

# Improving efficiency of transcritical CO<sub>2</sub> cycles through a magnetic refrigeration subcooling system

Laura Nebot-Andrés<sup>\*, a</sup>, Manuel Gesù Del Duca<sup>b</sup>, Ciro Aprea<sup>b</sup>, Andrej Žerovnik<sup>c</sup>,  
Jaka Tušek<sup>c</sup>, Rodrigo Llopis<sup>a</sup>, Angelo Maiorino<sup>b</sup>

<sup>a</sup> Thermal Engineering Group, Mechanical Engineering and Construction Department,  
Jaume I University, Spain

<sup>b</sup> Department of Industrial Engineering, Università di Salerno, Via Giovanni Paolo II, 132, 84084 Fisciano,  
Salerno, Italy

<sup>c</sup> Faculty of Mechanical Engineering, University of Ljubljana, Aškerčeva 6, 1000 Ljubljana, Slovenia

\*Corresponding author: lnebot@uji.es, +34 964 718133

## Abstract

Subcooling methods for transcritical CO<sub>2</sub> plants are being studied in order to improve the behaviour of these systems in hot climates, where basic configurations are not competitive enough. To achieve important improvements in the transcritical CO<sub>2</sub> performance, it is necessary to perform the subcooling with a refrigeration cycle working with a Coefficient of Performance higher than that of the CO<sub>2</sub> system without subcooling. Magnetic refrigeration devices can achieve high Coefficient of Performance values when the temperature difference between the hot sink and cold source is small, and therefore they meet the requirements to be applied as a CO<sub>2</sub> subcooling method. This work presents the coupling of two refrigeration technologies: vapour compression and magnetocaloric refrigeration, which has not yet been presented in the literature. The magnetic refrigeration system is, based on the experimental results of the existing prototype, analysed semi-empirically and further evaluated, as a subcooling method for a transcritical CO<sub>2</sub> cycle in a wide range of ambient conditions. Gas-cooler pressure, subcooling degree and operating parameters of the magnetic refrigerator were optimized for each condition to obtain the maximum Coefficient of Performance. We show that subcooling with the existing prototype of magnetic refrigeration system can enhance the overall Coefficient of Performance of transcritical CO<sub>2</sub> cycle by up to 9%.

## Keywords

Carbon Dioxide, Transcritical, Magnetic refrigeration, Subcooling, Modelling, Optimization

## Nomenclature

AMR	active magnetic regenerator
APP	temperature approach, K
BP	back-pressure valve
COP	coefficient of performance
EXV	electronic expansion valve
GWP	global warming potential
h	specific enthalpy, kJ·kg <sup>-1</sup>
$\dot{m}$	mass flow rate, kg·s <sup>-1</sup>
MCM	magnetocaloric material
MR	magnetic refrigeration
MRS	magnetic refrigeration subcooling
p	absolute pressure, bar

1	Pc	power consumption, kW
2	$\dot{Q}$	cooling capacity, kW
3	SUB	degree of subcooling produced in the subcooler, K
4	T	temperature, °C

5

## 6 **Greek symbols**

7	$b$	biases of the hidden and output neurons
8	$\rho$	density, kg·m <sup>-3</sup>
9	$\tau$	blow period, s
10	$\eta$	efficiency
11	$\nu$	specific volume, m <sup>3</sup> ·kg <sup>-1</sup>
12	$\Delta$	increment
13	$\Delta t$	temperature difference
14	$w$	synaptic weights

15

## 16 **Subscripts**

17	C	cold
18	CO <sub>2</sub>	corresponding to the CO <sub>2</sub> cycle
19	dis	compressor discharge
20	env	corresponding to the environment
21	gc	gas-cooler
22	ht	heat transfer fluid
23	H	hot
24	i	isentropic/ideal/internal
25	in	inlet
26	mag	corresponding to the magnetic system
27	main	corresponding to the main cycle
28	max	maximum
29	min	minimum
30	MR	corresponding to the magnetic refrigerator
31	0	evaporating level
32	o	outlet
33	s	isentropic
34	span	temperature span
35	sub	subcooler
36	suc	suction
37	v	volumetric

## 38 **1. Introduction**

39 Nowadays, carbon dioxide is the reference refrigerant used in most of centralized refrigeration applications  
 40 for commercial purposes. Despite being a natural refrigerant used since the beginning of refrigeration, its  
 41 use has been boosted in recent years due to regulations that restrict the use of most refrigerants in some  
 42 applications of refrigeration (European Commission, 2014; UNEP/TEAP, 1999). Carbon dioxide is a  
 43 refrigerant that meets the limitations because its global warming potential (GWP) equals to 1 and it can be

1 used in safety conditions since it is neither toxic nor flammable. Despite its great potential, its performance  
2 under certain working conditions can be significantly lower compared to fluorinated refrigerants used so  
3 far.

4 In recent years, complex cycles have been developed with the aim of improving its energy performance,  
5 especially for areas where the heat rejection temperature is high because the energy behaviour of CO<sub>2</sub>  
6 systems drops significantly when the heat rejection temperature is higher than 15 °C (Hafner A. et al.,  
7 2014). The adoption of natural refrigerants such as CO<sub>2</sub> and NH<sub>3</sub> in complex systems for commercial  
8 application outperform other systems in extreme warm climates (Purohit et al., 2018). Researchers have  
9 been working in different lines of research, as the use of ejectors (Hafner A. et al., 2014; Lawrence and  
10 Elbel, 2016; Lawrence and Elbel, 2019), the combination with other systems (Aprea et al., 2015; Arora et  
11 al., 2011; Karampour and Sawalha, 2016) and the parallel compression (Sarkar and Agrawal, 2010).  
12 Another line of research that has been intensively developed in recent years is CO<sub>2</sub> subcooling (Llopis et  
13 al., 2018). Different subcooling methods have been studied and they can be classified in two types:  
14 dedicated or integrated subcooling. The aim of the subcooling is to enhance the specific cooling capacity  
15 and the Coefficient of Performance (COP) of the refrigeration system. It reduces the optimum gas-cooler  
16 pressure, which leads to a reduction of the power consumption of the compressor and an increase of the  
17 refrigerant mass flow rate.

18 Among the integrated systems, the best known is the use of the internal heat exchanger (IHX), which  
19 subcools the CO<sub>2</sub> at the exit of the gas-cooler and reheats the vapour before the compressor suction point  
20 (Rigola et al., 2010). The integrated mechanical subcooling system performs the subcooling of the CO<sub>2</sub>  
21 since a part of the CO<sub>2</sub> flow that is extracted from the main cycle is expanded and then taken to the  
22 subcooler, which is placed downstream of the gas-cooler. This flow is then recompressed by an auxiliary  
23 compressor and reinjected to the main cycle (Nebot-Andrés et al., 2020a).

24 The dedicated methods have the same operating principle but subcooling is caused by a backup cycle,  
25 external to the main CO<sub>2</sub> cycle. The dedicated mechanical subcooling (Beshr et al., 2016; Bush et al.,  
26 2017; Llopis et al., 2016; Nebot-Andrés et al., 2020b) is the most studied one and performs the subcooling  
27 using an external vapour compression cycle that is thermally coupled to the main cycle through the  
28 subcooler and works with a refrigerant different from CO<sub>2</sub>. Since the results obtained theoretically and  
29 experimentally for this cycle were very promising, other dedicated systems have been developed. For  
30 example, for smaller cooling capacity systems, the use of thermoelectric modules providing the subcooling  
31 has been studied (Astrain et al., 2019). Experimentally, increments with regard to the cycle without  
32 subcooling of 6.3% at 25°C of ambient temperature and 9.9% at 30°C were obtained for an evaporative  
33 temperature of -10 °C (Sánchez et al., 2020).

34 Similar to the dedicated subcooling systems presented to date, there is a possibility of applying new  
35 technologies to the CO<sub>2</sub> cycle to perform the subcooling. Magnetic refrigeration (MR) is an emerging  
36 technology that uses a solid as a refrigerant, taking advantage of the magnetocaloric effect (MCE). The  
37 MCE is a feature of some magnetic materials, which heat up when they are subjected to an external  
38 magnetic field, and cool down after the removal of the applied field. They are called magnetocaloric  
39 materials (MCMs). The MCE value can be represented as the isothermal entropy change  $\Delta s_{iso}$  or the  
40 adiabatic temperature change  $\Delta T_{ad}$  of the MCM induced by the augmenting or the diminishing of the  
41 external magnetic field which the material is subjected to. If the conditions are kept adiabatic, the  
42 temperature of MCM increases by the  $\Delta T_{ad}$ . On the other hand, if the conditions are kept isothermal, the  
43 specific entropy of MCM decreases by the  $\Delta s_{iso}$ .

44 Magnetic refrigeration devices can achieve higher efficiencies than those of vapour compression systems,  
45 as long as they work with a relatively low temperature difference between the heat sink and cold source  
46 (Aprea et al., 2016; Dall'Olio et al., 2021; Eriksen et al., 2015; Kamran et al., 2020; Lozano et al., 2016;  
47 Masche et al., 2021; Masche et al., 2022). Furthermore, magnetic refrigeration systems are considered as  
48 environmental-friendly technologies since they use a solid substance as a refrigerant in substitution of the

1 ozone depleting and greenhouse effect gases used in vapour compression applications. The use of active  
2 magnetic regenerators (AMR), which is nowadays a key element of MR devices, was proposed by Barclay  
3 (1982) and it allowed overcoming the limitations regarding the small adiabatic temperature change  
4 showed by the most commonly used MCMs. The AMR cycle is based on four operational steps: an  
5 adiabatic magnetisation, an isofield cooling, an adiabatic demagnetisation and an isofield heating. During  
6 the first step, the MCM in the AMR is subjected to the external magnetic field that increases its  
7 temperature due to MCE. Then, keeping the magnetic field at a constant value, a fluid flows through the  
8 material absorbing heat from it, which is rejected in the hot heat exchanger. In the adiabatic  
9 demagnetisation step, the external magnetic field is removed, and the MCM cools down. At the last step of  
10 the cooling cycle, with no external magnetic field, the fluid flows in a counter-flow direction through the  
11 material, expelling heat to it. Next, the fluid absorbs heat in the cold heat exchanger. At this point, the  
12 steps are repeated. To date around 100 prototype of magnetic refrigeration devices have been developed  
13 around the world (Greco et al., 2019).

14 As presented in Llopis et al. (2018), the CO<sub>2</sub> systems will improve their COP as long as the COP of the  
15 auxiliary cycle is higher than the COP of the cycle without subcooling. The dedicated cycle performs heat  
16 rejection to the same hot sink as the CO<sub>2</sub> cycle and the cold source is defined by the temperature at the  
17 exit of the gas-cooler and the subcooling degree. That means that temperature difference between the hot  
18 sink and the cold source of the auxiliary cycle are rather small. Since the MR devices can operate with  
19 high COP values at small temperature span, they can be well suited for subcooling of the CO<sub>2</sub> system. In  
20 addition, MR uses environmentally friendly materials, and the environmental impact of the system would  
21 remain low.

22 The objective of this work is to evaluate the possibilities of merging these two technologies from a first law  
23 general approach and to quantify the improvements that could be obtained. The main objective is to  
24 perform the thermodynamic analysis of a CO<sub>2</sub> cycle subcooled by a MR and analyze the behavior of the  
25 main energetic parameters of the coupling for different ambient temperatures. Optimum conditions must  
26 be found in order to optimize the operation of this cycle, where following parameters that must be  
27 controlled are gas-cooler pressure, subcooling degree, mass flow rate and operating frequency of the MR.  
28 The results presented in this work correspond to the evaluation of the cycle for a range of environmental  
29 temperatures between 25 °C and 35 °C, always at the operating conditions that maximize the COP, and  
30 for an evaporation temperature of -15 °C. The overall COP as well as an in-depth study of the operating  
31 parameters of the MR system is presented. A comparison with the cycle without subcooling is also shown  
32 and the improvements that can be obtained are quantified.

## 33 **2. Methods**

34 A transcritical CO<sub>2</sub> refrigeration cycle as shown in Figure 1 is considered to evaluate the benefits of  
35 performing the CO<sub>2</sub> subcooling by a MR. The CO<sub>2</sub> system performs the expansion process in two stages  
36 with a liquid receiver in between both (points 4 to 7). The first expansion device is a back-pressure valve  
37 (BP, points 4 to 5) and controls the discharge pressure, while the second expansion valve (EXV, points 6  
38 to 7) controls the evaporating process. The liquid receiver is placed between both valves (points 5 to 6).  
39 Subcooling is performed in a subcooler (points 3 to 4) placed after the gas-cooler (2 to 3), which thermally  
40 connects the CO<sub>2</sub> cycle and the heat transfer fluid of the MR device.

41 The magnetic refrigeration system is made of a single-layer Gadolinium AMR with water as heat transfer  
42 (regenerative) fluid. It should be noted that magnetocaloric materials act as a refrigerant in the MR  
43 technology and water is only used as a heat transfer medium. Other heat transfer medium would in  
44 general also be possible, but water is cheap, environmentally friendly and has good heat transfer  
45 properties. Both cycles, the CO<sub>2</sub> transcritical and the MR one, perform the heat rejection to the same hot  
46 sink, which is considered to be the environmental temperature.

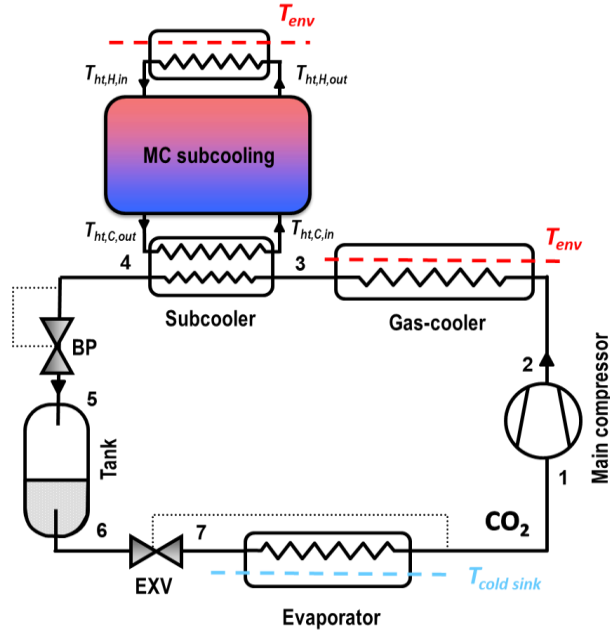


Figure 1. CO<sub>2</sub> transcritical refrigeration plant with magnetocaloric subcooling.

## 2.1. Transcritical CO<sub>2</sub> refrigeration cycle

The CO<sub>2</sub> cycle is evaluated by fixing the evaporation temperature and considering different environmental temperatures (hot sink). To evaluate the performance of the cycle, pressure drop through the heat exchangers and heat losses to the environment are neglected. Pressure at the gas-cooler and the degree of subcooling can be modified to reach the maximum COP.

The temperature at the exit of the gas-cooler is obtained by considering an approach temperature to the environment to be only 2 K (Eq. (1)), which is justified by the high heat transfer rates of CO<sub>2</sub> in transcritical conditions (Kim et al., 2004) and it is also supported by the data obtained from previous experimental studies (Llopis et al., 2016).

$$T_{gc,o} = T_{env} + APP \quad (1)$$

The subcooling degree is the temperature difference between the gas-cooler exit and the subcooler exit temperatures, as presented by Eq. (2). A constant 10 K superheating degree is considered in the evaporator (see Eq. (3)), which is controlled by the EXV. All the thermodynamic properties of the CO<sub>2</sub> are obtained using Refprop 9.1. (Lemmon et al., 2013).

$$SUB = T_{gc,o} - T_{sub,o} \quad (2)$$

$$T_{suc} = T_0 + 10 \quad (3)$$

Compressor is modelled using the curves for the isentropic and volumetric efficiency (Eq. (4) and coefficients from Table 1), based on data provided by the manufacturer. The CO<sub>2</sub> compressor, semihermetic with a displacement of 3.48 m<sup>3</sup>·h<sup>-1</sup> and nominal power of 4 kW is considered to run always at nominal speed of 1450 rpm.

$$\eta_v = \eta_i = a_0 + a_1 \cdot P_0 + a_2 \cdot P_{gc} + a_3 \cdot \frac{P_{gc}}{P_0} + a_4 \cdot v_{suc} \quad (4)$$

1

Table 1. Isentropic and volumetric efficiency coefficients.

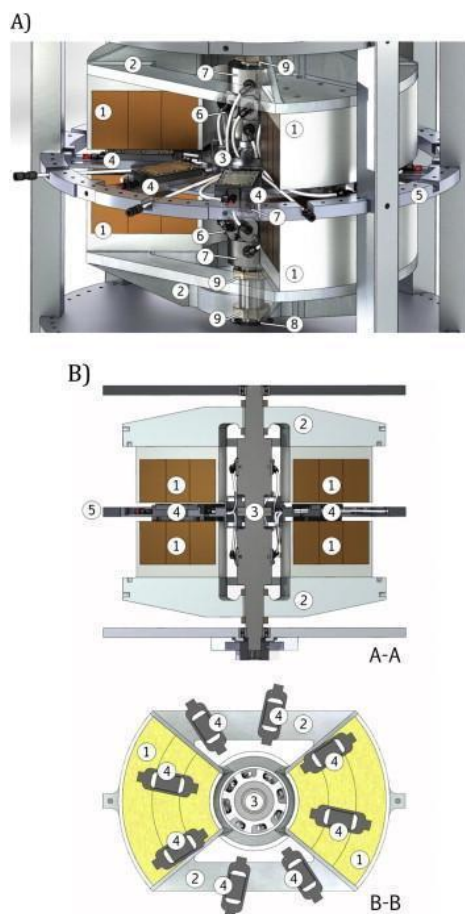
	$\eta_v$	$\eta_i$
$a_0$	1.0473	0.7634
$a_1$	0.0031	-0.0021
$a_2$	-0.0030	0.0013
$a_3$	0.0012	-0.0571
$a_4$	-11.1282	0.5425

2

3

## 2.2. Magnetic refrigeration subcooling cycle (MRS)

4 The simulation of the magnetic refrigeration subcooling (MRS) cycle was carried out by considering  
 5 performance data obtained during the experimental characterization of a magnetic refrigerator prototype,  
 6 named 8MAG (Aprea et al., 2014). The considered device is a rotary permanent magnet magnetic  
 7 refrigerator where the MCM is stationary, and the magnet is rotating. It is characterized by an octagonal  
 8 shape of the magnetic system and a total number of 8 packed-bed AMRs, supported by an aluminium  
 9 structure (or magnetocaloric wheel) with 45° spacing. Figure 2 shows a schematic view of the MR  
 10 prototype.



11

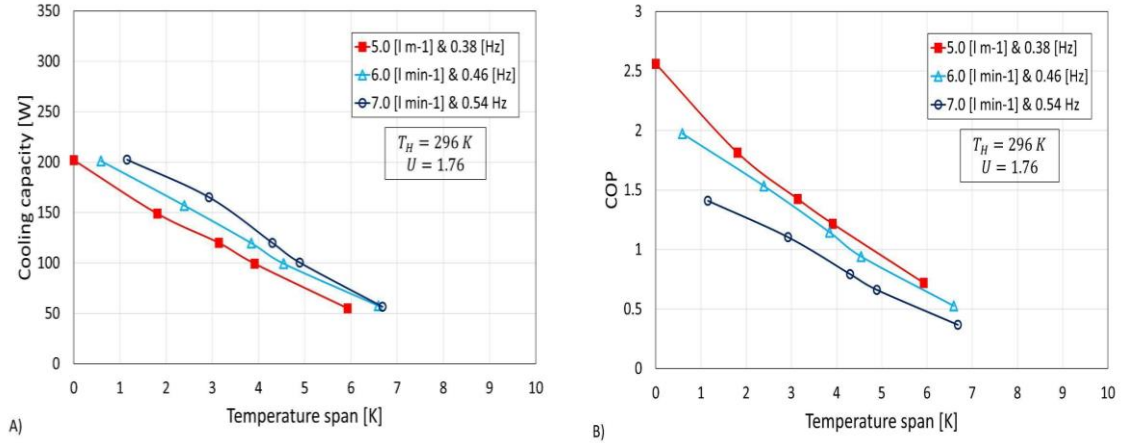
12 Figure 2. A. MR prototype core details: 1) Permanent magnet assembly. 2) Mounting support. 3) Shaft-rotary valve  
 13 combination. 4) AMR. 5) Magnetocaloric wheel. 6) Fluid manifold to/from regenerators. 7) Fluid manifold to/from heat  
 14 exchanger. 8) Bearings. 9) Adjustable rings. B. Longitudinal (A–A) and axial section (B–B) of the device core. (Aprea  
 15 et al., 2014).

1 Each AMR has a height of 20 mm, a length of 45 mm, and a width of 35 mm, with 31.5 cm<sup>3</sup> of available  
 2 volume for the MCM. AMRs were filled with a total mass of 1.2 kg of Gadolinium spheres (400-500  
 3 microns) with a purity of 99.82%, which means that each AMR had 150 g of Gadolinium with an average  
 4 porosity of 0.35. The driving system, consisting of a brushless DC gear-motor rotating the magnets, allows  
 5 operating with a variable speed in the frequency range between 0.1 Hz and 1 Hz and for every full rotation  
 6 of the magnetic system, each pair of regenerators undergoes two AMR cycles. Therefore, the MR  
 7 frequency ( $f_{MR}$ ) is twice the frequency of the magnetic system ( $f_{mag}$ ). The MR was tested at different  
 8 operating conditions in terms of mass flow rate of the heat transfer fluid ( $\dot{m}_{ht,MR}$ ), rotational frequency of  
 9 the magnets ( $f_{mag}$ ), temperature of the environment ( $T_{env}$ ) and temperature span ( $\Delta T_{span}$ ) to  
 10 characterize its performance, i.e. cooling capacity and COP. The temperature span is defined and  
 11 calculated as the difference between the heat transfer fluid exiting the hot side of the AMR ( $T_{ht,H,o}$ ) and  
 12 the heat transfer fluid exiting the cold side of the AMR ( $T_{ht,C,o}$ ), according to Eq. 5.  
 13

$$\Delta T_{span} = T_{ht,H,o} - T_{ht,C,o} \quad (5)$$

14

15 In Figure 3 the experimental cooling capacity and COP of the MR prototype are shown.



16 Figure 3. Experimental results of the MR prototype: a) cooling capacity and b) COP (Aprea et al. (2016)).

17 To integrate the experimental data of the MR prototype into the coupled model of the CO<sub>2</sub> system with a  
 18 MRS, an Artificial Neural Network (ANN) was developed according to the modelling technique used by  
 19 Aprea et al. (2017). The scheme of the ANN-based model is depicted in Figure 4. It is characterized by a  
 20 multi-layer ANN with four input nodes, one hidden layer and two output nodes. The model provides the  
 21 COP of the magnetic refrigerator ( $COP_{MR}$ ), as well as the cooling capacity ( $\dot{Q}_{MR}$ ), depending on the  
 22 temperature span ( $\Delta T_{span}$ ), the rotational frequency of the magnets ( $f_{mag}$ ), the mass flow of the heat  
 23 transfer fluid ( $\dot{m}_{ht,MR}$ ) and the temperature of the environment ( $T_{env}$ ). It should be noted that due to  
 24 some experimental limitations, the minimum mass flow rate of the heat transfer fluid was 0.08 kg·s<sup>-1</sup>.  
 25 However, the results were extrapolated by using the ANN model also to lower mass flow rates assuming  
 26 the same trends observed experimentally in the range of mass flow rates between 0.08 kg·s<sup>-1</sup> and 0.12  
 27 kg·s<sup>-1</sup>.

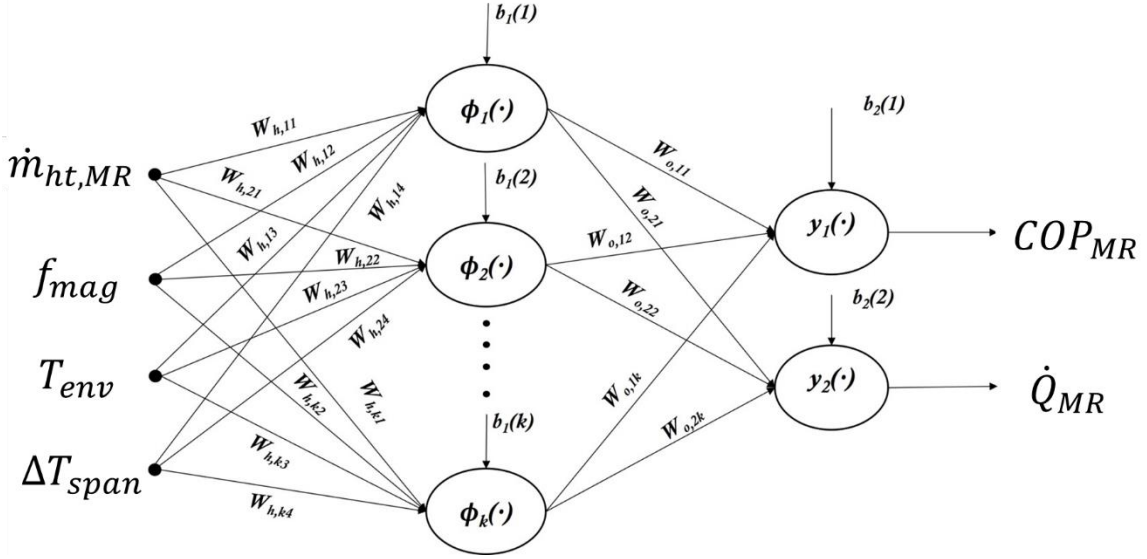


Figure 4. Scheme of the ANN model for the MR system.

1  
2

3 The outputs of the ANN model are mathematically represented by Eq.(6-8), as follows:

$$\begin{aligned}
 COP_{MR}(\dot{m}_{ht,MR}, f_{mag}, T_{env}, \Delta T_{span}) \\
 = COP_{MR,min} + \frac{COP_{MR,max} - COP_{MR,min}}{2} \cdot (1 \\
 + \sum_{k=1}^{N_h} (w_{1k} \cdot \tanh \tanh (v_h(\dot{m}_{ht,MR}, f_{mag}, T_{env}, \Delta T_{span}))) \\
 + b_{j=1}) \quad (6)
 \end{aligned}$$

$$\begin{aligned}
 \dot{Q}_{MR}(\dot{m}_{ht,MR}, f_{mag}, T_{env}, \Delta T_{span}) \\
 = \dot{Q}_{MR,min} + \frac{\dot{Q}_{MR,max} - \dot{Q}_{MR,min}}{2} \cdot (1 \\
 + \sum_{k=1}^{N_h} (w_{2k} \cdot \tanh \tanh (v_h(\dot{m}_{ht,MR}, f_{mag}, T_{env}, \Delta T_{span}))) \\
 + b_{j=2}) \quad (7)
 \end{aligned}$$

4 where:

$$\begin{aligned}
 v_h(\dot{m}_{ht,MR}, f_{mag}, T_{env}, \Delta T_{span}) \\
 = w_{k1} \cdot \frac{\dot{m}_{ht,MR,min} + \dot{m}_{ht,MR,max} - 2\dot{m}_{ht,MR}}{\dot{m}_{ht,MR,min} - \dot{m}_{ht,MR,max}} + w_{k2} \\
 \cdot \frac{f_{mag,min} + f_{mag,max} - 2f_{mag}}{f_{mag,min} - f_{mag,max}} + w_{k3} \\
 \cdot \frac{T_{env,min} + T_{env,max} - 2T_{env}}{T_{env,min} - T_{env,max}} + w_{k4} \\
 \cdot \frac{\Delta T_{span,min} + \Delta T_{span,max} - 2\Delta T_{span}}{\Delta T_{span,min} - \Delta T_{span,max}} + b_k \quad (8)
 \end{aligned}$$

5 In the Eq. (6-8),  $w_{1k}$  and  $w_{2k}$  represent the parameters of the ANN (synaptic weights) connecting the k-th  
6 hidden neurons to the first and second output, i.e. COP and cooling capacity, respectively. On the other



1 hand,  $w_{k1}$ ,  $w_{k2}$ ,  $w_{k3}$  and  $w_{k4}$  represent the synaptic weights connecting each input node to the k-th  
 2 hidden neuron. The terms  $b_k$ ,  $b_{j=1}$  and  $b_{j=2}$  are the biases of the hidden and output neurons,  
 3 respectively. The subscripts *min* and *max* are referring to the minimum and maximum value of each  
 4 input and output collected during the experimental tests and they were used to normalize inputs and  
 5 outputs before the training phase of the ANN (Jayalakshmi and Santhakumaran, 2011).

6 The learning phase was performed in MatLab environment by using the Levenberg-Marquadt algorithm  
 7 (Suratgar et al., 2007). The size of the hidden layer  $N_h$  can be chosen following trial-and-error procedures  
 8 (Mohanraj et al., 2012; Sheela and Deepa, 2013) or some empirical rules (Hunter et al., 2012), but an  
 9 iterative trial-and-error routine was implemented in this case, using the determination coefficient ( $R^2$ ) to  
 10 identify the optimal structure of the ANN, calculated as follows:

$$R^2 = 1 - \frac{\sum_{j=1}^{N_o} \sum_{m=1}^p (Y_{j,m} - y_{j,m})^2}{\sum_{j=1}^{N_o} \sum_{m=1}^p (Y_{j,m})^2} \quad (9)$$

11 where  $Y_{j,m}$  is the target value of the j-th output for the m-th example,  $y_{j,m}$  is the output value predicted by  
 12 the ANN of the j-th output for the m-th example,  $N_o$  is the number of output nodes and  $p$  is the number of  
 13 experimental data pattern. The training routine was performed changing the number of hidden neurons  
 14 between 9 and 50, leading to an optimal ANN structure characterized by 27 hidden neurons, which  
 15 provided an  $R^2$  equal to 0.9960, with an average deviation of 4.8 W and 0.04 for the cooling capacity and  
 16 COP, respectively. To show the results of the training process, the values of the synaptic weights before  
 17 and after the training are shown in Appendix A.1, as well as an excerpt of the computer code pertaining  
 18 Eq. 6-8 (Appendix A.2).

19 To integrate the ANN model of the MR prototype into the CO<sub>2</sub> model, some additional information about  
 20 the temperatures of the heat transfer fluid were also required. In detail, the temperature of the heat  
 21 transfer fluid exiting the hot side of the AMR ( $T_{ht,H,o}$ ), is obtained considering an approach temperature  
 22 with the environmental temperature, as stated in Eq. (10). In several numerical models of AMR cycles  
 23 (Aprea and Maiorino, 2010; Nakashima et al., 2022; Nielsen et al., 2009; Trevizoli et al., 2016; Tušek et  
 24 al., 2011), the external heat exchangers were considered as ideal, i.e., with an approach temperature  
 25 equal to 0. However, in this semi-empirical approach, it was fixed to be equal to 2 K since the maximum  
 26 approach temperature measured during the experimental tests was equal to 1.8 K (Aprea et al. (2016)).

$$T_{ht,H,o} = T_{env} + APP \quad (10)$$

27 To ensure the heat transfer in the subcooler and being able to subcool the CO<sub>2</sub>, the heat transfer fluid  
 28 entering the subcooler must always be at a lower temperature than the desired one in the CO<sub>2</sub> subcooler  
 29 exit. Thus, the  $T_{ht,C,o}$  is defined as:

$$T_{ht,C,o} = T_{sub,o} - \Delta T \quad (11)$$

30 The  $\Delta T$  in Eq. (11) is considered to be equal to 2 K.

31 In the end, for the simulation of the MR prototype, some boundary conditions for the input parameters  
 32 were considered based on the experimental results (Aprea et al., 2016). They are presented in Table 2. In  
 33 detail, the mass flow rate of the heat transfer fluid and the rotational frequency of the magnets, which  
 34 represent the operative parameters of the MR device, are varied by steps of 0.005 kg·s<sup>-1</sup> and 0.1 Hz,  
 35 respectively.

36 Table 2. Boundary values of the input parameters of the ANN model for the MR system.

Input parameter	Boundary values
-----------------	-----------------

$\dot{m}_{ht,MR} (kg \cdot s^{-1})$	0.02-0.12
$f_{mag} (Hz)$	0.2-0.8
$T_{env} (^{\circ}C)$	16-35
$\Delta T_{span} (K)$	0-11

1

2

### 2.3. Overall system

3

4

5

6

7

8

9

10

11

12

13

14

15

16

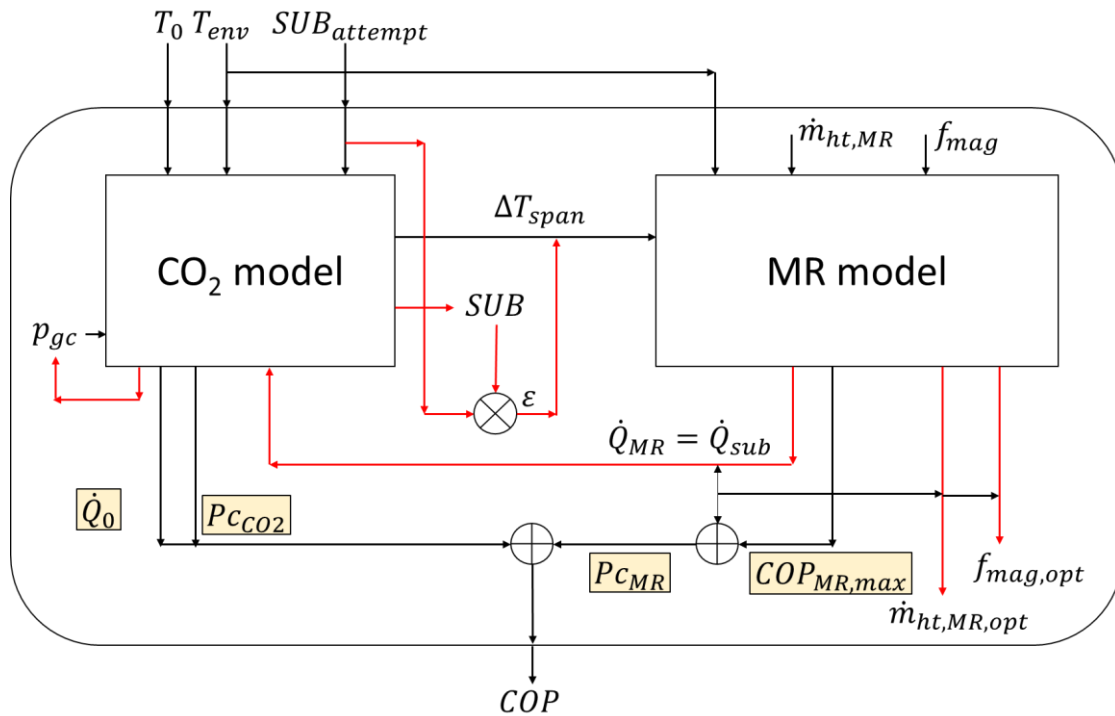
17

18

19

20

Both systems are thermally coupled through the subcooler (Figure 1) and the scheme of the overall model is shown in Figure 5. In detail, it is composed of two sub-models, namely the CO<sub>2</sub> and the MR sub-models as described in the previous Sections. The simulation procedure is as follows. Environmental and evaporation temperatures are the input values of the overall model. The gas-cooler pressure is the input value of the CO<sub>2</sub> model, and it follows an iteration procedure to find the optimal pressure for which the COP is maximum. The value of subcooling degree is also initialized to start the evaluation of the MR system performance. From an initial subcooling degree ( $SUB_{attempt}$ ), the temperature span is calculated and the ANN model simulating the behaviour of the MR prototype is run to find its maximum performance ( $COP_{MR,max}$ ), corresponding to a certain value of mass flow rate of the heat transfer fluid and rotational frequency of the magnets ( $\dot{m}_{ht,MR,opt}$  and  $f_{mag,opt}$ , respectively). As a result, the cooling capacity provided by the MR to the subcooler ( $\dot{Q}_{MR} = \dot{Q}_{sub}$ ) is also obtained. Then, from this data, a new subcooling degree value, which can be achieved with the provided cooling capacity, is recalculated, leading to a new value of the temperature span. This process is iteratively computed until the difference between the new subcooling degree and the previous one is less than a pre-defined value ( $\varepsilon = 0.01$ ) or the maximum number of iterations (equal to 30) is achieved. The latter means that no solution can be found, and the corresponding operating point is not feasible. The iterative process based on the bisection method is marked with red arrows. When the iteration is finished, the overall COP of the coupled system is obtained.

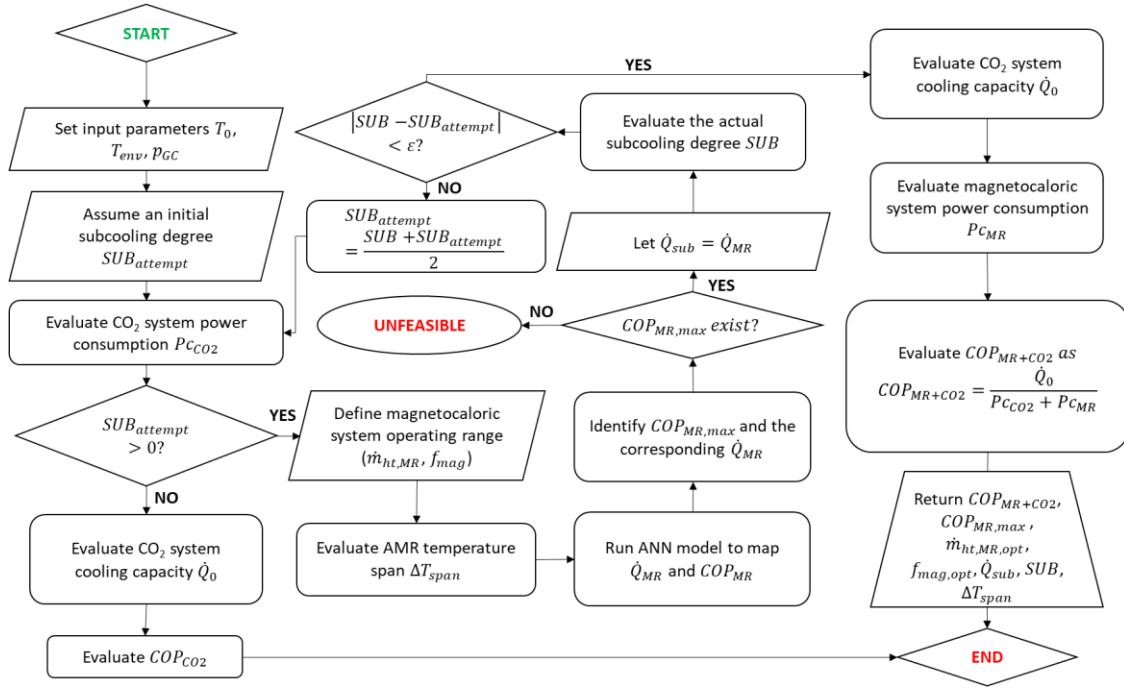


21

22

Figure 5. Scheme of the overall computational model.

1 The complete flow-chart of the coupled model is shown in Figure 6, where the detailed explanation of the  
 2 computational steps is provided.



3  
 4

Figure 6. Flow-chart of the coupled model

5 Power consumption of the CO<sub>2</sub> cycle is evaluated based on the isentropic efficiency, the difference  
 6 between discharge isentropic enthalpy and suction enthalpy and the mass flow rate, as shown in Eq. (12).

$$P_{CO_2} = \dot{m}_{CO_2} \frac{h_{dis,s} - h_{suc}}{\eta_i} \quad (12)$$

7

8 Power consumption of the MRS cycle is obtained from the relation between the COP of the MR system  
 9 and the cooling capacity needed in the subcooler. Cooling capacity in the subcooler is evaluated by Eq.  
 10 (13). The COP<sub>MR</sub> is referring to the overall electrical power absorbed by 8MAG, from the electrical motor  
 11 and the pump.

$$\dot{Q}_{sub} = \dot{m}_{CO_2} \cdot (h_{gc,o} - h_{sub,o}) \quad (13)$$

12

13 Then, power consumption of the MRS system is evaluated using Eq. (14). COP<sub>MR</sub> is evaluated by the ANN  
 14 model trained on the experimental data, as shown in Eq. (6). The experimental COP was calculated  
 15 considering the cooling capacity provided by the system and the power absorbed by the pump, used to  
 16 move the heat transfer fluid, and the electric motor used to move the magnets.

$$P_{AMR} = \frac{\dot{Q}_{sub}}{COP_{MR}} \quad (14)$$

17 The overall COP of the system of CO<sub>2</sub> with MRS is evaluated as expressed by Eq. (15).

$$COP = \frac{\dot{Q}_0}{P_{CCO_2} + P_{CMR}} = \frac{\dot{m}_{CO_2} \cdot (h_{suc} - h_{sub,o})}{P_{CCO_2} + P_{CMR}} \quad (15)$$

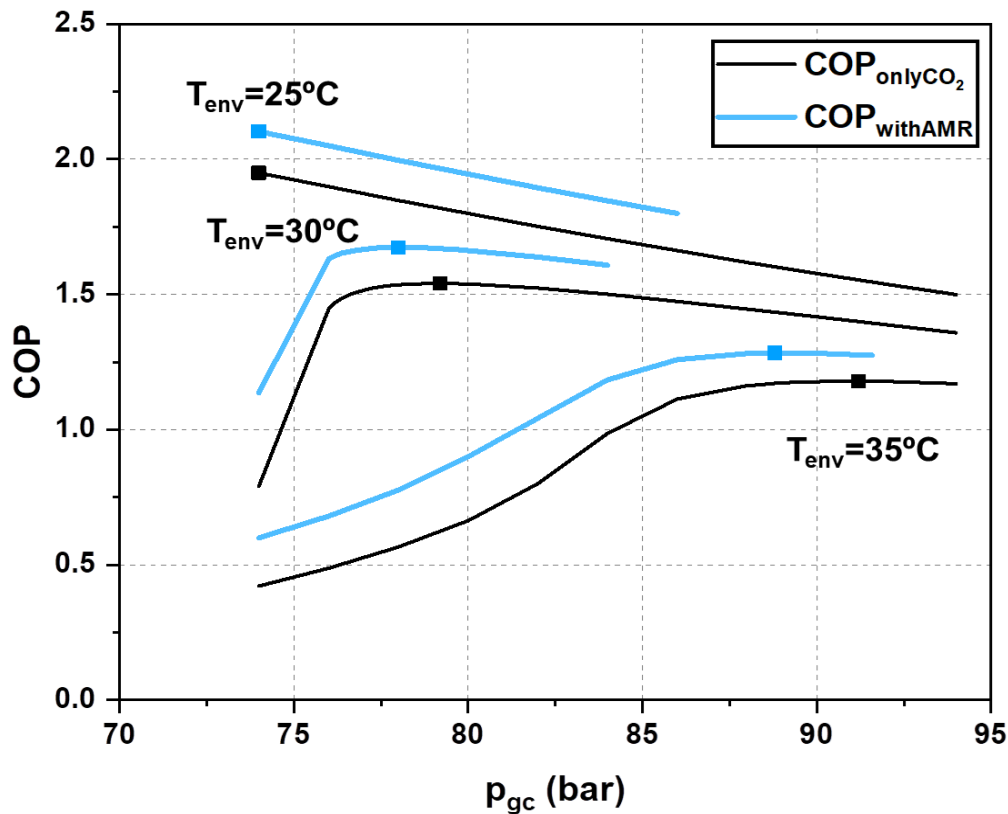
### 1 3. Results and discussion

2 This section explains the optimum operating conditions of the hybrid system and summarizes the main  
3 energy performances of the transcritical CO<sub>2</sub> cycle coupled with a MRS system.

#### 4 3.1. Optimum operating conditions

5 The aim of this section is to demonstrate the existence of a maximum COP that can be obtained by  
6 identifying the optimum gas-cooler pressure and optimum subcooling degree, which corresponds to the  
7 value that maximizes the performance of the MRS. First, the impact of the gas-cooler pressure on the  
8 performance of the coupled system is analysed. Then, the operating parameters of the MRS system,  
9 which are mass flow rate of the heat transfer fluid and rotating frequency of the magnets, are investigated  
10 with the aim to optimize its performance.

11 Figure 7 shows the overall COP as a function gas-cooler pressure obtained at the operating conditions  
12 corresponding to the maximum COP of the MR system, at an evaporating level of -15 °C and  
13 environmental temperatures ranging from 25 to 35 °C. The COP is calculated for the CO<sub>2</sub> cycle without  
14 subcooling (black) and for the hybrid system at maximum MR performance (blue). The points with  
15 maximum COP are marked with the squared points. As it can be observed, there is an optimum gas-cooler  
16 pressure for each condition. Considering the environmental temperature of 25 °C, it can be seen that the  
17 optimum gas-cooler pressure corresponds to the critical pressure. Then, when the environmental  
18 temperature increases, the optimum pressure also increases. When comparing to the system without  
19 subcooling, reduction in the optimal pressures can be observed, since the COP at the maximal point is  
20 higher and it is achieved at a lower pressure. For example, the optimal pressure for the base system at 35  
21 °C is 91.2 bar while if the subcooling is performed (in blue), the optimal pressure is 88.8 bar. This is one of  
22 the benefits of the subcooling of the CO<sub>2</sub>, working pressures can be reduced.

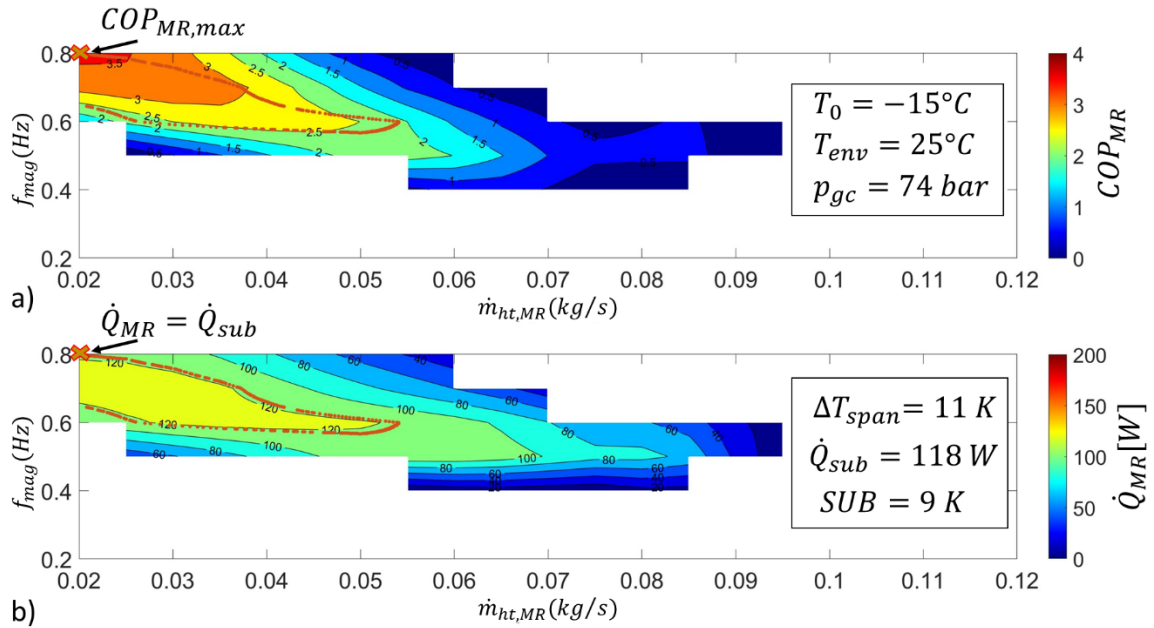


1

2 Figure 7. Overall COP evolution as a function of the gas-cooler pressure for different environmental temperatures  
3 and evaporation temperature of  $-15^\circ C$ .

4 The COP of the MR system depends on the mass flow rate of the heat transfer fluid and the MR  
5 frequency, which depends on the rotational frequency of the magnets in this case. According to the  
6 experimental data (Table 2), values of mass flow rate were changed between  $0.02$  to  $0.12 \text{ kg}\cdot\text{s}^{-1}$ , whereas  
7 values of rotational frequency of the magnets were varied between  $0.2$  and  $0.8 \text{ Hz}$ , with the aim to identify  
8 the optimal combination of operating parameters to maximize the performance of the MRS in terms of the  
9 COP for different operating conditions. In detail, simulations were performed for different environmental  
10 temperatures from  $25$  to  $35 \text{ }^\circ C$ , and for each of them, the optimal operating point was identified.

11 As an example, Figure 8 shows the behaviour of the COP (Fig. 8a) and the cooling capacity (Fig. 8b) of  
12 the MRS depending on the mass flow rate of the heat transfer fluid and the rotational frequency of the  
13 magnets for an evaporating temperature of  $-15 \text{ }^\circ C$ , with an environmental temperature of  $25 \text{ }^\circ C$  and a gas-  
14 cooler pressure of  $74 \text{ bar}$ . The temperature span and environmental temperature are the input values of  
15 the MRS model, which affect the possible combinations of mass flow rate of the heat transfer fluid and  
16 rotational frequency of the magnets. Therefore, for each environmental temperature, the temperature span  
17 is the only parameter which defines the possible coupling of operating parameters values. Among all these  
18 possible operating points, the one that provides the maximum COP of the MRS ( $COP_{MR,max}$ ) is selected.  
19 The cooling capacity supplied with the corresponding operating conditions is used to recalculate the new  
20 subcooling degree provided to the  $CO_2$  system. Considering the example shown in Figure 7, the maximum  
21 COP is  $3.6$  with a cooling capacity of  $118 \text{ W}$  and a provided subcooling degree of  $9 \text{ K}$ . This optimal  
22 condition of the MRS, marked with the cross symbol, is achieved with a mass flow rate equal to  $0.02 \text{ kg}\cdot\text{s}^{-1}$   
23 and a rotating frequency of the magnets of  $0.8 \text{ Hz}$ , corresponding to a frequency of the AMR cycle equal  
24 to  $1.6 \text{ Hz}$ .



1

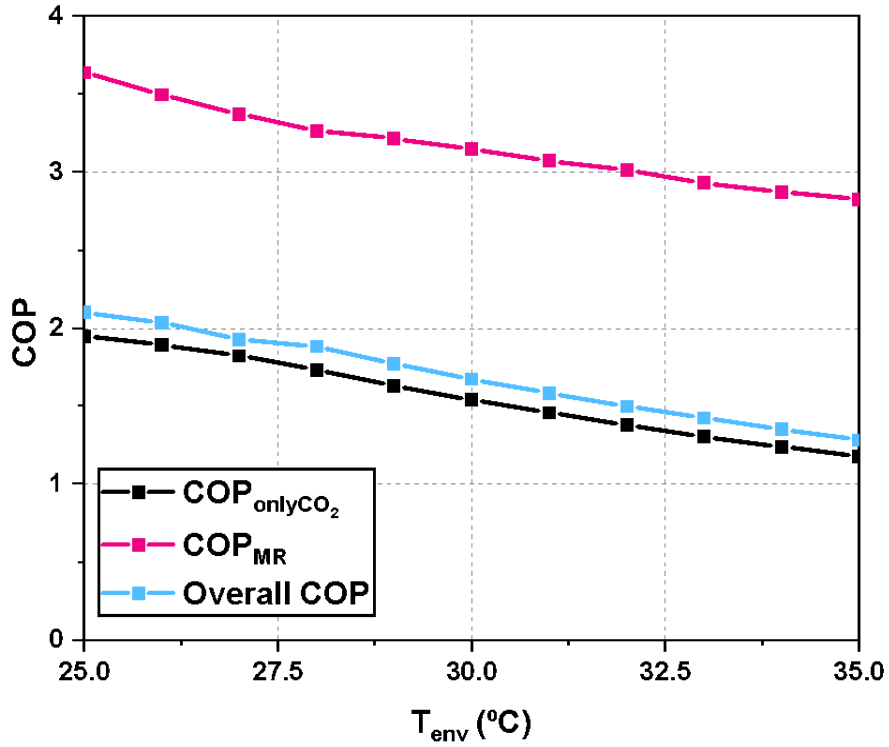
2 Figure 8. 2-D map of the performance of the magnetic refrigerator depending on the mass flow rate of the heat  
 3 transfer fluid and rotating frequency of the magnets with  $T_{env} = 25^\circ C$ ,  $T_0 = -15^\circ C$  and  $p_{gc} = 74 \text{ bar}$ :  $COP_{MR}$   
 4 and b) cooling capacity.  
 5

6 It is worth to highlight that the MRS could work also with different combinations of operating parameters  
 7 (see the orange line in both Figure 8a-8b). The latter are characterized by different values of temperature  
 8 span, and consequently of different subcooling degree. However, any other possible combinations lead to  
 9 decrease in the performance of the magnetic refrigeration system, and therefore they are not considered.

10 As illustrated in this section, for each condition, there is a value of gas-cooler pressure and  $COP_{MR}$   
 11 (consequently of subcooling degree) that maximizes the COP of the overall system. Therefore, from now  
 12 on, all the data presented in the following sections are optimized in terms of gas-cooler pressure and  
 13 subcooling degree.

### 14 3.2. COP and performance improvements

15 The overall COP for the optimal gas-cooler pressure and optimum  $COP_{MR}$  are shown in Figure 9. The  
 16 COP of the MR corresponding to these optimum points is also presented in this figure. COP of each cycle  
 17 decreases when the environmental temperature increases. Introducing the subcooling allows to obtain an  
 18 overall COP higher than the COP of the single  $CO_2$  cycle because the COP of the MR device is higher.



2

3

Figure 9. Overall COP and COP of each of the systems individually at optimum conditions.

4

As it has been said in the introduction, the hybrid of the MR technology with the transcritical CO<sub>2</sub> cycle has the objective of improving the energy performance of the cycle, as it is sought with other subcooling systems. In this work, the optimal operating parameters of the combination of these two technologies have been analysed and the energy performance of the global cycle has been determined. Figure 10 shows the increment in COP achieved by the MRS-CO<sub>2</sub> system with respect to the CO<sub>2</sub> system without subcooling for all the evaluated range. The increments are calculated as shown in Eq. (16).

5

6

7

8

9

$$\Delta COP (\%) = \frac{COP - COP_{onlyCO_2}}{COP_{onlyCO_2}} \quad (16)$$

10

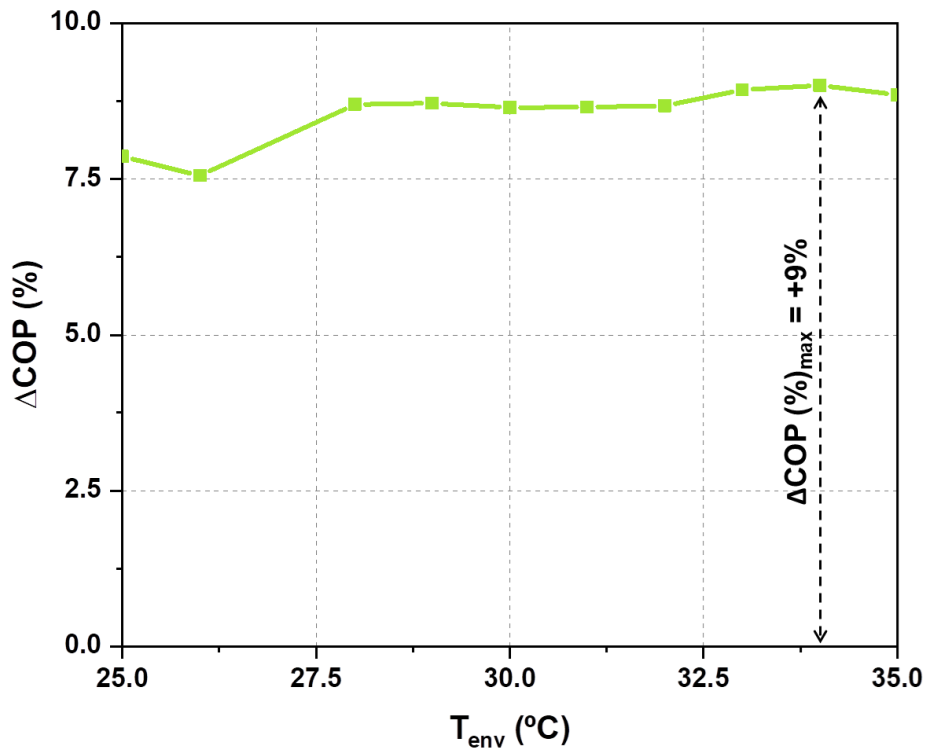
11

As it can be observed, from 25 °C to 27 °C there is a reduction in the improvement achieved and then it increases up to values of around 8.8% for the entire temperature range from 28 °C to 35 °C with a maximum COP increment of 9.0% obtained at 34 °C.

12

13

14



1

2

Figure 10. Increments in COP with respect to base system.

3

Subcooling should provide higher increments at higher ambient temperature (Nebot-Andrés et al., 2022), which is not true in this case. This is because the MR system is designed to work at lower ambient temperatures and its performance at temperatures above 30 °C is decreasing. This is closely related to the properties of the MCM used in the MR. In the following section, the operation of the MR will be analysed in detail to really understand what happens at these temperatures.

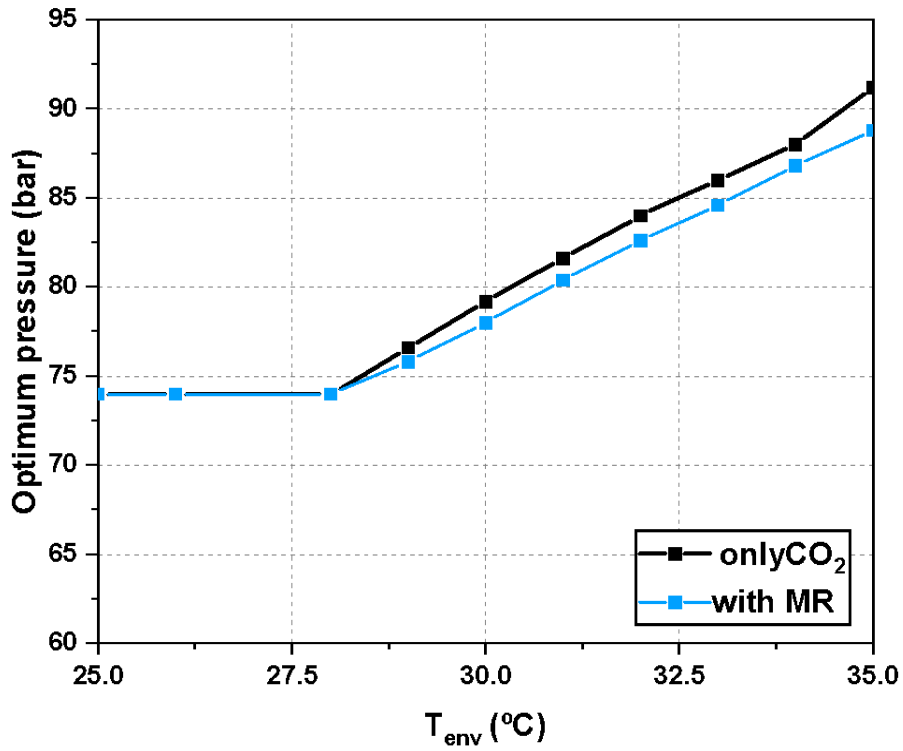
7

8

Figure 11 shows the optimum gas-cooler pressure for the MCS-CO<sub>2</sub> system and the system without subcooling depending on the environmental temperature. As it can be seen, optimum gas-cooler pressure increases with the environmental temperature, which is the normal behavior concerning CO<sub>2</sub> systems (Sarkar et al., 2004). The subcooling of CO<sub>2</sub> leads to a reduction of the optimal pressure, as it can be seen in Figure 11. Figure 12 sums up all the pressure reductions achieved by the use of the MRS. Optimal pressure between 25 and 28 °C is equal to the critical pressure and it is the same for both systems (with and without MR system), so the pressure reduction is equal to 0. It can be seen that when the temperature is higher than 29 °C, the subcooling allows pressure reductions of up to 2.4 bar.

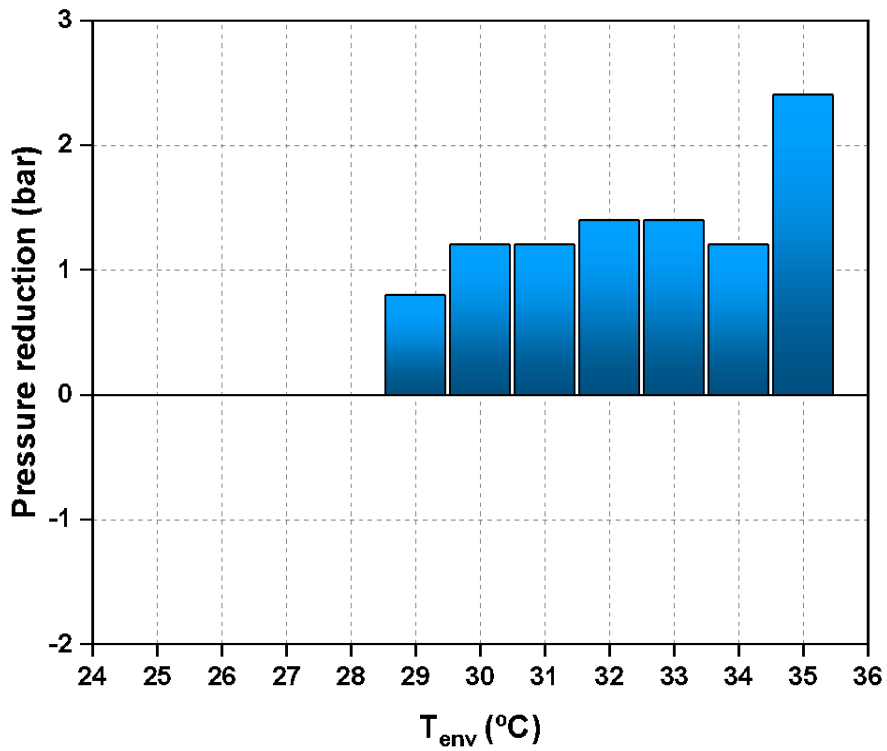
15





1

2 Figure 11. Optimum gas-cooler pressures for the MRS-CO<sub>2</sub> system (blue) and the system without subcooling (black).



3

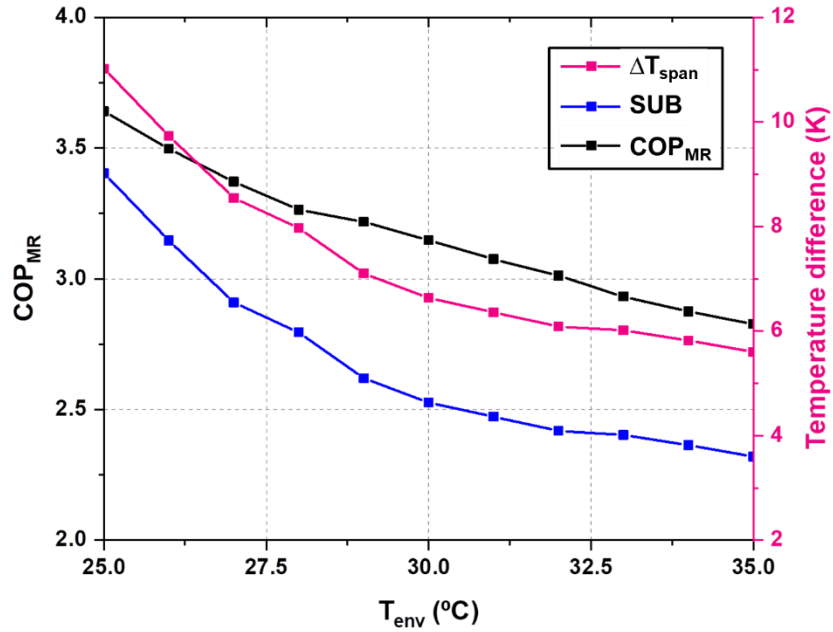
4

Figure 12. Pressure reduction with respect to the CO<sub>2</sub> system without subcooling.

5

6

1 Figure 13 shows the maximum COP of the MR system and the corresponding temperature span and  
 2 subcooling degree for different environmental temperature. The MRS is always considered to work with  
 3 operating parameters which maximise its COP, as shown in Section 3.1. The simulations were performed  
 4 for different environmental temperatures (between 25 °C and 35 °C), which allowed to analyse the  
 5 behavior of the optimal operating parameters according to different ambient conditions. In detail, the  
 6 behavior of mass flow rate of the heat transfer fluid, the rotating frequency of the magnets, and the  
 7 temperature span, and consequently the subcooling degree, are investigated considering optimal working  
 8 conditions (i.e. maximum COP of the MRS).



9

10 Figure 13. COP of the magnetocaloric system operating in optimal conditions and corresponding temperature span  
 11 and subcooling degree.

12 From Figure 13 it is seen how the increase of the environmental temperature leads to decrease the  
 13 maximum COP of the MRS, and subsequently also the overall COP of the coupled system.

14 Combining Eq. (10) and Eq. (11) with Eq. (5), the temperature span can be described as a function of the  
 15 environmental temperature, the subcooler outlet temperature and the approach considered on the  
 16 subcooler, as described in Eq. (17)

$$\Delta T_{span} = T_{env} + APP - T_{sub,o} + \Delta T \quad (17)$$

17 Using Eq. (2) and Eq. (1), the Eq. (18) is obtained, where it can be observed that the temperature span  
 18 only depends on the subcooling degree and the temperature approach considered in subcooler.

$$\Delta T_{span} = SUB + \Delta T \quad (18)$$

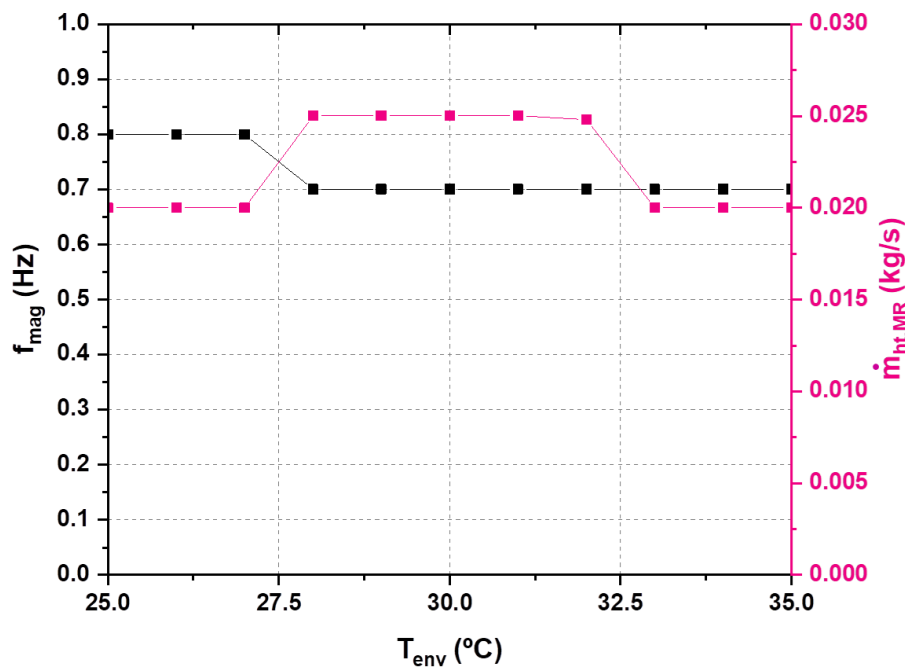
19 As it can be seen from Eq. (18), the temperature span is directly related to the subcooling degree. This is  
 20 important because higher subcooling degree is favourable for the CO<sub>2</sub> system, but on the other hand,  
 21 increasing the temperature span results in smaller COP of the MR device. It is therefore necessary to find  
 22 the compromise between both parameters that maximizes the performance of the overall system.

23 As the COP of the MRS, the temperature span and subcooling degree decrease as the environmental  
 24 temperature increases as well. From a point of view, the temperature span behaves accordingly to the  
 25 data in the literature (Aprea et al., 2014; Lozano et al., 2016), showing a negative slope for environmental

1 temperatures higher than 25 °C, since the system is working away from the Curie temperature of  
 2 Gadolinium, which is 20 °C. On the other hand, the behavior of the optimal subcooling degree seems to  
 3 be different from the classical behavior reported in the literature for CO<sub>2</sub> systems improved with subcooling  
 4 methods (Nebot-Andrés et al., 2020a; Nebot-Andrés et al., 2021b). Indeed, most of them are  
 5 characterized by a larger optimal subcooling degree for higher environmental temperatures since the CO<sub>2</sub>  
 6 without subcooling has lower performance at high hot sink temperatures. Therefore, there is more room  
 7 for improvements. In detail, the MCM adopted in the MRs can strongly limits the performance of the  
 8 overall system. Indeed, the MRS should optimally work around the Curie temperature of the MCM (which  
 9 is the temperature where the magnetocaloric effect achieves the maximum value), but its performance can  
 10 quickly drop down when the working temperature range is far from the Curie temperature. The maximum  
 11 performance can be achieved when the systems work as close as possible to the Curie temperature of the  
 12 material, which is the temperature where the magnetocaloric effect achieves its maximum value since the  
 13 material undergoes the most significant phase transition.

14 Therefore, to improve performance at environmental temperatures close to 30 °C or higher, it would be  
 15 necessary to design AMRs made of a MCM that has a Curie temperature close to the working point, or to  
 16 design a multi-layer regenerator which could provide optimal performance for a wider range of working  
 17 conditions (Maiorino et al., 2021). With this, it would be possible to further increase the COP  
 18 improvements presented in Figure 10.

19 Figure 14 shows the optimal values of the rotating frequency of the magnets and mass flow rate of the  
 20 heat transfer fluid that allows obtaining the maximum COP of the MRS with different environmental  
 21 temperatures. As it can be seen, almost all the points correspond to the upper limit of the working  
 22 frequency and the minimum value of the mass flow rate. It means that it is necessary to work very close  
 23 to the limits of the experimental apparatus to achieve the optimal conditions. Therefore, it is required  
 24 to design a MRS dedicated to the specified CO<sub>2</sub> system with the aim to further improve the overall  
 25 performance and to reduce the mechanical stress on the device.



26

27 Figure 14. Optimal values of the rotating frequency of the magnets and mass flow rate of the heat transfer fluids to  
 28 achieve the maximum COP of the MRS with different environmental temperatures.

29

### 3.3. Comparison with other methods

Table 3 shows the COP at 25 °C of environmental temperature and an evaporation temperature of -15 °C for different CO<sub>2</sub> systems: cycle with parallel compression, cycle with integrated mechanical subcooling (IMS) and with dedicated mechanical subcooling (DMS). All the presented values are optimized and obtained from experimental studies. The COP of the MRS cycle obtained in this work is also included. As it can be seen, the COP values obtained with the MRS system are higher than with the other systems, even if the basis of comparison is not the same, because the parallel compression, the DMS and the IMS are tested experimentally.

Table 3. COP and working conditions of different CO<sub>2</sub> systems at 25 °C of heat rejection temperature and evaporation temperature of -15 °C.

Method	Heat rejection temperature (°C)	T <sub>0</sub> (°C)	COP	p <sub>gc</sub> (bar)	SUB (K)	Type	Reference
Parallel compression	24.8	-14.9	1.84	74.4	-	E, O	(Nebot-Andrés et al., 2021a)
Dedicated mechanical subcooling	25.3	-15.5	1.95	74.9	14.3	E, O	(Nebot-Andrés et al., 2021b)
Integrated mechanical subcooling	25.2	-15.6	1.87	74.5	21.3	E, O	(Nebot-Andrés et al., 2020a)
Magnetic refrigeration subcooling	25.0	-15.0	2.10	74.0	9.01	S	This work

E = Experimental, S = Semiempirical, O = Optimized cycle

Performing experimental test of the MRS can lead to lower COP values, but on the other hand designing and optimizing the MR device for this specific application should increase the performance values (Dall'Olio et al., 2021; Masche et al., 2022). The results presented in this work only show the performance that can be achieved with the current state of two existing technologies. This allows establishing a starting point and demonstrating the future potential of this hybridization. Even if these results are not directly comparable with other technologies experimental results, this path can be promising for future investigations and shows that it is a promising technology and that it is worth focusing on its technological development in order to optimize and improve it.

#### 4. Conclusions

This paper presents for the first time a new hybrid system consisting of a CO<sub>2</sub> transcritical refrigeration plant with a magnetic refrigeration system providing subcooling. Magnetic refrigeration is an emerging technology that provides high COPs when working with small temperature span. The temperature differences of the hot sink and cold source of the dedicated subcooling methods are very small, which makes magnetic refrigeration a potential technology for the energy improvement of CO<sub>2</sub> cycles. The evaluation of the fusion of these two technologies has been performed based on previous experimental data.

The study covered in transcritical conditions environmental temperatures from 25 °C to 35 °C for the evaporating temperature of -15 °C. The studied cycle is a transcritical CO<sub>2</sub> single stage refrigeration system with the expansion process performed in two stages with a liquid receiver between both. The magnetic refrigeration system is simulated based on the experimental data obtained from a single-layer packed-bed Gadolinium magnetic refrigerator prototype.

The analysis of the cycle has allowed affirming that there are several variables that must be optimized in this type of systems: the degree of subcooling (related to the temperature span in the magnetic refrigerator), the gas-cooler pressure and the heat transfer fluid mass flow ratio and frequency of the AMR that allow to obtain maximum COP of the magnetic refrigerator. These optimal parameters have been identified and once known, the main energetic parameters of the system have been obtained.

1 Finally, values of COP are obtained for different ambient conditions reaching values from 2.10 to 1.28 at  
2 25.0 °C and 35.0 °C of environmental temperature, respectively. The optimum COP is compared with the  
3 COP of the system without subcooling and increments up to 9.0% are obtained for an environmental  
4 temperature of 34 °C with respect to the cycle without subcooling.

5 Optimum gas-cooler pressure is equal to the critical pressure when the environmental temperature is  
6 lower than 28 °C and, above this temperature it is increasing with temperature, following the same trend  
7 as other CO<sub>2</sub> systems. However, this pressure is lower than the optimal pressure for the CO<sub>2</sub> system  
8 without subcooling, which is another advantage of the use of this technology.

9 However, these improvements can be even higher because the magnetic refrigerator prototype was not  
10 designed for this purpose. By designing a specific prototype to work in this application and optimizing the  
11 magnetic refrigerator by using magnetocaloric materials with optimal Curie temperatures required for  
12 subcooling system, a significant increase in efficiency could be obtained. The possibility of recovering  
13 expansion work to drive the MR system and improve overall performance can also be explored.

14 In conclusion, a new subcooling method that produces significant improvements in COP and gives a new  
15 application to magnetic refrigeration technology has been presented. Finally, we can affirm that the use of  
16 the magnetic refrigeration subcooling cycle as a path to improve the performance of CO<sub>2</sub> transcritical  
17 plants needs to be considered and should be studied deeper. Additionally, it needs to be mentioned that  
18 these results are semi-empirical and experimental research with the design of a new prototype would be  
19 required to quantify exactly the improvements that can be achieved. What is important to conclude is that  
20 the technology proposed in this work allows opening a new door to research and, with more future  
21 developments, very competitive results can be reached.

## 22 **Acknowledgements**

23 The authors thank the Ministerio de Educación, Cultura y Deportes - Spain (grant FPU16/00151) and the  
24 Ministerio de Universidades – Spain (grant EST21/00293) for financing this research work.

## 25 **References**

26 Aprea, C., Greco, A., Maiorino, A., 2015. The application of a desiccant wheel to increase the energetic  
27 performances of a transcritical cycle. *Energy Conversion and Management* 99, 222-230.

28 Aprea, C., Greco, A., Maiorino, A., 2017. An application of the artificial neural network to optimise the  
29 energy performances of a magnetic refrigerator. *International Journal of Refrigeration* 82, 238-251.

30 Aprea, C., Greco, A., Maiorino, A., Masselli, C., 2016. The energy performances of a rotary permanent  
31 magnet magnetic refrigerator. *International Journal of Refrigeration* 61, 1-11.

32 Aprea, C., Greco, A., Maiorino, A., Mastrullo, R., Tura, A., 2014. Initial experimental results from a rotary  
33 permanent magnet magnetic refrigerator. *International Journal of Refrigeration* 43, 111-122.

34 Aprea, C., Maiorino, A., 2010. A flexible numerical model to study an active magnetic refrigerator for near  
35 room temperature applications. *Applied Energy* 87, 2690-2698.

36 Arora, A., Singh, N.K., Monga, S., Kumar, O., 2011. Energy and exergy analysis of a combined  
37 transcritical CO<sub>2</sub> compression refrigeration and single effect H<sub>2</sub>O-LiBr vapour absorption system.  
38 *International Journal of Exergy* 9, 453-471.

39 Astrain, D., Merino, A., Catalán, L., Aranguren, P., Araiz, M., Sánchez, D., Cabello, R., Llopis, R., 2019.  
40 Improvements in the cooling capacity and the COP of a transcritical CO<sub>2</sub> refrigeration plant operating with  
41 a thermoelectric subcooling system. *Applied Thermal Engineering* 155, 110-122.

42 Barclay, J.A., 1982. Use of a ferrofluid as the heat-exchange fluid in a magnetic refrigerator. *Journal of*  
43 *Applied Physics* 53, 2887-2894.

44 Beshr, M., Bush, J., Aute, V., Radermacher, R., 2016. Steady state testing and modeling of a CO<sub>2</sub> two-  
45 stage refrigeration system with mechanical subcooler, *Refrigeration Science and Technology*, pp. 893-  
46 900.

- 1 Bush, J., Beshr, M., Aute, V., Radermacher, R., 2017. Experimental evaluation of transcritical CO<sub>2</sub>  
2 refrigeration with mechanical subcooling. *Science and Technology for the Built Environment* 23, 1013-  
3 1025.
- 4 Dall'Olio, S., Masche, M., Liang, J., Insinga, A.R., Eriksen, D., Bjørk, R., Nielsen, K.K., Barcza, A., Vieyra,  
5 H.A., Beek, N.V., Bez, H.N., Engelbrecht, K., Bahl, C.R.H., 2021. Novel design of a high efficiency multi-  
6 bed active magnetic regenerator heat pump. *International Journal of Refrigeration* 132, 243-254.
- 7 Eriksen, D., Engelbrecht, K., Bahl, C.R.H., Bjørk, R., Nielsen, K.K., Insinga, A.R., Pryds, N., 2015. Design  
8 and experimental tests of a rotary active magnetic regenerator prototype. *International Journal of*  
9 *Refrigeration* 58, 14-21.
- 10 European Commission, 2014. Regulation (EU) No 517/2014 of the European Parliament and of the  
11 Council of 16 April 2014 on fluorinated greenhouse gases and repealing Regulation (EC) No 842/2006.
- 12 Greco, A., Aprea, C., Maiorino, A., Masselli, C., 2019. A review of the state of the art of solid-state caloric  
13 cooling processes at room-temperature before 2019. *International Journal of Refrigeration* 106, 66-88.
- 14 Hafner A., Hemmingsen A. K. T., Ven A. van de, 2014. R744 refrigeration system configurations for  
15 supermarkets in warm climates, 3rd IIR International Conference on Sustainability and the Cold Chain.  
16 Proceedings. International Institute of Refrigeration, London, UK.
- 17 Hunter, D., Yu, H., Pukish Iii, M.S., Kolbusz, J., Wilamowski, B.M., 2012. Selection of proper neural  
18 network sizes and architectures-A comparative study. *IEEE Transactions on Industrial Informatics* 8, 228-  
19 240.
- 20 Jayalakshmi, T., Santhakumaran, A., 2011. Statistical Normalization and Backpropagation for  
21 Classification. *International Journal of Computer Theory and Engineering* 3, 1793-8201.
- 22 Kamran, M.S., Ahmad, H.O., Wang, H.S., 2020. Review on the developments of active magnetic  
23 regenerator refrigerators – Evaluated by performance. *Renewable and Sustainable Energy Reviews* 133.
- 24 Karampour, M., Sawalha, S., 2016. Integration of heating and air conditioning into a CO<sub>2</sub> trans-critical  
25 booster system with parallel compression Part I: Evaluation of key operating parameters using field  
26 measurements, *Refrigeration Science and Technology*, pp. 323-331.
- 27 Kim, M.H., Pettersen, J., Bullard, C.W., 2004. Fundamental process and system design issues in CO<sub>2</sub>  
28 vapor compression systems. *Progress in Energy and Combustion Science* 30, 119-174.
- 29 Lawrence, N., Elbel, S., 2016. Experimental study on control methods for transcritical CO<sub>2</sub> two-phase  
30 ejector systems at off-design conditions, *Refrigeration Science and Technology*, pp. 511-518.
- 31 Lawrence, N., Elbel, S., 2019. Experimental investigation on control methods and strategies for off-design  
32 operation of the transcritical R744 two-phase ejector cycle. *International Journal of Refrigeration*.
- 33 Lemmon, E.W., Huber, M.L., McLinden, M.O., 2013. REFPROP, NIST Standard Reference Database 23,  
34 v.9.1. National Institute of Standards, Gaithersburg, MD, U.S.A.
- 35 Llopis, R., Cabello, R., Sánchez, D., Torrella, E., 2015. Energy improvements of CO<sub>2</sub> transcritical  
36 refrigeration cycles using dedicated mechanical subcooling. *International Journal of Refrigeration* 55, 129-  
37 141.
- 38 Llopis, R., Nebot-Andrés, L., Cabello, R., Sánchez, D., Catalán-Gil, J., 2016. Experimental evaluation of a  
39 CO<sub>2</sub> transcritical refrigeration plant with dedicated mechanical subcooling. *International Journal of*  
40 *Refrigeration* 69, 361-368.
- 41 Llopis, R., Nebot-Andrés, L., Sánchez, D., Catalán-Gil, J., Cabello, R., 2018. Subcooling methods for CO<sub>2</sub>  
42 refrigeration cycles: A review. *International Journal of Refrigeration* 93, 85-107.
- 43 Lozano, J.A., Capovilla, M.S., Trevizoli, P.V., Engelbrecht, K., Bahl, C.R.H., Barbosa, J.R., Jr., 2016.  
44 Development of a novel rotary magnetic refrigerator. *International Journal of Refrigeration* 68, 187-197.
- 45 Maiorino, A., Del Duca, M.G., Tomc, U., Tušek, J., Kitanovski, A., Aprea, C., 2021. A numerical modelling  
46 of a multi-layer LaFeCoSi Active magnetic regenerator by using Artificial Neural Networks. *Applied*  
47 *Thermal Engineering* 197.
- 48 Masche, M., Liang, J., Dall'Olio, S., Engelbrecht, K., Bahl, C.R.H., 2021. Performance analysis of a high-  
49 efficiency multi-bed active magnetic regenerator device. *Applied Thermal Engineering* 199.

- 1 Masche, M., Liang, J., Engelbrecht, K., Bahl, C.R.H., 2022. Performance assessment of a rotary active  
2 magnetic regenerator prototype using gadolinium. *Applied Thermal Engineering* 204.
- 3 Mohanraj, M., Jayaraj, S., Muraleedharan, C., 2012. Applications of artificial neural networks for  
4 refrigeration, air-conditioning and heat pump systems - A review. *Renewable and Sustainable Energy*  
5 *Reviews* 16, 1340-1358.
- 6 Nakashima, A.T.D., Peixer, G.F., Lozano, J.A., Barbosa, J.R., Jr., 2022. A lumped-element magnetic  
7 refrigerator model. *Applied Thermal Engineering* 204.
- 8 Nebot-Andrés, L., Calleja-Anta, D., Sánchez, D., Cabello, R., Llopis, R., 2022. Experimental assessment  
9 of dedicated and integrated mechanical subcooling systems vs parallel compression in transcritical CO<sub>2</sub>  
10 refrigeration plants. *Energy Conversion and Management* 252, 115051.
- 11 Nebot-Andrés, L., Catalán-Gil, J., Sánchez, D., Calleja-Anta, D., Cabello, R., Llopis, R., 2020a.  
12 Experimental determination of the optimum working conditions of a transcritical CO<sub>2</sub> refrigeration plant  
13 with integrated mechanical subcooling. *International Journal of Refrigeration* 113, 266-275.
- 14 Nebot-Andrés, L., Sánchez, D., Calleja-Anta, D., Cabello, R., Llopis, R., 2020b. Experimental  
15 determination of the optimum working conditions of a commercial transcritical CO<sub>2</sub> refrigeration plant with  
16 a R-152a dedicated mechanical subcooling. *International Journal of Refrigeration*.
- 17 Nebot-Andrés, L., Sánchez, D., Calleja-Anta, D., Cabello, R., Llopis, R., 2021a. Experimental  
18 determination of the optimum intermediate and gas-cooler pressures of a commercial transcritical CO<sub>2</sub>  
19 refrigeration plant with parallel compression. *Applied Thermal Engineering* 189.
- 20 Nebot-Andrés, L., Sánchez, D., Calleja-Anta, D., Cabello, R., Llopis, R., 2021b. Experimental  
21 determination of the optimum working conditions of a commercial transcritical CO<sub>2</sub> refrigeration plant with  
22 a R-152a dedicated mechanical subcooling. *International Journal of Refrigeration* 121, 258-268.
- 23 Nielsen, K.K., Bahl, C.R.H., Smith, A., Bjørk, R., Pryds, N., Hattel, J., 2009. Detailed numerical modeling  
24 of a linear parallel-plate Active Magnetic Regenerator. *International Journal of Refrigeration* 32, 1478-  
25 1486.
- 26 Purohit, N., Sharma, V., Sawalha, S., Fricke, B., Llopis, R., Dasgupta, M.S., 2018. Integrated supermarket  
27 refrigeration for very high ambient temperature. *Energy* 165, 572-590.
- 28 Rigola, J., Ablanque, N., Pérez-Segarra, C.D., Oliva, A., 2010. Numerical simulation and experimental  
29 validation of internal heat exchanger influence on CO<sub>2</sub> trans-critical cycle performance. *International*  
30 *Journal of Refrigeration* 33, 664-674.
- 31 Sánchez, D., Aranguren, P., Casi, A., Llopis, R., Cabello, R., Astrain, D., 2020. Experimental  
32 enhancement of a CO<sub>2</sub> transcritical refrigerating plant including thermoelectric subcooling. *International*  
33 *Journal of Refrigeration* 120, 178-187.
- 34 Sarkar, J., Agrawal, N., 2010. Performance optimization of transcritical CO<sub>2</sub> cycle with parallel  
35 compression economization. *International Journal of Thermal Sciences* 49, 838-843.
- 36 Sarkar, J., Bhattacharyya, S., Gopal, M.R., 2004. Optimization of a transcritical CO<sub>2</sub> heat pump cycle for  
37 simultaneous cooling and heating applications. *International Journal of Refrigeration* 27, 830-838.
- 38 Sheela, K.G., Deepa, S.N., 2013. Review on methods to fix number of hidden neurons in neural networks.  
39 *Mathematical Problems in Engineering* 2013.
- 40 Suratgar, A.A., Tavakoli, M.B., Hoseinabadi, A., 2007. Modified Levenberg–Marquardt Method for Neural  
41 Networks Training. *International Journal of Computer and Information Engineering* 1, 1745 - 1747.
- 42 Trevizoli, P.V., Nakashima, A.T., Barbosa, J.R., Jr., 2016. Performance evaluation of an active magnetic  
43 regenerator for cooling applications – part II: Mathematical modeling and thermal losses. *International*  
44 *Journal of Refrigeration* 72, 206-217.
- 45 Tušek, J., Kitanovski, A., Prebil, I., Poredoš, A., 2011. Dynamic operation of an active magnetic  
46 regenerator (AMR): Numerical optimization of a packed-bed AMR. *International Journal of Refrigeration*  
47 34, 1507-1517.
- 48 UNEP/TEAP, 1999. The implications to the Montreal Protocol of the inclusion of HFCs and PFCs in the  
49 Kyoto Protocol, United States.

# 1 Appendix

## 2 A.1. Change in the synaptic weights after the training process

3 Table A.1 Values of the synaptic weights associated with the synapses connecting the inputs and hidden  
4 neurons before the training process

	$\dot{m}_{ht,MR}$	$f_{mag}$	$t_{env}$	$\Delta T_{span}$	b
N <sub>h1</sub>	-0.94339	-1.28974	3.093512	-0.73143	3.724436
N <sub>h2</sub>	-2.25921	-0.88536	1.305825	1.98148	3.728728
N <sub>h3</sub>	2.110337	-1.38509	2.122081	-1.49498	-3.49973
N <sub>h4</sub>	-2.66862	2.502554	-0.85602	-1.51053	3.138845
N <sub>h5</sub>	2.49411	-1.62092	1.256638	1.209305	-3.41745
N <sub>h6</sub>	0.802151	-1.92516	-2.54259	1.19148	-2.73693
N <sub>h7</sub>	-3.50477	0.587505	0.948588	0.722832	2.494405
N <sub>h8</sub>	2.41485	-0.04833	-2.9224	0.396487	-2.23194
N <sub>h9</sub>	-0.03053	0.628823	-0.12381	3.451798	2.838863
N <sub>h10</sub>	1.023528	-2.35545	1.201627	-2.5227	-2.096
N <sub>h11</sub>	2.027893	-1.19315	2.865414	0.096072	-2.10232
N <sub>h12</sub>	0.738652	3.090258	1.620318	-0.92976	-2.43767
N <sub>h13</sub>	-3.16404	-0.99316	1.196257	0.68544	2.350195
N <sub>h14</sub>	1.571177	0.9847	1.275998	2.674245	-2.10178
N <sub>h15</sub>	3.180997	-1.29248	0.597859	1.428929	-1.69083
N <sub>h16</sub>	-1.40471	1.244473	2.669367	-1.24769	1.99745
N <sub>h17</sub>	-1.89351	1.361074	2.286541	0.876853	1.063878
N <sub>h18</sub>	-2.16302	1.726045	-1.14477	-1.51649	1.295272
N <sub>h19</sub>	-1.58251	1.705661	-3.04074	-0.62023	0.682558
N <sub>h20</sub>	-1.76695	-2.60392	-1.68331	1.013477	0.814549
N <sub>h21</sub>	0.476023	1.893735	1.085511	2.796804	-1.07793
N <sub>h22</sub>	1.211608	-2.57687	1.241809	1.823479	-0.08729
N <sub>h23</sub>	0.321331	-2.00663	-1.98461	-2.18223	-0.52207
N <sub>h24</sub>	1.460218	0.816902	2.242903	2.09573	-0.49559
N <sub>h25</sub>	-1.85653	2.723629	1.495791	1.172397	0.292429



$N_{h26}$	2.575687	-0.91501	2.706861	-0.07508	0.152312
$N_{h27}$	0.339961	-3.67111	-0.41728	0.949812	-1.28354

1

2 Table A.2 Values of the synaptic weights associated with the synapses connecting the inputs and hidden  
3 neurons after the training process

	$\dot{m}_{ht,MR}$	$f_{mag}$	$t_{env}$	$\Delta T_{span}$	b
$N_{h1}$	1.047987	-0.39389	-2.33432	1.483472	-3.65012
$N_{h2}$	0.535301	0.529454	-0.83574	-2.00213	-2.41984
$N_{h3}$	-2.17166	-1.20087	-0.16584	1.82792	3.029251
$N_{h4}$	-1.32379	1.799162	-1.1659	-0.97174	2.066108
$N_{h5}$	-1.26513	-0.01766	0.864803	-2.50552	2.752963
$N_{h6}$	0.827974	-3.1634	0.316578	-0.95732	-1.99569
$N_{h7}$	-1.13837	-2.00935	-1.20627	-0.10869	-1.51453
$N_{h8}$	-2.80814	-1.77112	0.112152	-0.03931	-0.19889
$N_{h9}$	-0.58041	3.428734	-0.05467	0.221148	1.450539
$N_{h10}$	3.312933	-0.25925	-2.03276	-0.47667	-0.09995
$N_{h11}$	0.586848	1.07881	2.282147	1.628111	0.489131
$N_{h12}$	-0.51293	-1.40238	-1.08912	-2.21064	-0.39503
$N_{h13}$	-0.60408	1.392884	-0.74947	-1.69252	-0.48795
$N_{h14}$	-1.04731	1.258227	-1.48563	-1.99034	-0.65495
$N_{h15}$	-0.80131	-0.15356	1.068742	-2.17088	-0.76183
$N_{h16}$	-0.42187	4.398275	0.030929	-1.03656	2.825442
$N_{h17}$	-0.53025	-1.16838	2.129564	-0.77052	-1.80935
$N_{h18}$	0.299357	-3.00427	0.019773	0.994305	-2.10648
$N_{h19}$	1.510134	-1.87113	2.684765	0.059701	0.538973
$N_{h20}$	0.174994	2.142209	0.274178	0.525565	-0.93244
$N_{h21}$	-1.36397	0.296641	1.428942	-3.18693	-1.13571
$N_{h22}$	0.826656	2.728666	-0.20276	-0.52838	3.244822
$N_{h23}$	1.86905	1.000462	-0.09844	-1.34913	2.702464
$N_{h24}$	-1.55088	0.404929	0.316511	2.822485	-2.4374

$N_{h25}$	1.644039	2.541612	-0.04219	-0.30804	3.304448
$N_{h26}$	-3.60392	-0.14224	-1.90892	-0.1066	-3.21727
$N_{h27}$	-0.86623	-0.36737	-0.33185	-3.18254	-3.9562

1

2 Table A.3 Values of the synaptic weights associated with the synapses connecting the hidden neurons  
3 and the outputs before the training process

	$COP_{MR}$	$\dot{Q}_{MR}$
$N_{h1}$	0.543383	0.177871
$N_{h2}$	-0.1638	0.142339
$N_{h3}$	-0.37106	-0.36432
$N_{h4}$	0.57606	0.240787
$N_{h5}$	0.767952	0.547431
$N_{h6}$	0.082809	0.160361
$N_{h7}$	0.05424	0.072979
$N_{h8}$	0.05097	-0.03919
$N_{h9}$	-0.29244	-0.26873
$N_{h10}$	-0.13399	-0.07782
$N_{h11}$	-0.14925	-0.03439
$N_{h12}$	-0.06616	-0.04763
$N_{h13}$	-0.23715	-0.26993
$N_{h14}$	-0.06278	0.028326
$N_{h15}$	0.100867	-0.08765
$N_{h16}$	0.212676	0.28105
$N_{h17}$	0.175697	0.084387
$N_{h18}$	-0.13984	-0.18189
$N_{h19}$	-0.59876	-0.52349
$N_{h20}$	-0.18878	-0.12269
$N_{h21}$	-0.17251	-0.15902
$N_{h22}$	-0.18978	-0.19275
$N_{h23}$	-0.09427	-0.09198

$N_{h24}$	-0.07012	0.049085
$N_{h25}$	-0.24129	-0.19088
$N_{h26}$	-0.24537	-0.19736
$N_{h27}$	-0.16594	-0.23573
b	-1.5185	-1.2217

1

2 Table A.4 Values of the synaptic weights associated with the synapses connecting the hidden neurons  
3 and the outputs after the training process

	$COP_{MR}$	$\dot{Q}_{MR}$
$N_{h1}$	-0.47318	-0.60077
$N_{h2}$	0.192569	-0.10631
$N_{h3}$	-0.11322	-0.19806
$N_{h4}$	-0.02559	-0.22609
$N_{h5}$	-0.15269	-0.38231
$N_{h6}$	-0.43459	-0.32155
$N_{h7}$	0.194892	0.280153
$N_{h8}$	-0.17975	-0.01721
$N_{h9}$	-0.78529	-0.74655
$N_{h10}$	-0.19511	-0.24228
$N_{h11}$	0.091006	-0.13448
$N_{h12}$	-0.02618	-0.16726
$N_{h13}$	0.471035	0.605773
$N_{h14}$	-0.21237	-0.38835
$N_{h15}$	-0.14282	-0.38472
$N_{h16}$	-1.87491	-2.14543
$N_{h17}$	-0.30491	-0.19888
$N_{h18}$	-3.24064	-3.52381
$N_{h19}$	-0.05518	-0.07082
$N_{h20}$	-0.6865	-0.50486
$N_{h21}$	0.109005	0.300677

$N_{h22}$	0.53005	2.264142
$N_{h23}$	-0.64937	-0.59772
$N_{h24}$	-0.17163	-0.1306
$N_{h25}$	0.033121	-1.15469
$N_{h26}$	-0.35571	-0.53411
$N_{h27}$	0.210589	0.358907
$b$	-2.0587	-2.4501

1

2

1 **A.2. Section of the computer code to run the ANN**

2 The code to run the ANN according to Eq.6-8 is divided into three parts characterized by three different  
3 functions. The first one is needed to normalize the input variables, as follows:

```
function [i_n]=Test_normalization(ingressi,map_i)

Ni=size(ingressi,1);

pattern=size(ingressi,2);

maxinput=map_i(2,:);

mininput=map_i(1,:);

i_n=zeros(Ni,pattern);

for i=1:1:Ni

    for l=1:1:pattern

        i_n(i,l)=-1+2*(ingressi(i,l)-mininput(i))/(maxinput(i)-mininput(i));

    end

end
```

4 In the code, the variable *map\_i*, taken as an input of the function, contains the maximum and minimum  
5 values of each input. This section returns the normalized input values (*i\_n*).

6 The second part is the core of the ANN computation, and it represents the forward pass to propagate the  
7 input throughout the neurons. It is shown below.

```
function [output]=tanlin_ANN(in,Wh,Wo)
```

```
HL=size(Wh,1);
```

```
IL=size(Wh,2)-1;
```

```
OL=size(Wo,1);
```

```
pattern=size(in,2);
```

```
H=zeros(HL,pattern);
```

```
output=zeros(OL,pattern);
```

```
for p=1:1:pattern
```

```
    for k=1:1:HL
```

```
        v=0;
```

```
        for i=1:1:IL
```

```
            v=v+Wh(k,i)*in(i,p);
```

```
        end
```

```
        v=v+Wh(k,IL+1);
```

```
        H(k,p)=2/(1+exp(-2*v))-1;
```

```
    end
```

```
    for j=1:1:OL
```

```
        v=0;
```

```
        for k=1:1:HL
```

```
            v=v+Wo(j,k)*H(k,p);
```

```
        end
```

```
        v=v+Wo(j,HL+1);
```

```
        output(j,p)=v;
```

```
    end
```

```
end
```

- 1 This code allows to evaluate the normalized outputs of the ANN considering the normalized inputs
- 2 calculated with the previous code and the parameters of the ANN, i.e. the synaptic weights ( $Wh$  and  $Wo$ ).
- 3 The latter are shown in Tab. A.2 ( $Wh$ ) and Tab. A.4 ( $Wo$ ).
- 4 In the end, since the previous step returns the normalized output values, there is a last section needed to
- 5 obtain the real values of the outputs, as follows:

```
function [output]=denormalize(output_n,map_t)  
  
pattern=size(output_n,2);  
  
No=size(output_n,1);  
  
output=zeros(No,pattern);  
  
maxtarget=map_t(2,:);  
  
mintarget=map_t(1,:);  
  
for j=1:1:No  
  
    for p=1:1:pattern  
  
        output(j,p)=(output_n(j,p)*(maxtarget(j)-mintarget(j))+mintarget(j)+maxtarget(j))./2;  
  
    end  
  
end
```

- 6
- 7 In the code, the variable  $map\_t$ , taken as an input of the function, contains the maximum and minimum
- 8 values of each output.
- 9