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The time-varying risk–return trade-off and its explanatory and predictive factors

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ABSTRACT

We analyze the intertemporal dimension of the risk–return trade-off and determine the drivers that better explain and predict its evolution. To this end, we propose a novel estimate of the relationship between return and risk where we generate time variation in the trade-off without conditioning the estimates to any state variable. We compare this dynamic approach with time-invariant or state-dependent estimates and observe that our dynamic method reasonably aligns with the constant (state dependent) methods but it offers a much broader picture of the risk–return trade-off. We also link its evolution to a set of macroeconomic, systematic and sentiment or uncertainty risk factors. We find that the risk–return relationship is positive during expansionary periods but it decreases during recessionary periods where occasionally even turns out negative. Our main conclusions hold for the consideration of hedging components, different MV-GARCH models or window lengths and several proxies of market returns and risk.

1. Introduction

Most asset pricing models are based on the risk–return relation. The rationale behind the risk–return trade-off is that the market rewards investors for bearing additional risk, which is measured generally by the relation $\frac{\partial E(R_t)}{\partial V_t}$, being $E(R_t)$ the expected reward and V_t the proxy for risk. Given the real-world importance of understanding the risk–return fundamental, literature on the topic has developed several theoretical models and empirical tests. To date, theory supports both positive and negative risk–return trade-offs and, unfortunately, empirical studies have found neither a conclusive answer to its sign nor to its evolution through time. Furthermore, little attention has been paid to the analysis of the factors that can explain or predict the temporal changes in the risk–return trade-off. This study contributes to the literature by presenting a new conditional methodology to obtain time-varying estimates of the temporal relationship between return and risk, while shedding light on the main drivers of the risk–return relation.

Regarding theoretical models, one of the most widely used, whether we consider the academic field or the investment industry, is the intertemporal capital asset pricing (ICAPM) model (Merton, 1973). According to this model, there is a positive and constant linear relation between the market risk premium and the conditional variance of excess returns, so that periods of high excess stock returns coincide with periods of high stock market volatility, implying a constant risk–return relation. More recent models, however, show that the risk–return relationship is ambiguous and may depend on the investment opportunities and/or the economic conditions (Abel, 1988; Whitelaw, 2000). Since theoretical models support both positive and negative risk–return trade-offs, empirical studies have analyzed this intertemporal relationship, without reaching a conclusive answer regarding its sign or

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shape.[1] While some articles find a positive and significant relationship between the expected excess market return and conditional variance (Bali & Peng, 2006; Bollerslev et al., 2013; Ghysels et al., 2005; Guo & Whitelaw, 2006; Kanas, 2012; Kinnunen, 2014; Ludvigson & Ng, 2007) other studies identify that this relationship is negatively significant (Aslanidis et al., 2016; Bekaert & Wu, 2000; Brandt & Kang, 2004; Ghysels et al., 2014; Lettau & Ludvigson, 2010). Moreover, there is a strand of literature that suggests that this relationship is not significant (Lee et al., 2001; Nelson, 1991; Theodossiou & Lee, 1995), enhancing the discussion further still. This mixed and inconclusive empirical evidence makes the intertemporal trade-off between return and risk one of the major puzzles in the finance literature.

In the last decades, literature has started relaxing the strong assumption on a time-invariant relationship between return and risk, documenting a highly significant, strong nonlinear dependence of returns on past market volatility (Adrian et al., 2019), what represents a promising venue to disentangle the dynamics of the risk–return trade-off. Also, given that these time-varying risk–return models are fairly novel, relatively little is known about how the risk–return trade-off varies empirically over the business cycle or with key macroeconomic indicators. In fact, there is an ongoing debate on whether the risk–return trade-off is pro[2] or counter cyclical,[3] or even differs depending on the market analyzed.[4] However, aside from business cycles, the role of other macro, systematic or sentiment variables in the temporal variation of the risk–return trade-off remains largely unexplored.

Motivated by the above literature that suggests that the risk–return trade-off is time-varying our objective is twofold. Firstly, we propose a new conditional methodology using market level data to obtain time-varying estimates of the temporal relationship between return and risk. Several studies estimate time-varying risk–return trade-off measures in which this relation depends on state variables related to the economic trend (Guo et al., 2013). The main drawback of this methodology is that the reported empirical evidence is potentially driven by the conditioning variables used as state variables. We circumvent this problem and present a conditional estimate of the risk–return trade-off by estimating conditional slopes (λ_t) between the first two moments of the market portfolio's return. To develop this method, we take advantage of the fact that all the information needed to find the coefficients in a linear regression can be found in the covariance matrix of all variables (including explanatory and response variables). Using this definition of the regression coefficients as function of covariances, we can use conditional volatility models to add the time-variation to the risk–return trade-off without needing any state variable. Compared to previous studies, our approach has the main advantage that it is exclusively the data regarding aggregate market returns and risk itself the ones that determine the dynamics of the relationship between return and risk[5] instead of using state variables to generate the time-variation in the trade-off.

In this sense, we follow a three-step approach. First, we estimate the return and the risk of the market portfolio at each point of time by using the weighted-average return and a weighted-covariance matrix between the individual components in the market portfolio. This approach has the advantage of avoiding auxiliary parametric assumptions on the dynamics of conditional moments. Second, we use the market return and risk series to estimate a multivariate GARCH model (DCC-GARCH) that allows us to obtain the time-varying covariance matrices between the variables in our regression of interest. Our implicit assumption is that the distributions of all variables (market return, market risk and hedge component) and the correlations between them are time-varying, providing new insights about how these elements interrelate through time. Finally, we calculate the conditional slopes ($\lambda_{t,i}$) between return and risk by using the time-varying covariance matrices from the multivariate GARCH model and using them to solve for the expressions of the coefficients in the linear regression.

Results show that, in terms of magnitude, the estimates of the risk–return trade-off are in average around 2.74, with a minimum and maximum value of −3.81 and 10.19 respectively (see panel A in Table 1). Therefore, in our setting, the risk–return relation can take different values (either positive or negative) depending on the period considered and the information available. Positive values are observed for more than the 80% of the sample period while negative values are especially prominent during recession periods. These results support findings in previous literature (Bali, 2008) and are reasonably aligned with the estimates of time-invariant (value of 2.06) or state-dependent risk–return relationships as appreciated in Fig. 1 (minimum value of −0.48 and maximum value of 10.21).

Secondly, besides obtaining time-varying estimates of the risk–return trade-off and analyzing its sign or shape, our ultimate goal lies in identifying those factors that enable us not only to explain but also to predict its dynamics. Understanding the drivers of this relationship is crucial for investment decisions since it may allow us to find the factors or combinations of them rewarded by the market and take positions in our portfolio in an early stage. To this end, we use information provided by different sources to link the risk–return relation with a wide variety of factors that theoretically or empirically have been proven to have explanatory and predictive power regarding the return or the volatility of assets. Our set of factors is categorized into three subgroups: a) macroeconomic, b) risk factors linked to the performance and characteristics of stocks (systematic risk factors) and, c) indexes representative of market sentiment and economic, financial or political uncertainty (henceforward sentiment factors)[6].

Regarding the individual explanatory power of each factor, DEF (Difference between Moody's yield on Baa corporate bonds and the 10-year government bond yield used as the proxy of credit risk), $CISS$ (New Composite Indicator of Systemic Stress),

[1] Although GARCH-M models are the most used (Engle et al., 1987) there are other estimation techniques such as instrumental variables (Whitelaw, 1994); MIDAS models (Ghysels et al., 2005) or regime-switching models (Salvador et al., 2014; Whitelaw, 2000), just to mention a few, that try to uncover this pervasive temporal relationship at the aggregate level.

[2] Bliss and Panigirtzoglou (2004), Ghysels et al. (2014), Rossi and Timmermann (2010), Salvador et al. (2014) reveal a procyclical behavior.

[3] Antell and Vaihekoski (2020), Kim and Lee (2008), Lundblad (2007), Nyberg (2012) find evidence of a counter-cyclical risk–return relation.

[4] Liu (2017) finds that results are different depending on the market analyzed (countercyclical for U.S. and procyclical for European markets).

[5] We acknowledge that to implement the multivariate GARCH model we need a distributional assumption regarding the aggregate market returns and aggregate volatility.

[6] See Table 2 and Section 4.1 for a detailed description of the proxies for macroeconomic, systematic and sentiment risk factors.

Table 1
Summary statistics for univariate estimates of the risk–return trade-off.

Panel A. FULL SAMPLE				
Series	Mean	Std	Max	Min
$\lambda_{t,AGGR}$	2.74***	2.40	10.19	−3.81
Panel B. EXPANSIONARY PERIOD				
Series	Mean	Std	Max	Min
$\lambda_{t,AGGR}$	3.01***	2.28	10.19	−2.60
Panel C. RECESSIONARY PERIOD				
Series	Mean	Std	Max	Min
$\lambda_{t,AGGR}$	0.71*	2.36	7.01	−3.81
Panel D. SIGNIFICANCE TEST for regression $\lambda_{t,AGGR} = \alpha_{i,0} + \alpha_{i,1} USREC_t + \epsilon_t$				
	$\alpha_{i,0}$		$\alpha_{i,1}$	
$\lambda_{t,AGGR}$	3.01*** (0.13)		−2.29*** (0.37)	

Table 1 presents the main summary statistics of the risk–return trade-off series ($\lambda_{t,AGGR}$) obtained using the estimates of aggregate volatility as market risk for the full period (Panel A), expansionary periods (Panel B) and recessionary periods (Panel C) according to the NBER. We use the one-sample t-test to check the significance for the mean of the series. The sample period spans from January 1990 to September 2020. Panel D shows the estimates (standard error in parenthesis) for regressions of the risk–return trade-off series ($\lambda_{t,AGGR}$) on a dummy that takes the value of 1 for a recession period according to the NBER and 0 otherwise. ***, ** and * represent significance at 1%, 5% and 10% significance level.

$USSLIND$ (Leading index for USA), $NFCI$ (Chicago Fed National Financial Conditions Index), $CAPE$ (Cyclically adjusted Price Earnings Ratio) and $TERM$ (Difference between 10-year Treasury Constant Maturity and 3-month Treasury Constant Maturity) have higher R-squared with values of 39.24%, 34.53%, 31.49%, 37.22%, 23.33% and 18.36%, respectively; the systematic factors, however, have marginal explanatory power. Within the sentiment factors, $UMCSENT$ (consumer sentiment) and $MUNC$ (macroeconomic uncertainty) stand out with an R-squared of 35.60% and 15.98% respectively. This result can be considered as a sign of the importance of taking into account not only macroeconomic factors but also sentiment factors to understand their role in the risk–return relation. Focusing on multivariate regressions, and particularly on each subgroup of factors, macroeconomic factors have greater explanatory power (50%), followed by sentiment factors (39%). Systematic factors have been ignored from the multivariate analysis given their lower significance and explanatory power. Taking into account these two set of factors together, their explanatory power increases up to 53%, meaning that in a multivariate setting macroeconomic factors play a key role in explaining the risk–return trade-off but sentiment or uncertainty factors' contribution is important too.

With respect to the predictive accuracy of the risk factors, it seems that considering only macroeconomic factors leads to better results; however, the differences in performance are not significant (Diebold & Mariano, 2002) meaning that sentiment factors or the combination of both are important as well. Also, significant improvements are obtained for all models when estimating the out-of-sample R-squared (Welch & Goyal, 2008) relative to the constant model. These results show the importance of both macroeconomic and sentiment factors, not only for explaining the behavior and dynamics of the risk–return trade-off, but also for its forecast.

This study also includes a comprehensive robustness analysis: a) considering hedging components, b) using the VIX index as proxy of volatility, c) estimating different specifications of the multivariate GARCH model to compute the time-varying risk–return trade-off, d) using different window lengths in the estimates of market risk, and e) using directly information on market indexes to estimate the market return and risk.[7] Results concerning the robustness section reveal that minor differences arise across the different approaches implemented, reinforcing the idea that conclusions are robust and are not biased by an aggregate effect of individual assets or the approach applied (Avramov & Chordia, 2006; Gagliardini et al., 2016; Giglio & Xiu, 2012; Kim et al., 2021).

The remainder of this paper is organized as follows: Section 2 describes the methodology and data used to estimate the time-varying risk–return trade-off; Section 3 shows the main patterns of the estimates of the risk–return relation; Section 4 analyzes the potential sources linked to the risk–return trade-off and the predictive power of each group of factors; Section 5 performs several robustness checks; and finally, Section 6 summarizes the results and concludes.

2. Methodology and data

Merton (1973) analytically derives the intertemporal capital asset pricing model (ICAPM) in a continuous-time economy in which the investment opportunity set is time-varying. Merton's ICAPM states that the intertemporal risk–return relation for the aggregate market is given by:

$$E_t[r^m_{t+1}] = \lambda_t Var_t[r^m_{t+1}] + \gamma_t Cov_t[r^m_{t+1}, s_{t+1}] \tag{1}$$

[7] In the robustness section, we also retrieve the market return and market variance using the information on two market indexes: the Fama and French Market factor and the S&P500 index. The evidence is similar to the baseline results.

where the first term on the right-hand side of Eq. (1) captures the volatility and the second term captures the hedging component of the risk premium. Eq. (1) has been the focus of much of the existing literature examining how the expected return and conditional volatility of the aggregate market as well as its hedge-related risk are correlated through time. Some studies assume away the hedge component, $Cov_t[r^m_{t+1}, s_{t+1}]$, following Merton's argument that the hedge component is negligible if the investment opportunity set is static or if investors have logarithmic utility.

As Eq. (1) is written in aggregate terms (e.g., the market return r^m_{t+1}), most previous studies attempt to infer the nature of λ_t from aggregate time series data, such as market returns or other financial instruments. However, as stated in the introduction, the results from these time-series analysis have reported inconclusive evidence.

Since literature has focused primarily on a constant risk–return relation, with widely diverging results, we start our analysis by investigating this special case as a benchmark. In other words, we first assume that λ_t and γ_t are constant. Under this approach, the equation to be estimated is as follows:

$$\underbrace{E_t[r^m_{t+1}]}_{y_t} = \alpha + \lambda \underbrace{Var_t[r^m_{t+1}]}_{x_{1,t}} + \gamma \underbrace{Cov_t[r^m_{t+1}, s_{t+1}]}_{x_{2,t}} + \varepsilon_t \tag{2}$$

This equation (estimated in most of the previous empirical studies) will be our starting point. However, unlike previous studies, we are going to express the coefficients of this linear regression as a function of the covariance matrix of all variables (including explanatory and response variables). Traditionally, the ordinary least squares estimates $\hat{\beta} = (\hat{\alpha}, \hat{\lambda}, \hat{\gamma})$ of the coefficients in model (2) are obtained by solving the system of linear equations: $X_t' X_t \hat{\beta} = X_t' y_t$ where X_t is a matrix of explanatory variables and y_t is a column vector for the response variable. Alternatively, we can solve an equivalent system of equations $\frac{1}{n} X_t' X_t \hat{\beta} = \frac{1}{n} X_t' y_t$ and, by applying Gaussian elimination, our final specification to estimate the risk–return trade-off[8] is Eq. (3), where instead of using unconditional covariances these are replaced by their conditional counterparts given the information set Ω_t available in t.

$$\hat{\beta}_t = \begin{bmatrix} \hat{\lambda}_t \\ \hat{\gamma}_t \end{bmatrix} = \begin{bmatrix} Var_t(x_{1,t}|\Omega_t) & Cov_t(x_{1,t}, x_{2,t}|\Omega_t) \\ Cov_t(x_{2,t}, x_{1,t}|\Omega_t) & Var_t(x_{2,t}|\Omega_t) \end{bmatrix}^{-1} \begin{bmatrix} Cov_t(x_{1,t}, y_t|\Omega_t) \\ Cov_t(x_{2,t}, y_t|\Omega_t) \end{bmatrix} \tag{3}$$

Therefore, our methodology departs from Eq. (2) in which time-invariant unconditional moments are now replaced by time-varying conditional moments, so that the estimated coefficients λ and γ become λ_t and γ_t and they will change through time as new information arrives to the market. This conditional model reduces to the conventional regression model if the joint distribution of expected returns, risk and hedging component is constant through time.

To compute the time-varying coefficients (β_t) in Eq. (3) we implement a three-step procedure. First, we estimate the time-series of return and risk for the market portfolio. We retrieve daily and monthly prices from the CRSP (Center for Research in Security Prices) dataset comprising all the NYSE, NYSE America and NASDAQ stocks available from January 1990 to September 2020. We use monthly prices to compute individual monthly returns for each stock and construct a weighted-average portfolio using all stocks in our sample as a proxy for the return of the market portfolio $E_t[r^m_{t+1}]$. Similarly, we use daily prices and a window of 60 days[9] to compute the value of individual variances and covariances for all stocks at the end of every month of our sample period. With these estimates, we construct a weighted-average estimate of the market's variance using this expression:

$$V_t = \omega_t' \Sigma_t \omega_t \tag{4}$$

where ω_t represents the weights of every individual stock within the weighted-average portfolio (which are proxied by the corresponding market cap) at the end of every month and Σ_t is the estimated covariance matrix at the end of every month using the last 60 day observations. In both estimates (market return and variance) we drop observations for which no market equity data are available or we miss the variance of daily returns.[10]

Second, after having computed the monthly market returns and variances, we propose a multivariate GARCH model[11] to obtain the conditional time-varying estimates of the covariance matrix between all variables in Eq. (2). This multivariate GARCH model includes as response variables $E_t[r^m_{t+1}]$, $Var_t[r^m_{t+1}]$ and $Cov_t[r^m_{t+1}, s_{t+1}]$ and allows us to retrieve the time-varying estimates for all covariances in the right-hand side of Eq. (3). In the baseline results, we use the DCC model (Engle, 2002) as it is parsimonious in the number of parameters and simple to estimate.

Third, once we have obtained the conditional estimates for $Var_t(x_1)$, $Cov_t(x_1, x_2)$, $Var_t(x_2)$, $Cov_t(x_1, y)$ and $Cov_t(x_2, y)$ from the multivariate GARCH model, we solve Eq. (3) for every t and retrieve the estimates for our parameters of interest β_t, paying special attention to λ_t as the estimate of the time-varying risk–return trade-off.

8 Although the covariances provide no information about the intercept α of the regression, it can be estimated from the mean of the data as $\hat{\alpha} = \bar{y} - X\hat{\beta}$.

9 Different window lengths are considered in the robustness section.

10 As a robustness check, we also use data for the VIX index as an estimate of market's volatility $Var_t[r^m_{t+1}]$. We also provide a robustness check where both the market return and market risk are obtained directly through well-known market indexes.

11 Different asymmetric multivariate GARCH specifications are used in the robustness section.

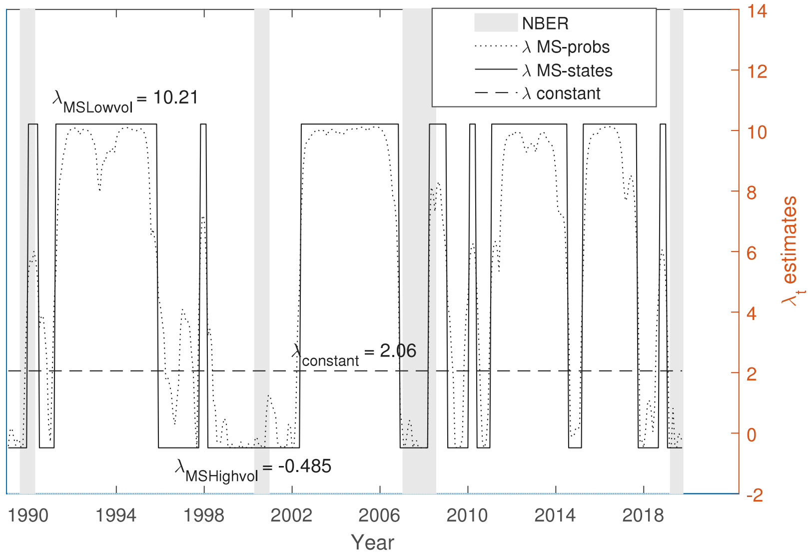


Fig. 1. Constant and state-dependent risk–return trade-off.
Fig. 1 depicts the time series of monthly risk–return trade-off using different approaches available in the literature. The dashed line represents the estimates assuming a constant relationship between return and risk while the solid line (weighted by market state) and dotted lines (weighted by state probabilities) represents the state-dependent relationship between return and risk where the changes in regime are governed by a hidden state variable following a Markov process. The sample period spans from January 1990 to September 2020. Gray areas correspond to periods of recession according to the NBER. The horizontal axis represents the time period on a monthly basis, while the vertical axis represents the level of the risk–return trade-off.

3. Estimates of the time-varying risk–return trade-off

Assuming that the hedge component is negligible, Merton (1980) and numerous subsequent studies have investigated empirically whether there is a constant positive relationship between the expected excess market return and the conditional market volatility:

$$\underbrace{E_t[r^m_{t+1}]}_{y} = \alpha + \lambda \underbrace{Var_t[r^m_{t+1}]}_{x_1} + \varepsilon_t \tag{5}$$

In a first generation of models that explore the potential gains that can be obtained by allowing for nonlinearities in the risk–return trade-off, authors such as Salvador et al. (2014) or Ghysels et al. (2014) allow for state-dependence in the parameters of this regression:

$$\underbrace{E_t[r^m_{s_t,t+1}]}_{y_{s_t}} = \alpha_{s_t} + \lambda_{s_t,t} \underbrace{Var_t[r^m_{s_t,t+1}]}_{x_{1,s_t}} + \varepsilon_{s_t,t+1} \tag{6}$$

where the parameters of the model are conditioned to a hidden state (s_t) variable following a Markov process.

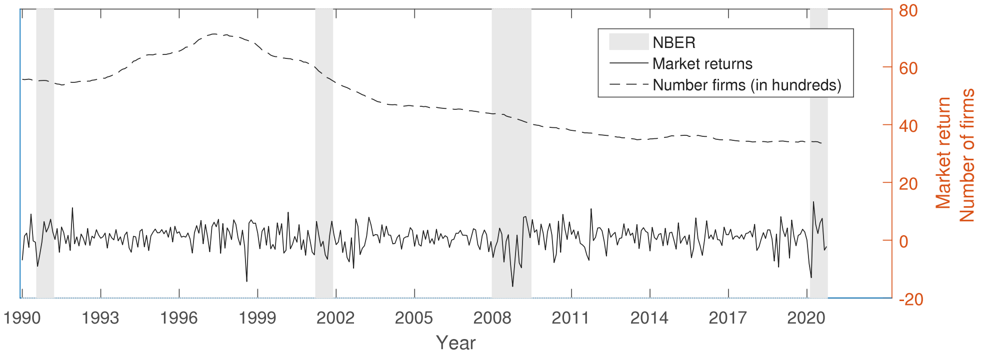
Fig. 1 shows the estimates of the time-invariant and the state-dependent risk–return trade-off[12] when using the time-series of market returns and risk estimated as outlined in Section 2. The level of the risk–return relation in the time-invariant case reveals a value of 2.06 for the whole sample period. The state-dependent estimates uncover a procyclical behavior of the risk–return trade-off with values ranging from −0.48 in periods associated with high volatility states to values of 10.21 in periods related to low volatility states. Notice that low and high volatility periods follow closely (although not perfectly) the NBER recession periods in shaded areas.

The next step in exploring the role of potential non-linearities in the risk–return trade-off is to allow for time-varying parameters. According to the approach developed in Section 2, the expression to compute the time-varying coefficients will be reduced, if the hedge component is ignored, to:

$$\hat{\beta}_t = \hat{\lambda}_t = Var_t(x_1)^{-1} Cov_t(x_1, y) \tag{7}$$

We obtain the estimates for the time-varying risk–return trade-off in Eq. (7) by implementing the three-step procedure described in Section 2. In the first step, we estimate the market return and risk series using Eq. (4). Fig. 2 shows the plots for the estimated market returns and risk series (it also displays the evolution of the VIX series as an alternative proxy for market risk) with NBER

[12] Notice that the evolution of the state-dependent risk–return trade-off corresponds to the weighted average of the parameters λ_{s_t} using the probabilities of being in state s_t.



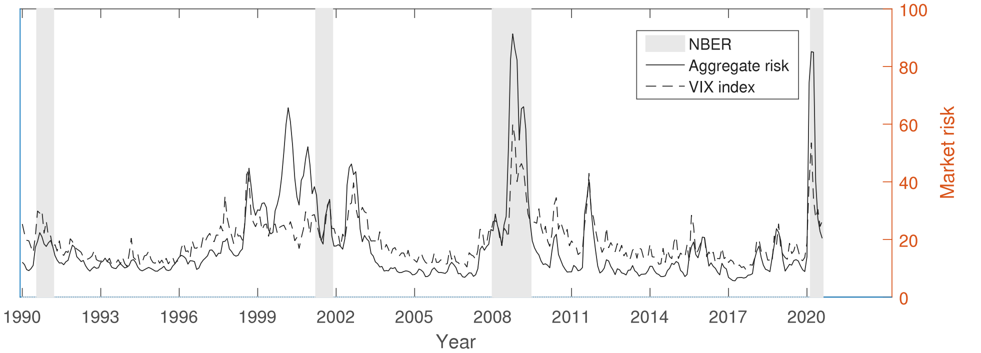


Fig. 2. Market return and risk.
Fig. 2 displays the time series of monthly market returns and risk. Top plot shows in solid line the aggregate market returns computed as a weighted average for the firms in the NYSE, NYSE America and NASDAQ from January 1990 to September 2020 (given availability of market equity data and 60-day variance estimates in the Center for Research in Security Prices). The number of firms (in hundreds) used to compute the aggregate series is plotted in the dashed line. The bottom plot represents the estimates of market risk. The solid line represents estimates using a weighted-average of individual stock variances and covariances using Eq. (4) while the dotted line shows the estimates of the market volatility according to the VIX index from January 1990 to September 2020. Gray areas correspond to periods of recession according to NBER.

recession periods in shaded areas. We observe that the estimated market return and risk series show very different patterns. While the risk series are quite persistent and exhibit strong cyclical patterns, the market return process is much less predictable and its link to business cycles is fuzzier. Also note that we use a large number of firms to construct our proxies for market return and risk, always above the 3,400 firms and using up to 7,000 firms in certain points of the sample period.

In the second step, we estimate a bivariate DCC model using the estimates for market return and risk as the dependent variables. Therefore, the model to estimate is as follows:

$$\begin{bmatrix} y_t = E_t[r^m_{t+1}] \\ x_{1_t} = Var_t[r^m_{t+1}] \end{bmatrix} = \begin{bmatrix} b_{0,r} \\ b_{0,v} \end{bmatrix} + \begin{bmatrix} \varepsilon_{r,t+1} \\ \varepsilon_{v,t+1} \end{bmatrix} \text{ where } \varepsilon_t \sim N(0, H_t) \tag{8}$$

where $E_t[r^m_{t+1}]$ is the proxy for market return, $Var_t[r^m_{t+1}]$ is the proxy for market risk, $b_{0,r}$, $b_{0,v}$ are parameters to estimate and the errors of the model ε_t follow a bivariate normal distribution with zero mean and conditional covariance matrix H_t. The DCC model uses the popular decomposition of the covariance matrix, $H_t = D_t R_t D_t$, where H_t is the conditional covariance matrix, R_t the conditional correlation matrix and $D_t = diag(h^{1/2}_{1,t}, h^{1/2}_{2,t})$ is the diagonal matrix of conditional standard deviations of the returns at time t (computed using a GARCH(1,1) model). The correlation matrix R_t follows an ARMA(1,1) model so it assumes that the correlation of asset returns is not constant and is conditional upon the information set available up to t. In summary, $R_t = diag(Q_t)^{-1/2} Q_t diag(Q_t)^{-1/2}$ where the time-varying conditional covariance matrix, Q_t, is defined as $Q_t = \bar{Q}(1 - a - b) + a \cdot diag(Q_{t-1})^{1/2} + \hat{\varepsilon}_{t-1}\hat{\varepsilon}'_{t-1} diag(Q_{t-1})^{1/2} + b \cdot Q_{t-1}$, the unconditional covariance matrix $\bar{Q}$ are the standardized residuals, and a and b are parameters to be estimated.

Finally, once we obtain the estimated covariance matrices $\widehat{H_t}$, we just use the estimates $\widehat{Var}(x_{1,t})$ and $\widehat{Cov_t}(x_{1,t}, y_t)$ from the bivariate DCC model to solve Eq. (7).

Table 1 contains the main summary statistics of the risk–return trade-off series using aggregate volatility as estimates of market risk ($\lambda_{t,AGGR}$) for the full period (Panel A), expansionary periods (Panel B) and recessionary periods (Panel C) according to the NBER. As can be appreciated, the mean value for the full period for $\lambda_{t,AGGR}$ is equal to 2.74, diminishing to 0.71 when considering recessionary periods (Panel C) and increasing up to 3.01 in expansionary periods (Panel B). Focusing on maximum and minimum

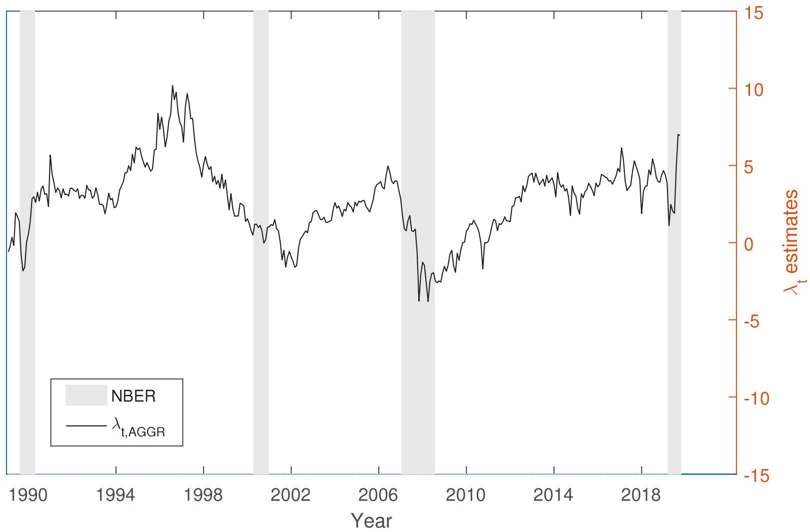


Fig. 3. Time-varying risk–return trade-off.
Fig. 3 depicts the monthly risk–return trade-off time series ($\lambda_{t,AGGR}$) obtained by the univariate framework in Eq. (7) using the aggregate volatility as market risk. The sample period covers from January 1990 to September 2020. Gray areas correspond to periods of recession according to the NBER. The horizontal axis represents the time period on a monthly basis, while the vertical axis represents the level of the risk–return trade-off.

values, the risk–return relation using aggregate volatility reaches its maximum level of 10.19 in July 1997 while the lowest value of −3.81 corresponds to March, 2009.

Fig. 3 displays the temporal evolution of this time-varying risk–return trade-off ($\lambda_{t,AGGR}$) together with the cycle of the economy. The relationship between return and risk stays on the positive side during the 87.53% of the sample period. However, the level of the risk–return measure seems to change due to the economic circumstances in such a way that during recessionary periods the risk–return relation drops, reaching negative values during the financial crisis of 2007–2009. This dynamic is in line with the evidence from the state-dependent estimates and it is similar to previous studies such as Ghysels et al. (2014), Liu (2017), Rossi and Timmermann (2010), Whitelaw (1994, 2000), where investors generally demand a positive (but time-varying) risk–return trade-off for investing in risky assets over time, except in periods under extreme crisis situations in which investors are even willing to accept a negative risk premium, as in the financial crisis period started in 2007. Notice, however, that during the recessionary period associated with the Covid-19 pandemic, the relationship becomes positive. The proposed time-varying risk–return trade-off allows us to understand how the relationship evolves through time and the details regarding its magnitude, extending the limited description on the dynamics of the risk–return trade-off depicted in Fig. 1.

To test whether differences among periods of expansion and recession are significant, we estimate the following equation:

$$\lambda_{t,AGGR} = \alpha_{i,0} + \alpha_{i,1} USREC_t + \epsilon_{i,t} \tag{9}$$

where $\lambda_{t,AGGR}$ is the risk–return coefficient and $USREC_t$ is a dummy variable that takes a value of 1 for the NBER recessionary periods and the value of 0 for expansionary periods. Panel D in Table 1 displays the results of this regression. Although the average risk–return trade-off (a_0) is positive, the difference in the level of the coefficient of risk–return between recessionary and expansionary periods (a_1) is negatively significant. Overall, results point that during recession periods the relationship between return and risk is lower than in the other periods emphasizing further the time-varying nature of the risk–return trade-off and its potential link to business cycles.

4. Drivers of the risk–return trade-off dynamics

4.1. Explanatory factors

Since we are interested in determining the drivers of the risk–return relationship, and given that there is no theoretical model to determine those factors that may affect the risk–return relation (λ_t), its shape or sign, we consider a wide variety of variables used in literature that may explain return, volatility and changes in the investment opportunity set (Chen et al., 1986). Additionally, we also consider sentiment factors (Baker & Wurgler, 2006; Pastor & Veronesi, 2012) that include market sentiment and uncertainty variables. Based on previous empirical papers, Table 2 identifies three groups of key risk factors and the individual variables used as proxies with a detailed description and the data source.

The first group refers to macroeconomic risk factors (Panel A). Particularly, as a proxy of credit risk we use the TED, DEF (default) and $TERM$ spreads (Bali & Engle, 2010; Brunnermeier & Pedersen, 2009; González-Urteaga et al., 2019; Stock

Table 2
Description of the potential risk–return trade-off determinants.

Panel A.Traditional Macroeconomic risk factors	
TED	Spread between 3-month LIBOR based on US dollars and the 3-month Treasury Bill used as indicator of credit risk (series TEDRATE obtained from FRED of St Louis)
DEF	Difference between Moody's yield on Baa corporate bonds and the 10-year government bond yield used as the proxy of credit risk (series BAA and AAA from FRED of St Louis)
TERM	Difference between 10-Year Treasury Constant Maturity and 3-Month Treasury Constant Maturity ($BC3MONTH$) (series $BC10YEAR$ and $BC3MONTH$ from FRED of St Louis)
EINF	Ten-year Expected Inflation estimate expressed as monthly percentage series (obtained from Federal Reserve Bank of Cleveland)
IRP	IRP is a measure of the premium investors require for the possibility that inflation may rise or fall more than they expect (obtained from Federal Reserve Bank of Cleveland)
RRP	Real Risk Premium is a measure of the compensation investors require for holding real (inflation-protected) bonds over some period, given the fact that future short-term rates might be different from what they expect (obtained from Federal Reserve Bank Cleveland)
Δ IPI	Percentage change of US industrial production used as the proxy of economic growth (series G17IP Federal Reserve; Central Bank USA)
CFNAI	Chicago Fed National Activity Index. A zero value for the index indicates that the national economy is expanding at its historical trend rate of growth; negative values indicate below-average growth; and positive values indicate above-average growth (retrieved from FRED St Louis)
CAPE	Cyclically Adjusted Price Earning Ratio used as the proxy of real activity (series from Robert Shiller's website)
USSLIND	Leading Index for the U.S.predicts the six-month growth rate of the state's coincident index (retrieved from FRED St Louis)
NFCI	Chicago Fed National Financial Conditions Index provides comprehensive weekly update on U.S. financial conditions in money markets, debt and equity markets and the traditional and "shadow" banking systems (retrieved from FRED St Louis)
RECPRO	It represents smoothed recession probabilities for the United States (retrieved from FRED St Louis)
ADS	The ADS (Aruoba-Diebold-Scotti) business conditions index is based on the framework developed in Aruoba et al. (2009) to track real business conditions (retrieved from Federal Reserve Bank Philadelphia)
CISS	New Composite Indicator of Systemic Stress (CISS) (retrieved from European Central Bank)
Panel B. Systematic risk factors	
SMB	Size factor of Fama and French (series obtained from authors' website)
HML	Book-to-Market factor of Fama and French (series obtained from authors' website)
CMA	Investment factor of Fama and French (series obtained from authors' website)
RMW	Profitability factor of Fama and French (series obtained from authors' website)
MOM	Momentum factor of Fama and French (series obtained from authors' website)
QMJ	Self-financing excess returns of long/short Quality Minus Junk (obtained from Applied Quantitative Research's website (AQR))
BaB	Self-financing excess returns of long/short Low Beta Minus High Beta assets (obtained from Applied Quantitative Research's website (AQR))
Panel C. Sentiment and uncertainty risk factors	
UMCSENT	University of Michigan: Consumer Sentiment (retrieved from FRED St Louis)
SENT	Sentiment index (see Baker and Wurgler, 2006)
EPU	Economic Policy Uncertainty Index for U.S. is based on newspapers in the U.S. (see Baker et al. (2016), (2016)
MUNC	Measure of Macro Uncertainty for $h = 1$ months ahead (Jurado et al., 2015)
VRP	Monthly Variance risk premium used as a proxy of economic uncertainty (retrieved from Zhou's website).

Table 2 presents summary details of the variables used as potential drivers of the risk–return trade-off categorized into three subgroups: macroeconomic (Panel A), systematic (Panel B) and sentiment and uncertainty (Panel C) factors. Left column shows the abbreviation used in the main text (or in the regressions) to refer to the variables. Right column shows a short definition of the variables.

& Watson, 2003). To account for inflation risk we employ the expected inflation for a one-year horizon and the Inflation Risk Premium, denoted as $EINF$ and IRP respectively (Chen et al., 1986). We use RRP (Real Risk Premium) as a measure of the compensation investors require for holding real (inflation-protected) bonds over some period (Haubrich et al., 2012), and the ΔIPI (monthly growth rate of the Industrial Production Index) and $CFNAI$ (Chicago Fed National Activity Index) as a measure of real economic activity (González-Urteaga et al., 2019; Liu, 2017). The most popular predictor of future equity returns is the aggregate dividend yield (Campbell & Cochrane, 1999). We use $CAPE$ (Cyclically Adjusted Price Earning Ratio) as a proxy of this aggregate dividend yield. Additionally, we have also included indexes from the FRED that represent the U.S. economic trend. Concretely, $USSLIND$ predicts the six-month growth rate of the U.S. coincident index[13] and $NFCI$ provides information regarding U.S. financial conditions. Other variables considered in this macroeconomic subgroup are $RECPRO$ as a measure of the probability of a U.S. recession, the ADS index (Aruoba et al., 2009) that tracks real business conditions, and the $CISS$ (New Composite Indicator of Systemic Stress) index published by the European Central Bank.

The second group (systematic factors) contains aggregate risk factors of the Fama and French (2015) five factor model, except the market risk premium factor. It also includes the QMJ (Quality Minus Junk) factor and the BaB (Betting Against Beta) factor of Asness et al. (2014, 2019) (Panel B).

The third group are proxies of market sentiment and economic, financial or political uncertainty that can influence investors' behavior in their willingness or attitude towards risk (Panel C). The development of behavioral finance underlines the fact that changes in investors' sentiment and the perception of the degree of uncertainty may affect their decisions and, therefore, the performance of either macroeconomic or financial markets indicators. In empirical finance, these variables are rarely used alone as explanatory variables. Instead, they are frequently added to a set of standard explanatory variables to see whether their integration improves or deteriorates forecasting performance (Algaba et al., 2020). Concretely, we use $UMCSENT$ (consumer sentiment index) and $SENT$ (sentiment index) of Baker and Wurgler (2006) as proxies of sentiment; EPU (economic policy uncertainty index for U.S.) as a proxy of policy uncertainty (Baker et al., 2016); the $MUNC$ index as a measure of macroeconomic uncertainty (Jurado et al., 2015); and finally, we use VRP (variance risk premium) as a proxy for economic uncertainty (Bali & Zhou, 2016; Zhou, 2018).

4.2. Empirical results

Once the time-varying risk–return coefficients have been estimated, we are interested in analyzing how market rewards investors at different points in time depending on a set of potential variables categorized into three subgroups defined in the previous section (see Table 2). Unlike previous research that focus on analyzing the drivers of expected returns or volatility, our approach lies in studying the drivers of the risk–return fundamental. To this end, we link the risk–return dynamics calculated using the aggregate specification of market volatility ($\lambda_{j,t}$ for $j = AGGR$), to these potential drivers as follows:

First, we perform a univariate analysis where $\lambda_{j,t}$ is the dependent variable and each factor is used separately as independent variable using Eq. (10),

$$\lambda_{j,t} = \alpha + \phi Y_t + \varepsilon_t \tag{10}$$

where Y_t represents each one of the variables included into the three categories at month t, and ε_t is the residual.

Second, we implement a multivariate analysis as displayed in Eq. (11):

$$\lambda_{j,t} = \gamma_j + \sum_{m=1}^{M} \psi_m Y_{m,t} + \sum_{c=1}^{C} \psi_c Y_{c,t} + \sum_{s=1}^{S} \psi_s Y_{s,t} + \varepsilon_{j,t} \tag{11}$$

where $Y_{m,t}$, $Y_{c,t}$ and $Y_{s,t}$ represent the sub-categories of macroeconomic, systematic and sentiment factors respectively, being M, C and S the number of factors included in each category, and $\varepsilon_{j,t}$ is the residual of the regression. We analyze the cases where the regressions are estimated separately for each group of factors and for all factors together.

As listed in Table 2, there are plenty of variables that can be used in the study. However, since a high degree of cross-correlation between them could cause multicollinearity problems in a multivariate regression analysis, we proceed as follows: first, we regress each factor separately as in Eq. (10); second, we perform a multivariate regression analysis as in Eq. (11) considering the factor that separately has higher R-squared jointly with those factors that are independent of this one and have significant explanatory power. In case the factor with greater explanatory power is highly correlated with the remaining factors, we take the next factor with higher explanatory power that combined with other uncorrelated variables leads to higher explanatory power in a multivariate setting.[14] Also, once the main variable (higher individual R-squared) has been selected, we orthogonalize the other variables that show high correlation with the main one[15] to combine as many variables as possible to get the greater explanatory power without potential distortions.

Table 3 Panel A reports the estimates for the univariate regressions in Eq. (10). The second, fifth and eighth columns show the estimated value of the parameter ϕ in Eq. (10) while the third, sixth and ninth columns show the corresponding explanatory power (R-squared) for macroeconomic, systematic and sentiment factors respectively. Focusing on those factors that are significant and covariate positively with the risk–return trade-off, we differentiate two main results. On one hand, we find factors such as ΔIPI, $CFNAI$, $CAPE$, $USSLIND$, ADS, $UMCSENT$ and $SENT$ that covariate positively. These factors can be related to an improvement of the business conditions, economic growth, overall economic conditions or the sentiment of the investors or

[13] A coincident index is a single summary statistic that tracks the current state of the economy. The index is computed from a number of data series that move systematically with overall economic conditions. An increase in the index indicates an expansion of economic activity while a decline indicates a contraction in economic activity.

[14] To keep this article to a reasonable length, the correlation matrix of the different variables is not attached, but it is available upon request.

[15] That is, we obtain the residuals of each variable on the main one. These orthogonalized variables are identified by placing an R in front of the name of the variable, for instance RDEF refers to the residuals of DEF on the main variable.

Table 3
Univariate and multivariate regression analysis.

Panel A. Univariate regression

Macroeconomic factors			Systematic factors			Sentiment factors		
Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared
TED	−0.06 (0.86)	−0.26%	SMB	**−0.072*** (0.08)	0.55%	UMCSENT	**0.12***** (0.00)	35.60%
DEF	**−2.03******* (0.00)	39.24%	HML	−0.01 (0.80)	−0.25%	SENT	**1.04***** (0.00)	6.54%
TERM	**−0.95******* (0.00)	18.36%	RMW	−0.0.56 (0.96)	0.01%	EPU	**−0.01***** (0.00)	5.78%
EINF	**0.67******* (0.00)	3.78%	CMA	**−0.15**** (0.02)	1.30%	MUNC	**−8.38***** (0.00)	15.98%
IRP	**2.97*** (0.08)	0.57%	MOM	**0.06**** (0.01)	1.37%	VRP	0.01 (0.72)	−0.24%
RRP	2.55 (0.12)	0.40%	QMJ	−5.44 (0.26)	0.01%			
Δ IPI	**0.33**** (0.01)	1.71%	BaB	5.10 (0.12)	0.39%			
CFNAI	**0.40******* (0.00)	3.44%						
CAPE	**0.18******* (0.00)	23.33%						
USSLIND	**1.75******* (0.00)	31.49%						
NFCI	**−2.53******* (0.00)	27.22%						
RECPRO	**−0.04******* (0.00)	16.14%						
ADS	**0.29******* (0.00)	5.13%						
CISS	**−10.49******* (0.00)	34.53%						

Panel B. Multivariate regression

Macroeconomic factors			Sentiment factors			All factor		
Variable	Coefficient (*p*-value)	R-squared (Nobs)	Variable	Coefficient (*p*-value)	R-squared (Nobs)	Variable	Coefficient (*p*-value)	R-squared (Nobs)
CISS	**−9.50******* (0.00)	50%	UMCSENT	**0.12******* (0.00)	39%	CISS	**−9.50******* (0.00)	53%
TERM	**−0.75******* (0.00)		RMUNC	**−4.25******* (0.00)		TERM	**−0.52******* (0.00)	
RDEF	**−1.00******* (0.00)					RDEF	**−1.00******* (0.00)	
						RUMCSENT	**0.05******* (0.00)	
						RRMUNC	−0.94 (0.38)	

Table 3 presents the estimates (p-values in parenthesis) and the R-squares for a battery of regressions of the estimated risk–return trade-off ($\lambda_{t,AGGR}$) using the aggregate volatility as proxy of market risk on a set of factors (described in Table 2). Panel A shows the results for univariate regressions of the estimated risk–return trade-off ($\lambda_{t,AGGR}$) on individual macroeconomic (left columns), systematic (middle columns) and sentiment (right columns) factors. Panel B displays results for multivariate regressions including macroeconomic factors (left columns), sentiment factors (middle columns) and all factors together (right columns). ***, ** and * represent significance at 1%, 5% and 10% significance level.

consumers, in such a way that, the higher their value, the higher the return investors expect to achieve by taking on extra risks. In summary, when these situations occur, the ICAPM model works and the risk–return trade-off is positive. Investors participate ("participation effect") in the market whenever they perceive that the expected return offsets the risk taken.

Special mention deserves other factors related with inflation expectations, such as $EINF$ and IRP, also correlated positively. A possible explanation is related with the positive effect that certain level of inflation can have on consumption or investment opportunities, so that investors are willing to take more risks in exchange of higher returns. Notice, however, that persistent or structural inflation linked to an economic crisis might just lead to opposite responses, that is, to negative risk–return trade-offs.

We find other factors that covary negatively and significantly with the risk–return relation, concretely DEF, $TERM$, $NFCI$, $RECPRO$, $CISS$, EPU and $MUNC$, that indicate credit risk, stress across the markets, politic or macroeconomic uncertainty. Since assets do poorly under turbulent markets or uncertainty, investors' fear accentuates choosing to exit the market. This phenomenon is known as the "flight-to-safety effect" (Baele et al., 2020) and it is just the opposite effect to the "participation effect".

As far as the individual explanatory power of each variable is concerned, it is worth noting that among macroeconomic factors, the factor with higher value is DEF (39.24%), followed by $CISS$ (34.53%), $USSLIND$ (31.49%) and $NFCI$ (27.22%). Notice that sentiment factors contain relevant information as well, concretely $UMCSENT$ (35.60%) and $MUNC$ (15.98%). So, taken together, on an individual basis our results reveal the importance of considering both macroeconomic and sentiment factors.

Table 3 Panel B exhibits the results of the multivariate regression for three sub-categories: macroeconomic factors, sentiment factors and all factors[16]. Within each sub-category, we find in the column variable the factors considered in the multivariate analysis, followed by the estimated coefficient and the explanatory power of each regression.

Regarding the macroeconomic sub-group, given that the univariate analysis reveals that DEF has the greatest explanatory power (39.24%), we should take this variable as reference. However, after analyzing the degree of correlation between DEF and the remaining macroeconomic factors, we conclude that none of these factors are independent of DEF. Moreover, $CISS$ is independent of $TERM$, and considering these two factor in a multivariate setting leads to a higher R-squared, surpassing the explanatory power of considering only the DEF factor. Therefore, it seems reasonable to select the variables $CISS$ and $TERM$ instead of DEF. Also, after orthogonalizing the other variables that show high correlation with $CISS$ and $TERM$, we include in the macroeconomic multivariate regression $RDEF$ (residuals of DEF on the main ones $CISS$ and $TERM$).

We proceed in the same way for the multivariate regression that considers sentiment risk factors. According to the univariate analysis, the $UMCSENT$ factor has greater explanatory power and the remaining factors are highly correlated with each other, but after orthogonalizing our regression also includes $RMUNC$ (residuals of $MUNC$ on $UMCSENT$).

Then, following our approach, the multivariate regression for all sub-categories is implemented for variables $CISS$, $TERM$, $RDEF$ and the orthogonalized sentiment factors ($UMCSENT$ and $RMUNC$) respect to the macroeconomic factors. Looking at the explanatory power for both sub-groups, we find that it is 50% and 39% for macroeconomic and for sentiment factors respectively, increasing up to 53% when both groups of factors are jointly considered.

This suggests that the goodness of fit of the multivariate regression improves by considering a combination of macroeconomic and sentiment factors, although macroeconomic factors (concretely the $CISS$ factor related to a Systemic Stress Indicator and the $TERM$ spread) provide a greater contribution towards explaining the dynamics of the risk–return trade-off.

Finally, Table 4 reports the results obtained when analyzing the predictive ability of each model. Our objective with this analysis is to identify those subgroups of factors that consistently obtain more accurate forecasts of the market risk–return relation. To do this, we estimate one-period out-of-sample forecasts using rolling window regressions from January 2010 to February 2020[17]. Table 4 shows that the best forecast is obtained when macroeconomics factors are considered. This conclusion is consistent regardless of the evaluation model used: MAE (Mean absolute Error) or MSE (Mean Squared Error). Notice, however, that the differences between models (macroeconomic versus sentiment, macroeconomic versus all or sentiment versus all) are negligible, as shown by the Diebold and Mariano (2002) test (see the third row of Table 4). Last row of Table 4 displays the out-of-sample R-squared (Welch & Goyal, 2008) for the different models. This test measures the level of decline in the mean square percentage error (MSPE) between the predictive regression model (macroeconomic, sentiment or all) and the historical average model. When the R-squared is greater than zero, it indicates that the model predicts better than the constant model. Also, the higher the R-squared, the better the forecasts of the models compared to the constant model. In this latter case, the best results are obtained when all types of factors are considered.

Summing up, although macroeconomic factors play an important role as explanatory and predictive variables regarding the risk–return dynamic, sentiment factors are important too.

5. Robustness checks

In this section, we present a battery of robustness tests on the baseline results. We report evidence on the dynamics of the risk–return trade-off, the drivers of the relationship and the predictive ability of the factors using different approaches. Our main purpose is to test whether our main conclusions hold. To this end, Section 5.1 considers the hedge component in the estimate of the risk–return trade-off[18]; Section 5.2 uses the VIX index as a proxy for market risk ($\lambda_{VIX,t-1}$); Section 5.3 reports the results when estimating the multivariate GARCH model using asymmetric GARCH models ($\lambda_{AGGR,t}^{assym-GARCH}$) and asymmetric GARCH models with asymmetric correlations ($\lambda_{AGGR,t}^{assym-DCC-GARCH}$); Section 5.4 uses different window lengths (30 days ($\lambda_{AGGR,t}^{w=30}$) and 90 days ($\lambda_{AGGR,t}^{w=90}$)) to compute the stock daily variance; and finally, Section 5.5 considers two market indexes (the S&P500 index and the Fama and French excess return on the market) instead of individual stocks to compute the aggregate market returns and market risk. All the evidence regarding this section is displayed in Appendix B.

16 Since the systematic factors have negligible explanatory power and provide no relevant information, results are not reported to keep this article to a reasonable length. Nevertheless, they are available upon request.

17 Since $USSLIND$ time series ends at February 2020 our out-of-sample analysis has been adapted accordingly. This series was used to compare the results obtained with different combinations of variables in multivariate regressions.

18 See Appendix A for details regarding the methodology employed

Table 4
Out-of-sample forecast of risk–return relation.

Out-of-sample forecast $\lambda_{t,AGGR}$			
	Macroeconomic	Sentiment	All factors
MAE	0.99	1.00	1.02
MSE	1.43	1.48	1.58
Diebold–Mariano	Macro versus Sent	Macro versus All	Sent versus All
	−0.11 (0.91)	−0.42(0.67)	−0.42(0.67)
Adjusted R-square	50.41% ** (0.01)	50.02% * (0.07)	62.46% *** (0.00)

Table 4 shows the out-sample forecast accuracy of the risk–return trade-off (estimated using the aggregate volatility as a proxy for market risk) when using macroeconomic factors (left column), sentiment factors (middle column) and all factors together (right columns). The out-sample period covers the period from January 2010 until February 2020, for a total of 122 months. The first row displays the Mean Absolute Error between the actual and predicted risk–return trade-off while the second row displays the corresponding Mean Squared Error. The Diebold and Mariano test indicates if the forecast accuracy (MSE) of two competing models are the same (p-value in parentheses). Finally, last row shows the adjusted R-squares for each model computed as in Welch and Goyal (2008), using the vector of errors from the constant model and the vector of errors from the suggested model. Critical values are obtained using the F-statistic (p-value in parentheses) with ***, ** and * representing significance at 1%, 5% and 10% level.

5.1. Multivariate model with the hedging component

This section shows the estimates obtained when taking into account the hedging component in Eq. (2). A brief explanation of the state variables considered to capture shifts in investment opportunities (DY, $TERM$, DEF and $BOND$) and the methodology employed is displayed in Appendix A.

Table B.1 contains the main summary statistics of the risk–return trade-off series considering the selected state variables as hedge component ($\lambda_{t,DY}$, $\lambda_{t,TERM}$, $\lambda_{t,DEF}$, $\lambda_{t,BOND}$) and using the aggregate portfolio as market risk. Results for the full period are displayed in Panel A, while expansionary periods and recessionary periods according to the NBER are presented in Panels B and C, respectively. As can be appreciated, the mean value for the full period lies within the narrow range of 3.55 to 4.63, increasing up to a range of 4.01 to 5.35 during expansionary periods and decreasing to average values from −0.98 to −0.05 during recessionary periods. Under this framework, the maximum values for the risk–return trade-off occur in July 1997 while the minimum values are exclusive for October 2008. Panel D displays the estimates of regressing the different time-series of estimates λ_t on a dummy variable that takes a value of 1 for the NBER recessionary periods and the value of 0 for expansionary periods. We obtain consistent results where in all cases the average risk–return coefficient (a_0) is significantly positive while the impact of recessions on the level of risk–return (a_1) shows a significant negative value.

The temporal evolution of the time-varying risk–return trade-off obtained from the multivariate framework together with the cycle of the economy can be appreciated in Fig. B.1. Similar to the evolution reported for the univariate case (without the hedging component), the relationship between return and risk shows positive values for the 83.5%–86% of the sample period, depending on the specification. We also observe a decrease in the level of risk–return trade-off strongly associated with recession periods, such as the financial crisis 2007–2009, while in the recessionary period associated with the COVID-19 pandemic, the relationship between return and risk shows a very strong positive relationship in line with previous results (see Fig. 3).

All these results are in line with the evidence reported in the previous section and suggest that the estimates of the time-varying risk–return relation λ_t is robust with respect to the inclusion of the hedge component. This evidence is consistent with studies such as Bali (2008), Bali and Engle (2010), Jiang and Lee (2014), Pastor et al. (2008) that also find that the hedge component does not affect substantially the intertemporal risk–return relation.

The multivariate approach in Eq. (3) also allows us to estimate the time-varying coefficients for the hedge-related risk γ_t. Maio and Santa-Clara (2012) state that the interpretation of the sign of the hedge-related risk coefficients γ_t depends on the ability of the state variable to forecast expected return or volatility. On one hand, when considering a state variable that predicts future expected returns, the risk price for intertemporal risk (γ_t) should be positive and the asset should earn a risk premium. The intuition is that the asset does not provide a hedge against future negative shocks in the returns of aggregate wealth (reinvestment risk), as it offers low returns when future aggregate returns are also expected to be low. On the other hand, when considering a state variable that forecasts the future variance of market return, the risk price for intertemporal risk (γ_t) “should be” negative and an asset that covaries positively with changes in the state variable (and is thus positively correlated with the future market volatility) earns a negative premium. The economic implication of this last condition is that such an asset provides a hedge for reinvestment risk, as it pays high returns when future aggregate volatility is also high. Thus, such an asset should earn a lower risk premium[19].

Table B.2 displays the main summary statistics of the time-varying hedge-related risk coefficients for the different alternatives ($\gamma_{t,DY}$, $\gamma_{t,TERM}$, $\gamma_{t,DEF}$, $\gamma_{t,BOND}$) and using the aggregate portfolio as market risk. Results for the full period are displayed in Panel A. The mean values are in all cases positive ranging from a value of 0.06 for the case $TERM$ spread to a value of 0.32 for the case

[19] If we assume instead that the state variable forecasts negative expected market returns (or negative market volatility), then the intertemporal risk price must be negative (positive), and the arguments are just the opposite.

of DEF. The magnitude of the hedge-related risk is relatively important in certain periods with maximum values up to 5.28 for the $TERM$ spread or 2.29 for the long-term $BOND$. For expansionary periods, this positive relationship in the hedge-related risk is increased in all cases with average values spanning from 0.19 to 0.38. Interestingly, the average values during recession periods show negative values ranging from −1.03 in the case of the $TERM$ spread to 0.04 in the case of the long-term $BOND$, suggesting a strong negative relationship at certain points with values of −6.21 or −1.59 in the term spread ($TERM$) and long-term $BOND$ respectively. Panel D displays the estimates of regressing the time-series of hedge-related risk coefficients γ_t on a dummy variable that takes a value of 1 for the NBER recessionary periods and a value of 0 for expansionary periods. We obtain consistent results in all specifications where the average hedge risk (a_0) is significantly positive while the impact of recessions on the price of this hedge-related risk (a_1) shows a significant negative impact. This last result might show the effect that our state variables tend to co-move positively with market variance during recession periods.

Fig. B.2 displays the temporal evolution of the time-varying risk coefficients for the hedge component together with the cycle of the economy. The temporal variation of the coefficients depends strongly on the proxy selected as the hedge component. We can observe certain sub-periods (e.g.1991–1994; 2010–2015) where all series align relatively close, but generally the values for the coefficients diverge in most of the sample period. The relationship of these time-varying hedge-risk (γ_t) coefficients with business cycles is less obvious than in the case of the λ_t estimates. However, there is a dominant presence of positive values, showing values above zero for the 72.33%, 79.67%, 75.61% and 79.94% of the sample period in the $\gamma_{t,DY}$, $\gamma_{t,TERM}$, $\gamma_{t,DEF}$ and $\gamma_{t,BOND}$ specification. All the state variables that we selected in our multivariate approach are shown in the predictability literature to provide good forecasts of expected returns so it is not surprising that the coefficients for the hedge-related risk show positive values during most of the sample period, highlighting the premium demanded to the market portfolio for the reinvestment risk (Maio & Santa-Clara, 2012).

Tables B.3–B.6 summarize the results considering as state variables DY ($\lambda_{t,DY}$), $TERM$ ($\lambda_{t,TERM}$), DEF ($\lambda_{t,DEF}$) and $BOND$ ($\lambda_{t,BOND}$), respectively, to analyze the individual explanatory power of each factor.

When comparing these results with those showed in Table 3, we observe that the main conclusions hold. With regard to the significance of the regression parameters, the results do not change except for the TED, RMW, QMJ and BaB factors, which are now significant, and the SMB factor that becomes not significant.

If we focus on the individual explanatory power of each factor, there are marginal changes depending on the state variable considered, but note that $CISS$ is the factor with greater explanatory power (with values between 43.06% and 44.92%). In the case of systematic factors, they also have very low explanatory power compared to the results obtained with macroeconomic or sentiment factors. Regarding sentiment factors, $UMCSENT$ and $MUNC$ remain as the factors with higher R-squared.

In the multivariate regressions (panel B of each table), results hardly differ from those obtained with the model that neglects the hedging component, reinforcing previous results. Focusing on macroeconomic factors, their explanatory power ranges between 47% ($\lambda_{t,TERM}$) and 51% ($\lambda_{t,BOND}$ and $\lambda_{t,DY}$). Similar conclusions are reached for sentiment factors with values ranging between 21% ($\lambda_{t,DEF}$) and 31% ($\lambda_{t,BOND}$). The combination of macroeconomic and sentiment factors does not increase the explanatory power relative to macroeconomic factors alone.

Finally, Table B.7 shows the results obtained when analyzing the predictive ability of each model. According to MSE and MAE, better predictions are generally achieved with the regressions models that incorporate sentiment factors. However, as found before, the differences between the different models regarding predictive accuracy are not significant (see Diebold and Mariano Test). The results obtained for the out-sample R-squares (Welch & Goyal, 2008) show improvements for all subsets of risk factors with respect to the model without them. Therefore, it seems that both macroeconomic and sentiment factors have a key role in the predictive ability.

Summing up, all results are consistent with the main conclusions reached in the analysis without the hedging component. The drivers that move this relationship and their predictive ability are robust to the consideration of the hedging component.

5.2. Alternative measures of market risk: the VIX index

Comparing the results for different volatility proxies involves keeping in mind the idiosyncrasy of each proxy. The VIX index[20] measures the expected volatility over the following 30 days. It is an ex-ante measure of the market volatility, while aggregate volatility is a contemporary measure, rather than ex-ante. So, it means that we have to consider the value of VIX lagged one period to compare it to the baseline results. In some way, we are analyzing whether VIX is a good contemporary volatility estimate at one month.

First row in Panels A, B and C of Table B.8 shows the main summary statistics for the risk–return trade-off estimates for the whole sample, expansionary and recessionary periods respectively when considering the VIX index as proxy for market volatility. The value for the whole period is 2.95, while for expansion periods this value increases up to 3.06 and decreases to 2.05 for recession periods. The maximum and minimum values are 12.14 (May, 1990) and −1.37 (February, 2009). That is, there is a procyclical behavior of the risk–return trade-off. These results are in line with those obtained when considering aggregate volatility as proxy of market risk (see Table 1), but the values obtained here are not so extreme. Also, looking at Fig. B.3, we can appreciate that the dynamics of

[20] The VIX index, retrieved from the Chicago Board Options Exchange (CBOE), was first published in January 1990, so our sample period spans from January 1990 to September 2020. It is set by investors and expresses their consensus view about expected future stock market volatility and is based on S&P500 index option prices (Whaley, 2000).

this risk return trade-off together with the NBER cycle of economy follow a similar pattern to that found in Fig. 3. Finally, Panel D in Table B.8 corroborates that the differences between business cycles are significant.

Regarding the estimates for the univariate regressions in Eq. (10), conclusions are in line with the previous results (see Table B.9 and compare with Table 3). The macroeconomic and sentiment factors that covariate positively/negatively coincide with those identified when using aggregate volatility but the factors TED (negatively correlated) and RRP (positively correlated) are now added. Slightly differences arise regarding the systematic factors. Concretely, the SMB and CMA factors become not significant. Also, some results relative to the individual explanatory power are noteworthy. First, among the factors that covariate positively, factors such as $EINF$, IRP, ΔIPI, $CFNAI$, $USSLIND$ and ADS have now greater explanatory power, and others such as $CAPE$, $UMCSENT$ and $SENT$ have lower explanatory power. Second, as far as factors that covariate negatively is concerned, the explanatory power of all variables increases except for the $TERM$ factor. It is noteworthy the increase in $CISS$ (from 34.53% to 57.10%) and $MUNC$ (from 15.98% to 36.82%), which become now the factors with greater explanatory power within the macroeconomic and sentiment factors respectively. Third, the explanatory power of the systematic factors remains in marginal values and some of them have become not significant as commented before.

Some aspects regarding the multivariate analysis are important to note. The goodness-of-fit measured by the R-squared slightly improves when combining macroeconomic and sentiment factors, see in Table B.9 panel B that the difference is scant (66% versus 64%) . Note, however, that there is a significant increase in the explanatory power obtained when using macroeconomic factors (64% according Table B.9) compared with that obtained with aggregate volatility $\lambda_{t,AGGR}$ (50% according Table 3). The most likely explanation of these differences, in the absence of more conclusive data, is that the VIX considers additional information that the option markets are pricing in, which is not included in the other proxy (aggregate volatility in Section 4). In particular, implied volatility (VIX) takes into account forthcoming known events such as earnings announcements, macroeconomic indicators or political events among others. Since our proxies for volatility are related but not identical, results in the univariate/multivariate analysis, albeit with slightly differences, show a similar picture.

Table B.16 (Panel A) displays the results relative to the predictive ability using VIX. It seems that sentiment factors have better forecasting accuracy (0.77 MAE and 0.94 MSE) than macroeconomic factors (0.92 MAE and 1.29 MSE) or macroeconomic factors combined with investor's sentiment (0.84 MAE and 1.04 MSE). Notwithstanding, according to the Diebold and Mariano test, differences are not significant as found above. On the other hand, the out-sample R-squares show that all factors outperform the forecasts of the constant model, although SENT factors at higher confidence levels.[21]

Overall, despite the fact that some differences arise when using VIX as a proxy, results are consistent with those found for aggregate volatility. That is, drivers move in the same direction and both macroeconomic and sentiment factors are important as explanatory and predictive factors.

5.3. Asymmetric GARCH models

The study of the asymmetric behavior of volatility and correlation between financial assets in response to positive or negative innovations has been an important field of research in financial economics (Engle & Ng, 1993). This section analyzes the robustness of the results obtained in the estimation of the risk–return trade-off and its drivers when this feature is considered into the multivariate GARCH estimates (according to Eq. (8)).

Table B.8 shows the main summary statistics of the risk–return trade-off using multivariate GARCH models considering asymmetries only in individual variances ($\lambda_{AGGR,t}^{assym-GARCH}$) and when asymmetry is considered both in the individual variances and correlations ($\lambda_{AGGR,t}^{assym-DCC-GARCH}$). The results for the whole sample, expansionary and recessionary periods (see second and third rows in panels A, B and C), are in line with those obtained in previous estimates, that is, there exists a procyclical behavior of the risk–return trade-off and values move within a similar range. See also Fig. B.3 in which estimates are depicted and clearly follow a similar pattern.

Regarding the individual significance of the different risk factors and their sign, there are no important differences (see panel A in Table B.10 and B.11). Only emphasize that the R-squared for $MUNC$ and $UMCSENT$ are similar, and considering $MUNC$ in the multivariate analysis lead us to a higher R-squared than considering $UMCSENT$.[22] In line with previous results, the goodness-of-fit measured by the R-squared hardly improves when considering all factors. Macroeconomic factors seem to have greater explanatory power than sentiment factors, but both factors are relevant.

Finally, Table B.16 (Panel B for $\lambda_{AGGR,t}^{assym-GARCH}$ and Panel C for $\lambda_{AGGR,t}^{assym-DCC-GARCH}$) shows the results for the predictive ability of these models. See that results are in line with those obtained with the aggregate specification or the VIX index.

Summing up, neither the dynamics of the risk–return trade-off nor the explanatory power of the models change significantly when considering the asymmetric behavior of volatility and/or correlations.

[21] This result is also observed in other robustness tests.

[22] Results obtained with $UMCSENT$ are not included in the paper (specifically in multivariate regression) but they are available upon request.

5.4. Alternative window lengths

This section analyzes the robustness of the results when considering different window lengths in the estimation of aggregate volatility. Concretely, we extend the previous analysis with a window length of 60 days (see Section 4.2) to 30 and 90 days. See in Table B.8 (rows four and five in panels A, B and C) and Fig. B.3 that previous results generally hold.

Specifically, rows 4 (30 days) and 5 (90 days) in panels A, B and C show the results for the full sample, expansionary and recessionary periods, respectively. Although there are differences in the value of the estimated coefficients, what is important is that again we obtain a procyclical behavior of the risk–return trade-off, and that the differences between the periods of recession and expansion are in all cases significant. Notice also in Fig. B.3 that these estimates move in a similar way than the other estimates of the risk–return relation.

Also, as appreciated in Table B.12 and B.13, the explanatory power of the risk factors for the univariate analysis are in line with previous results. The risk factors in the multivariate regressions remain the same regardless of the window length considered and are consistent with previous results. As far as the out-of-sample forecast accuracy is concerned (see Panel D and E in Table B.16), we find that the combination of sentiment and macroeconomic factors stands out although the differences are not significant according to the Diebold and Mariano test as found in previous sections. Finally, we get similar evidence to previous sections for the out-sample R-squares.

5.5. Evidence using market indexes: S&P500 and Fama and French

This section shows the results when instead of computing the market return or volatility from individual stocks we use market indexes. In this case, we have considered two well-known indexes: the S&P500 index and the Fama and French (F&F) market factor. In both cases, we use the monthly return of the index as a proxy for the market return. We retrieve the market risk at the end of every month by computing the sample variance of the last 60 day observations (similar to the methodology outlined in the baseline results).

The last two rows of each panel in Table B.8 show the descriptive statistics of the risk–return trade-off series using these two market indexes ($\lambda_{SP500,t}$, $\lambda_{FF,t}$). Although there are minor differences in the values obtained compared to previous sections, it should be noted that the behavior of the trade-off remains procyclical. Thus, the mean value for the SP&500 (F&F) is 2.97 (1.71), during expansionary periods this value increases up to 3.36 (1.99) and for recessions it decreases to −0.16 (−0.59). The difference in the level of the coefficient of risk–return between recessionary and expansionary periods is significant as can be seen from the results in panel D for both indexes.

Regarding the sign of the correlation of each factor, it coincides with that obtained in Section 4. The explanatory power, however, differs. Now, for the S&P 500, within the macroeconomic and sentiment factors $CAPE$ (67%) and $UMCSENT$ (57%) stand out as the factors with higher explanatory power. Notice also that systematic factors have hardly explanatory power as encountered before. Considering the presence of high correlations between variables, the multivariate regression finally includes $CAPE$, $NFCI$ and the $EINF$ residuals with an R-squared of 80%. Notice that the explanatory power of the multivariate model for macroeconomic factors and all factors is similar (see Table B.14). This result is consistent with that obtained previously.

If we focus on the F&F index, the results are very similar to those mentioned above. The difference in this case is that the two macroeconomic factors with greater explanatory capacity are $CAPE$ (47%) and $USSLIND$ (48%), and although $UMCSENT$ remains as the sentiment factor with higher R-squared, it drops from 57% (S&P500) to 35%. The systematic factors are also not relevant. This implies that now the factors included in the multivariate regression analysis change, but notice that the explanatory power hardly varies (77%). These results once again show that both macroeconomic and sentiment factors are important in explaining the dynamics of the risk–return trade-off, but macroeconomic factors seem to be more relevant as explanatory factors[23].

Concerning the out-of-sample forecast, it should be noted that in the case of F&F, macroeconomic factors seem to make a difference in forecast (the differences are significant relative to sentiment factors), but the remaining results corroborate the conclusions reached in the rest of the robustness analyses.

6. Conclusions

This paper investigates the time-varying dynamics of the risk–return trade-off and which factors better explain or predict this relationship. We propose a novel methodology where we use conditional slopes between the first two moments of the market portfolio to overcome the limitations of the time-invariant or state-dependent approaches. Compared to previous literature, the main advantage of our approach is that we are able to generate the time-variation in the relationship between return and risk by using exclusively data on market returns and risk instead of ‘ad-hoc’ state variables. Our estimates show that the risk–return trade-off is time-varying and procyclical, reaching even negative values in crisis periods.

We also analyze the explanatory and predictive power of three group of factors (macroeconomic, systematic and sentiment) on the estimates of the time-varying risk–return trade-off. Results show that macroeconomic (particularly $CISS$, $TERM$, DEF, $USSLIND$, $CAPE$ and $NFCI$) and sentiment factors ($UMCSENT$ and $MUNC$) have higher explanatory power, while the

[23] Since the datasets used to compute the risk–return trade-off are similar but not identical, results in the univariate/multivariate analysis differ but provide a consistent picture.

impact of systematic factors is negligible. The combined explanatory power of these factors reaches 53% and it is always above 47% for all specifications of the risk–return trade-off. Finally, regarding to the predictive ability of the factors, the results confirm the fundamental role played for both macroeconomic and sentiment factors.

Overall, these results are robust for several alternatives to determine the risk–return trade-off within our novel framework. We can extrapolate our main conclusions to the consideration of hedging components, different MV-GARCH models or window lengths and for several proxies of market returns and risk.

CRediT authorship contribution statement

Nuria Alemany: Conceptualization, Investigation, Resources, Writing – original draft, Writing – review & editing, Visualization. **Vicent Aragó:** Conceptualization, Investigation, Resources, Writing – original draft, Writing – review & editing, Software, supervision, Validation. **Enrique Salvador:** Conceptualization, Investigation, Resources, Methodology, Software, Data curation, Writing – original draft, Writing – review & editing.

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Appendix A. Methodology and data in a multivariate framework with hedge component

To estimate the complete model in Eq. (2), besides the estimates of market return and risk, we also need to choose the state variables $Cov_t[r^m_{t+1}, s_{t+1}] = x_{2,t}$ in the ICAPM relation. Since theory provides relatively little guidance in this regard, we base our choice of factors on previous literature. The four state variables we select as the variable $x_{2,t}$ in Eq. (2) are the dividend yield (DY), the term spread ($TERM$) computed as the ten-year over the three-month yield spread in the Treasury market, the default spread (DEF) computed as the yield spread between Baa- and Aaa-rated corporate bonds and the long-term US Treasury bond ($BOND$) return (Guo et al., 2013; Scruggs, 1998; Smith & Whitelaw, 2009). The use of these financial variables has a long history in the literature, including the prediction of future equity returns and volatilities. Data on these variables comes from the Federal Reserve Economic Data (FRED).

Fig. A.1. Hedge component variables.
Fig. A.1 shows the time series of monthly returns for the four state variables used as hedge component in Eq. (2). The top left plot represents the dividend yield, the top right plot shows the TERM spread, the bottom left plot is the DEF spread while the bottom right plot displays the US Long-Term Bond yield. The sample period covers from January 1990 to September 2020. Gray areas correspond to periods of recession according to NBER. The correlation in each plot is computed for each variable and the market risk using the aggregate portfolio.

Fig. A.1 plots the evolution of the variables chosen as hedge component (DEF, TERM, DY and BOND), with the shaded areas denoting business recessions dated by the NBER. The relationship between business cycles and dividend yield (DY) or default spread

(DEF) is clear with large increases in both series during recession periods. However, the other state variables (TERM and BOND) follow a pattern poorly correlated to business cycles. Also, the relationship between the state variables and our estimates of market risk is weak, except for the case of the default spread which shows a strong positive correlation. These patterns confirm that the selected state variables can serve as good proxies of the hedge component in the multivariate model of Eq. (2).

To obtain the parameters for this multivariate model, we stick to the same three-step procedure followed previously. However, different to the univariate case, we also include the hedge component when estimating the multivariate GARCH model as follows:

$$\begin{bmatrix} y_t = E_t[r^m_{t+1}] \\ x_{1_t} = Var_t[r^m_{t+1}] \\ x_{2_t} = Cov_t[r^m_{t+1}, s_{t+1}] \end{bmatrix} = \begin{bmatrix} b_{0,r} \\ b_{0,v} \\ b_{0,s} \end{bmatrix} + \begin{bmatrix} \varepsilon_{r,t+1} \\ \varepsilon_{v,t+1} \\ \varepsilon_{s,t+1} \end{bmatrix} \text{where } \varepsilon_t \sim N(0, H_t) \tag{12}$$

where $E_t[r^m_{t+1}]$ is the proxy for market return, $Var_t[r^m_{t+1}]$ is the proxy for market risk, $Cov_t[r^m_{t+1}, s_{t+1}]$ is the proxy for the hedge component, $b_{0,r}$, $b_{0,v}$, $b_{0,s}$ are parameters to estimate and the errors of the model ε_t follow a multivariate normal distribution with zero mean and conditional covariance matrix H_t, which is specified as a DCC model.

Under this multivariate framework, once we obtain the estimated covariance matrices $\widehat{H}_t$, we use the estimates $\widehat{Var}(x_{1,t})$, $\widehat{Var}(x_{2,t})$, $\widehat{Cov_t}(x_{1,t}, x_{2,t})$, $\widehat{Cov_t}(x_{1,t}, y_t)$ and $\widehat{Cov_t}(x_{2,t}, y_t)$ from the multivariate DCC model to solve Eq. (3).

Appendix B. Robustness section results

This robustness section exhibits the Tables and Figures of the different approaches implemented. Concretely, we show the results under a multivariate framework with hedge component, using the VIX index, different GARCH specifications, different window lengths and market indexes.

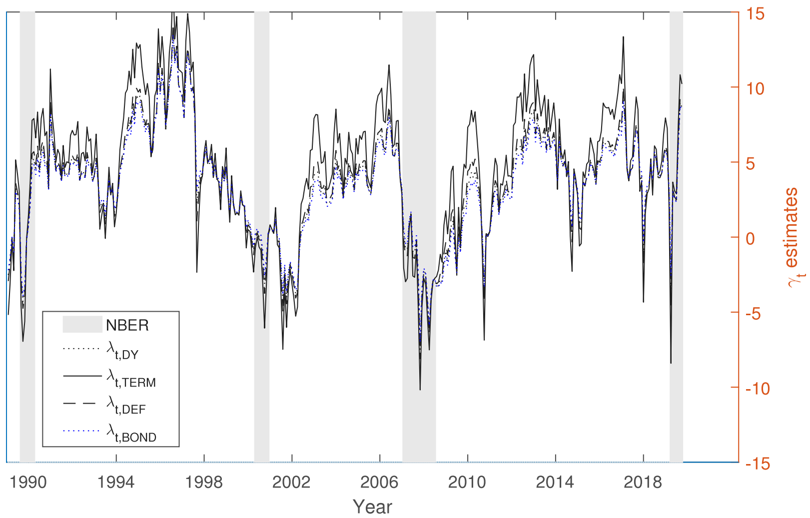

Fig. B.1. Time-varying risk–return trade-off from multivariate model.
Fig. B.1 depicts the time series of monthly risk–return trade-off under the multivariate framework in Eq. (3) for different proxies of the hedging component (DY, TERM, DEF and BOND) and using the aggregate volatility as the market risk. The solid line represents the estimates of the time-varying risk–return trade-off using TERM as the hedging component; the dotted line uses DEF as the hedging component; the dashed line uses DY as the hedging component while the blue dotted line uses BOND as the hedging component. The sample period covers from January 1990 to September 2020. Gray areas correspond to periods of recession according to the NBER. The horizontal axis represents the time period on a monthly basis, while the vertical axis represents the level of the risk–return trade-off.

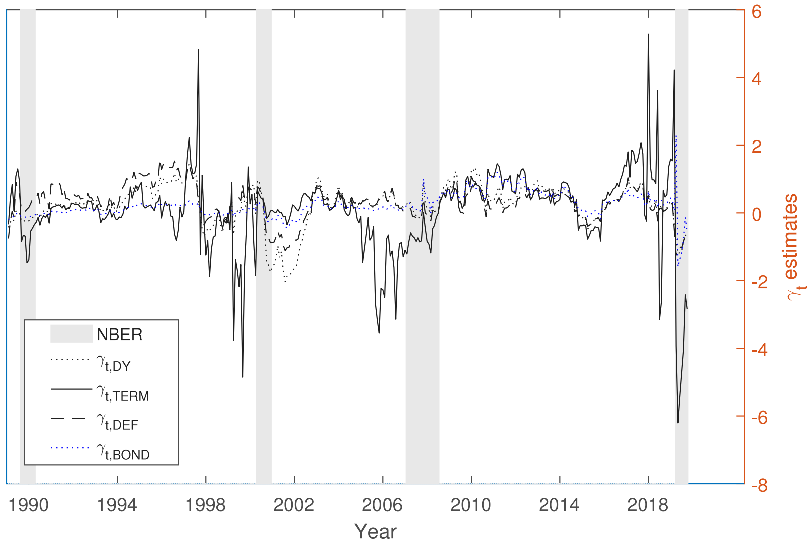

Fig. B.2. Time-varying hedging parameter from multivariate model.
Fig. B.2 depicts the time series of monthly hedging parameters under the multivariate framework of Eq. (3) for different proxies of the hedging component (DY, TERM, DEF and BOND) and using the aggregate volatility as the market risk. The solid line represents the estimates of the hedging parameter using TERM; the dotted line uses DEF as the hedging component; the dashed line uses DY as the hedging component while the blue dotted line uses BOND as the hedging component. The sample period covers from January 1990 to September 2020. Gray areas correspond to periods of recession according to the NBER. The horizontal axis represents the time period on a monthly basis, while the vertical axis represents the estimates for the hedging parameter.

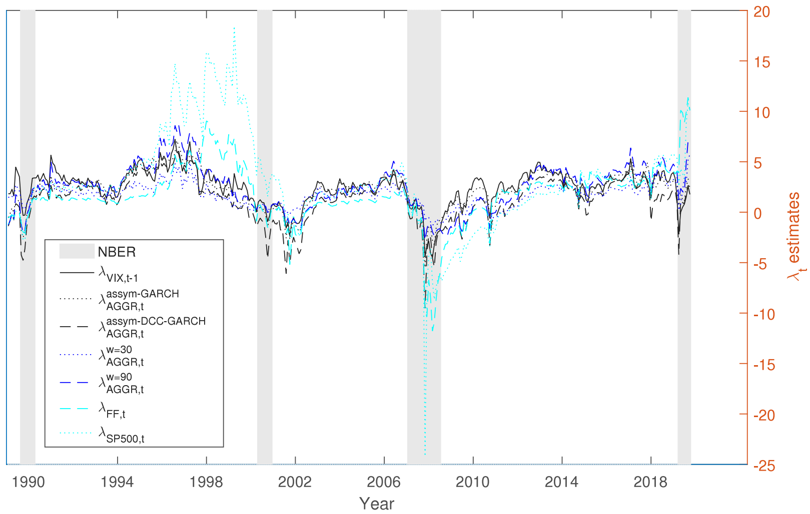

Fig. B.3. Time-varying risk–return trade-off for alternative specifications.
Fig. B.3 depicts the monthly risk–return trade-off time series obtained by the univariate framework in Eq. (7) for alternative specifications to the ones displayed in the baseline results. The solid line represents the estimates of the time-varying risk–return trade-off using the VIX index ($\lambda_{VIX,t-1}$) as market risk. The dotted line and dashed line represents the estimates of the risk–return trade-off using the aggregate portfolios but using asymmetric GARCH models ($\lambda^{assym-GARCH}_{AGGR,t}$) and asymmetric DCC models ($\lambda^{assym-DCC-GARCH}_{AGGR,t}$) as the parameterization in the multivariate GARCH model, respectively. The blue dotted line and the blue dashed line displays the evolution of the risk–return trade-off when using a window of 30 days ($\lambda^{w=30}_{AGGR,t}$) or 90 days ($\lambda^{w=90}_{AGGR,t}$) to compute the daily stock variance in the aggregate portfolio. Finally, the cyan dotted line represents the risk–return trade-off series for the SP500 index ($\lambda_{SP500,t}$) while the cyan dashed line the series for the F&F market factor ($\lambda_{FF,t}$). The sample period covers from January 1990 to September 2020. Gray areas correspond to periods of recession according to the NBER. The horizontal axis represents the time period on a monthly basis, while the vertical axis represents the level of the risk–return trade-off.

Table B.1
Summary statistics for multivariate estimates of the risk–return trade-off.

Series	Mean	Std	Max	Min
Panel A. FULL SAMPLE				
$\lambda_{t,DY}$	3.64***	3.49	13.83	−7.49
$\lambda_{t,TERM}$	4.63***	4.69	16.94	−10.18
$\lambda_{t,DEF}$	3.73***	3.64	14.20	−8.11
$\lambda_{t,BOND}$	3.55***	3.37	13.50	−6.99
Panel B. EXPANSIONARY PERIOD				
Series	Mean	Std	Max	Min
$\lambda_{t,DY}$	5.35***	4.18	16.94	−7.48
$\lambda_{t,TERM}$	4.27***	3.28	14.20	−5.01
$\lambda_{t,DEF}$	4.13***	3.17	13.84	−4.36
$\lambda_{t,BOND}$	4.01***	3.08	13.50	−3.95
Panel C. RECESSIONARY PERIOD				
Series	Mean	Std	Max	Min
$\lambda_{t,DY}$	−0.98	4.73	10.81	−10.18
$\lambda_{t,TERM}$	−0.40	3.73	9.25	−8.12
$\lambda_{t,DEF}$	−0.20	3.52	8.97	−7.49
$\lambda_{t,BOND}$	−0.05	3.36	8.73	−6.99
Panel D. SIGNIFICANCE TEST for regression $\lambda_{t,i} = \alpha_{i,0} + \alpha_{i,1} USREC_t + \epsilon_{i,t}$				
	$\lambda_{t,DY}$	$\lambda_{t,TERM}$	$\lambda_{t,DEF}$	$\lambda_{t,BOND}$
$\alpha_{i,0}$	5.35***	4.27***	4.13***	4.01***
	(0.23)	(0.18)	(0.18)	(0.17)
$\alpha_{i,1}$	−6.34***	−4.67***	−4.33***	−4.05***
	(0.70)	(0.55)	(0.53)	(0.52)

Table B.1 presents the main summary statistics of the risk–return trade-off series in the multivariate models using a battery of alternatives to model the hedging component in Eq. (3). $\lambda_{t,DY}$ represents the risk–return trade-off estimates when using Dividend Yield as the hedging component; $\lambda_{t,TERM}$ represents the risk–return trade-off estimates when using the TERM spread as the hedging component; $\lambda_{t,DEF}$ represents the risk–return trade-off estimates when using the Default spread as the hedging component and $\lambda_{t,BOND}$ represents the risk–return trade-off estimates when using the Long-term US Bond as the hedging component. Results for the full period are reported in Panel A, while results for expansionary periods and recessionary periods according to NBER are reported in panel B and C, respectively. We use the one-sample t-test to check the significance for the mean of the series. The sample period spans from January 1990 to September 2020. Panel D shows the estimates (standard errors in parenthesis) for regressions of the risk–return trade-off series $\lambda_{t,DY}$; $\lambda_{t,TERM}$; $\lambda_{t,DEF}$ and $\lambda_{t,BOND}$ on a dummy that takes the value of 1 for a recession period according to NBER and 0 otherwise. ***, ** and * represent significance at 1%, 5% and 10% significance level.

Table B.2
Summary statistics for multivariate estimates of the hedging component.

Panel A. FULL SAMPLE				
Series	Mean	Std	Max	Min
$\gamma_{t,DY}$	0.24***	0.61	1.45	−2.03
$\gamma_{t,TERM}$	0.06	1.20	5.28	−6.21
$\gamma_{t,DEF}$	0.32***	0.51	1.60	−2.03
$\gamma_{t,BOND}$	0.23***	0.36	2.29	−1.59
Panel B. EXPANSIONARY PERIOD				
Series	Mean	Std	Max	Min
$\gamma_{t,DY}$	0.29***	0.59	1.45	−2.03
$\gamma_{t,TERM}$	0.19***	1.02	5.28	−4.85
$\gamma_{t,DEF}$	0.38***	0.49	1.60	−1.11
$\gamma_{t,BOND}$	0.26***	0.31	1.20	−0.45
Panel C. RECESSIONARY PERIOD				
Series	Mean	Std	Max	Min
$\gamma_{t,DY}$	−0.16*	0.60	0.97	−1.54
$\gamma_{t,TERM}$	−1.03***	1.79	4.22	−6.21
$\gamma_{t,DEF}$	−0.11	0.49	0.77	−1.25
$\gamma_{t,BOND}$	0.04	0.59	2.28	−1.59
Panel D. SIGNIFICANCE TEST for regression $\gamma_{t,i} = \alpha_{i,0} + \alpha_{i,1} USREC_t + \epsilon_{i,t}$				
	$\gamma_{t,DY}$	$\gamma_{t,TERM}$	$\gamma_{t,DEF}$	$\gamma_{t,BOND}$
$\alpha_{i,0}$	0.29***	0.19***	0.38***	0.26***
	(0.03)	(0.06)	(0.03)	(0.02)
$\alpha_{i,1}$	−0.45***	−1.22***	−0.49***	−0.22***
	(0.09)	(0.18)	(0.08)	(0.06)

Table B.2 presents the main summary statistics of the time-varying hedging components coefficients (γ_t) in Eq. (3) in the multivariate models using a battery of alternatives to model the hedging component in Eq. (3). $\gamma_{t,DY}$ represents the time-varying hedging components coefficients when using Dividend Yield as the hedging component; $\gamma_{t,TERM}$ represents the time-varying hedging components coefficients when using the TERM spread as the hedging component; $\gamma_{t,DEF}$ represents the time-varying hedging components coefficients when using the Default spread as the hedging component and $\gamma_{t,BOND}$ represents the time-varying hedging components coefficients when using the Long-term US Bond as the hedging component. Results for the full period are reported in Panel A, while results for expansionary periods and recessionary periods according to NBER are reported in panel B and C, respectively. We use the one-sample t-test to check the significance for the mean of the series. The sample period spans from January 1990 to September 2020. Panel D shows the estimates (standard errors in parenthesis) for regressions of the time-varying hedging components coefficients trade-off series $\gamma_{t,DY}$; $\gamma_{t,TERM}$; $\gamma_{t,DEF}$ and $\gamma_{t,BOND}$ on a dummy that takes the value of 1 for a recession period according to NBER and 0 otherwise. ***, ** and * represent significance at 1%, 5% and 10% significance level.

Table B.3
Univariate and multivariate regression analysis (using $\lambda_{t,DY}$).

Panel A. Univariate regression

Macroeconomic factors			Systematic factors			Sentiment factors		
Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared
TED	**−1.30***** (0.00)	1.51%	SMB	−0.04 (0.55)	−0.17%	UMCSENT	**0.13***** (0.00)	20.74%
DEF	**−3.01***** (0.00)	40.36%	HML	0.01 (0.94)	−0.27%	SENT	**0.92***** (0.00)	2.20%
TERM	**−0.98***** (0.00)	9.49%	RMW	**−0.13*** (0.07)	0.62%	EPU	**−0.01***** (0.00)	7.98%
EINF	**0.64**** (0.01)	1.46%	CMA	**−0.27***** (0.00)	2.20%	MUNC	**−13.96***** (0.00)	20.98%
IRP	**6.98***** (0.00)	0.66%	MOM	**0.08**** (0.04)	0.93%	VRP	0.00 (0.64)	−0.21%
RRP	**4.41*** (0.06)	0.66%	QMJ	**−21.75***** (0.00)	2.36%			
Δ IPI	**0.61***** (0.00)	2.84%	BaB	**11.79**** (0.01)	1.41%			
CFNAI	**0.66***** (0.00)	4.38%						
CAPE	**0.18***** (0.00)	11.20%						
USSLIND	**2.60***** (0.00)	33.21%						
NFCI	**−4.15*****) (0.00)	34.72%						
RECPRO	**−0.07** (0.00)	22.66%						
ADS	**0.54***** (0.00)	8.60%						
CISS	**−17.24***** (0.00)	44.07%						

Panel B. Multivariate regression

Macroeconomic factors			Sentiment factors			All factor		
Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared
CISS	**−16.36***** (0.00)	51%	MUNC	**−13.95***** (0.00)	30%	CISS	**−16.36***** (0.00)	51%
TERM	**−0.67***** (0.00)		RUMCSENT	**0.09***** (0.00)		TERM	**−0.67***** (0.00)	
RDEF	**−1.22***** (0.00)					RDEF	**−1.22***** (0.00)	
						RMUNC	−1.69 (0.30)	
						RRUMCSENT	0.00 (0.96)	

Table B.3 presents the estimates (p-values in parenthesis) and the R-squares for a battery of regressions of the estimated risk–return trade-off $\lambda_{t,DY}$ using the multivariate regression in Equation (3) (with the dividend yield as the hedge component and the aggregate volatility as proxy of market risk) on a set of factors (described in Table 2). Panel A shows the results for univariate regressions of the estimated risk–return trade-off $\lambda_{t,DY}$ on individual macroeconomic (left columns), systematic (middle columns) and sentiment (right columns) factors. Panel B displays results for multivariate regressions including macroeconomic factors (left columns), sentiment factors (middle columns) and all factors together (right columns). ***, ** and * represent significance at 1%, 5% and 10%.

Table B.4
Univariate and multivariate regression analysis (using $\lambda_{t,TERM}$).

Panel A. Univariate regression								
Macroeconomic factors			Systematic factors			Sentiment factors		
Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared
TED	**−2.88***** (0.08)	4.52% .	SMB	0.04 (0.61)	−0.20%	UMCSENT	**0.13***** (0.00)	12.45%
DEF	**−3.76***** (0.00)	34.83%	HML	0.02 (0.75)	−0.24%	SENT	0.43 (0.31)	0.00%
TERM	**−0.86***** (0.15)	3.81%	RMW	**−0.22**** (0.02)	1.25%	EPU	**−0.02***** (0.00)	8.34%
EINF	0.49 (0.15)	0.28%	CMA	**−0.37***** (0.00)	2.41%	MUNC	**−18.52***** (0.00)	20.44%
IRP	**11.35***** (0.00)	2.94%	MOM	**0.09*** (0.09)	0.52%	VRP	0.00 (0.55)	−0.18%
RRP	**6.66**** (0.04)	0.90%	QMJ	**−44.34***** (0.00)	5.79%			
Δ IPI	**0.88***** (0.00)	3.37%	BaB	**18.38***** (0.35)	1.98%			
CFNAI	**0.88***** (0.00)	4.45%						
CAPE	**0.18***** (0.00)	5.45%						
USSLIND	**3.21***** (0.00)	27.94%						
NFCI	**−5.63***** (0.00)	35.28%						
RECPRO	**−0.10***** (0.00)	24.06%						
ADS	**0.79***** (0.00)	10.01%						
CISS	**−23.27***** (0.00)	44.49%						
Panel B. Multivariate regression								
Macroeconomic factors			Sentiment factors			All factor		
Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared
CISS	**−22.73***** (0.00)	47%	MUNC	**−18.52***** (0.00)	24%	CISS	**−22.73***** (0.00)	47%
TERM	**−0.41**** (0.01)		RUMCSENT	**0.08***** (0.00)		TERM	**−0.41**** (0.01)	
RDEF	**−1.25**** (0.00)					RDEF	**−1.25**** (0.00)	
						RMUNC	0.48 (0.83)	
						RRUMCSENT	**0.03*** (0.08)	

Table B.4 presents the estimates (p-values in parenthesis) and the R-squares for a battery of regressions of the estimated risk–return trade-off $\lambda_{t,TERM}$ using the multivariate regression in Equation (3) (with the TERM spread as the hedge component and the aggregate volatility as proxy of market risk) on a set of factors (described in Table 2). Panel A shows the results for univariate regressions of the estimated risk–return trade-off $\lambda_{t,TERM}$ on individual macroeconomic (left columns), systematic (middle columns) and sentiment (right columns) factors. Panel B displays results for multivariate regressions including macroeconomic factors (left columns), sentiment factors (middle columns) and all factors together (right columns). ***, ** and * represent significance at 1%, 5% and 10% significance level.

Table B.5
Univariate and multivariate regression analysis (using $\lambda_{t,DEF}$).

Panel A. Univariate regression

Macroeconomic factors			Systematic factors			Sentiment factors		
Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared
TED	−1.64*** (0.00)	2.32%	SMB	−0.18 (0.78)	−0.25%	UMCSENT	0.13*** (0.00)	18.11%
DEF	−3.10*** (0.00)	39.30%	HML	0.01 (0.88)	−0.27% .	SENT	0.79** (0.02)	1.38%
TERM	−0.92*** (0.00)	7.60%	RMW	−0.14** ((0.05)	0.79%	EPU	−0.01*** (0.00)	8.13%
EINF	0.59** (0.02)	1.09%	CMA	−0.28*** (0.00)	2.30%	MUNC	−14.62*** (0.00)	21.17%
IRP	7.75*** (0.00)	2.22%	MOM	0.08** (0.05)	0.82%	VRP	0.00 (0.58)	−0.19%
RRP	4.76* (0.06)	0.73%	QMJ	−26.23*** (0.00)	3.25%			
Δ IPI	0.66** (0.00)	3.06%	BaB	13.01*** (0.00)	1.61%			
CFNAI	0.69*** (0.00)	4.49%						
CAPE	0.18*** (0.00)	9.36%						
USSLIND	2.68*** (0.00)	32.38%						
NFCI	−4.38*** (0.00)	35.46%						
RECPRO	−0.08*** (0.00)	23.59%						
ADS	0.59*** (0.00)	9.24%						
CISS	−18.15*** (0.00)	44.92%						

Panel B. Multivariate regression

Macroeconomic factors			Sentiment factors			All factor		
Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared
CISS	−17.38*** (0.00)	50%	MUNC	−14.62*** (0.00)	21.17%	CISS	−17.38*** (0.00)	50%
TERM	−0.58*** (0.00)		RUMCSENT	0.09*** (0.00)		TERM	−0.58*** (0.00)	
RDEF	−1.20*** (0.00)					RDEF	−1.19*** (0.00)	
						RMUNC	−1.09 (0.52)	
						RRUMCSENT	−0.01 (0.54)	

Table B.5 presents the estimates (p-values in parenthesis) and the R-squares for a battery of regressions of the estimated risk–return trade-off $\lambda_{t,DEF}$ using the multivariate regression in Equation (3) (with the default spread as the hedge component and the aggregate volatility as proxy of market risk) on a set of factors (described in Table 2). Panel A shows the results for univariate regressions of the estimated risk–return trade-off $\lambda_{t,DEF}$ on individual macroeconomic (left columns), systematic (middle columns) and sentiment (right columns) factors. Panel B displays results for multivariate regressions including macroeconomic factors (left columns), sentiment factors (middle columns) and all factors together (right columns). ***, ** and * represent significance at 1%, 5% and 10% significance level.

Table B.6
Univariate and multivariate regression analysis (using $\lambda_{t,BOND}$).

Panel A. Univariate regression

Macroeconomic factors			Systematic factors			Sentiment factors		
Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared
TED	−1.05*** (0.03)	0.97%	SMB	−0.05 (0.41)	−0.1%	UMCSENT	0.13*** (0.00)	22.94%
DEF	−2.92*** (0.00)	40.84%	HML	0.00 (0.09)	−0.27	SENT	1.01*** (0.00)	2.9%
TERM	−1.02*** (0.00)	11.02%	RMW	−0.11* (0.09)	0.51%	EPU	−0.01*** (0.00)	7.78%
EINF	0.68** (0.00)	1.79%	CMA	−0.25*** (0.00)	2.09%	MUNC	−13.35*** (0.00)	20.60%
IRP	6.37** (0.01)	1.69%	MOM	0.08** (0.03)	1.01%	VRP	0.00 (0.68)	−0.22%
RRP	4.15* (0.07)	0.61%	QMJ	−18.51*** (0.00)	1.78%			
Δ IPI	0.57*** (0.00)	2.66%	BaB	10.79 (0.02)	1.24%			
CFNAI	0.62*** (0.00)	4.27%						
CAPE	0.19*** (0.00)	12.78%						
USSLIND	2.53*** (0.00)	33.53%						
NFCI	−3.96*** (0.00)	33.87%						
RECPRO	−0.07*** (0.00)	21.77%						
ADS	0.51*** (0.00)	8.06%						
CISS	−16.44*** (0.00)	43.06%						

Panel B. Multivariate regression

Macroeconomic factors			Sentiment factors			All factor		
Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared
CISS	−15.49*** (0.00)	51%	UMCSENT	0.13*** (0.00)	31%	CISS	−15.49*** (0.00)	51%
TERM	−0.72*** (0.00)		RMUNC	−9.34*** (0.00)		TERM	−0.72*** (0.00)	
RDEF	−1.23*** (0.00)					RDEF	−1.23*** (0.00)	
						RUMCSENT	0.01 (0.28)	
						RRMUNC	−1.75 (0.26)	

Table B.6 presents the estimates (p-values in parenthesis) and the R-squares for a battery of regressions of the estimated risk–return trade-off $\lambda_{t,BOND}$ using the multivariate regression in Equation (3) (with the long-term US bond as the hedge component and the aggregate volatility as proxy of market risk) on a set of factors (described in Table 2). Panel A shows the results for univariate regressions of the estimated risk–return trade-off $\lambda_{t,BOND}$ on individual macroeconomic (left columns), systematic (middle columns) and sentiment (right columns) factors. Panel B displays results for multivariate regressions including macroeconomic factors (left columns), sentiment factors (middle columns) and all factors together (right columns). ***, ** and * represent significance at 1%, 5% and 10% significance level.

Table B.7
Out-of-sample forecast of the risk–return relation.

	Macroeconomic	Sentiment	All factors
Panel A. Out-of-sample forecast $\lambda_{t,DY}$			
MAE	1.74	1.53	1.66
MSE	4.77	3.78	4.22
Diebold/Mariano	Macro versus Sent	Macro versus All	Sent versus All
	0.89(0.37)	1.36 (0.17)	−0.61 (0.54)
Adjusted R-square	**59.64%**(0.01)**	**44.03%*(0.08)**	**60.63%*****(0.00)
Panel B. Out-of-sample forecast $\lambda_{t,TERM}$			
MAE	2.67	2.49	2.54
MSE	10.95	9.93	9.89
Diebold/Mariano	Macro versus Sent	Macro versus All	Sent versus All
	0.62 (0.53)	1.53 (0.12)	0.03 (0.97)
Adjusted R-square	**54.77%**(0.01)**	**37.61%*(0.09)**	**55.48%***(0.00)**
Panel C. Out-of-sample forecast $\lambda_{t,DEF}$			
MAE	1.88	1.66	1.78
MSE	5.47	4.46	4.84
Diebold/Mariano	Macro versus Sent	Macro versus All	Sent versus All
	0.88(0.38)	1.41 (0.16)	−0.53 (0.60)
Adjusted R-square	**58.74%**(0.01)**	**42.86%*(0.08)**	**59.59%***(0.00)**
Panel D. Out-of-sample forecast $\lambda_{t,BOND}$			
MAE	1.63	1.44	1.55
MSE	4.26	3.34	3.80
Diebold–Mariano	Macro versus Sent	Macro versus All	Sent versus All
	0.88(0.38)	1.35(0.17)	−0.62(0.53)
Adjusted R-square	**60.02%**(0.01)**	**44.86%*(0.08)**	**61.20%***(0.00)**

Table B.7 shows the out-sample forecast accuracy of the risk–return trade-off (estimated using aggregate market volatility as a proxy for market risk and the dividend yield, the TERM spread, the default spread and the long-term US bond as hedge component) when using macroeconomic factors (left column), sentiment factors (middle column) and all factors together (right columns). The out-sample period covers the period from January 2010 until February 2020, for a total of 122 months. The first row displays the Mean Absolute Error between the actual and predicted risk–return trade-off while the second row displays the corresponding Mean Squared Error. The Diebold and Mariano test indicates if the forecast accuracy (MSE) of two competing models are the same (P-value in parentheses). Finally, last row shows the adjusted R-squares for every model computed as in Welch and Goyal (2008), using the vector of errors from the constant model and the vector of errors from suggested model. Critical values are obtained using the F-statistic (p-value in parentheses). ***, ** and * represent significance at 1%, 5% and 10% significance level.

Table B.8
Statistics for alternative estimates of the risk–return trade-off.

Series	Mean	Std	Max	Min
Panel A. FULL SAMPLE				
$\lambda_{VIX,t-1}$	2.95***	2.52	12.14	−1.37
$\lambda_{AGGR,t}^{assym-GARCH}$	2.19***	2.58	8.79	−18.43
$\lambda_{AGGR,t}^{assym-DCC-GARCH}$	1.87***	2.89	8.26	−21.89
$\lambda_{AGGR,t}^{w=30}$	2.83***	2.74	11.61	−12.14
$\lambda_{AGGR,t}^{w=90}$	4.18***	5.06	17.31	−23.99
$\lambda_{SP500,t}$	2.97***	4.62	18.43	−24.19
$\lambda_{FF,t}$	1.71***	2.93	11.37	−11.77
Panel B. EXPANSIONARY PERIOD				
Series	Mean	Std	Max	Min
$\lambda_{VIX,t-1}$	3.06***	2.50	12.14	−0.94
$\lambda_{AGGR,t}^{assym-GARCH}$	2.56***	1.42	5.99	−7.63
$\lambda_{AGGR,t}^{assym-DCC-GARCH}$	2.30***	1.58	5.61	−9.46
$\lambda_{AGGR,t}^{w=30}$	3.07***	1.89	8.86	−4.13
$\lambda_{AGGR,t}^{w=90}$	4.83***	3.86	17.31	−10.07
$\lambda_{SP500,t}$	3.36***	4.28	18.43	−6.83
$\lambda_{FF,t}$	1.99***	2.17	9.05	−5.18
Panel C. RECESSIONARY PERIOD				
Series	Mean	Std	Max	Min
$\lambda_{VIX,t-1}$	2.05***	2.44	8.42	−1.37
$\lambda_{AGGR,t}^{assym-GARCH}$	−0.67	5.84	8.79	−18.43
$\lambda_{AGGR,t}^{assym-DCC-GARCH}$	−1.48	6.49	8.26	−21.89
$\lambda_{AGGR,t}^{w=30}$	0.96	5.90	11.61	−12.14
$\lambda_{AGGR,t}^{w=90}$	−0.84	9.04	16.39	−23.99
$\lambda_{SP500,t}$	−0.16	5.99	10.84	−24.19
$\lambda_{FF,t}$	−0.59	5.87	11.37	−11.77

Panel D. SIGNIFICANCE TEST for regression
$\lambda_{t,i} = \alpha_{i,0} + \alpha_{i,1} USREC_t + \epsilon_{i,t}$

Series	$\alpha_{i,0}$	$\alpha_{i,1}$
$\lambda_{VIX,t-1}$	**2.80*****	**−2.91*****
	(0.00)	(0.00)
$\lambda_{AGGR,t}^{assym-GARCH}$	**2.56*****	**−3.23*****
	(0.00)	(0.00)
$\lambda_{AGGR,t}^{assym-DCC-GARCH}$	**2.30*****	**−3.77*****
	(0.00)	(0.00)
$\lambda_{AGGR,t}^{w=30}$	**3.07*****	**−2.10*****
	(0.00)	(0.00)
$\lambda_{AGGR,t}^{w=90}$	**4.83*****	**−5.66*****
	(0.00)	(0.00)
$\lambda_{SP500,t}$	**3.36*****	**−3.52*****
	(0.00)	(0.00)
$\lambda_{FF,t}$	**1.19*****	**−2.59*****
	(0.00)	(0.00)

Table B.8 presents the main summary statistics of the risk–return trade-off series using five different alternatives: when using the VIX index as a proxy for market risk ($\lambda_{VIX,t-1}$); when estimating the multivariate GARCH model using asymmetric GARCH models for the individual variances ($\lambda_{AGGR,t}^{assym-GARCH}$) and when using asymmetric variances and asymmetric correlations ($\lambda_{AGGR,t}^{assym-DCC-GARCH}$); when using a different window length of 30 and 90 days to compute the stock daily variance ($\lambda_{AGGR,t}^{w=30}$ and $\lambda_{AGGR,t}^{w=90}$); finally, when using two well-known market indexes to estimate the return and risk of the market: the SP500 index ($\lambda_{SP500,t}$) and the F&F Market factor($\lambda_{FF,t}$). Results for the full period are reported in Panel A, while results for expansionary periods and recessionary periods according to NBER are reported in panels B and C, respectively. We use the one-sample t-test to check the significance for the mean of the series. The sample period spans from January 1990 to September 2020. Panel D shows the estimates (p-values in parenthesis) for regressions of the risk–return trade-off series $\lambda_{VIX,t-1}$; $\lambda_{AGGR,t}^{assym-GARCH}$; $\lambda_{AGGR,t}^{assym-DCC-GARCH}$; $\lambda_{AGGR,t}^{w=30}$ and $\lambda_{AGGR,t}^{w=90}$ on a dummy that takes the value of 1 for a recession period according to NBER and 0 otherwise. *** represent significance at 1% significance level.

Table B.9
Univariate and multivariate regressions using VIX_{t-1}.

Panel A. Univariate regression

Macroeconomic factors			Systematic factors			Sentiment factors		
Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared
TED	**−1.00***** (0.00)	3.85%	SMB	−0.01 (0.66)	−0.22%	UMCSENT	**0.07***** (0.00)	22.03%
DEF	**−1.72***** (0.00)	51.62%	HML	0.04 (0.14)	0.32%	SENT	**0.49***** (0.00)	2.52%
TERM	**−0.44***** (0.00)	7.26%	RMW	−0.07 (0.97)	0.7%	EPU	**−0.01***** (0.00)	12.59%
EINF	**0.55***** (0.00)	4.65%	CMA	−0.09 (0.04)	0.89	MUNC	**−9.29***** (0.00)	36.82%
IRP	**5.07***** (0.00)	4.30%	MOM	**0.03**** (0.07)	0.62%	VRP	0.00 (0.26)	0.08%
RRP	**3.55***** (0.00)	2.11%	QMJ	−11.97 (0.26)	2.87%			
Δ IPI	**0.40**** (0.01)	4.89%	BaB	5.19 (0.30)	0.1%			
CFNAI	**0.41***** (0.02)	7.05%						
CAPE	**0.09***** (0.00)	9.82%						
USSLIND	**1.50***** (0.00)	43.67%						
NFCI	**−2.36***** (0.00)	43.99%						
RECPRO	**−0.04***** (0.00)	33.59%						
ADS	**0.31***** (0.00)	11.47%						
CISS	**−9.88***** (0.00)	57.10%						

Panel B. Multivariate regression

Macroeconomic factors			Sentiment factors			All factor		
Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared
CISS	**−9.55***** (0.00)	64%	MUNC	**−9.30***** (0.00)	43%	CISS	**−9.55***** (0.00)	66%
TERM	**−0.25***** (0.00)		RUMCSENT	**0.04**** (0.04)		TERM	**−0.25***** (0.00)	
RDEF	**−0.76***** (0.00)					RDEF	**−0.76***** (0.00)	
						RMUNC	**−2.87***** (0.00)	
						RRUMCSENT	−0.01 (0.20)	

Table B.9 presents the estimates (p-values in parenthesis) and the R-squares for a battery of regressions of the estimated risk–return trade-off using VIX as proxy of market risk on a set of factors (described in Table 2). Panel A shows the results for univariate regressions of the estimated risk–return trade-off using the VIX index lagged one period as proxy of market risk on different macroeconomic (left columns), systematic (middle columns) and sentiment (right columns) factors. Panel B displays results for multivariate regressions including macroeconomic factors (left columns), sentiment factors (middle columns) and all factors together (right columns). ***, ** and * represent significance at 1%, 5% and 10% significance level.

Table B.10
Univariate and multivariate regressions using asymmetric GARCH.

Panel A. Univariate regression

Macroeconomic factors			Systematic factors			Sentiment factors		
Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared
TED	**−0.77***** (0.00)	2.02%	SMB	−0.04 (0.20)	0.18%	UMCSENT	**0.07***** (0.00)	23.24%
DEF	**−1.66***** (0.00)	45.75%	HML	0.00 (0.85)	−0.26%	SENT	**0.45***** (0.00)	1.89%
TERM	**−0.50***** (0.00)	9.30%	RMW	**−0.06**** (0.08)	0.55%	EPU	**−0.00***** (0.00)	6.56%
EINF	**0.24*** (0.07)	0.61%	CMA	**−0.11**** (0.01)	1.41%	MUNC	**−7.84***** (0.00)	24.78%
IRP	**4.61**** (0.08)	3.31%	MOM	**−0.03*** (0.08)	0.57%	VRP	0.00 (0.79)	−0.25%
RRP	**3.19***** (0.01)	1.55%	QMJ	**−8.66***** (0.02)	1.29%			
Δ IPI	**0.33***** (0.00)	3.20%	BaB	4.71 (0.05)	0.73%			
CFNAI	**0.36***** (0.00)	5.00%						
CAPE	**0.09***** (0.00)	11.21%						
USSLIND	**1.52***** (0.00)	41.37%						
NFCI	**−2.27***** (0.00)	38.65%						
RECPRO	**−0.04***** (0.00)	27.28%						
ADS	**0.25***** (0.00)	6.55%						
CISS	**−8.72***** (0.00)	42.09%						

Panel B. Multivariate regression

Macroeconomic factors			Sentiment factors			All factor		
Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared
CISS	**−8.27***** (0.00)	52%	MUNC	**−7.84***** (0.00)	35%	CISS	**−6.37***** (0.00)	53%
TERM	**−0.34***** (0.00)		RUMCSENT	**0.05***** (0.00)		TERM	**−0.41***** (0.00)	
RDEF	**−0.93***** (0.00)					RDEF	**−0.93***** (0.00)	
						RMUNC	**−2.20**** (0.01)	
						RRUMCSENT	0.01 (0.45)	

Table B.10 presents the estimates (p-values in parenthesis) and the R-squares for a battery of regressions of the estimated risk–return trade-off using asymmetric GARCH models on a set of factors (described in Table 2). Panel A shows the results for univariate regressions of the estimated risk–return trade-off using asymmetric GARCH models on different macroeconomic (left columns), systematic (middle columns) and sentiment (right columns) factors. Panel B displays results for multivariate regressions including macroeconomic factors (left columns), sentiment factors (middle columns) and all factors together (right columns). ***, ** and * represent significance at 1%, 5% and 10% significance level.

Table B.11
Univariate and multivariate regressions using asymmetric DCC-GARCH models.

Panel A. Univariate regression

Macroeconomic factors			Systematic factors			Sentiment factors		
Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared
TED	−0.92 (0.00)	1.97%	SMB	−0.01 (0.76)	−0.25%	UMCSENT	**0.08***** (0.00)	22.83%
DEF	**−2.06***** (0.00)	47.26%	HML	0.02 (0.56)	−0.18%	SENT	**0.62***** (0.00)	2.48%
TERM	**−0.64***** (0.00)	10.05%	RMW	**−0.09**** (0.05)	0.79%	EPU	**−0.01***** (0.00)	9.88%
EINF	**0.43***** (0.00)	1.69%	CMA	**−0.16***** (0.00)	2.09%	MUNC	**−9.27***** (0.00)	23.22%
IRP	**6.01***** (0.00)	3.81%	MOM	0.03 (0.18)	0.22%	VRP	0.00 (0.23)	0.12%
RRP	**4.60***** (0.01)	2.27%	QMJ	**−15.54***** (0.02)	3.10%			
Δ IPI	**0.40***** (0.00)	3.15%	BaB	6.77 (0.02)	1.12%			
CFNAI	**0.44***** (0.00)	5.11%						
CAPE	**11.59***** (0.00)	11.02%						
USSLIND	**1.76***** (0.00)	37.98%						
NFCI	**−2.73***** (0.00)	37.57%						
RECPRO	**−0.05***** (0.00)	26.13%						
ADS	**0.37***** (0.00)	10.18%						
CISS	**−11.26***** (0.00)	47.18%						

Panel B. Multivariate regression

Macroeconomic factors			Sentiment factors			All factor		
Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared
CISS	**−10.69***** (0.00)	56%	MUNC	**−9.27***** (0.00)	33%	CISS	**−10.69***** (0.00)	56%
TERM	**−0.43***** (0.00)		RUMCSENT	**0.06***** (0.00)		TERM	**−0.48***** (0.00)	
RDEF	**−0.93**** (0.00)					RDEF	**−0.93**** (0.00)	
						RMUNC	1.14 (0.24)	
						RRUMCSENT	0.00 (0.91)	

Table B.11 presents the estimates (p-values in parenthesis) and the R-squares for a battery of regressions of the estimated risk–return trade-off using asymmetric DCC-GARCH models on a set of factors (described in Table 2). Panel A shows the results for univariate regressions of the estimated risk–return trade-off using asymmetric GARCH models on different macroeconomic (left columns), systematic (middle columns) and sentiment (right columns) factors. Panel B displays results for multivariate regressions including macroeconomic factors (left columns), sentiment factors (middle columns) and all factors together (right columns). ***, ** and * represent significance at 1%, 5% and 10% significance level.

Table B.12
Univariate and multivariate regressions using aggregate market variance with a 30-day window.

Panel A. Univariate regression								
Macroeconomic factors			Systematic factors			Sentiment factors		
Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared
TED	**−0.39**** (0.02)	1.30%	SMB	−0.03 (0.11)	0.41%	UMCSENT	**0.04***** (0.00)	22.37%
DEF	**−1.01***** (0.00)	43.53%	HML	−0.01 (0.43)	−0.01%	SENT	**0.22***** (0.00)	1.33%
TERM	**−0.34***** (0.00)	10.94%	RMW	−0.03 (0.24)	0.00%	EPU	**−0.00***** (0.00)	4.89%
EINF	**0.28***** (0.00)	2.88%	CMA	**−0.09***** (0.00)	2.60%	MUNC	**−2.95***** (0.00)	8.79%
IRP	0.40 (0.62)	−0.23%	MOM	**0.03**** (0.02)	1.31%	VRP	0.00 (0.97)	−0.27%
RRP	1.17 (0.13)	0.36%	QMJ	**−4.27*** (0.06)	0.70%			
Δ IPI	**0.20***** (0.00)	2.88%	BaB	**2.96*** (0.05)	0.74%			
CFNAI	**0.25***** (0.00)	6.04%						
CAPE	**0.05***** (0.00)	7.63%						
USSLIND	**0.71***** (0.00)	24.78%						
NFCI	**−1.28***** (0.00)	31.27%						
RECPRO	**−0.02***** (0.00)	16.46%						
ADS	**0.16***** (0.00)	7.38%						
CISS	**−5.23***** (0.00)	38.75%						
Panel B. Multivariate regression								
Macroeconomic factors			Sentiment factors			All factor		
Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared
CISS	**−4.91***** (0.00)	50%	UMCSENT	**0.04***** (0.00)	24%	CISS	**−4.91***** (0.00)	54%
TERM	**−0.25***** (0.00)		RMUNC	**−1.37***** (0.00)		TERM	**−0.25***** (0.00)	
RDEF	**−0.57***** (0.00)					RDEF	**−0.57***** (0.00)	
						RUMCSENT	0.00 (0.87)	
						RRMUNC	**2.37***** (0.00)	

Table B.12 presents the estimates (p-values in parenthesis) and the R-squares for a battery of regressions of the estimated risk–return trade-off (using a window length of 30 days to compute the aggregate market variance in Equation (4)) on a set of factors (described in Table 2). Panel A shows the results for univariate regressions of the estimated risk–return trade-off using asymmetric GARCH models on different macroeconomic (left columns), systematic (middle columns) and sentiment (right columns) factors. Panel B displays results for multivariate regressions including macroeconomic factors (left columns), sentiment factors (middle columns) and all factors together (right columns). ***, ** and * represent significance at 1%, 5% and 10% significance level.

Table B.13
Univariate and multivariate regressions using aggregate market variance with a 90-day window.

Panel A. Univariate regression

Macroeconomic factors			Systematic factors			Sentiment factors		
Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared
TED	−0.28 (0.38)	−0.01%	SMB	**−0.07*** (0.08)	0.58%	UMCSENT	**0.10***** (0.00)	31.12%
DEF	**−1.79***** (0.00)	35.98%	HML	−0.02 (0.56)	−0.02%	SENT	**0.79***** (0.00)	4.56%
TERM	**−0.85***** (0.00)	18.16%	RMW	−0.06 (0.18)	0.21%	EPU	**−0.01***** (0.00)	5.35%
EINF	**0.35**** (0.03)	1.00%	CMA	**−0.15***** (0.00)	1.74%	MUNC	**−7.06***** (0.00)	13.51%
IRP	1.83 (0.24)	0.11%	MOM	**0.06**** (0.02)	1.30%	VRP	0.00 (0.78)	−0.25%
RRP	1.60 (0.29)	0.00%	QMJ	−5.80 (0.19)	0.20%			
Δ IPI	**0.29***** (0.00)	1.56%	BaB	4.74 (0.11)	0.41%			
CFNAI	**0.36***** (0.00)	3.24%						
CAPE	**0.15***** (0.00)	19.98%						
USSLIND	**1.55***** (0.00)	29.76%						
NFCI	**−2.31***** (0.00)	27.26%						
RECPRO	**−0.04***** (0.00)	15.81%						
ADS	**0.25***** (0.00)	4.63%						
CISS	**−9.55***** (0.00)	34.25%						

Panel B. Multivariate regression

Macroeconomic factors			Sentiment factors			All factor		
Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared
CISS	**−8.66***** (0.00)	48%	UMCSENT	**0.10***** (0.00)	34%	CISS	**−8.66***** (0.00)	50%
TERM	**−0.68***** (0.00)		RMUNC	**−3.51***** (0.00)		TERM	**−0.68***** (0.00)	
RDEF	**−0.76***** (0.00)					RDEF	**−0.76***** (0.00)	
						RUMCSENT	**0.03***** (0.00)	
						RRMUNC	−0.04 (0.97)	

Table B.13 presents the estimates (p-values in parenthesis) and the R-squares for a battery of regressions of the estimated risk–return trade-off (using a window length of 90 days to compute the aggregate market variance in Equation (4)) on a set of factors (described in Table 2). Panel A shows the results for univariate regressions of the estimated risk–return trade-off using asymmetric GARCH models on different macroeconomic (left columns), systematic (middle columns) and sentiment (right columns) factors. Panel B displays results for multivariate regressions including macroeconomic factors (left columns), sentiment factors (middle columns) and all factors together (right columns). ***, ** and * represent significance at 1%, 5% and 10% significance level.

Table B.14
Univariate and multivariate regressions using the S&P500 index.

Panel A. Univariate regression

Macroeconomic factors			Systematic factors			Sentiment factors		
Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared
TED	0.72 (0.29)	0%	SMB	−0.02 (0.64)	0%	UMCSENT	**0.28***** (0.00)	57%
DEF	**−3.07***** (0.00)	24%	HML	−0.06 (0.26)	0%	SENT	**3.09***** (0.00)	16%
TERM	**−2.28***** (0.00)	29%	RMW	**−0.13**** (0.03)	1%	EPU	**−0.01***** (0.00)	5%
EINF	**1.90***** (0.00)	8%	CMA	**−0.13*** (0.08)	1.74%	MUNC	**−10.73***** (0.00)	7%
IRP	**5.70*** (0.08)	1%	MOM	**0.09**** (0.01)	2%	VRP	**0.03***** (0.00)	2%
RRP	**7.13**** (0.02)	1%	QMJ	**−13.48**** (0.02)	1%			
Δ IPI	**0.42*** (0.08)	1%	BaB	−2.97 (0.46)	0%			
CFNAI	**0.56***** (0.00)	2%						
CAPE	**0.60***** (0.00)	67%						
USSLIND	**2.87***** (0.00)	23%						
NFCI	**−3.58***** (0.00)	15%						
RECPRO	**−0.07***** (0.00)	11%						
ADS	**0.44***** (0.00)	3%						
CISS	**−15.91***** (0.00)	21%						

Panel B. Multivariate regression

Macroeconomic factors			Sentiment factors			All factor		
Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared
CAPE	**0.56***** (0.00)	80%	UMCSENT	**0.28***** (0.00)	57%	CAPE	**0.56***** (0.00)	80%
NFCI	**−2.08***** (0.00)					NFCI	**−2.08***** (0.00)	
REINF	**1.83***** (0.00)					REINF	**1.81***** (0.00)	
						RUMCSENT	0.00 (0.85)	

Table B.14 presents the estimates (p-values in parenthesis) and the R-squares for a battery of regressions of the estimated risk–return trade-off (using the S&P500 index to compute the aggregate market return and variance in Equation (2)) on a set of factors (described in Table 2). Panel A shows the results for univariate regressions of the estimated risk–return trade-off using the S&P500 index on different macroeconomic (left columns), systematic (middle columns) and sentiment (right columns) factors. Panel B displays results for multivariate regressions including macroeconomic factors (left columns), sentiment factors (middle columns) and all factors together (right columns). ***, ** and * represent significance at 1%, 5% and 10% significance level.

Table B.15
Univariate and multivariate regressions using the Fama and French market returns.

Panel A. Univariate regression

Macroeconomic factors			Systematic factors			Sentiment factors		
Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared
TED	−1.06** (0.01)	1%	SMB	−0.02 (0.64)	0%	UMCSENT	0.14*** (0.00)	35%
DEF	−1.89*** (0.00)	22%	HML	−0.06 (0.26)	0%	SENT	0.78*** (0.00)	2%
TERM	−1.34*** (0.00)	25%	RMW	−0.13** (0.03)	1%	EPU	−0.01** (0.02)	1%
EINF	0.01 (0.97)	0%	CMA	−0.13* (0.08)	1%	MUNC	−5.48*** (0.00)	4%
IRP	2.97* (0.08)	0%	MOM	0.09** (0.01)	2%	VRP	0.00 (0.75)	0%
RRP	−0.02 (0.99)	0%	QMJ	−13.48** (0.02)	1%			
Δ IPI	0.29* (0.06)	1%	BaB	−2.97 (0.46)	0%			
CFNAI	0.42*** (0.00)	2%						
CAPE	0.32*** (0.00)	47%						
USSLIND	2.48*** (0.00)	48%						
NFCI	−2.92*** (0.00)	24%						
RECPRO	−0.06*** (0.00)	22%						
ADS	0.31*** (0.00)	4%						
CISS	−12.00*** (0.00)	30%						

Panel B. Multivariate regression

Macroeconomic factors			Sentiment factors			All factor		
Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared	Variable	Coefficient (*p*-value)	R-squared
USSLIND	2.28*** (0.00)	77%	UMCSENT	0.14*** (0.00)	35%	USSLIND	2.28*** (0.00)	77%
TERM	−1.01*** (0.00)					TERM	−1.01*** (0.00)	
RCAPE	0.19*** (0.00)					RCAPE	0.19*** (0.00)	
						RUMCSENT	−0.01 (0.22)	

Table B.15 presents the estimates (p-values in parenthesis) and the R-squares for a battery of regressions of the estimated risk–return trade-off (using the excess returns on the F&F market portfolio to compute the aggregate market return and variance in Equation (2)) on a set of factors (described in Table 2). Panel A shows the results for univariate regressions of the estimated risk–return trade-off on different macroeconomic (left columns), systematic (middle columns) and sentiment (right columns) factors. Panel B displays results for multivariate regressions including macroeconomic factors (left columns), sentiment factors (middle columns) and all factors together (right columns). ***, ** and * represent significance at 1%, 5% and 10% significance level.

Table B.16
Out-of-sample forecast of the risk–return trade-off for alternative estimates.

Panel A. Out-of-sample forecast $\lambda_{t,VIX_{t-1}}$			
	Macroeconomic	Sentiment	All factors
MAE	0.92	0.77	0.84
MSE	1.29	0.94	1.04
Diebold/Mariano	Macro versus Sent	Macro versus All	Sent versus All
	1.06 (0.29)	1.59(0.11)	−0.51 (0.61)
Adjusted R-square	**67.38%***(0.00)**	**54.51%*(0.07)**	**69.84%***(0.00)**
Panel B. Out sample forecast $\lambda_{AGGR,t}^{assym-GARCH}$			
	Macroeconomic	Sentiment	All factors
MAE	0.98	0.72	0.92
MSE	1.36	0.78	1.16
Diebold/Mariano	Macro versus Sent	Macro versus All	Sent versus All
	1.28 (0.20)	1.06 (0.29)	−1.22 (0.22)
Adjusted R-square	**63.57%***(0.00)**	**44.33%*(0.08)**	**65.61%***(0.00)**
Panel C. Out sample forecast $\lambda_{AGGR,t}^{assym-DCC-GARCH}$			
	Macroeconomic	Sentiment	All factors
MAE	1.46	0.95	1.08
MSE	1.83	1.33	1.65
Diebold/Mariano	Macro versus Sent	Macro versus All	Sent versus All
	1.02 (0.31)	1.08 (0.28)	−0.89 (0.37)
Adjusted R-square	**64.50%***(0.00)**	**41.76%*(0.08)**	**64.94%***(0.00)**
Panel D. Out sample forecast $\lambda_{AGGR,t}^{w=30}$			
	Macroeconomic	Sentiment	All factors
MAE	0.72	0.77	0.71
MSE	0.73	0.9	0.72
Diebold/Mariano	Macro versus Sent	Macro versus All	Sent versus All
	−0.90 (0.37)	0.26 (0.79)	0.93 (0.35)
Adjusted R-square	**57.73%***(0.00)**	**40.09%*(0.09)**	**57.71%***(0.00)**
Panel E. Out sample forecast $\lambda_{AGGR,t}^{w=90}$			
	Macroeconomic	Sentiment	All factors
MAE	1.24	1.19	1.19
MSE	2.14	1.96	2.00
Diebold/Mariano	Macro versus Sent	Macro versus All	Sent versus All
	0.30 (0.76)	1.46 (0.14)	−0.09(0.93)
Adjusted R-square	**56.62%**(0.01)**	**44.59%*(0.08)**	**58.99%***(0.00)**
Panel F. Out sample forecast $\lambda_{SP500,t}$			
	Macroeconomic	Sentiment	All factors
MAE	1.64	1.77	1.60
MSE	4.02	4.78	3.82
Diebold/Mariano	Macro versus Sent	Macro versus All	Sent versus All
	−0.89 (0.37)	1.20 (0.23)	1.10(0.0.27)
Adjusted R-square	**84.69%**(0.00)**	**68.13%**(0.05)**	**84.60%***(0.00)**
Panel G. Out sample forecast $\lambda_{FF,t}$			
	Macroeconomic	Sentiment	All factors
MAE	0.96	1.57	0.99
MSE	1.43	3.84	1.57
Diebold/Mariano	Macro versus Sent	Macro versus All	Sent versus All
	−1.75*(0.08)	−1.08 (0.28)	1.60(0.0.11)
Adjusted R-square	**84.59%***(0.00)**	**52.26%*(0.07)**	**84.33%***(0.00)**

Table B.16 shows the out-sample forecast accuracy of the risk–return trade-off when using macroeconomic factors (left column), sentiment factors (middle column) and all factors together (right columns). We use seven different alternatives to estimate the risk–return trade-off series: when using the VIX index as a proxy for market risk ($\lambda_{VIX,t-1}$) in panel A; when estimating the multivariate GARCH model using asymmetric GARCH models for the individual variances ($\lambda_{AGGR,t}^{assym-GARCH}$) and when using asymmetric variances and asymmetric correlations ($\lambda_{AGGR,t}^{assym-DCC-GARCH}$) in Panel B and C respectively; when using a different window length of 30 and 90 days to compute the stock daily variance ($\lambda_{AGGR,t}^{w=30}$ and $\lambda_{AGGR,t}^{w=90}$) in Panel D and E; finally, in panel F and G when using the S&P500 index ($\lambda_{S\&P500,t}$) and the returns on the F&F market portfolio ($\lambda_{FF,t}$) to obtain the aggregate market return and risk. The out-sample period covers the period from January 2010 until February 2020, for a total of 122 months. Within each panel, the top row displays the Mean Absolute Error between the actual and predicted risk–return trade-off while the second row displays the corresponding Mean Squared Error. The Diebold and Mariano test indicates if the forecast accuracy (MSE) of two competing models are the same (p-value in parentheses). Finally, last row shows the adjusted R-squares for each model computed as in Welch and Goyal (2008), using the vector of errors from the constant model and the vector of errors from the suggested model. Critical values are obtained using the F-statistic (p-value in parentheses). ***, ** and * represent significance at 1%, 5% and 10% significance level.

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