis is performed over the period of the recent financial crises and could thus prove which models work better in periods of market uncertainty.

5.3.- Hedging effectiveness

To analyze hedging effectiveness we consider four different measures. The first two measures are based on the variance of the loss distribution of the hedge portfolio. The first approach is the variance of the hedged portfolio (Ederington, 1979) for each model compared with an unhedged portfolio, that is $HR_t = 0$ for all t . The variance of the hedged portfolio is:

$$Var(x_t | \Omega_{t-1}) = Var((\Delta S_t - HR_t * \Delta F_t) | \Omega_{t-1}) \quad (25)$$

Another commonly used approach is to analyze the economic benefits of the hedging (Kroner and Sultan, 1993) by constructing the investor's utility function based on the return and risk of the hedge portfolio. This measure is motivated by the fact that dynamic strategies are most costly to implement since they require a frequent updating of the hedge portfolio. In line with studies such as Park and Switzer (1995) and Meneu and Torró (2003), the utility function is constructed in a mean/variance context:

$$E[U(x_t | \Omega_{t-1})] = E[x_t | \Omega_{t-1}] - \lambda Var[x_t | \Omega_{t-1}] \quad (26)$$

where λ is the investor's level of risk aversion (normally $\lambda = 4$) and the hedged portfolio returns are also assumed to present an expected value equal to 0 (Alizadeh et al., 2008, Lee, 2010).

The third metric proposed is based on the VaR measure (Jorion, 2000). The VaR of the hedged portfolio at the confidence level q is given by the smallest number l such that the probability that the loss L exceeds l is no larger than $(1-q)$. In our case, this is calculated by the sample quantiles using the empirical distribution of the hedge portfolio returns.

$$VaR_q = \inf \{l \in \mathfrak{R} : P(L > l) \leq 1 - q\} \quad (27)$$

The last effective measure is based on the ES of the hedged portfolio (Artzner et al., 1999). ES is an alternative to VaR in that it is more sensitive to the shape of the loss distribution in the tail of the distribution. The ES at the $q\%$ level is the expected return on the portfolio in the worst $q\%$ of the cases.

$$ES_q = E(x|x < \mu) \quad (28)$$

where μ ²⁵ is determined by $\Pr(x < \mu) = q$ and q is the given threshold, while x is a random variable that represents profit during a specified period.

Table 4 summarizes the hedging strategy effectiveness for all the series used in the study. It shows the effectiveness measures both for in-sample and out-sample analysis and for all linear and non-linear models proposed as well as the effectiveness achieved by using a constant OLS strategy and by the unhedged portfolio.

[INSERT TABLE 4]

Panel A presents the effectiveness analysis for the in-sample period in all countries considered. The highest effectiveness considering the reduction of variance of the hedge portfolio is observed in the MRS-ASYM-BEKK in the UK, Germany and Europe. That is, non-linear models outperform the effectiveness of the rest of the models in terms of variance reduction. Another interesting result arises here. The effectiveness of the OLS strategy outperforms in all cases (except the UK) the linear GARCH hedging strategies²⁶. This result is the same as those found in studies such as Lien (2009), Lien and Tse (2002), Cotter and Hanly (2006) and Park and Jei (2010). These authors find that constant strategies present better effectiveness than do dynamic strategies. However, when we consider non-linear strategies, these more complex models outperform the rest of the policies. Generally, the introduction of non-linearities in the models lets us achieve a greater fit to the data because of the identification of different regimes in the volatility process and the more accurate estimation. Therefore, this non-linear specification outperforms both the linear models and constant strategies. The utility analysis reaches a similar conclusion because these first two measures are both based on the variance of the hedge portfolio loss distribution. However, as Park and Jei (2010) remark, this measure could present problems when the return distribution deviates from normality.

If we consider tail-based measures, we obtain most of the greater risk reduction in the non-linear models but using this metric the evidence is less clear than it is with the variance reduction. For VaR metrics, we find that MRS-BEKK performs best for the UK at 1% and 10% significance levels, Germany at 1% and Europe at all levels. The asymmetric non-linear model (MRS-ASYM-BEKK) performs best for the UK at 5% and Germany at 10%. However, for Germany at 5% significance the asymmetric linear

²⁵ Note that μ is the value at risk.

²⁶ Lien (2009) shows that variance-based metrics reflect the reduction of the unconditional volatility of the hedge portfolio. Therefore, OLS strategies reach the greatest variance reduction by definition, whereas the linear GARCH strategies achieve a reduction on the conditional variance.

GARCH achieves the best hedging performance. The result for the ES, which reflects the expected loss when we consider only the worst scenarios, again non-linear models performs better than linear models in most cases. However, there are some cases where linear models outperform non-linear ones, such as Germany at 1% significance. Using these last two metrics, the dominance of non-linear models is again evident outperforming in almost all cases linear and constant models²⁷.

Panel B presents the effectiveness analysis for the out-sample analysis. We evaluate the four effectiveness metrics unconditionally, using 92 forecasted hedged portfolio returns over the 92 periods out-sample. The highest effectiveness considering the reduction of variance of the hedge portfolio is observed in the MRS-ASYM-BEKK in the UK and the MRS-BEKK in Europe and Germany. Non-linear models outperform the effectiveness of the rest of the models in terms of variance reduction in the out-sample analysis. The utility results are similar. With this evidence, it seems clear that more complex non-linear models lead to better forecasts of the hedge ratio and a greater risk reduction using variance-based metrics. However, if we compare linear GARCH models to constant strategies we find a greater variance reduction for constant strategies. This result reveals an issue widely discussed in the empirical literature. Most of the literature comparing dynamic (i.e. linear GARCH models) with constant strategies obtain a better performance from the latter (Lien, 2009; Lien and Tse, 2002; Cotter and Hanly, 2006; Park and Jei, 2010). However, when non-linear dynamics models through regime switching are introduced, a better performance compared with constant and linear GARCH models is achieved. The tail loss distribution measures also reflect the higher performances of non-linear models in most cases. VaR measures show that MRS-BEKK presents the highest effectiveness in Europe and Germany at 1%, while the MRS-ASYM-BEKK is the best strategy in the UK, Germany and Europe at 5% and 10% levels. For the UK at 1%, the linear BEKK model is most effective. The ES results show similar conclusions to those of the VaR results in the out-sample analysis. This metric also shows the greater effectiveness of non-linear models (the symmetric case for Europe at all levels, Germany at 1% and 10%, and the asymmetric model for the UK at all levels and Germany at 5%)²⁸.

This implies that non-linear models exhibit a higher hedging effectiveness than do constant and dynamic linear models using variance-based metrics. The evidence with tail loss metrics also supports the more complex models in most cases, although in a few scenarios linear models beat them. This greater out-sample effectiveness of non-linear models may be because they offer more accurate forecasting than do more parsimonious models (Marcucci, 2005). When the dynamic relationship between spot and futures returns is characterized by regime shifts, allowing the hedge ratio to be dependent upon the state of the market, one can obtain more efficient hedge ratios and hence, superior hedging performance compared with other methods in the literature.

²⁷ Cotter and Hanly (2006) find that some performance metrics (especially VaR) yield different results in terms of the best hedging model compared with the traditional variance reduction criterion.

²⁸ Alizadeh and Nomikos (2004) and Alizadeh et al. (2008) also find a general outperforming of regime-switching models regarding other strategies in their studies but in a few scenarios, the more complex models they propose are beaten.

5.4. Specification test

To test robustness, this section performs several specification tests to check the adequacy of the QML estimations of the multivariate models. For this reason, we analyze the properties of the standardized residuals ($\epsilon_{i,t} = \varepsilon_{i,t} / \sqrt{h_{ii,t}}$) for $i=s,f$ and the product of the standardized residuals for the models proposed.

Table 5 displays the main results of these specification tests. The first part of the table shows summary statistics for the standardized residuals of the estimated models. The mean value is around zero in all cases with a standard deviation close to one. A reduction in the skewness and kurtosis of the residuals is observed compared with the original series. The Ljung–Box test performed over the standardized residuals reveals a lack of serial autocorrelation in both levels and their cross products. This also removed the heteroskedasticity problem present in the original series.

[INSERT TABLE 5]

These results confirm the consistency of the estimations of our models even for deviations from normality (Capiello and Fearnley, 2000).

6. Conclusions

This paper analyzes hedging effectiveness using non-linear GARCH models in some of the main European stock indexes. It presents MRS-BEKK specifications that assume non-linear dynamics between spot and futures returns to overcome the traditional linear GARCH limitations and reflect properly the characteristics of the financial data.

The estimation of the models reveals that significant differences exist in the variance equation parameters between states. This reflects the fact that the volatility process is not defined by a unique process as proposed by linear GARCH models but by two different volatility processes observed during high and low volatility periods. The consideration of one instead of two volatility processes leads to poor estimations of volatility and this may influence the estimated hedge ratios. Differences in volatility between low and high volatility states are observed in terms of the (asymmetric) impact of shocks and past variance on the volatility formation in each state. Another interesting result is related to the state governing the process in each period. Usually, high volatility states are present in contexts of market uncertainty such as the dot-com bubble or the last financial crisis.

The volatility estimations and forecasts are also different between linear and non-linear models. These differences affect the effectiveness reached by each strategy as our empirical results demonstrated. Non-linear models generally outperform the rest of the models in both in-sample and out-sample analysis. The presented results are robust across countries and for most of the effectiveness measures proposed.

Because the out-sample analysis was performed during the last financial crisis it seems that non-linear models improve the rest of the models during these periods of market jitters. The reason of these results is that the use of these more complex models let us extract more information from data (they let us distinguish between calm and uncertain

periods). Thus the resulting estimator for conditional hedge ratios have a simple behavior (MRS based (out-of-sample) hedge ratios are much more stable than that of BEKK models). This finding is consistent with the finding that OLS hedge ratio outperforms any other conditional hedge ratios.

These results also support empirically previous works which find a theoretical superiority of non-linear models regarding static and linear GARCH models.

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