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# The low risk anomaly in Spain

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**Abstract**

This paper finds the low risk anomaly in the IBEX-35 during January 2002 – December 2021. This indicates an inverse relationship between risk and return which is contrary to financial theory. The results are obtained from observing the monthly returns of listed companies and creating risk-sorted quintile portfolios which are equally weighted. The methodology used is in line with existing literature using Sharpe ratio and Jensen's alpha as performance metrics.

**Keywords: low risk anomaly, CAPM, idiosyncratic risk, systematic risk**

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# 1. Introduction

In this paper we investigate whether the low risk anomaly exists in the Spanish market. The fundamental financial theory of the paper is based on the relationship between risk and return that is established in the capital asset pricing model (CAPM). The first section takes a look at the background of this relationship through discussing the Markowitz mean-variance model, the behavioural aspects of risk-averse investors and the empirical evidence that rejects the suggested positive relation between risk and return.

In section 2 we take a look at how risk is computed and the appropriate proxy for our study; standard deviation. Moreover, the discussion is developed on whether the focal point should be systematic risk since idiosyncratic risk can be diversified away. Furthermore, we discuss existing literature on the low risk anomaly and the possible explanations driving this effect. Section 3 takes a look at the CAPM theoretical framework as well as empirical evidence rejecting the model.

The methodology discussed in section 4 is in line with existing research such as Baker, Bradley and Wurgler (2011). We look at both beta and total risk of the monthly returns of companies listed in the IBEX-35 from 2002 – 2021.

### 1.1 Background

A fundamental theory in finance is the relationship between risk and return. The common belief is that higher risk is rewarded by higher expected return. The risk and return trade-off can be described as the first fundamental law of finance (Ghysels et al., 2005).

The risk and return relationship has been studied from different aspects including a behavioural aspect. The foundation of modern finance theory is strongly based on a branch of applied microeconomics. Utility theory has been used to understand investor's behaviour and ranking of risky alternative choices which is based on the axioms by Von Neumann and Morgenstern (1947). The five axioms of cardinal utility provide a set of assumptions which assist in identifying how an individual ranks the combination of risky assets based on preferences.

Risk aversion is important to consider in decision making under uncertainty. A risk averse agent is one who avoids zero-mean risks. Assuming that investors are risk averse implies that their utility functions are concave and increasing. This means that more wealth is preferred to less, positive marginal utility of wealth, moreover, as more wealth is acquired the marginal utility of wealth decreases (Copeland, 1988 pg. 88). A further definition of risk-aversion has been defined by Pratt (1964) and Arrow (1971) which is an approximation of the degree of absolute risk aversion. It is referred to as absolute risk aversion as it shows how an individual's absolute risk aversion changes for a given level of wealth.

Markowitz (1959) studies the relationship between risk and return in his theoretical framework. The foundations of portfolio theory are based on the Markowitz's portfolio choice model. This model makes assumptions on the behaviour of an optimizing investor. Markowitz (1995), explains the importance of uncertainty in the analysis of an optimising investor. It is an important factor because an investor who is certain about future returns would invest in one type of stock which, will certainly, have the highest future return. However, a common investment strategy is a diversified portfolio which indicates the importance of uncertainty. An investor is considered risk averse and aims to maximise the expected return at the lowest risk. The concept of efficient portfolios was derived from this assumption. An efficient portfolio is one that minimizes risk given expected return as well as maximizes expected return for a given level of risk. Thus, the Markowitz model (1959) is referred to as the mean-variance efficient portfolio. Clarke, de Silva & Thorley (2006) found empirical evidence that challenges the mean-variance theory. They found that minimum variance portfolios compiled using 1,000 largest US stocks, between the period 1968-2005, had a 25% reduction in volatility. However, these portfolios delivered comparable or higher average returns than the market portfolio.

The capital asset pricing model (CAPM) by Sharpe (1964) and Lintner (1965) builds further on Markowitz's portfolio choice model. The CAPM makes additional assumptions to further the concept of mean-variance efficient portfolio. The first assumption is that investors have homogenous beliefs. Secondly, there is the possibility of lending and borrowing in the market at a risk-free rate. These assumptions are further discussed in section 3.1.

However, the theory behind risk and return has been challenged by compelling empirical evidence. There a number of existing anomalies which challenge the fundamental concept of market efficiency. The efficient hypothesis states that markets correctly prices securities using the most recent information available thus the price reflects the security's intrinsic value. The implication of the price reflecting the intrinsic value of the security indicates that there cannot be any abnormal profits earned. Therefore, implying that active trading will not result in abnormal returns. However, it has been proven that investment strategies such as the value, size and momentum strategies generate significantly higher returns than the market portfolio.

### 1.2 Research Question

The research question is whether the low risk anomaly exists in Spain's IBEX-35 market which will ultimately indicate if the market is efficient. The null hypothesis in this case is that IBEX-35 is efficient. The market does not exhibit the low risk anomaly. If the null hypothesis is rejected, this implies that lower risk portfolios could yield higher returns.

$H_0$: IBEX-35 is efficient. There is no low risk anomaly present in the market.

$H_1$: IBEX-35 is not efficient. The low risk anomaly is present in the market.

### 1.3 Results

In this paper, we find that the low risk anomaly is present in the Spanish stock market during 2002 – 2021. This confirms that low risk stocks yield higher returns as discovered in existing literature such as Ang.et al, 2006. However, this result is not consistent when using all the performance metrics. We find that Jensen's alpha presents contradictory results to the Sharpe ratio. Jensen's alpha shows that, during the period 2002-2021, high risk stocks outperform low risk stocks which is in line with financial theory. The sample is divided into a subsample to test whether the results obtained are sample dependent in line with the study by Bali et al. (2005). The results obtained from the subsample, 2002-2015, confirm the existence of the low risk anomaly in terms of Jensen's alpha and Sharpe ratio.

## 2. Literature Review

### 2.1 Proxy for risk

Total risk is portioned into two parts; systematic risk and unsystematic risk (also referred to as idiosyncratic risk). The former is a measure of how an asset covaries with the economy and the latter is independent of the macro factors. Theory places an importance on pricing systematic risk in the market with the belief that investors are able to diversify idiosyncratic risk. However, Goyal and Santa-Clara (2001) highlight the importance of idiosyncratic risk in their study. They found a significant positive relationship between average stock variance and return on the market during the period 1963-08 to 1999-12. Thus, there is a trade-off between risk (measured as total risk) and return. Furthermore, their findings show that idiosyncratic risk is an explanatory factor for most of the average stock risk variation through time and it is also the driving factor behind forecasting the stock market. In their sample they found that the idiosyncratic component of total risk accounted for over 80% of total stock variance; this figure is over 70% of variation through time (Goyal and Santa-Clara, 2001). Moreover, results from a recent study by Asness et al. (2020) suggest that volatility (total risk) is the main driver of the low risk anomaly.

Furthermore, Blitz & Vliet (2007) propose the question "low volatility or low beta?". In their response, they state that volatility and beta are closely related metrics. The beta of a stock to the market index is computed as the volatility multiplied with its correlation with the market index, divided by the market volatility. Thus, defining the low risk anomaly in terms of volatility or beta is a choice based on the added value of the correlations. Additionally, Baker, Bradley and Wurgler (2011) found that there is a high corelation between beta and total risk. Thus, the measurement selected will have lower significance. In their study they found slightly stronger results for total risk than for beta however, the difference found was relatively small. In a recent study, Liu et al. (2018) confirms the weaker results for beta. They further discussed that idiosyncratic risk is significant if the portfolios are not diversified enough.

Risk plays an essential role in finance, thus, having an accurate measure of risk is important. Markowitz (1959), uses variance (standard deviation) as the measure of risk in his model. However, it is argued that the measure of risk in the mean-variance model is only appropriate if the investor has a quadratic utility function or if the joint distribution of returns follow a normal distribution (Jia & Dyer, 1996). However, many studies follow Ang et al. (2006) calculation of idiosyncratic volatility which uses the standard deviation of daily stock returns in one month. A recent study by Burckett & Scherer (2020), takes a further look at the misunderstanding of risk and the bias associated with the calculation of standard deviation. They highlight the effect that various techniques used to measure risk have on the decision-making criteria.

Furthermore, there are arguments that standard deviation and beta might not even be the best measures of risk. A proposed alternative measure is downside risk which looks at the returns that fall below a benchmark. Although Markowitz (1959) uses standard deviation in his mean-variance model, he argues that semi-variance is a more appropriate measure as it considers downside risk as well. Semi-variance is calculated in the same way as the variance however it takes into account the negative deviations from the mean. However, academics have continued using variance as a measure of risk even though semi-variance is considered a more accurate measure. Markowitz (1959) states that initially when the semi-variance was introduced the efficient set took longer to compute as opposed to using variance as a measure. Thus, contributing to the popularity of mean-variance. Generally, estimates of downside risk come with high estimation errors and computational burdens while variance and volatility modelling are much more developed. This may explain the popularity of variance as a measure of risk.

### 2.2 The low risk anomaly

There have been a number of anomalies that have been unfavourable to CAPM's mean-variance efficiency. Banz (1981) and Reinganum (1981) found that smaller firms have higher returns than could be explained by the CAPM. A possible explanation for the small firm effect is that due to transactions costs shares of small firms are held, on average, in relatively undiversified portfolios. Thus, the lack of diversification results in investors needing to be compensated for bearing total risk rather than systematic risk (Levy, 1978 and Mayshar (1979,1981,1983)). Moreover, Reinganum (1981) found that there was no relation between beta and average return during the period between 1964 – 1979. Furthermore, Keim (1983) observed a seasonality in stock returns, which was known as the January-effect, as stocks seemed abnormally high in January. There are two main hypotheses used to explain this effect. The first one is the year-end tax loss selling hypothesis where shares that have declined in value over the previous year are sold. The second hypothesis by Rozeff and Kinney (1976) is known as the information hypothesis which states that January is an important month for financial and information events. Baker, Bradely and Wurgler (2011) state that the low-risk anomaly could be considered the greatest anomaly in finance. This anomaly has also been referred to as “the mother of all inefficiencies” (Zaremba, 2015).

The positive relationship between risk and return has been difficult to identify in data. Early evidence by Black, Jensen and Scholes (1972) found that expected returns on portfolios with lower beta were consistently higher in the post-war period between 1946-1966. Whereas, portfolios with higher levels of beta recorded consistently lower expected returns. More recent empirical evidence of this phenomenon is present. Research conducted by Ang et al. (2006) analyses U.S data between 1963 – 2000. The findings are that firms that have high idiosyncratic risk have low average returns. In their results they controlled for various cross-sectional effects which made their findings robust. They controlled for size, book-to-market ratios, volume, leverage and liquidity risk.

There is empirical evidence of the existence of the low risk anomaly at a global level. A number of studies by Blitz and Vliet (2007), Ang et al. (2009), Frazzini and Pedersen (2011), Baker, Bradley and Taliaferro (2013) all present evidence of the low risk anomaly at an international level. Blitz and Vliet (2007) look at data from 1986 – 2006 and find that the annual alpha, risk-adjusted performance measurement, has a spread of 12% between the global low and global high volatility docile portfolios. They observe this volatility effect in the US, European as well as the Japanese markets in isolation. The findings in the Japanese market are confirmed by a study conducted by Iwasawa and Uchiyama (2014).

A further study conducted by Baker, Bradley and Wurgler (2011) grouped portfolios based on risk. Instead of idiosyncratic volatility, they used beta and total risk as a proxy for risk. Using data from 1968, they found that investing one dollar in the lowest risk portfolio would increase to 59.55 dollars by the end of 2008. However, the same investment would have reduced to 58 cents if invested in the riskiest portfolio. Baker, Bradley and Wurgler (2013) decompose the low-risk anomaly into two components; micro and macro effects. According to them, the low risk and high return pattern can come from the macro selection of lower risk countries and industries or from the micro selection of low-risk stocks within those industries and countries. The sample analysed consisted of 29 US industries and 31 developed countries. Baker, Bradley and Wurgler (2013) found that both micro and macro selection contribute to the existence of the low risk anomaly.

The low risk anomaly has been found in other asset classes. The fundamentals of risk-return trade-off is not limited to stock market; it affects any investing decision involving risk. A study by Carvalho et al. (2014) found the anomaly within the fixed income market. They found that portfolios containing bonds with the lowest risk outperform riskier bonds in terms of alpha and Sharpe ratios. These results were found in government and corporate bonds. Interestingly, they also considered bonds denominated in different currencies and this did not alter their results. A concern raised about currency risk in the stock market is that volatilities and correlations of stocks can vary when returns on stocks are measured in a distinct currency (Blitz, Vliet and Baltussen, 2019). Furthermore, Blitz & de Groot (2014) found this effect in commodities and Frazzini & Pederson (2011) found the low risk anomaly in credit markets.

### 2.3 Explanations to the low risk anomaly

According to Ang (2014), the low risk anomaly is explained by the combination of three effects. The first effect is the negative relationship between volatility and future returns. The second effect is the negative relationship between realized beta and future returns. The third effect is that the minimum variance portfolios outperform the market. Moreover, Jensen et al. (1972) argued that the positive correlation between risk and return is "too flat" than predicted by the CAPM. Thus, giving way to abnormal returns of low-beta stocks. Furthermore, Miller (1977) proposes a situation where the riskiest stocks will have returns that are below the capital market line which connects the risk-free interest rate and the market portfolio. He states that if investors have their own estimates of expected risk and return and systematic risk and uncertainty occur jointly, then the price of risky assets will be increased and their yield lowered. Results deduced from this is that the return on high beta stocks (risky stocks) will be lowered below what CAPM predicts. A further explanation to the low risk anomaly is shown by Trainor (2012). In his study, an explanatory factor is the mathematical compounding of calculating beta and returns. He found that during the period where the market exhibits low volatility, high beta outperforms low beta. However, overtime as the market becomes excessively volatile high beta produces worse results. Furthermore, Trainor (2012) states that the compounding problem is not able to explain the reason why the monthly average returns of higher beta portfolios do not yield higher returns.

There is research that provides behavioural explanations to the existence of this anomaly. Hou and Loh (2016) found that the existing explanations to the anomaly only account for less than 10% of the anomaly whereas, a larger portion is explained by investors' lottery preferences and market frictions. They also state that some of the existing studies differ in empirical methodologies thus, making it difficult to directly compare. The lottery preference was explored by Bali, Cakici and Whitelaw (2011). They found that investors are over investing best performing stocks, ones that achieved higher daily returns in the previous month. According to Hou and Loh (2016), this explains 29%-61% of the anomaly.

Blitz & Vliet (2007) provide possible explanations behind the low risk anomaly. Possible explanations are based on structural inefficiencies of the financial markets such as leverage restrictions and inefficient two-step investment processes. Access to leverage is necessary in order to take advantage of the low risk stocks' attractive returns. Blitz & Vliet (2007) state that in practice investors are unwilling or not allowed to apply leverage at a scale that is needed to exploit this effect. They further give an example that a low risk stock portfolio that has volatility which is two thirds of that of the market portfolio, the leverage needed is 50% in order to obtain the same level of volatility as the market. Thus, this opportunity presented by the low risk stocks is not easily arbitraged away. This dates back to Black (1972) who identified borrowing restrictions as an argument for the good performance of low beta stocks. However, Ang, 2014 p. 342, states that the leverage constraints only explain the poor performance of the high risk stock whereas the abnormal positive returns on low risk stocks is still unexplained.

Furthermore, constraints to short-selling could have an explanatory power to the low risk anomaly. In investing, short-selling is the process where an investor sells a security that they do not own with the hopes that they can buy it back later on for a lower price. The expectation is that the stock they are short selling, will decline in price after selling which will allow them to buy it back at a lower price. Miller (1977) states that the absence of sufficient short-sellers flattens the risk-return relationship. A further explanation is that in the absence of short-selling the price of a security experiences an increase if investors have heterogenous expectations. If there is insufficient or no short selling the demand for a specific stock will be heavily influenced by the minority of investors who have optimistic expectations. Furthermore, Linter (1969) shows in his model with heterogenous expectations as well as unrestricted short selling that the results obtained are similar to the traditional CAPM.

Moreover, the low risk anomaly can be explained by the focus on relative performance instead of absolute performance (Blitz, Vliet and Baltussen, 2019). They further state that investors do not only care about absolute returns, as assumed by CAPM, but on beating the market average. Blitz and Vliet (2007) as well as Baker, Bradley and Wurgler (2011) argue that in the case where CAPM would hold, benchmark-relative investors would find low risk stocks unattractive as they would have high tracking error and lower expected return. Furthermore, Brennan et al., (2012) identified two types of investors; absolute and relative return-oriented investors. They found that the implication of this distinction is that the security market line will partially flatten depending on the number of relative-return investors as opposed to the number of absolute-return investors.

The two-step investment approach is explained as inefficient decentralized investment approach (Blitz & Vliet, 2007). An example provided is that a common practice in the professional investment industry is for the investment committee to make the asset allocation decision. Second step is for the capital to be allocated to managers in charge of buying securities within the different asset classes. A study conducted by Binsbergen et al. (2007) shows that this investment approach may result in inefficient portfolios. This represents benchmark driven investing which is biased towards high beta or high volatility portfolios, which, assuming that CAPM holds, are the easiest way to make above average returns. However, this may lead to the high risk stocks being overpriced and the low risk stocks underpriced.

Furthermore, behavioural biases of private investors have an explanatory power to the existence of this anomaly. According to behavioural portfolio theory private investors think in terms of a two-layer portfolio. Shefrin & Statman (2000) identify two layers; a low aspiration layer and a high aspiration layer. The former is designed to avoid poverty and the latter is designed for a chance at wealth. Blitz & Vliet (2007) provide an example where a private investor makes a risk-averse decision in a particular asset allocation (this represents the first layer). However, the same investor behaves differently and becomes risk-neutral or risk-seeking within a different asset class (second layer). Thus, investors will perceive risky stocks similar to lottery tickets as a result, overpaying for the risky stocks. If investors deviate from the risk-averse behaviour this could result in high-risk stocks to be overpriced and subsequently underprice low risk stocks.

Baker, Bradly and Wurgler (2011) suggest that benchmarking is partly explanatory to the existence of the anomaly. This stresses the point raised earlier by (Blitz & Vliet, 2007) that beating the benchmark leads to a bias towards high risk stocks. In their study, Iwasawa & Uchiyama (2014) found that foreign institutional investors exhibit the same bias towards high beta stocks as their aim is to beat the benchmark. They found that this is the main explanation to the low risk anomaly in the Japanese market.

There have been arguments on whether the low risk anomaly is a distinct effect or whether it is a result of interest rate exposure. A study by De Franco, Monnier and Rulik (2017) investigate this hypothesis. They find that low risk stocks do have a statistically significant exposure to interest rate risk, however, it only explains a small portion of the stocks' alpha. Furthermore, Blitz, Vliet and Baltussen (2019) explain that interest rate risk cannot be a driving factor for the alpha of low risk stocks because the anomaly was already present during periods where interest rates were stable or increasing. Furthermore, arguments were raised on whether the low risk anomaly is a statistically false positive (Blitz, Vliet and Baltussen, 2019). A concern highlighted by Harvey (2017) is p-hacking where the true significance of data is misreported or selectively reporting data. However, Blitz, Vliet and Baltussen (2019) state that this is an unlikely explanation to the anomaly because research on the anomaly was discovered while testing CAPM predictions and not originating from performing tests on different alpha factors. Additionally, they raise the point that the anomaly does not point to a slight error in the CAPM but it completely rejects the positive relation between risk and return. Furthermore, the anomaly has been tested on a wide range of samples as well as across different asset classes.

There has been evidence used to reject the existence of the low-risk anomaly. Bali and Cakici (2008) found in their research that there is a positive correlation between risk and return. Their argument is that the existence of the low risk anomaly is a result of the difference in methodology applied. Through recreating the study by Ang et al. (2006), Bali and Cakici (2008) used daily volatility frequency sampling and found a negative correlation between risk and return. However, when they changed the sampling method and used monthly volatility frequency, they were unable to find the negative correlation. Instead, they found that the correlation between expected returns and volatility was weak. Furthermore, Fu (2009) argues that the risk measures used in most studies are backward-looking and are inaccurate future predictors. His argument is that past idiosyncratic volatility is a poor measure for future idiosyncratic volatility. He uses a different model, EGARCH which is a dynamic model, and found that there is a positive relationship between conditional volatility and expected returns.

### 2.4 The low risk anomaly in practice

Blitz, Vliet and Baltussen (2019) discuss the practical approach in which investors could leverage the low risk anomaly when investing. They first identify that investors aiming to profit from this anomaly typically use the long-only approach. A long position in investing means that an investor will buy stocks with the speculation that they will increase in value. Whereas, a short position means that an investor will sell stocks as they speculate that their value will drop. A common strategy considered in theoretical studies is long-short portfolios (Blitz, Vliet and Baltussen, 2019). Where a long portfolio of stocks consisting only of the good factor indices (i.e., Fama-French factors for value, size and profitability) and conversely a matching short portfolio which encompasses all the unfavourable factor characteristics. The reasons provided for the popularity of the long-only strategy are the management fees which tend to be lower. Whereas, in the long-short strategy having a short position in risky securities is more expensive (Blitz, Vliet and Baltussen, 2019).

Furthermore, there is a large dispersion on the observed turnover levels that are required by asset managers and index providers for low risk strategies. In their study, Li et al. (2014) state that high turnover and implementation costs are restricting investors trying to profit from this anomaly. Blitz, Vliet and Baltussen (2019) highlight in their paper that using the low risk investing strategies requires active investing. The riskiness of stocks does not remain constant throughout time, thus, a buy-and-hold strategy is not applicable in this case and turnover is required.

## 3. CAPM Theoretical Model

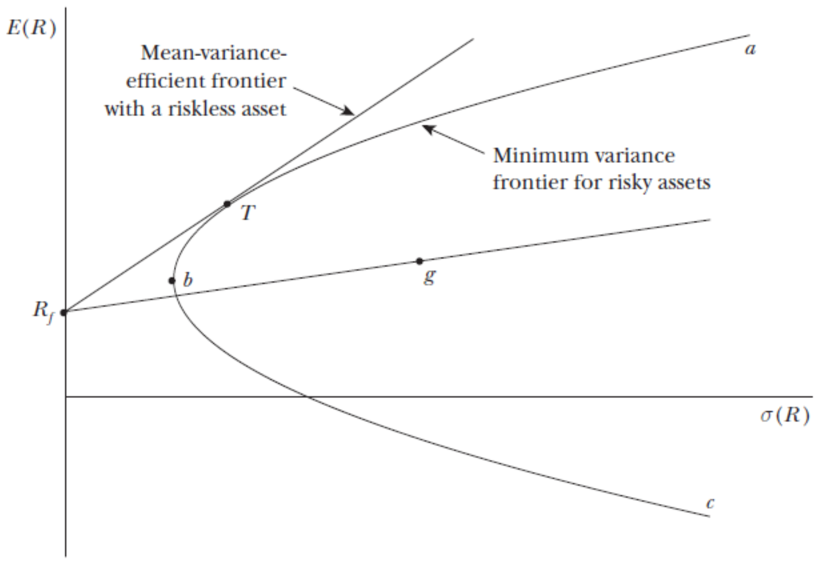
The capital asset pricing model is fundamental in pricing theory. It was initially developed by Sharpe (1964) and Treynor (1961) and was developed further by Mossin (1966), Lintner (1965) and Black (1972). This model extends the concept of market equilibrium as a way to determine the market price for a risky asset as well as the appropriate measure of risk. It shows the relationship between the equilibrium rates of return on risky assets and their covariance with the market portfolio. (Copeland,1988 pg.195).

### 3.1 CAPM Assumptions

The model makes a set of assumptions on investor behaviour and the market opportunity set. The first assumption is that investors are risk-averse with the aim of maximising their expected utility of wealth. CAPM is a single-period model implying that investors maximise their end-of-period wealth. Secondly, investors are deemed to be price-takers meaning that they have homogenous expectations about asset returns. Investors also have access to costless information that is available simultaneously to all. This implies that decision making is based on an identical opportunity set with access to the same information. Thirdly, the market is frictionless where investors can borrow or lend at a risk-free rate. This assumption led to the development of the linear efficient set referred to as the Capital Market line. Fourthly, all assets are considered marketable and perfectly divisible. Thus, excluding the opportunity of human capital. Lastly, markets do not have imperfections nor restrictions on short selling (Copeland,1988 pg.194). These assumptions are set in hypothetical terms and have been criticised for being overly simplifying by researches such as Fama and French (1993).

Figure 1 indicates the graphical break-down of CAPM. The x-axis shows portfolio risk which is measured by standard deviation whereas the y-axis shows expected return. The curve labelled *abc* is the minimum variance frontier which represents a combination of expected return and risk for portfolios of risky assets that offer the highest return at a given level of risk. The portfolios that lie on this curve do not include riskless assets. This means that investors wanting high expected returns should take a higher volatility. For example, an investor who prefers portfolio a because of the higher expected return should also be willing to accept a higher volatility. Furthermore, in the absence of risk-free borrowing or lending, portfolios that lie on the upper half of the minimum variance frontier (above point b) are efficient as they maximise expected return for a given level of risk. This is an important element of the CAPM model. Proof of the model entails that the market portfolio is efficient. This is the basis of the homogenous expectations assumption (Copeland,1988 pg.195). Moreover, if there is borrowing and lending at the risk-free rate $R_f$, the efficient frontier collapses into a straight line. An investor who invests in a portfolio with zero-variance will get a return equal to the point $R_f$ which is the risk-free rate of return. Point T represents the optimal mix of risky securities which is what Tobin (1958) found and referred to as the separation theorem.

**Figure 1: Investment Opportunities**




Source: Fama & French (2004)

### 3.2 Empirical Tests of the CAPM

CAPM is a linear model which is expressed in terms of the following expression in its *ex ante* form:

$$ER_i = R_f + \beta_i(ER_m - R_f)$$

Where, $ER_i$ is the expected return, $R_f$ is the risk-free rate and $\beta_i$ is the systematic risk which is multiplied by the risk premium $(ER_m - R_f)$.

However, in order to empirically test the CAPM it should be converted to its *ex post* form *(Eq. 2)* which converts expectations (since we cannot fully measure expectations) to a form that can use observed data. The assumption behind this conversion is that on average the realised rate of return of an asset is equal to the expected rate of return (Copeland, 1988 pg. 212). An argument is raised by Miller (1977) on expectations and realised returns. He states that prices do not reflect the investors' expectations but reflect the minority of investors who hold the particular stock. Thus, *ex post* investment results are not an accurate measure of an investor's ex ante expectations.

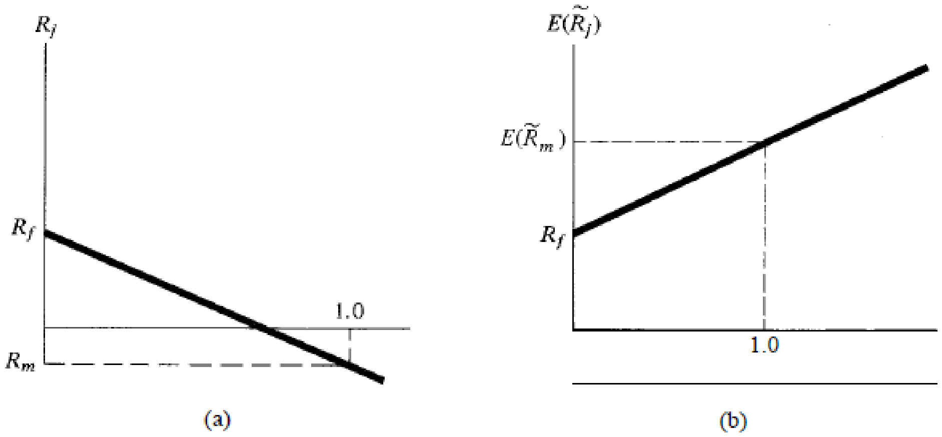
***Equation 2***

$$R_{it} - R_{ft} = a_i + \beta_i(R_{mt} - R_{ft}) + \varepsilon_{it}$$

$R_{it} - R_{ft}$ is the excess returns on an asset, where beta ($\beta_i$) is the systematic risk which indicates the sensitivity towards the market. Beta is multiplied by the risk premium $(R_{mt} - R_{ft})$. Alpha ($a_i$) is referred to as Jensen's alpha which captures the risk-adjusted excess returns compared to the market. The idiosyncratic risk is measured by the variance of $\varepsilon_{it}$.

Another important distinction between the two forms of the CAPM is that the *ex post* empirical model can have a negative slope whereas the *ex ante* empirical model cannot. In the case of a negative slope this means that the security market line (which is a representation of the trade-off between risk and return) is downward sloping as shown in Figure 2 (a). On the contrary, Figure 2 (b) represents the *ex ante* form of CAPM which requires that the expected return on the market should be higher than the risk-free rate of return. This is so the assets with more risk have higher expected returns.

**Figure 2: Ex post CAPM and Ex ante CAPM**




Source: Copeland (1988)

There are three implications of the relationship between expected return and market beta that are considered in tests of the CAPM. The first implication is the linear relationship between expected returns on all assets and their betas. Thus, there is no other variable that can be an explanatory factor. However, there are arguments that this relationship might not be linear and that other factors may also be explanatory to the expected return (Copeland, 1988 pg.212). Secondly, the expected return on the market portfolio is higher than the expected return of assets that do not have a correlation with the market return, meaning that the beta premium is positive. Thirdly, the expected returns of assets that are uncorrelated with the market are equal to the risk-free interest rate. Therefore, the beta premium is denominated as the expected market return minus the risk-free rate (Fama & French, 2004).

### 3.3 CAPM extensions

Many of the assumptions made by the model are rigid and do not hold in the real world. The model has been extended to take into account the violation of some of the assumptions in the real world. Black (1972) solved the problem of the absence of a riskless asset where investors are unable to borrow and lend at a risk-free rate. Instead, investors use a zero-beta portfolio which contains risky assets that are uncorrelated with the market portfolio. There still is a portfolio that has the minimum variance of all the portfolios that are uncorrelated with the market portfolio.

The existence of nonmarketable assets is also a violation of the fundamental CAPM assumptions. An asset that is not perfectly divisible and does not have a market value, for example human capital, results in investors holding different portfolios of non-diversifiable risky assets. If this assumption is not violated investors choose between holding the risk-free asset or the market portfolio. Mayers (1972) highlighted two important implications of the introduction of nonmarketable risky assets. Firstly, there will be a variety of portfolios of risky assets that individual investors can hold since the asset has differing amounts of risk. Secondly, the market price of risk is objectively determined which is independent of the individual’s risk preference.

The CAPM was extended from a single time period to a model in continuous time. This version, referred to as ICAPM, was derived by Merton (1973) which assumes that trading is an event that takes place over time. Additionally, that the distribution of asset returns is lognormally distributed. In the CAPM model investors are concerned with their end of period wealth however, in the ICAPM model the life of the investment is considered.

If the assumption of homogenous expectations is relaxed then investors do not view the same opportunity sets as they have heterogenous expectations about future returns. Thus, investors will select different portfolios. Lintner (1969) has shown that the CAPM is not significantly altered by the existence of heterogenous expectations. The difference is that the expected returns and covariances are expressed in terms of complex weighted averages of investor expectations. However, if investors have heterogenous expectations this implies that the market portfolio might not be efficient which makes the model non-testable.

### 3.4 CAPM Limitations

#### Roll's Critique

An application of the securities market line is that it can be used as a benchmark for performance of securities. Abnormal returns are defined as those that fall outside the prediction of the security market line. Roll (1977) criticizes the asset pricing theory's tests by taking an exception to the interpretation of the performance measurement of abnormal returns. Additionally, he takes a different look at the empirical tests of CAPM. Firstly, Roll concludes that a legitimate test of CAPM is the mean-variance efficiency of the market portfolio. Secondly, if performance measurement is relative to an index that is *ex post* inefficient, then it is possible to have any ranking of portfolio performance depending on the selection of the inefficient index (Copeland,1988 pg.218).

The implication of these statements is that regardless of the efficiency of markets which make CAPM valid, the cross-section security market line cannot be used as a way of measuring the *ex post* performance of portfolio selection. Overall, Roll states that the market portfolio is not the only portfolio than can be used as an index. It is possible to select any efficient portfolio as an index. In the absence of a risk-free asset, it is still possible to derive the security market line as the combination of the market-portfolio and a zero-beta portfolio which is has no correlation with the market index.

#### Fama and French

Fama and French (1992), state that the empirical record of the CAPM is poor which stem from the difficulty of implementing valid tests of the model. In their results they found that stock risk is multidimensional and is not only explained by systematic risk. The first dimension they discuss is risk proxied by size and the second dimension is proxied by book-to-market equity which is the ratio of the book value of equity to its market value. They state that this effect is more powerful than the size effect. Furthermore, Fama and French (1993) developed a three-factor model which they argue it captures the anomalies that are not explained by CAPM. The three factor model states that there are three factors that can explain the expected excess return on a portfolio. The first factor is referred to as the market premium which can be computed as Ri – Rf. The second factor is the size premium which is computed as the difference between the return on small stock portfolio and big stock portfolio. The third factor is the book-to-market factor which is computed as the difference between returns of high book-to-market stock portfolios and low book-to-market stock portfolios.

## 4. Data & Methodology

The data collected is from the big Spanish firms listed on the IBEX-35. The data is downloaded from Infobolsa Netstation, a private database accessed through Universitat Jaume I. We use data from January 2002, the first year that Spain started using the Euro as their official currency, until December 2021. The relevant data is the monthly prices for all stocks. Using these monthly prices, the average return is calculated using the last closing price as the monthly price. We also use daily returns to compute our measure of risk.

This paper tests the low risk anomaly in the Spanish market by looking at the total risk as a measure of risk instead of just systematic risk. The importance of total risk is highlighted by Goyal and Santa-Clara (2001). In their study, they measure the average variance of a stock using the daily stock returns within the month. This paper looks at short term volatility therefore, we use the bi-monthly standard deviation as a measure of total risk at each month. Thus, the standard deviation calculated at each month takes a look at short term volatility.

***Equation 3***

$$\sigma_i = \sqrt{\frac{\sum_{i=1}^{n}(Ri - \overline{R})^2}{n-1}}$$

The use of monthly sampling is in line with the statistical evidence found by Bali and Cakici (2008) showing that monthly sampling is a better proxy for future expected volatility rather than daily sampling.

The return on stocks is estimated as the bi-monthly arithmetic mean. Each month takes into account the previous month’s return. The stocks at each month are then sorted based on lowest risk to highest risk. At each month the stocks are divided into quintile portfolios which are equally weighted. The average return is calculated at each month using the returns in every quintile. Thus, there is an average return for each month of the time series. Furthermore, the overall mean for each quintile portfolio is calculated using the monthly returns from 2002 until 2021. The overall standard deviation for each quintile portfolio is also calculated.

We then calculate the Sharpe ratio for each portfolio. William Sharpe (1966) developed the Sharpe ratio which is a commonly used to measure risk-adjusted performance of security portfolios. The ratio is interpreted as the additional excess return gained from adding one extra unit of risk. It is calculated as the portfolio's return minus the risk-free rate divided by the standard deviation of returns.

***Equation 4***

$$SR_p = \frac{(R_p - R_f)}{\sigma_p}$$

Where $R_p - R_f$ is the mean excess return and $\sigma_p$ measures the total risk of the portfolio. In this case, the risk-free rate was not included in the calculation as it was close to zero meaning that it would not make a significant statistical difference on results obtained.

We obtained the market return by downloading the IBEX-35 monthly index return from January 2002 until December 2021.Furthermore, we calculated a linear regression for the returns of each quintile portfolio on the IBEX-35 index returns in order to obtain Jensen's alpha as well as the beta which represents the systemic risk of each risk-sorted portfolio.

Jensen's alpha is computed as:

***Equation 5***

$$a_p = r_p - [r_f + \beta_p(r_m - r_f)]$$

Where $r_p$ is the return of the portfolio, the risk-free rate is represented by $r_f$. $\beta_p$ is the beta of portfolio p which is computed as the sum of each stock's weight in the portfolio multiplied with the beta of the stock.

Beta is calculated as:

***Equation 6***

$$\beta_i = \frac{Cov(r_i , r_m)}{Var(r_m)}$$

Where the beta of each stock, $i$ , is calculated using the IBEX-35 monthly returns, $r_m$ .

In line with Bali et al. (2005), we extend the study by testing whether the results are sample dependent. A subsample is created just looking at data from January 2002 until December 2015. The Sharpe ratio, Jensen's alpha as well as beta are re-calculated in order to compare with the results observed in the main sample.

To broaden our research, we could have tested if the weighting of our portfolios changes the result by value-weighting the quintile portfolios. This is in line with Baker, Bradley and Taliaferro (2013) who applied this method as short-selling contraints could have an explanatory factor to the low risk anomaly. Furthermore, an additional metric that we could have looked at is the information ratio. This ratio looks at the excess portfolio returns as compared to the returns of a benchmark, index, to the risk associated with those returns. The risk is computed as the standard deviation of the difference between the two returns. According to Baker, Bradley and Wurgler (2011) the information ratio can help to identify whether benchmarking can explain the low risk anomaly.

## 5. Results & Analysis

The main results are summarised in Table 1. In the analysis, we look at three performance measurements namely; Sharpe ratio, Jensen's Alpha and Beta (which is the systematic risk). The portfolios are numbered based on increasing total volatility with portfolio 1 being the low risk portfolio and portfolio 5 being the highest risk portfolio. Each portfolio has an equal weighting.

The Sharpe ratio is the risk-adjusted returns which allows for a direct comparison between the portfolios as it measures excess return for every unit of risk. Based on the results, portfolio 1 has a substantially higher ratio than portfolio 5 (0.3842 and 0.1807). This means that for the same unit of risk portfolio 1 gives better return than portfolio 5. Portfolios 2, 3 and 4 also outperform portfolio 5 (0.2740, 0.2438 and 0.2238 respectively). Thus, providing evidence of the low risk anomaly.

However, when observing the average return that is not risk-adjusted we find that portfolio 5 has the highest return compared to portfolio 1 (6.54% and 4.98% respectively). The average return is a gross measure of performance which considers all risk in a portfolio not just a unit of risk as in the Sharpe ratio. Thus, looking at risk as a whole and not a unit, these results are in line with financial theory. Portfolio 5 is the riskiest portfolio which explains the high average return.

Using Jensen's alpha as a performance measurement we observe different results. This measure represents the risk-adjusted average returns of a portfolio given the portfolio's beta and the average market return. Jensen's alpha shows the excess returns earned that are not explained by the index. Thus, a positive alpha indicates that one can "beat the market" by earning excess returns than CAPM predicted results. Portfolio 5 has a higher alpha of 5.89% and subsequently the highest beta of 3.6113. This indicates that portfolio 5 has higher systematic risk and investing in this portfolio will be rewarded with higher return which is shown by the high alpha. Moreover, portfolio 5's high beta shows that the portfolio is more exposed to the index, meaning that if the market moves by 1 (favourably or negatively) the portfolio will move in the same direction by 3.6113. Thus, making the portfolio more volatile. Whereas, Portfolio 1 has an alpha of 4.79% and the lowest beta. This is aligned with what financial theory states, high risk is rewarded by high return. However, even though portfolio 1 has a low alpha, it still outperforms portfolios 2 and 3.

Table 1: Performance metrics for risk-sorted portfolios for the full sample 2002-2021

| Performance Measures | Portfolio 1 | Portfolio 2 | Portfolio 3 | Portfolio 4 | Portfolio 5 |
|---|---|---|---|---|---|
| Average Return | 4.98% | 4.48% | 4.75% | 5.35% | 6.54% |
| Standard Deviation | 12.96% | 16.34% | 19.50% | 23.89% | 36.19% |
| Sharpe Ratio | 0.3842 | 0.2740 | 0.2438 | 0.2238 | 0.1807 |
| Jensen's Alpha | 4.79% | 4.16% | 4.37% | 4.88% | 5.89% |
| Beta | 1.0345 | 1.7662 | 2.1234 | 2.6040 | 3.6113 |

Own elaboration

The different conclusions reached by observing the Sharpe Ratio and Jensen's alpha lead back to the golden question, does total volatility (idiosyncratic risk) matter? This is a relevant question as the Sharpe Ratio was calculated in terms of total risk and Jensen's alpha given beta (market risk). Goyal & Santa-Clara (2001) emphasize the importance of idiosyncratic risk and its higher explanatory power. However, Bali et al. (2005) extended this study by adding two additional years to the sample (2000 and 2001). He found that the Goyal & Santa-Clara's results are sample specific. He further states that it could also be driven by the small stocks that are traded on Nasdaq. Additionally, the weighting scheme also plays a significant role. Angelidis & Tessaromantis (2009) hypothesize in their study that the sample dependency of the idiosyncratic risk and returns is indicative of the existence of two different stock market regimes. The difference is the means and standard deviation. A study conducted by Schwert (1989) found compelling evidence of the two different stock regimes. He found a low variance regime with positive average returns and a high variance regime with negative average returns which is less frequently observed.

Applying the sample dependency hypothesis, we extend our study to test whether reducing the sample time series makes a significant change to the results. The subsample of returns used is from January 2002 – December 2015 and the summarised results are in table 2. Based on the results, the Sharpe ratio still indicates that portfolio 1 performs better than portfolio 5. In fact, the low risk anomaly has a stronger presence in this subsample as the gap slightly widened between the two portfolios. In the first sample, 2002-2021, the difference in the Sharpe ratio between the two portfolios was 0.20 (20%). However, in the reduced sample

(2002 – 2015), the difference increased by 3%. In comparison to the first sample results, the overall average portfolio returns increased for portfolios 1, 2, 3 and 4 however, the average return for portfolio 5 decreased. Thus, the conclusion based on the Sharpe ratio is consistent in both samples.

Additionally, Jensen's alpha results are not consistent with what was observed in the first sample. In this case, portfolio 1 outperforms portfolio 5. The alpha in portfolio 5 reduces by 2% whereas, the beta increased. Thus, the portfolio is slightly more volatile however, the return that is not explained by the index is lower. On the other hand, portfolio 1 has a higher alpha than portfolio 5 even though it has a lower beta. This contradictory to common theory and provides evidence of the low risk anomaly. Moreover, portfolio 2 also outperforms portfolio 5 presenting a higher alpha. However, portfolio 4 is the best performing portfolio which is riskier than portfolio 1 thus, challenging the low risk anomaly evidence. This could be a result of the effect of weighting in the portfolios as stated by Bali et al. (2005).

Table 2: Performance metrics for risk-sorted portfolios for the subsample period 2002-2015

| Performance Measures | Portfolio 1 | Portfolio 2 | Portfolio 3 | Portfolio 4 | Portfolio 5 |
|---|---|---|---|---|---|
| Average Return | 5.02% | 5.26% | 4.79% | 6.39% | 5.44% |
| Standard Deviation | 13.58% | 16.72% | 19.32% | 25.08% | 38.67% |
| Sharpe Ratio | 0.3699 | 0.3143 | 0.2478 | 0.2546 | 0.1407 |
| Jensen's Alpha | 4.76% | 4.78% | 4.20% | 5.65% | 4.43% |
| Beta | 0.9614 | 1.7638 | 2.1566 | 2.7051 | 3.7354 |

Own elaboration

## 6. Conclusion

The aim of this study is to observe whether the low risk anomaly is present in the Spanish stock market. The data used is the stocks traded in the IBEX-35 from the period January 2002 to December 2021. The methodology used is in line with existing literature which uses CAPM's beta and volatility as proxies for risk. Additional performance metrics used are Sharpe ratio and Jensen's alpha. The portfolios are risk-sorted with portfolio 1 being the lowest risk portfolio and 5 being the highest risk portfolio. All portfolios are quintile portfolios which are equally weighted. We find that the low risk anomaly is present in the Spanish market, from 2002 to 2021, when observing the Sharpe ratio. The lowest risk portfolio (portfolio 1) has a higher Sharpe ratio than the riskiest portfolio (portfolio 5). Thus, for the same unit of risk the lowest risk portfolio has higher excess returns. However, when observing Jensen's alpha, we find that the riskiest portfolio has a higher beta and a higher alpha outperforming the lowest risk portfolio.

We tested whether our results were sample dependent by creating a subsample from January 2002 to December 2015 and applied the same performance metrics. In this case, both the Sharpe ratio and Jensen's alpha gave consistent results indicating the existence of the low risk anomaly. The results are even stronger in this sample confirming the anomaly. This could be an indication the low risk anomaly is no longer strong when taking into account the more recent years as the main sample indicates weaker results.

Financial literature has still not accounted for the larger part of the existence of the low risk anomaly. Many researchers state that it could be caused by benchmarking and short-selling constraints. There is an implication that the security market line is flatter with the slope depending on how constrained the leverage is for investors. However, there could be more explanatory factors as this topic is still undergoing research.

Our results are subject to the limitations of the CAPM model as the fundamentals of this research were based on the application of the model. Furthermore, the results did not undergo many other robustness checks.

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## 8. Appendix

**Table 3: Articles investigating the low risk anomaly**

| Year | Author | Proxy for risk | Findings |
|---|---|---|---|
| 2006 | Ang et.al. | Idiosync. Vol | confirms LR |
| 2009 | Ang et al. | Idiosync. Vol | confirms LR |
| 2006 | Clarke et al. | Total Vol | confirms LR |
| 2007 | Blitz & Vliet | Idiosync. Vol | confirms LR |
| 2008 | Bali & Cakici | Idiosync. Vol | LR not present |
| 2011 | Bali,Cakici & Whitelaw | Idiosync. Vol | LR not present |
| 2011 | Frazzini & Pederson | Beta | confirms LR |
| 2011 | Baker, Bradley & Wurgler | Beta, Total Vol | confirms LR |
| 2013 | Baker, Bradley & Taliaferro | Beta | confirms LR |
| 2014 | Iwasawa & Uchiyama | Beta | confirms LR |

Source: Own elaboration

**Table 4: Early Empirical tests of CAPM**

| Year | Author | Title |
|---|---|---|
| 1970 | Friend & Blume | Measurement of Portfolio Performance under Uncertainty |
| 1972 | Black, Jensen & Scholes | The Capital Asset Pricing Model: Some Empirical Tests |
| 1972 | Miller & Scholes | Rates of Return in Relation to Risk: A Re-examination of Some Recent Findings |
| 1973 | Blume & Friend | A New Look at the Capital Asset Pricing Model |
| 1973 | Fama & Macbeth | Risk, Return and Equilibrium: Empirical Test |
| 1977 | Basu | Investment Performance of Common Stocks in Relation to Their Price-Earnings Ratios: A Test of the Efficient Markets Hypothesis. |
| 1981 | Reinganum | Misspecification of Capital Asset Pricing: Empirical Anomalies Based on Earnings Yields and Market Values. |
| 1979 | Litzenberger & Ramaswamy | The Effect of Personal Taxes and Dividends and Capital Asset Prices: Theory and Empirical Evidence. |
| 1981 | Banz | The Relationship between Return and Market Value of Common Stocks. |
| 1982 | Gibbons | Multivariate Tests of Financial Models: A New Approach |

| | | |
|---|---|---|
| 1982 | Stambaugh | On the Exclusion of Assets from Tests of the Two-Parameter Model: A Sensitivity Analysis |
| 1985 | Shanken | Multivariate Tests of the Zero-beta CAPM |

Source: Own elaboration

A common finding in these empirical studies is that the intercept term in the model is different to zero and the slope is not as steep as predicted by the model. This implies that the returns on high beta securities are lower than CAPM predicts and low beta returns are higher than the model predictions. Furthermore, more factors that could have explanatory powers either than beta were discovered.

**Table 5: Anomalies**

| **Year** | **Author** | **Title** | **Results** |
|---|---|---|---|
| 1981 | Banz | The Relationship between Return and Market Value of Common Stocks | Smaller firms have higher returns than can be explained by the model. (Size effect) |
| 1981 | Reinganum | Misspecification of Capital Asset Pricing: Empirical Anomalies Based on Earnings Yields and Market Values | Size effect |
| 1983 | Keim | Size-Related Anomalies and Stock-Market Seasonality: Further Empirical Evidence | Seasonality in stock effects. January effect |
| 1988 | Reinganum | The anatomy of A stock market winner | Association between book-to-market equity and average returns |
| 1993 | Jegadeesh & Titman | Returns to buying winners and selling losers: Implications for stock market efficiency | Momentum effect |

Source: Own elaboration