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On interpolating sequences for Bloch type spaces

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When we deal with H^∞, it is known that c_0-interpolating sequences are interpolating and it is sufficient to interpolate idempotents of ℓ_∞ in order to interpolate the whole ℓ_∞. We will extend these results to the frame of interpolating sequences for Bloch type spaces $\mathcal{B}_v^\infty$ and study the connection between the interpolating operators on $\mathcal{B}_v^\infty$ and $\mathcal{B}_v^0$. Furthermore, for some particular weights v, we will provide examples of interpolating sequences for $\mathcal{B}_v^\infty$ whose constant of separation is as close to 0 as desired.

1. Introduction and background

Let $\mathbb{D}$ be the open unit disk of the complex plane $\mathbb{C}$ and $H(\mathbb{D})$ the space of complex analytic functions on $\mathbb{D}$. In this paper we will investigate sequences $(z_n) \subset \mathbb{D}$ which are interpolating for the derivative of functions in Bloch type spaces (see [1], [4]). It is also possible to study these sequences for Bloch type spaces that do not take into account the derivative of the function. For classical Bloch spaces, this has been done in [5].

Let v be a weight, that is, a strictly positive continuous function on $\mathbb{D}$ and suppose that v is typical: v is radial ($v(z) = v(|z|)$ for any $z \in \mathbb{D}$), non-increasing and $\lim_{|z|\to 1} v(z) = 0$.

The Bloch type spaces $\mathcal{B}_v^\infty$ and $\mathcal{B}_v^0$ are defined by:

$$\mathcal{B}_v^\infty = \{f \in H(\mathbb{D}) : \|f\|_{\mathcal{B}_v^\infty} := |f(0)| + \sup_{z\in\mathbb{D}} v(z)|f'(z)| < +\infty\}$$
$$\mathcal{B}_v^0 = \{f \in \mathcal{B}_v^\infty : \lim_{|z|\to 1^-} v(z)|f'(z)| = 0\}.$$

It is clear that $\mathcal{B}_v^0$ is a closed subspace of $\mathcal{B}_v^\infty$. We will also consider $\widetilde{\mathcal{B}}_v^\infty$ and $\widetilde{\mathcal{B}}_v^0$, the closed subspaces of $\mathcal{B}_v^\infty$ and $\mathcal{B}_v^0$ respectively consisting in functions f satisfying $f(0) = 0$. It is also clear that $\widetilde{\mathcal{B}}_v^0$ is a closed

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subspace of $\widetilde{\mathcal{B}}_v^\infty$. For typical weights v, it is well-known that the closed unit ball of $\widetilde{\mathcal{B}}_v^0$ is dense with respect to the compact-open topology ($co-$topology) in the closed unit ball of $\widetilde{\mathcal{B}}_v^\infty$. Indeed, for f in the closed unit ball of $\widetilde{\mathcal{B}}_v^\infty$, the functions $f_r(z) = f(rz)$ belong to the closed unit ball of $\widetilde{\mathcal{B}}_v^0$ for $0 < r < 1$ and $f_r \xrightarrow{co} f$.

Recall that H^∞ is the classical space of bounded analytic functions $f : \mathbb{D} \to \mathbb{C}$ endowed with the supremum norm $\|\cdot\|_\infty$. If $v(z) = 1 - |z|^2$, then $\mathcal{B}_\infty^v$ and $\mathcal{B}_0^v$ become the classical Bloch space $\mathcal{B}$ and the little Bloch space $\mathcal{B}_0$ respectively. It is well-known that H^∞ is properly contained in $\mathcal{B}$. This will remain true if we deal with weights $\mathcal{O}(1 - |z|^2)$, but it is not true in general. Take $v_\alpha(z) = (1 - |z|)^\alpha$ for $0 < \alpha < 1$ and $f(z) = (1 - z)^\beta$ for $0 < \beta < 1 - \alpha$. Then f belongs to H^∞ but $|f'(z)|v_\alpha(z) \to \infty$ when $z \to 1$, so $f \notin \mathcal{B}_v^\infty$.

Let us first adapt some results due to Bierstedt and Summers done for the weighted Banach spaces of analytic functions H_v^∞ and H_v^0 to the frame of Bloch type spaces [3]. Denote by $i : \widetilde{\mathcal{B}}_v^\infty \to (\widetilde{\mathcal{B}}_v^\infty)^{**}$ the natural inclusion of $\widetilde{\mathcal{B}}_v^\infty$ into its bidual $(\widetilde{\mathcal{B}}_v^\infty)^{**}$ given by $i(f)(u) = u(f)$ for $f \in \widetilde{\mathcal{B}}_v^\infty$ and $u \in (\mathcal{B}_v^\infty)^*$. In [7] it has been pointed out that the closed unit ball of $\widetilde{\mathcal{B}}_v^\infty$ is $co-$compact, so using the Dixmier-Ng Theorem [13] we obtain that the space:

$$^*\widetilde{\mathcal{B}}_v^\infty = \{\ell \in (\widetilde{\mathcal{B}}_v^\infty)^* : \ell|_{B_{\widetilde{\mathcal{B}}_v^\infty}} \text{ is } co-\text{continuous}\}$$

endowed with the norm induced by the dual space $(\widetilde{\mathcal{B}}_v^\infty)^*$ is a Banach space and the map $f \in \widetilde{\mathcal{B}}_v^\infty \mapsto i(f)|_{^*\widetilde{\mathcal{B}}_v^\infty} \in (^*\widetilde{\mathcal{B}}_v^\infty)^*$ is an onto isometric isomorphism. In particular, $^*\widetilde{\mathcal{B}}_v^\infty$ is a predual of $\widetilde{\mathcal{B}}_v^\infty$.

Consider the evaluation functionals δ_z' given by $\delta_z'(f) = f'(z)$ for any $z \in \mathbb{D}$, which clearly belong to $^*\widetilde{\mathcal{B}}_v^\infty$. By the Hahn-Banach theorem it follows that the linear span of $\{\delta_z' : z \in \mathbb{D}\}$ is norm dense in $^*\mathcal{B}_v^\infty$. Therefore, $^*\mathcal{B}_v^\infty$ is separable since it is sufficient to consider evaluations δ_z' for $z = p + iq$, where $p, q \in \mathbb{Q}$.

Following the argument given by Bierstedt and Summers [3] we show:

Proposition 1.1. *The space $^*\widetilde{\mathcal{B}}_v^\infty$ is isometrically isomorphic to $(\widetilde{\mathcal{B}}_v^0)^*$ and the restriction map $\widetilde{R}$: $^*\widetilde{\mathcal{B}}_v^\infty \to (\widetilde{\mathcal{B}}_v^0)^*$ given by $\widetilde{R}(\ell) = \ell|_{\widetilde{\mathcal{B}}_v^0}$ is an onto isometric isomorphism. In particular, $(\widetilde{\mathcal{B}}_v^0)^{**}$ is isometrically isomorphic to $\widetilde{\mathcal{B}}_v^\infty$.*

Proof. The map $\widetilde{R}$ is well-defined since for any $\ell \in {}^*\widetilde{\mathcal{B}}_v^\infty \subset (\widetilde{\mathcal{B}}_v^\infty)^*$ it follows that $\|\widetilde{R}(\ell)\| \leq \|\ell\|$. First we prove that $\widetilde{R}$ is surjective. Consider $\ell \in (\widetilde{\mathcal{B}}_v^0)^*$. Notice that $\widetilde{\mathcal{B}}_v^0$ is isometrically isomorphic to a closed subspace of $C_0(\mathbb{D})$, the space of continuous functions on the closed unit disk which vanish in the boundary, via $f \mapsto vf'$. By the Hahn-Banach theorem and the Riesz representation theorem, there is a bounded Radon measure μ on $\mathbb{D}$ such that:

$$\ell(f) = \int_{\mathbb{D}} vf'd\mu \quad \text{for } f \in \widetilde{\mathcal{B}}_v^0.$$

Define $\tilde{\ell}(f) = \int_{\mathbb{D}} vf'd\mu$ for all $f \in \widetilde{\mathcal{B}}_v^\infty$ which is clearly well-defined and satisfies $\tilde{\ell}|_{\widetilde{\mathcal{B}}_v^0} = \ell$. It follows from the Lebesgue bounded convergence theorem that $\tilde{\ell}|_{\widetilde{\mathcal{B}}_v^\infty}$ is $co-$continuous, so $\widetilde{R}$ is surjective. Since the closed unit ball of $\widetilde{\mathcal{B}}_v^0$ is $co-$dense in the closed unit ball of $\widetilde{\mathcal{B}}_v^\infty$, we conclude that $\widetilde{R}$ is an isometry. □

Corollary 1.2. *The space $(\mathcal{B}_v^0)^*$ is isometrically isomorphic to $^*\mathcal{B}_v^\infty$ and $(^*\mathcal{B}_v^\infty)^*$ is isometrically isomorphic to $\mathcal{B}_v^\infty$. In particular, the space $(\mathcal{B}_v^0)^{**}$ is isometrically isomorphic to $\mathcal{B}_v^\infty$.*

Proof. Notice that $\mathcal{B}_v^0$ is isometrically isomorphic to $(\widetilde{\mathcal{B}}_v^0 \times \mathbb{C}, \|\cdot\|_1)$, so $(\mathcal{B}_v^0)^*$ is isometrically isomorphic to $^*\mathcal{B}_v^\infty := (^*\widetilde{\mathcal{B}}_v^\infty \times \mathbb{C}, \|\cdot\|_\infty)$. The dual of this space is isometrically isomorphic to $(\widetilde{\mathcal{B}}_v^\infty \times \mathbb{C}, \|\cdot\|_1)$ which in turn is isometrically isomorphic to $\mathcal{B}_v^\infty$ and we conclude that $(\mathcal{B}_v^0)^{**}$ is isometrically isomorphic to $\mathcal{B}_v^\infty$. □

2. Interpolating sequences for Bloch type spaces

Recall that the pseudohyperbolic distance for $z, w \in \mathbb{D}$ is given by:

$$\rho(z,w) = \left|\frac{z-w}{1-\bar{z}w}\right|.$$

A sequence $(z_n) \subset \mathbb{D}$ is said to be separated if there exists $\delta > 0$ such that:

$$\rho(z_n, z_k) \geq \delta \quad \text{for any } n \neq k, \tag{2.1}$$

and we define its constant of separation as $r := \inf_{n\neq k} \rho(z_n, z_k)$.

A sequence $(z_n) \subset \mathbb{D}$ is said to be interpolating for H^∞ if for any $(a_n) \in \ell_\infty$ there exists $f \in H^\infty$ such that $f(z_n) = a_n$ for any $n \in \mathbb{N}$. The most important result on interpolating sequences for H^∞ is the classical Carleson's Theorem [6], which states that $(z_n) \subset \mathbb{D}$ is interpolating for H^∞ if and only if (z_n) is uniformly separated, that is, if there exists $\delta > 0$ such that $\inf_{k\in\mathbb{N}} \prod_{n\neq k} \rho(z_n, z_k) \geq \delta$.

A sequence $(z_n) \subset \mathbf{D}$ is said to be interpolating for the Bloch type space $\mathcal{B}_v^\infty$ if for any $(a_n) \in \ell_\infty$ there exists $f \in \mathcal{B}_v^\infty$ such that $v(z_n)f'(z_n) = a_n$ for any $n \in \mathbb{N}$. We define the interpolating operator $T : \mathcal{B}_v^\infty \to \ell^\infty$ by $T(f) = (v(z_n)f'(z_n))$, which is clearly well-defined and linear. Notice that (z_n) is interpolating for $\mathcal{B}_v^\infty$ if and only if T is surjective. If $(z_n) \subset \mathbf{D}$ satisfies $|z_n| \to 1$, then the interpolating operator $T|_{\mathcal{B}_v^0}$ maps $\mathcal{B}_v^0$ into c_0 since $f'(z_n)v(z_n) \to 0$ when $n \to \infty$.

For H^∞ and Bloch type spaces, we can also consider c_0-interpolating sequences just by considering sequences (a_n) in c_0 instead of ℓ_∞. Notice that interpolating sequences (z_n) for Bloch type spaces satisfy $|z_n| \to 1$ since they do not have accumulation points in $\mathbb{D}$. The connection between c_0-interpolating sequences and interpolating sequences has been studied in the context of uniform algebras (see [8]). In particular, the authors proved that c_0-interpolating sequences for H^∞ are indeed interpolating for H^∞. We will show that this result remains true if we deal with $\mathcal{B}_v^\infty$.

In the proof of the next theorem we will use the following result (see [12], Theorem 5, p. 82): let X, Y be Banach spaces and $T : X \to Y$ a linear and bounded operator. Then T is bounded below if and only if T^* is surjective. Furthermore, T is surjective if and only if T^* is bounded below.

Theorem 2.1. *Let v be a typical weight on $\mathbb{D}$. If $(z_n) \subset \mathbb{D}$ is a sequence of distinct points, then the following statements are equivalent:*

(a) *The sequence (z_n) is interpolating for $\mathcal{B}_v^\infty$.*
(b) *There exists a constant $C > 0$ such that:*

$$\|(\xi_n)\|_1 \leq C \left\| \sum_{n=1}^{\infty} \xi_n v(z_n) \delta'_{z_n} \right\| \quad \textit{for any } (\xi_n) \in \ell_1.$$

(c) *The sequence (z_n) is c_0-interpolating for $\mathcal{B}_v^0$.*

Proof. Define $S : \ell_1 \to {}^*\mathcal{B}_v^\infty$ given by:

$$S((\xi_n)) = \sum_{n=1}^{\infty} \xi_n v(z_n) \delta'_{z_n},$$

which is clearly a well-defined, linear, continuous map. Condition (b) states that S is bounded below. We have $({}^*\mathcal{B}_v^\infty)^* = \mathcal{B}_v^\infty$ by Corollary 1.2 and it is easy that $S^* = T$, where T is the interpolating operator on $\mathcal{B}_v^\infty$.

(a) $\Leftrightarrow$ (b) It is clear since (a) states that the interpolating operator T is surjective and this is equivalent to S being bounded below.

(b) $\Leftrightarrow$ (c) Notice that ${}^*\mathcal{B}_v^\infty = (\mathcal{B}_v^0)^*$ by Corollary 1.2. Since $|z_n| \to 1$, T maps $\mathcal{B}_v^0$ into c_0 and $(T|_{\mathcal{B}_v^0})^* = S$. Hence (b) and (c) are equivalent by the result above. □

From Theorem 2.1 and its proof we have the following results:

Corollary 2.2. *We have:*

(a) *A sequence $(z_n) \subset \mathbb{D}$ is interpolating for $\mathcal{B}_v^\infty$ if and only if it is c_0-interpolating for $\mathcal{B}_v^0$.*
(b) *$(T|_{\mathcal{B}_v^0})^{**} = T$ and T is $w^* - w^*-$continuous.*

The inspiration to the next result comes from Theorem 2.4 in [8] and Proposition 7.7 in [4].

Theorem 2.3. *Let v be a typical weight and $(z_n) \subset \mathbb{D}$. Suppose that for any $(a_n) \in c_0$ there exists $f \in \mathcal{B}_v^\infty$ such that $v(z_n)f'(z_n) = a_n$ for any $n \in \mathbb{N}$. Then (z_n) is interpolating for $\mathcal{B}_v^\infty$.*

Proof. Let $a = (a_n) \in \ell_\infty$. By Goldstein's theorem, there exists a sequence $\{b^k\} \subset c_0$ such that $b^k \overset{w^*}{\to} a$ when $k \to \infty$. Consider the interpolating operator $T : \mathcal{B}_v^\infty \to \ell_\infty$ given by $T(f) = (f'(z_n)v(z_n))$ and take $A = T^{-1}(c_0) \subset \mathcal{B}_v^\infty$. Since A is closed in $\mathcal{B}_v^\infty$, it follows that A is a Banach space. The linear operator $T|_A : A \to c_0$ is surjective by the assumption. The induced operator $\tilde{T}|_A : A/\ker(T|_A) \to c_0$ is thus bounded, injective and surjective. Therefore by the Open Mapping theorem, there is $M > 0$ such that:

$$\inf_{h\in\ker(T|_A)} \|f + h\|_{\mathcal{B}_v^\infty} \le \frac{M}{2}\|b\|_\infty \ \text{ if } \ T|_A(f) = b \in c_0.$$

Hence for each $b \in c_0$ there is $f \in A$ such that $T|_A(f) = b$ and $\|f\|_{\mathcal{B}_v^\infty} \le M\|b\|_\infty$. In particular for any $k \in \mathbb{N}$ there exists $g_k \in \mathcal{B}_v^\infty$ such that $T(g_k) = b^k$ and $\|g_k\|_{\mathcal{B}_v^\infty} \le M\|b^k\|_\infty$. Since $\{b^k\}$ is weak-star convergent, it is bounded in c_0, so there is $C > 0$ such that $\|g_k\| \le C$ for all $k \in \mathbb{N}$. Since ${}^*\mathcal{B}_v^\infty$ is separable and $({}^*\mathcal{B}_v^\infty)^* = \mathcal{B}_v^\infty$, by Alaoglu's theorem there exists a subsequence (g_{k_m}) of (g_k) which w^*-converges to $g \in \mathcal{B}_v^\infty$. By Corollary 2.2, $(T|_{\mathcal{B}_v^0})^{**} = T$ and T is $w^* - w^*-$continuous, so:

$$a = w^* - \lim_{m\to\infty} b^{k_m} = w^* - \lim_{m\to\infty} T(g_{k_m}) = T(g). \quad \square$$

Recall that an idempotent (a_n) of ℓ_∞ is a sequence that satisfies $a_n^2 = a_n$ for any $n \in \mathbb{N}$, that is, $a_n = 0$ or $a_n = 1$ for any $n \in \mathbb{N}$. Hayman proved that it is sufficient to interpolate idempotent elements $(a_n) \in \ell_\infty$ to assure that the sequence $(z_n) \subset \mathbb{D}$ is interpolating for H^∞ (see [10]). We will prove that this result remains true when we deal with interpolating sequences for $\mathcal{B}_v^\infty$.

To prove Theorem 2.4, we will need the following result due to Beurling (see Theorem 4.3 in [2]): if X is a Banach space and $L : \ell_1 \to X$ is a linear operator such that $L^*(X)$ is dense in $\ell_\infty = (\ell_1)^*$, then $L^*(X^*) = \ell_\infty$.

Now we can state our main result:

Theorem 2.4. *Let v be a typical weight and $(z_n) \subset \mathbf{D}$ a sequence of distinct points. Then the following assertions are equivalent:*

(a) *(z_n) is interpolating for $\mathcal{B}_v^\infty$.*
(b) *(z_n) is c_0-interpolating for $\mathcal{B}_v^0$.*
(c) *(z_n) is c_0-interpolating for $\mathcal{B}_v^\infty$.*
(d) *(z_n) is interpolating for $\mathcal{B}_v^\infty$ when only considering idempotents of ℓ_∞.*

Proof. It remains only to prove that $(d) \to (a)$. Let us consider the interpolating operator $T : \mathcal{B}_v^\infty \to \ell^\infty$. By Theorem 2.1 and Corollary 2.2 we have that $(T|_{\mathcal{B}_v^0})^{**} = T$ and $(T|_{\mathcal{B}_v^0})^*$ maps ℓ_1 into ${}^*\mathcal{B}_v^\infty$. Therefore we need to prove that T has dense range. Indeed, consider $E \subseteq \mathbb{N}$ and denote by χ_E the sequence in ℓ^∞ given by:

$$\chi_E(n) := \begin{cases} 1 & \text{if } n \in E \\ 0 & \text{if } n \in \mathbb{N} \setminus E. \end{cases}$$

Define $S \subset \ell^\infty$ to be the set of functions on $\mathbb{N}$ of the form $\sum_{i=1}^m a_i \chi_{A_i}$ such that $m \in \mathbb{N}$, $a_i \in \mathbb{C}$ for any $i = 1, 2, \dots, m$ and $(A_i)_{i=1}^m$ are pairwise disjoint sets. The set S is dense in ℓ^∞ since S is the set of simple functions in $\ell_\infty = L^\infty(\mathbb{N}, c)$ where c is the cardinal measure. Let $x \in S$, that is, $x := \sum_{i=1}^m a_i \chi_{A_i} \in \ell^\infty$. By hypothesis, for any $i = 1, 2, \dots, m$ there are functions $f_i \in \mathcal{B}_v^\infty$ such that:

$$f_i'(z_n)v(z_n) = \chi_{A_i}(n) \quad \text{for all } n \in \mathbb{N}.$$

For $f := \sum_{i=1}^m a_i f_i \in \mathcal{B}_v^\infty$ we have that:

$$T(f) = (f'(z_n)v(z_n))_{n=1}^\infty = \left(\sum_{i=1}^m a_i f_i'(z_n) v(z_n) \right)_{n=1}^\infty = \left(\sum_{i=1}^m a_i \chi_{A_i}(n) \right)_{n=1}^\infty = x$$

and we are done. □

2.1. Examples of interpolating sequences for $\mathcal{B}_v^\infty$

Finally we turn to some examples of interpolating sequences for $\mathcal{B}_v^\infty$. Recall that a sequence $(z_n) \subset \mathbb{D}$ is said to be a Blaschke sequence if $\sum_{n=1}^\infty (1 - |z_n|) < \infty$. It is well-known that the sequence of zeros of a non-zero bounded analytic function on $\mathbb{D}$ satisfies the Blaschke condition (see [9]). It is also well-known that interpolating sequences for H^∞ satisfy the Blaschke condition. However, there exist interpolating sequences for $\mathcal{B}_v^\infty$ for some particular weights which does not satisfy this condition (an easy adaptation of Proposition 6.4 in [4]).

Proposition 2.5. *If $(z_n) \subset \mathbb{D}$ is interpolating for H^∞ and $z_0 \in \mathbb{D}$ is such that $z_0 \neq z_n$ for every $n \in \mathbb{N}$, then the sequence (w_n) defined by $w_1 = z_0$ and $w_{n+1} = z_n$ for $n \geq 1$ is also interpolating for H^∞.*

Proof. Let $(z_n) \subset \mathbb{D}$ be an interpolating sequence for H^∞. Since it satisfies the Blaschke condition, we can consider its Blaschke product $B : \mathbb{D} \to \mathbb{C}$, which is a bounded analytic function satisfying $B(z) = 0$ if and only if $z = z_n$ for some $n \in \mathbb{N}$. Consider $(a_n) \in \ell_\infty$. There exists a function $f \in H^\infty$ satisfying $f(z_n) = a_{n+1}$ for every $n \in \mathbb{N}$, so if we let $\alpha := B(z_0) \neq 0$ and define $g : \mathbb{D} \to \mathbb{C}$ by:

$$g(z) := \frac{(a_1 - f(z_0))}{\alpha} B(z) + f(z)$$

we have that $g \in H^\infty$ and $g(w_n) = a_n$ for all $n \in \mathbb{N}$. □

Madigan and Matheson proved that if a sequence is sufficiently separated for the pseudohyperbolic distance, then it is interpolating for the classical Bloch space $\mathcal{B}$ [11]. We prove that this condition is not necessary since we can find interpolating sequences for $\mathcal{B}_{v_\alpha}^\infty$, where $v_\alpha(z) = (1 - |z|^2)^\alpha$, $\alpha > 0$, and in particular for $\mathcal{B}$, as close as we want:

Proposition 2.6. *Let $\alpha > 0$. For any $\varepsilon > 0$ there exist interpolating sequences for $\mathcal{B}^\infty_{v_\alpha}$ whose constant of separation is less than ε.*

Proof. Let $\varepsilon > 0$. Consider an interpolating sequence (z_n) for H^∞, for example, $z_n := 1 - \frac{1}{2^n}$, and add a point $z_0 \notin (z_n)$ such that $\rho(z_0, z_1) < \varepsilon$. By Proposition 2.5 the sequence given by $\{z_0\} \cup (z_n)$ is interpolating for H^∞ hence interpolating for $\mathcal{B}^\infty_{v_\alpha}$ by Theorem 6.3 in [4]. □

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