Dimensionless tuning procedure of the Kalman filter for state-of-charge estimators

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Abstract*—This paper presents a solution to the challenging problem of estimating the state of charge (SOC) of batteries using a Kalman filter algorithm. The algorithm requires knowledge of the dynamical model and its parameters, along with the covariance matrices associated with measurement noise, process noise, and initial estimation error. However, determining the values for the process noise and initial estimation error matrices is often difficult. To address this issue, we propose a novel method to tune these matrices based on a new tuning parameter, the measurement noise variance, and the expected slope of the open circuit voltage vs. state of charge curve. We demonstrate the effectiveness of the proposed approach through extensive simulations involving various batteries and operating conditions in an electrical driving scenario. We assess the performance of the Kalman filter estimation under noisy environments, wrong initial conditions, and modeling errors, obtaining dimensionless performance indices that quantify its behavior. By analyzing the simulation results, we establish a general design procedure for the process covariance matrix, where the user only needs to set the desired limits for the state of charge error in noisy environments and the convergence time under wrong initializations. This design procedure is applicable to batteries of any type, sampling period, or measurement noise level, providing a practical and efficient solution for accurate state of charge estimation

***Index Terms*—State of charge estimation, battery, Kalman filter, process noise covariance matrix, tuning parameters, dimensionless design, measurement noise, open circuit voltage, SOC curve, modeling errors.**

I. INTRODUCTION

Lithium-ion batteries (LIBs) are the cornerstone for the successful development of electrical vehicles (EVs), which play a key role on the decarbonization of the mobility sector [1]. Beyond technical challenges, much of the industry's concern is about cost. Although LIBs have been experiencing a very significant cost decrease in the last decade, this trend has recently shown signs of reversing [2]. So, LIBs still represent one of the most relevant factors in the final cost of EVs.

One of the ways to reduce the size of a battery and, accordingly, its cost is to extend the profitable state of charge (SOC) range. However, since overcharging or overdischarging a battery is dangerous, it is of great importance to properly estimate SOC. Furthermore, this estimation is also required for a precise determination of the remaining driving autonomy, which is a relevant feature for the popularization of EV.

One of the most extended methods to estimate SOC, because of its simplicity, is the Coulomb counting (also known as ampere-hour counting or current integration). However, it is an open loop estimator very sensitive to different error sources, such as the initial SOC, actual battery capacity, or current measurement precision [3]. On the contrary, estimation by measuring the open circuit voltage (OCV) can be very accurate (when there is a good precision in the voltage measurement) but it generally requires a very long rest time, which makes it unpractical to use in real-time applications [4].

In this context, approaches which combine both an open loop model and voltage measurement, i.e. state observer based estimators, have been widely proposed in the literature [4], [5]. In particular, because of its popularity in many other fields, Kalman filters (KFs), and its modifications such as extended Kalman filters (EKFs) or unscented Kalman filters (UKFs), are some of the most used algorithms for the design of observers in this application [6]. For their adequate functioning, KF-based algorithms require a proper design of their parameters, i.e. the matrices for covariance of the process noise (Q_k), covariance of the observation noise (R_k) and initial covariance of the estimation error (P_0).

Some authors formulate some assumptions to try to calculate, as accurately as possible, the real values of these matrices [3], [7]. If these values are correct and some hypotheses are valid, the KF is guaranteed to be optimal. Nevertheless, these hypotheses (absence of modelling error, white noise and disturbances, uncorrelated noises, etc...) are too strong in virtually all cases, and, therefore, the effort on determining the real covariance matrices does not provide any clear advantage.

Even like that, despite not fulfilling the hypotheses for optimality under which the KF was developed, the algorithm is still valid and covariance matrices can be considered as tuning parameters of the filter, which determine the performance of the observer. Many works in the literature regard these matrices as parameters and offer some very broad recommendations [7] but, to the authors' knowledge, it does not exist a precise procedure for their design for any battery in a general case, and in fact this is considered a difficult task [6].

In this work we consider whether it is possible to define some criteria for the selection of the values to be used in the Q_k, R_k and P_0 matrices and applied to any battery. To do so, we propose the analysis of the behavior of the estimator in different scenarios of estimation error, for different batteries and with different observation noise levels. These scenarios

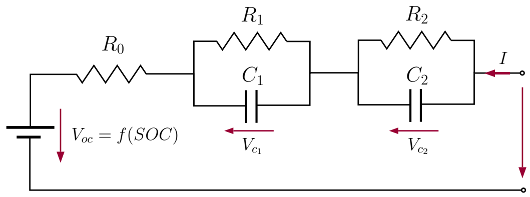

Fig. 1. Equivalent electrical circuit.

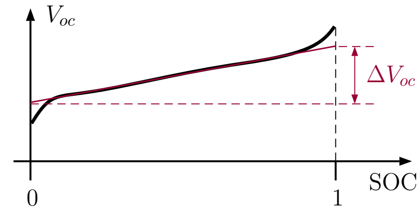

Fig. 2. Open circuit voltage vs state of charge.

include the study of: the speed of convergence as a function of SOC initialization errors, high-frequency errors due to voltage measurement noise, and steady-state errors due to modeling inaccuracies. Furthermore, we establish dimensionless techniques for the assignment of values to the covariance matrices required by the KF, as well as for the performance analysis. With the combination of both proposals we present an approach that allows us to predict the behavior of the estimator for any battery. Based on this, we propose a procedure for the design of estimators based on the KF that allow certain performances to be guaranteed.

Some known limitations of this work to be addressed in future works include, regarding the implementation of the algorithm: we neglected in the design phase the effect of P_0 matrix and the terms of Q_k matrix that refer to states different from the SOC; and, regarding the simulated scenarios: we did not analyze the effect of the modeling error on the passive elements of the equivalent circuit (resistances and capacitors), and we did not include noise in the current sensor.

The structure of the paper is as follows. In Section II we formulate the problem, including the battery model used and the KF algorithm implementation. Section III is devoted to introduce our proposal for the parameters tuning and the dimensionless analysis of the observer performance. In Section IV we validate the proposal with a benchmark including different battery models. Section V summarizes the design procedure and, finally, Section VI introduces some conclusions.

II. Problem Statement

A. Battery model

In this work, we consider an equivalent circuit model (ECM) with a series resistance and two RC branches (2RC) as the one shown in Fig. 1. This kind of model is extensively implemented for online SOC estimation as it offers a good compromise between simplicity and accuracy [6].

In Fig. 1, $I(t)$ is the charging current; $V(t)$ is the terminal voltage; V_{oc} is the OCV, which depends on SOC; R_0 is the series resistance; and the RC branches are used to model the time constants $\tau_1 = R_1 C_1$ and $\tau_2 = R_2 C_2$ that describe the slow and fast transient response caused by charge transfer and diffusion within the LIBs [5].

By applying elemental circuit principles, the behavior of the LIB is modelled by the following differential equations:

$$SOC(t) = SOC(0) + \frac{\eta}{C_{bat}} \int_0^t I(t)\, dt \tag{1}$$

$$\dot{V}_{c1}(t) = \frac{-1}{\tau_1} V_{c1}(t) + \frac{R_1}{\tau_1} I(t) \tag{2}$$

$$\dot{V}_{c2}(t) = \frac{-1}{\tau_2} V_{c2}(t) + \frac{R_2}{\tau_2} I(t) \tag{3}$$

$$V(t) = V_{oc}(SOC(t)) + R_0\, I(t) + V_{c1}(t) + V_{c2}(t) \tag{4}$$

where C_{bat} is the LIB capacity; η is the efficiency, and $V_{c1}(t)$ and $V_{c2}(t)$ are the voltages in both capacitors.

For the observer design phase, C_{bat}, η and all the passive ECM elements are considered constant in time and independent from SOC and temperature, although some robustness analyses will be pursued.

OCV presents a dependence with SOC that is usually known in commercial LIBs or can be derived, as a piecewise function, from a stepwise measurement of OCV after a sufficient rest time for different values of SOC. Fig. 2 shows a typical $OCV - SOC$ characteristic for a generic LIB.

Although this curve can slightly change throughout the LIB lifetime, it is considered constant in this work. However, if some degradation was observed in the LIB behaviour, the curve could be updated with new stepwise experiments. Finally, hysteresis effects are neglected.

B. State of charge estimation

As previously discussed, model-based state observers are widely used in the literature for SOC estimation, mainly because terminal voltage can be very easily measured. This kind of observers requires a discrete equivalent model for the system, described as:

$$x_{k+1} = A\, x_k + B\, u_k + w_k \tag{5}$$

$$y_k = g(x_k, u_k) + v_k \tag{6}$$

with

$$x_k = \begin{bmatrix} SOC_k \\ V_{c1,k} \\ V_{c2,k} \end{bmatrix},\ u_k \equiv I_k,\ y_k \equiv V_k \tag{7}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{-T_s}{\tau_1}} & 0 \\ 0 & 0 & e^{\frac{-T_s}{\tau_2}} \end{bmatrix} \quad B = \begin{bmatrix} \frac{T_s\, \eta}{C_{bat}} \\ R_1(1 - e^{\frac{-T_s}{\tau_1}}) \\ R_2(1 - e^{\frac{-T_s}{\tau_2}}) \end{bmatrix} \tag{8}$$

where T_s is the sample period; $I_k = I(k\, T_s)$; $V_k = V(k\, T_s)$; and w_k and v_k are included to denote the uncertainty in the state and measurement equations, respectively.

From (4) and the definition of the system states, the output equation $g(x_k, u_k)$ has the form:

$$g(x_k, u_k) = V_{oc}(x_{k,1}) + x_{k,2} + x_{k,3} + R_0\, u_k \tag{9}$$

With previous equations, the SOC is estimated by means of a model based observer:

$$\hat{x}_k^- = \hat{A}\hat{x}_{k-1} + \hat{B}\,u_{k-1} \tag{10}$$

$$\hat{y}_k^- = \hat{g}(\hat{x}_k^-, u_k) \tag{11}$$

$$\hat{x}_k = \hat{x}_k^- + L_k(y_k - \hat{y}_k^-) \tag{12}$$

$$S\hat{O}C_k = \begin{bmatrix}1 & 0 & 0\end{bmatrix}\hat{x}_k \tag{13}$$

where $\hat{A}$, $\hat{B}$, and $\hat{g}$ are introduced to consider potential modelling errors in the ECM parameters and L_k is the time-variant observer gain.

This observer must be initialized at a given initial value $\hat{x}_0$ that includes the estimate of the initial value of SOC and that of the voltages in the two capacitors included in the model. The most widespread alternative to obtain the observer gain is the EKF technique, in which the gain L_k is defined every sampling period using the following equations:

$$P_k^- = \hat{A}\,P_{k-1}\,\hat{A}^T + Q_k \tag{14}$$

$$L_k = P_k^-\,\hat{C}^T\left(\hat{C}_k\,P_k^-\hat{C}_k^T + R_k\right)^{-1} \tag{15}$$

$$P_k = (I - L_k\,\hat{C}_k)\,P_k^- \tag{16}$$

where P_k models the state error propagation:

$$P_k = E\{\tilde{x}_k\,\tilde{x}_k^T\}, \quad \tilde{x}_k = x_k - \hat{x}_k \tag{17}$$

Note that, as the measurement equation includes a nonlinear function, there is no C_k matrix for the real system. However, $\hat{C}_k$ can be obtained by using the partial derivative of $\hat{g}(x,u)$:

$$\hat{C}_k = \left[\left.\frac{\partial\hat{g}(x,u)}{\partial x_1}\right|_{x=\hat{x}_k^-,u=u_k} \quad 1 \quad 1\right] \tag{18}$$

C. Motivation

The EKF algorithm (14)-(16) requires an initial value for P_0 that should be equal to the covariance of the initial estimate error of the state, i.e., $P_0 = E\{(x_0 - \hat{x}_0)\,(x_0 - \hat{x}_0)^T\}$ [1]. Furthermore, the matrices Q_k and R_k must be defined as a function of the error propagation model in both the state equation and the output equation. In the state equation, Q_k refers to the uncertainty that is associated with the modeling error or the available current measurement. On the other hand, the term R_k refers to the measurement noise of the voltage sensor as well as the measurement noise due to the current sensor introduced through the term $R_0\,I_k$.

The KF is an observer that ensures that the estimate of the state is optimal in the sense of minimizing the total squared error $trace(P_k)$. However, this statement is only true under the following conditions: when the error propagation model coincides with the one indicated above; when the algorithm is initialized with the exact value of the initial error covariance; and when both disturbance w_k and measurement noise v_k are white noise type signals with zero mean and, together with the estimation error of the starting state $\tilde{x}_{k-1}$, are signals uncorrelated in time, that is, if

$$w_k \sim N(0, Q_k), \quad v_k \sim N(0, R_k), \tag{19}$$

and $E\{w_k v_k\} = E\{w_k \tilde{x}_{k-1}^T\} = E\{v_k \tilde{x}_{k-1}^T\} = 0$.

If these hypotheses are not fulfilled by the system the observer is no longer optimal and, then, performing an exhaustive analysis to determine matrices P_0, Q_k and R_k is not justified. However, the values that we assign to these matrices used in the algorithm (14)-(16) to obtain the gain matrix L_k can still be used as adjustment factors (tuning parameters) to achieve a certain behavior of the SOC estimator.

In this paper we aim to design the Q_k and R_k matrices to be used in the EKF for the SOC estimation with two goals: to ensure the estimator's properties can be guaranteed from the design phase, and to eliminate the need for an exhaustive analysis of the battery's dynamic model, the uncertainty, or the available signals.

III. Proposal

This section presents and justifies our proposal to establish the values of the matrices P_0, Q_k and R_k. Furthermore, a proposal for the analysis of the estimator's performance in different scenarios is presented. With this, the guaranteed (for the hypotheses established in the work) design proposal of observers is finally presented.

A. Parameter tuning

We present here our proposal for the design of the matricesR_k, Q_k, and P_0. As will be seen, each parameter will be a function of the previously assigned one.

1) Covariance of the measurement noise R_k: In this work it is proposed a constant matrix equal to the variance of the measurement noise of the voltage sensor, $R_k = R$. We assume that the noise is a zero mean bounded signal with maximum absolute value $V_{\max}$. Under the assumption of uniform distribution, we can obtain the variance of this signal with $R = \frac{V_{\max}^2}{9}$, that is also a valid approximation in the case of Gaussian distribution. In the output equation (9) for our implementation of the EKF, it can be seen that the output is also influenced by the uncertainty that may be present in the current measurement (ΔI), both in the form of noise and poor calibration or resolution errors. However, we assume that the term $R_0\,\Delta I$ is much smaller than the error introduced by the measurement noise of the voltage sensor.

2) Covariance of the process noise Q_k: Our proposal is to use a constant diagonal Q_k matrix of the form $Q_k = Q = \mathrm{diag}([q, 0, 0])$. This proposal is supported on the fact that the poles corresponding to states $x_{k,2}$ and $x_{k,3}$ ($V_{c1,k}$ and $V_{c2,k}$) are stable in open loop. Thus, their error will converge with error dynamics equal to the corresponding pole (that is, with time constants τ_1 and τ_2 respectively) even if they are not estimated in closed loop (i.e. if the estimate was not corrected with the help of measurements). Accepting these hypotheses, we propose for matrix Q the following expression

$$Q = \mathrm{diag}([q, 0, 0]), \qquad q = \gamma\,\frac{R}{(\Delta V_{oc})^2} \tag{20}$$

[1] Impossible to know since we don't know the actual initial state x_0.

being ΔV_{oc} the expected value of the partial derivative in (18). This value can be computed by approximating the SOC-Voc curve using a first order function of the form $V = a + b\,SOC$ as shown in Fig. 2 (with $b \equiv \Delta V_{oc}$). To obtain this value, the known curve of the battery can be used and an approximation by least squares can be made.

This proposal is based on the analysis of the a priori error propagation in the filter. Analyzing (15) (where the gain L_k is calculated), we see in the denominator the expected value of the squared error of the estimation of the output voltage ($E\{(y_k - \hat{y}_k^-)(y_k - \hat{y}_k^-)^T\}$), which is given by ($\hat{C}_k\,P_k^-\,\hat{C}_k^T + R$). This can be rewritten as

$$\begin{aligned} &E\{(y_k - \hat{y}_k^-)(y_k - \hat{y}_k^-)^T\} = \hat{C}_k\,P_k^-\,\hat{C}_k^T + R \\ &= \hat{C}_k \hat{A}\,P_{k-1}\,\hat{A}^T \hat{C}_k^T + \hat{C}_k\,Q\hat{C}_k^T + R \end{aligned}$$

In view of this equation, our proposal is that the term $\hat{C}_k\,Q\hat{C}_k^T$ (error in the voltage estimation due to the propagated uncertainty or due to the input disturbance variance) is expressed in relative terms w.r.t. measurement noise covariance R. With our proposed $Q = \mathrm{diag}([q, 0, 0])$ this term is

$$\hat{C}_k\,Q\hat{C}_k^T = q \left(\frac{\partial \hat{g}(x, u)}{\partial \hat{x}_1} \right)^2 \Bigg|_{x=\hat{x}_k^-, u=u_k} \quad (21)$$

The partial derivate of $g(x, u)$ changes over time but, for simplicity, we propose to use its expected value, calculated as the slope of the linear regression indicated in Fig. 2, ΔV_{oc}. With this simplification andthe proposed value for q in (20), we have

$$\hat{C}_k\,Q\hat{C}_k^T = q(\Delta V_{oc})^2 = \gamma\,R \quad (22)$$

3) Initial estimate of the state error covariance matrix P_0: From equations (14)-(16), it follows that the EKF requires an initialization for matrix P_k. If the original algorithm assumptions were met, this value should be equal to $E\{\tilde{x}_0 \tilde{x}_0^T\}$. Since they are not fulfilled in general, nor can the initial value of $\tilde{x}_0$ be known, the proposal is to initialize it taking into account the permanent regime value expected for P_k. In this sense, the value of P_k, if the matrices Q_k, R_k and $\hat{C}_k$ are constant, converges to a constant steady state value. In our case, $Q_k = Q$ and $R_k = R$ are constant matrices by design decision, and $\hat{C}_k$, when we approximate the partial derivate of $g(x, u)$ as ΔV_{oc} too.

This can be achieved by first obtaining the permanent value for $\bar{P}_k^-$ ($\bar{P}^-$) by solving the Ricatti algebraic equation

$$\hat{A}\bar{P}^-\hat{A}^T - \bar{P}^- - \hat{A}\bar{P}^-\hat{C}^T(\hat{C}\bar{P}^-\hat{C}^T + R)^{-1}\hat{C}\bar{P}^-\hat{A}^T + R = 0 \quad (23)$$

With this, our proposal for the initization of P_k is its steady state value for P_k ($P_0 = \bar{P}$) computed as[2]

$$\bar{P} = \hat{A}^{-1}(\bar{P}^- - Q)(\hat{A}^T)^{-1} \quad (24)$$

[2]This result is directly given by MATLAB command dlqe

B. Performance evaluation

The goal of the KF is to provide a SOC estimation with low errors in the face of different situations that we can associate to different frequency ranges (measurement noise, errors in the initial conditions or modeling errors). This paper proposes to analyze these phenomena separately using a different metric for each of them.

Next, we propose how to evaluate the estimation error $\tilde{SOC} = (SOC - \hat{SOC})$ under different scenarios.

1) Measurement noise: The measurement noise is considered a signal with abrupt variations between consecutive samples, so it can be considered a high frequency signal that also causes certain high frequency variations in the estimated SOC ($\hat{SOC}$). To analyze the isolated effect of the measurement noise on $\hat{SOC}$, it is proposed to test the estimator behavior in a situation in which the initial conditions of the observer coincide with reality ($\hat{x}_0 = x_0$) and where the model used does not contain errors ($\hat{A} = A$, $\hat{B} = B$, $w = 0$) but where there is measurement noise with a certain known variance R. Knowing R, Q and P_0 can be adjusted as indicated above. It is proposed to quantify this behavior through the maximum absolute value of SOC estimation error

$$\tilde{SOC}_{\max} = \max |\tilde{SOC}_k| \quad (25)$$

2) Initial conditions: When the observer is started, both an estimated value for the state estimate $\hat{x}_0$ and the matrix P_0 are required. During the initial transient, the state estimated by the observer (SOC and capacitor voltages) must converge to its real value in the shortest possible time. The time required for this convergence depends on the actual initial state of the system, on its estimate, and on the value of matrix P_0. The proposal for this matrix has already been discussed previously. Regarding the effect of the initial error, it is proposed to initialize the estimate of the state in $\hat{x}_0 = [0.5, 0, 0]$, that is, an intermediate value of the SOC and a null value in the capacitor voltages.

In order to analyze the isolated effect of the initialization error on $\tilde{SOC}$, it is proposed to subject the estimator to a situation in which the measurement noise and the modeling error are null, but the initial conditions of the battery are $x_0 = [1, 0, 0]$ (battery fully charged and at rest with discharged capacitors).It is proposed to quantify this behavior through the settling time at 98%

$$t_{s_{98}} = k \cdot Ts, \;\; k = \{\min k: \; \tilde{SOC}_k \leq 0.01\} \quad (26)$$

being 0.01 the 2% of the initial error $\tilde{SOC}_0 = 0.5$.

This scenario allows evaluating the performance of the observer at medium frequencies, which is where the response speed is defined. This index will make it possible to estimate the response times that would occur in other similar situations where there is a high initial error in estimating SOC.

3) Model uncertainty: Modeling error is one of the main sources of error when estimating the SOC. In particular, one of the most common modeling errors is battery capacity, as it can change over time due to aging.

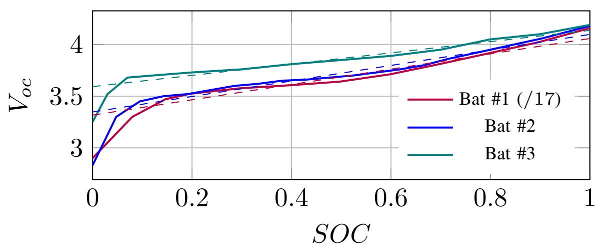

Fig. 3. Open circuit voltage vs. state of charge curve.

Bat.#	$\mathbf{C_{bat}}$ (Ah)	$\mathbf{\Delta V_{oc}}$ (V)	$\mathbf{R_0}$ ($m\Omega$)	$\mathbf{R_1}$ ($m\Omega$)	$\mathbf{C_1}$ (kF)	$\mathbf{R_2}$ ($m\Omega$)	$\mathbf{C_2}$ (kF)
1	20	12.6	304	170	109.4	90	0.66
2	42	0.75	1.31	0.69	930.4	0.72	50
3	4.4	0.54	44.1	18.6	69.2	4	0.138

TABLE I
PARAMETERS OF THE MODEL FOR THE DIFFERENT BATTERIES

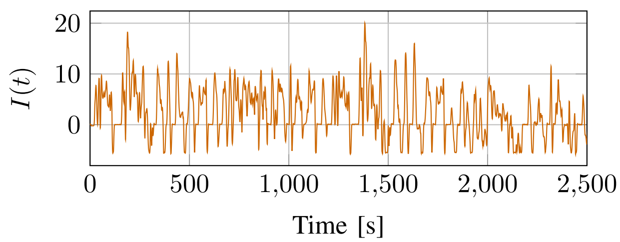

Fig. 4. One cycle current of the UDSS for battery #1.

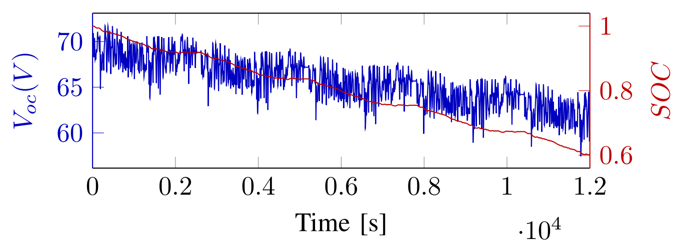

Fig. 5. Evolution of the Voc voltage and SOC in the experiment for bat. #1.

In order to analyze the isolated effect of the modeling error on $\tilde{SOC}$, it is proposed to subject the estimator to a situation in which the measurement noise is zero and the initial conditions are known, and where the observer model uses a capacity value different from the real one, that is, $\hat{B}_1 = \frac{T_s \eta}{\hat{C}_{bat}}$.

This behavior can be quantified through the maximum error

$$\tilde{SOC}_{\max} = \max |\tilde{SOC}_k| \tag{27}$$

which evaluates the observer's low frequency performance.

IV. Validation of the Proposal

In the following, we present different implementations of a *Simulink-Simscape* model consisting of three different batteries for which measurements have been taken with different noise levels and different sampling periods. For each scenario, the observer matrices have been defined with different values of the only adjustment parameter γ. Based on these data, and with these matrices, the SOC has been estimated against the three scenarios proposed above (measurement noise, initial conditions and model uncertainty) and the indices (25) -(27) have been obtained. Finally, a scaling of these indices is shown that will allow us to establish a dimensionless design procedure in the following section.

A. Benchmark setup

In this work we have modeled three batteries from different sources. Battery #1 is a complete battery pack from an electrical motorcycle whose electrical model has been identified according to the procedure described in [7]; battery #2 is the NMC pouch cell from [8]; and battery #3 is the NMC cylindrical cell from [7]. Each battery is defined by its $V_{oc} - SOC$ curve and electrical parameters as shown, respectively, in figure 3 (where the voltage shown for battery #1 is divided by 17, the number of cells in series) and in table I.

Each of the batteries has been tested in the following way. We assume that initially the battery is completely charged and waits in steady state (i.e., with null voltage in the capacitors) to be discharged. The discharge of the battery is performed by a current profile which repeats five times the Urban Dynamometer Driving Schedule (UDDS), extracted from [9] and scaled for each battery in such a way that the maximum discharge current corresponds to a 1C C-rate. This way, the SOC profile for each battery is the same. Fig. 4 and 5 show, as an example, the current profile cycle UDDS (that is repeated 5 times), the battery voltage and SOC evolution for battery #1.

As a result of this experiment, for each battery we get a current profile $I(t)$ (model input u), the real evolution of $SOC(t)$ (state x_1 to be estimated), and the voltage at the battery terminals $V(t)$ (noise free). To validate the proposal, a uniform random signal has been generated in the range $[-V_{\max}, V_{\max}]$ and it has been added to $V(t)$ to have the variable y of our noisy model. We have run experiments with 6 different noise levels for each battery within the range $[\frac{\Delta V_{oc}}{1000} \leq V_{\max} \leq \frac{\Delta V_{oc}}{10}]$. Likewise, these values have been sampled with three different periods: $T_s = \{0.1,\ s; 0.5,\ s; 2.5\ s\}$.

Each dataset that the observers have been experimented with is thus defined by the use of a particular battery, a noise level and a given sampling period.

B. Simulation results

For each of the datasets, the (10)-(16) estimation algorithm has been launched with different values of γ in the scenarios indicated in Section III-B. Assuming noise covariance R as known data and following the indications in III-A for the parameter tuning, different estimates of SOC are obtained.

For example, Fig. 6 shows the isolated effect of noise in the evolution of the estimated SOC in battery $\#2$ with a sampling period $T_s = 2.5\,\mathrm{s}$ and with a measurement noise $V_{max} = 0.075\,\mathrm{V}$, while Fig. 7 shows the effect of initial conditions (eliminating sensor noise) and Fig. 8 the effect of modeling error for the same filter parameters.

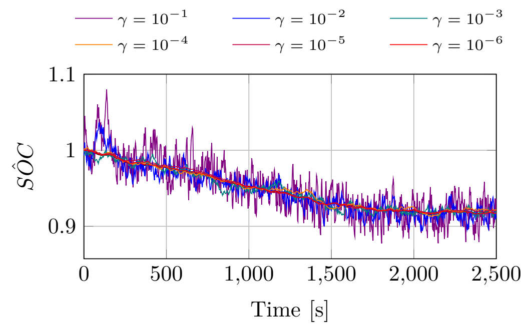

Fig. 6. Evolution of $\hat{SOC}$ in noisy environment for different filters.

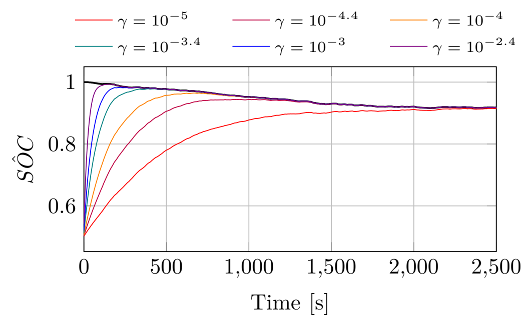

Fig. 7. Evolution of $\hat{SOC}$ under wrong initial conditions for different filters.

Once we have the evolution of the estimation errors with the different values of γ in the different scenarios, we have analyzed the dependence of $SOC_{\max}$ (obtained for the first scenario, *measurement noise*) vs. γ for the different batteries and with the different noise levels. Results show that the index $\tilde{SOC}_{\max} \cdot \frac{\Delta_{Voc}}{Vmax}$ roughly depends only on γ and not on the battery number or the sampling period. Indeed, Fig. 9 shows how this is true for the different batteries, with different sampling periods and different noise levels. It can be clearly seen that low values of γ lead us to an observer with a better behavior to deal with measurement noise.

Index $\tilde{SOC}_{\max} \cdot \frac{\Delta_{Voc}}{Vmax}$ can be understood as the ratio between:

- the SOC estimation error obtained by the observer
- the SOC estimating error that would be obtained from measuring OCV in steady state with null currents with a noisy sensor and using the inverse of curve $V_{oc} = f(SOC)$, i.e., $\hat{SOC} = f^1(V_{oc})$

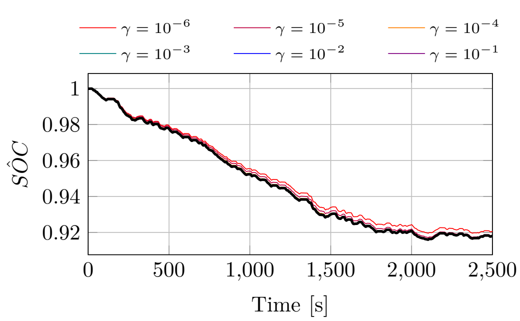

Fig. 8. Evolution of $\hat{SOC}$ under wrong C_{bat} parameter in the model for different filters.

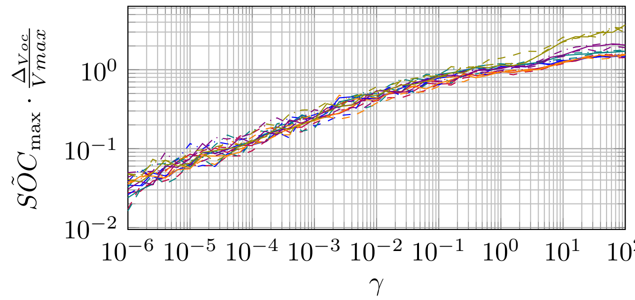

Fig. 9. Relative error $\tilde{SOC}_{\max} \cdot \frac{\Delta_{Voc}}{Vmax}$ vs γ under noisy measurements.

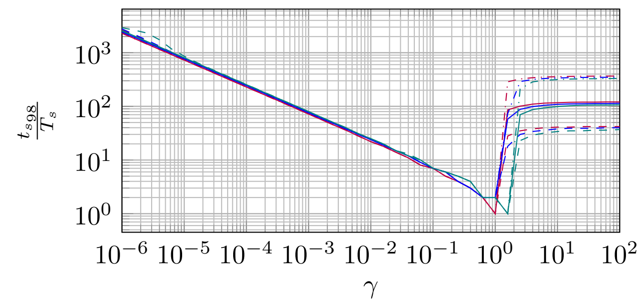

Fig. 10. Relative settling time $\frac{t_{s98}}{T_s}$ vs γ under wrong initial conditions.

With an analogous analysis, we have derived Fig. 10 that shows $\frac{t_{s,98}}{T_s}$ vs. γ, showing how independence of the results from battery number and measurement noise has also been achieved. Index $\frac{t_{s,98}}{T_s}$ shows the number of samples needed for the estimated SOC to converge to the true value. It can be seen that, for the range $\gamma \in (0, 1]$, higher values of γ have faster behaviors. When values of $\gamma > 1$ are used, undesirable dynamic behaviors are obtained because the discrete poles resulting from the closed-loop observer have a negative real part, which implies a fast oscillation of period $2T_s$ in the estimator and an inadmissible noise amplification. Note that, with the design proposal for Q, the noise level for which the observer has been designed does not affect the estimation result (the curves are superimposed).

Figures 9 and 10 allow to roughly predict what the effect of the measurement noise will be on the SOC estimate and what the convergence time of the algorithm will be for an observer determined by $\gamma \in (0, 1]$.

On the other hand, Fig. 11 shows for the different batteries (with different colors) and sampling periods (with different line type) the estimation error of SOC $\tilde{SOC}_{max}$ vs. γ when there are modeling errors. In particular, an error on the battery capacity is considered ($\hat{C}_{bat} = 1.1C_{bat}$). In this case, it can be seen that, the shorter the sampling period, the lower the estimation error obtained, and that, for each battery, the minimum error is obtained for a γ in the range $\gamma \in [10^{-4}, 10^{-2}]$ (with the optimal value being lower for higher battery capacities).

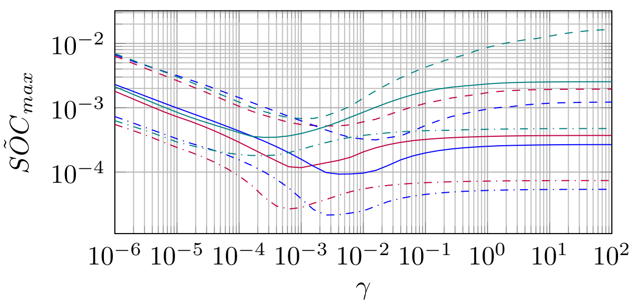

Fig. 11. Absolute error $\tilde{SOC}_{\max}$ vs γ under uncertainties in parameter C_{bat}.

Although in this work it was not possible to express this index in dimensionless terms (and therefore it cannot be used to accurately predict the error due to model uncertainty) it still allows us to see the range of values for γ that will lead us to a lower error due to uncertainty. Again, the proposed relative definition for Q means that error results in this situation are independent of the measurement noise.

V. Design procedure

From the previous figures we can establish a design procedure that includes assigning a value to γ in Q matrix in order to guarantee a certain performance in the observer:

1) Obtain, by any of the algorithms existing in the literature, a 2RC ECM for the battery, including the slope of the line that approximates the curve SOC-Voc (ΔV_{oc}).

2) Obtain the measurement noise of the voltage sensor ($V_{\max}$, $R = \frac{V_{\max}^2}{9}$) and set the sampling period T_s.

3) Establish the maximum admissible error value due to measurement noise ($SOC_{\max,d}$). Using Fig. 9 and the value $SOC_{\max,d} \cdot \frac{\Delta_{Voc}}{V_{\max}}$, obtain $\gamma_{\max}$, the maximum value admissible for γ. If there is no upper limit for $SOC_{\max,d}$, set $\gamma_{\max} = 1$.

4) Establish the minimum admissible settling time ($t_{s,98,d}$) for the response of the estimator and, using Fig. 10 and the value $\frac{t_{s,98,d}}{T_s}$ get a value for $\gamma_{\min}$, the minimum value allowed for γ. If there is no lower limit for $t_{s,98,d}$, set $\gamma_{\min} = 0$.

5) Choose one of the following design goals:

1) Minimize the error due to noise, which leads us to select the largest value $\gamma \in [\gamma_{\min}, \gamma_{\max}]$.
2) Minimize the settling time, which leads as to select the lowest value $\gamma \in [\gamma_{\min}, \gamma_{\max}]$.

If there is no specific goal in terms of error due to noise or settling time, choose a value in the range $\gamma \in [10^{-4}, 10^{-2}]$ to achieve low estimation errors due to uncertainty.

6) Start EKF with matrices $R_k = R$, $Q_k = diag([\frac{\gamma R}{\Delta V_{oc}^2}, 0, 0])$ and P_0 according to (23)-(24).

VI. Conclusions

In this work we have addressed the problem of estimating the state of charge of a battery from current and voltage measurements and using an Extended Kalman Filter algorithm that requires the knowledge of a dynamical model and its parameters (electrical parameters of the equivalent circuit and open circuit voltage vs. SOC curve) as well as some values for the covariance of the measurement noise, the input disturbance and initial estimation error.

Under the assumption that is hard for a user to know the last two matrix values, we propose to use those matrices as tuning parameters, and we propose to do it in relative terms w.r.t. to the measurement noise variance (assumed to be known or measurable) and using the slope of the linear approximation of the OVC-SOC curve as units conversion. In this work, we show an initial simple proposal that is reduced to only one tuning parameter.

In order to help the user to tune that proposed parameter, we have established three different scenarios to asses the behaviour at different frequencies: noisy environment, wrong initial conditions and modeling error in battery capacity. In each one we have obtained a characteristic metric. We have applied this procedure in a simulation with several batteries, sampling periods and noise levels, showing that the metrics related to noise attenuation and convergence speed can be expressed in a dimensionless way (independently of battery and conditions). That allows us to establish a dimensionless design procedure applicable to any battery.

Future research lines include expanding the proposal to include more tuning parameters when defining matrix Q (also in relative dimensionless terms) to include in the model the disturbances or modelling errors in state equations related to voltage in capacitors (i.e., positions 2 and 3 of the diagonal).

Acknowledgment

This work was supported by the Spanish State Research Agency under grants PID2021-125634OB-I00 and TED2021-130120B-C22, ERDF, EU; and the Universitat Jaume I through project number UJI-B2021-35.

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