The shift from passive to active investing: an application to the Spanish market using the Markowitz model

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ABSTRACT

This paper aims to increase the knowledge of the Markowitz model and through its practical application, create a series of efficient portfolios that can beat the IBEX-35 and/or the naïve strategy 1/N. These portfolios should provide the lowest possible risk for a given level of return.

For this purpose, we will select several assets from different sectors that will make up our investment portfolio. By solving the mathematical optimisation model with the model's boundary conditions, we will obtain the Markowitz corner portfolios. These will be compared with IBEX-35 and the naïve strategy 1/N to check if the objective has finally been achieved.

RESUMEN

El objetivo del presente trabajo consiste en aumentar los conocimientos sobre el modelo de Markowitz y mediante su aplicación práctica, crear una serie de carteras eficientes que consigan batir al IBEX-35 y/o a la naïve strategy 1/N. Estas carteras deberán proporcionar el menor riesgo posible para un determinado nivel de rentabilidad.

Para ello, deberemos seleccionar una serie de activos de distintos sectores que integrarán nuestra cartera de inversión. Al resolver el modelo matemático de optimización con las condiciones de contorno del modelo, obtendremos las carteras esquina de Markowitz. Estas las compararemos con el IBEX-35 y la naïve strategy 1/N para comprobar si finalmente se ha alcanzado el objetivo.

Keywords

Markowitz, portfolio management, naïve strategy 1/N, optimization, efficient frontier

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INTRODUCTION

First of all, I would like to explain the reasons why I decided to choose portfolio management through the Markowitz model as my final thesis, to be able to apply the knowledge acquired during my degree in finance and accounting.

My interest in portfolio management initially stemmed from its popularity in the world of finance. This interest grew when I realized that there are currently a large number of people without adequate financial knowledge who choose to invest their savings in equities. This is large because nowadays fixed income does not provide good returns.

"He who does not know his origin does not know his destiny".
Popular saying

As the above popular saying goes, one of the reasons why I have decided to conduct portfolio management using the Markowitz model is because it is vitally important to know the origins of portfolio management to understand and develop hypotheses and models after it. Hereupon, the Markowitz model will be the main core of this paper. Specifically, the aim is to demonstrate that active portfolio management can beat the Spanish stock market index (IBEX-35) and/or the naïve strategy 1/N (passive management). In other words, the investment portfolio created by applying this model provides a higher return at the same level of risk, or conversely, a lower risk for the same return.

Although there are authors such as Grinold (1989) who defend active management, stating that the manager's strategy adds value to investments, we must bear in mind that, in Markowitz's case, the set of efficient portfolios obtained show past behaviour, this does not mean that the model's solution is irrefutable, since markets can be affected by a multitude of factors, which means that they do not have a defined behaviour the other hand, if we were to take all these factors into account, it would be technically impossible to carry out the model, so we can take Markowitz's proposal as a reference for the future creation of investment portfolios.

The structure of the paper is divided into five sections.

The first of these consists of the theoretical explanation of the model. First of all, to put the reader in context, we will explain the diverse ways of managing an investment portfolio. We will then proceed to develop the Markowitz model in depth, explaining the hypotheses to be considered, how it is developed, which functions have to be maximised and minimised and under which restrictions, etc. All this is to obtain the portfolio composition that optimises the risk-return optimizes

Once the theory of the model has been explained, in the second section, we will extract the necessary data. To do this, we will start by defining the time horizon, the stock market and, the number of assets we are going to work with. With this established, we will create a database in Microsoft Excel consisting of the daily closing prices of the selected time horizon.

In the third section, after obtaining the database, we will develop the methodology used to find the optimal investment portfolio. Briefly, the procedures we will conduct will be the calculation of the annualized returns, volatilities, and correlations of all the assets of the IBEX-35. Considering all the above we will select the 12 assets that will form part of the optimal portfolio and we will create their annual variance and covariance matrix. Finally, the risk of the portfolio will be obtained.

In the fourth section, to check whether the objective of the paper is met, we will calculate the return and risk that would be extracted if both passive and active management were chosen, and then compare the results.

In the last section, we will bring together the conclusions drawn during the work and the objective whether of the work has been achieved, that is, whether active management beats passive management.

Finally, the bibliography used and the annexes necessary to complete the document will be presented.

I. THEORETICAL FRAMEWORK:

Firstly, before explaining in depth the active management of a portfolio using the Markowitz model, let us name the diverse ways in which an investment portfolio can be managed.

1. Ways of managing a portfolio:

There are two ways of managing a portfolio to achieve an investor's objectives, passive and active.

The **passive management** or passive strategy of a portfolio of securities is used when the hypothesis of market efficiency is assumed to be fulfilled, i.e., when we think that the quoted prices of the assets that are going to form part of our portfolio reflect all the information existing in the market. Given the existence of perfect information, from the point of view of portfolio managers, it will not make sense to try to predict the future interest rates of the assets, as this will not allow us to obtain more profitability than in the reference market. For this reason, the objective will be to follow a benchmark portfolio that reflects market movements and to create mirrors in the performance of that index. This portfolio will be composed of stocks from the index, which will have a high degree of diversification and will maintain our risk aversion, allowing us to keep the portfolio unchanged in the short term, altering it only to make necessary adjustments, such as a reinvestment of dividends, thus optimising transaction costs.

One of the most common ways to diversify an investment portfolio while maintaining our risk aversion as documented by Benartzi and Thaler (2001) is by using the naïve 1 / *N* portfolio diversification rule, which consists of distributing wealth equally by allocating to each of the *N* available assets the proportional share that corresponds to them by the fraction 1 / *N.* The reasons why this rule is so common are: firstly, because of its simplicity of implementation, since it is based neither on the estimation of the moments of return of the assets nor on optimisation, and secondly, because investors continue to use this simple rule to distribute their wealth among assets, despite the development of much more sophisticated theoretical models and advances in the methods for estimating the parameters of these models in the last 50 years (DeMiguel et al., 2009).

On the other hand, before starting to talk about the **active management or strategy** of a portfolio of securities, I would like to quote Grinold (1989), who defends the existence of a fundamental rule, with which he aims to shed some light on this type of management, defining it as follows: "the added value of the manager's strategy is equal to the product of the number of independent forecasts of extra returns made per year times the correlation between the forecasts and the actual results squared".

For Fabozzi and Fong (1994), active portfolio management or strategy, as its name suggests, consists of actively seeking market inefficiencies that allow us to obtain a profit. This search for new opportunities is conducted by making variations in the investment portfolio through the purchase and sale of undervalued or overvalued assets, seeking to achieve the highest possible return, which must always be capable of covering transaction costs and without exceeding the determined level of risk (that which the investor is willing to assume). All this is due to the belief that the hypothesis of market efficiency is not fulfilled, i.e. the assumption that asset prices do not reflect all the available information, and therefore the managers' aim by anticipating its movements through active management, The managers will behave differently depending on the quantity and quality of the information available to them, since not all investors will have the same information, which will allow them to obtain profits, since otherwise, all investors would invest in the same assets, thereby causing their prices to adjust.

According to Bodie, Kane and Marcus (1993), a portfolio can be actively managed in two diverse ways. The first of these is the one previously described by Fabozzi and Fong (1994), which is based on making profits by buying and selling undervalued or overvalued assets. The other way is to beat the market by forecasting the behaviour of future interest rates, based on historical interest rates and without considering market uncertainty, so we will have to be aware that past returns will not guarantee future returns.

Moreover, although the use of forecasting in portfolio management is advantageous, it has its critics, such as Choice (1990), who considers that there are two possible alternatives for successful portfolio management: the first consists of developing a model that is capable of predicting future interest rate movements in their entirety correctly or at least fairly accurately (such as the crystal ball model, which is very common among managers), the second alternative that the author proposes is a model that he has developed, called DDS (Duration Strategy), based on the development of a strategy that tries to seek the highest possible participation in markets with an upward trend.

After explaining what active management consists of and the diverse ways in which a portfolio can be actively managed, it is necessary to highlight the main tools used for traditional active management. Firstly, a macroeconomic analysis is conducted, which consists of a prior study of the global functioning of the economy, obtaining the necessary information to know where we have to invest. Next, fundamental analysis is conducted from which we will obtain which company to invest in and finally, technical analysis is conducted, based on the study of the evolution of a security, from which we will obtain when we have to invest (market timing).

2. Markowitz Model:

2.1 Biography:

Harry Max Markowitz was born in Chicago in 1927. During his professional career, he trained as an economist, specialising in investment analysis. In his working life, aside from immersing himself in numerous research and creative projects, he was also a professor at the City University of New York. In 1990 he was awarded the Nobel Prize in Economics together with Merton H. Miller and William F. Sharpe for their pioneering work in the theory of financial economics and their contributions to the analysis of investment portfolios and methods of corporate finance.

Markowitz attended high school in his hometown, and when he finished his studies he entered the University of Chicago, where he graduated with a degree in economics. According to his autobiography, he was fortunate to have great professors such as M. Friedman, T. Koopmans, J. Marschak and Leonard J. Savage, who had already worked on the problems of investment selection. In addition, during his studies, he was invited to become one of the student members of the Cowles Commission for Research in Economics.

In 1954 he obtained his doctorate with a thesis based on portfolio theory, which served as inspiration for his article "*Portfolio Selection"* published in the prestigious journal The Journal of Finance, considered to be the origin of portfolio selection, in which he studied the process of optimal selection of an investment portfolio. A few years later, in 1959, he published his book "*Portfolio Selection, Efficient Diversification of Investments"*, which gave impetus to his portfolio theory. In it, he pioneered a mathematical approach to diversifying an investment so that the composition of the portfolio maximises the return for a given level of risk or minimises the risk for a given level of return. This all began as a result of a casual conversation in which the possibility of applying mathematical methods to the stock market came up, inspiring him to the basic concepts of his portfolio theory in John Burr Williams'

"*Theory of Investment Value"* (1938), in which Williams proposed that the value of a stock should be equal to the present value of its future dividends.

This model has been phenomenally successful at the theoretical level, but it is worth noting the technical complexity of the model, which at the time led most portfolio managers and investment analysts to opt for the Sharpe model, as this simplified the calculations. Today the model can be solved without further complication thanks to advances in software and quantitative computing techniques.

Finally, some of his other publications are worth mentioning. Firstly, "*The SIMSCRIPT II programming language"*, produced together with Kiviat P.J. and Villanueva R. (1968), is a manual for users and programmers of a new programming language called *SIMSCRIPT II, the main objective of this design was the production of a readable language oriented towards debugging and efficient execution of large simulation models. Another publication of note is "Mean-variance analysis in portfolio choice capital markets"* (1987), in which the reader will find a complete treatment of the portfolio choice model, efficient solution algorithms, characteristics of viable solutions, aspects of several important exceptional cases and much more.

2.2 Model assumptions:

To develop and subsequently calculate the model, Markowitz makes the following assumptions:

- The model has a **single period** as its time horizon, therefore, at the time of analysis, all investments will have the same period, with a period of securities being purchased at the beginning of the period and sold at the end of the period, so that the securities will have immediate liquidity at the end of the period in question.
- The **assets** (N) that will be part of the portfolio are **known**.
- All selected **assets** are **risky**, using the variance or standard deviation as a measure of risk, which will always be greater than zero.
- Investors measure the return on assets by their average, i.e., the **expected return**.
- The **random variables** of the return on assets are known, and they must also be **distributed** according to **regular laws**.
- **Investors behave rationally**, preferring higher returns to lower returns, but they are **risk-averse** when making their decisions, i.e., between two portfolios of securities

with the same expected return, investors prefer the portfolio with the lower risk. In conclusion, investors will prefer a portfolio in which they earn low returns and do not have to expose their capital to elevated risk.

- Because investors are risk-averse, their isoutility functions or **indifference curves** are **increasing** (implying that the higher the risk, the higher the required return), and **convex** (the higher the risk, the higher the required return increases in proportion). These curves indicate the risk-return combinations that provide the same utility to the investor. The investor will choose the one that is further to the left and upwards because it will be the one that provides the highest return with the lowest risk.
- Credit or short selling is **not** allowed, i.e., **short selling positions are** not covered by this method, only longs. This implies that all ratios are positive or zero.
- The **budget** available to the investor to build the portfolio **must be fully invested.**
- **Perfect capital markets** are assumed, in which:

    - All information is equally available to all market participants and is free of charge.
    - Investors are price takers, i.e., they cannot influence price formation.
    - There are no taxes and no inflation in the economy.
    - There are no transaction costs in the purchase and sale of financial assets.
    - All actions are infinitely divisible, i.e., any proportion of the budget can be invested.

2.3 Model development:

Before starting to explain Markowitz's portfolio selection theory, i.e., the general analysis of portfolio management with N securities, we should mention the fact that it is based on the idea that a portfolio with higher return and lower risk (better performing portfolio) could be obtained through the appropriate combination of two or more securities.

In this model, Markowitz aims to obtain the best combinations among a set of given financial assets and then decide which one to invest in, solving the problem of how to distribute the investment budget among the different risky financial assets traded on the market. To do this, he makes a mathematical model assuming the rationality of investor behaviour, i.e., assuming that the investor desires profitability (profit obtained from an investment or operation compared to the resources used to generate it) and rejects risk (probability of an adverse event occurring that causes negative financial consequences). It aims to obtain an efficient portfolio for each investor, either by obtaining the maximum possible return for a given level of risk or the other way around, i.e., having the minimum possible risk for a given level of return. When it comes to optimising and finding the corner portfolios in the practical application of this model, we will focus on the second of the cases, using as a measure of profitability the expectation of the current value of the equity portfolio and as a measure of risk its variance.

The search to minimise the level of risk will be conducted by employing the diversification strategy, which consists of expanding the investment portfolio to balance the overall profitability of the portfolio, compensation being generated in one investment with the gains that may appear in another. To do this, it is necessary to analyse the historical behaviour of share prices and the relationship between them beforehand, thus reducing the impact of potential risk with an appropriate investment portfolio.

To develop Markowitz's portfolio selection theory, several steps must be followed to identify the optimal portfolio, i.e., the portfolio composition that maximises the investor's expected utility.

The first step is based on observation and experience, whereby the set of investment possibilities offered by the market must be determined. To obtain this with multiple assets and in the mean-variance environment, we must identify all the possible combinations of expected return and standard deviation that can be achieved given a number N of assets with risk, which requires an analysis of all the assets traded in the market, estimating for each security its expected return, its risk and its covariances with the rest of the securities and portfolios that can be formed. Once these estimates are known, all the possible combinations can be plotted on the expected return - standard deviation coordinate axes. In this way, we obtain the set of investment opportunities given by the market as each investor sees it.

In the second step, the efficient frontier must be determined, considering the investor's set of possibilities (previous step), the investor must consider which is the portfolio that mostly satisfies them, seeking the risk-return combination that maximises his expected utility.

Portfolios will be considered efficient when they are generating the highest return given a standard deviation (risk) and when they have a lower standard deviation given an expected return (return).

As we can see in the following image, any portfolio that is below or to the right of the frontier will be inefficient. This frontier is delimited at the bottom by the portfolio with the minimum variance, and at the top by the portfolio with the maximum level of profitability. Moreover, the **efficient frontier** must necessarily be concave.

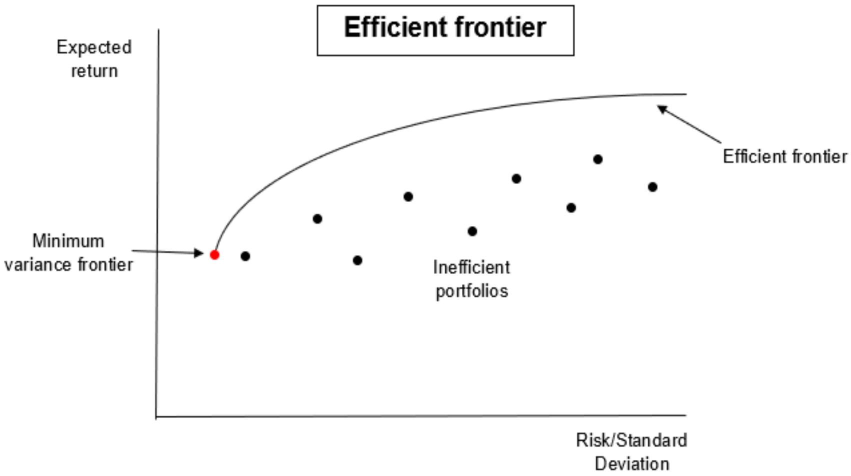

Figure 1: Markowitz Efficient Frontier (theoretical).
Source: own elaboration.

Once the efficient frontier has been obtained, the next step is to determine the investor's attitude to risk, i.e., the possibility that the expected returns will not be met and that this will lead to negative financial consequences for the investor.

The mean-variance criterion does not allow comparisons to be made between two efficient portfolios, since the riskier portfolio will also be the one that offers a higher return, so it will only be possible to know the choice of the optimal portfolio if we know the investor's specific degree of risk aversion.

Each investor has a different risk aversion, as there will be investors willing to take on more risk in exchange for a higher return, and there will be other investors who will settle for a lower return in exchange for bearing less risk, i.e. investors will demand a higher return for each increase in the level of risk they are willing to take on.

The last step is to determine the optimal portfolio for the investor using the data obtained in the previous steps, i.e. once the efficient frontier and the investor's particular risk preferences have been determined, we can obtain his optimal portfolio, which is the portfolio that combines mean - standard deviation (or mean-variance) that maximises his expected utility, i.e. the portfolio that best fits his personal risk preferences.

Finally, it should be noted that each investor will have a different optimal portfolio depending on their expectations of stock returns and their preferences in terms of risk-return trade-offs.

2.4 Approach to the Markowitz mathematical-financial model:

The boundary conditions to be met for the Markowitz mathematical-financial model of portfolio construction are as follows:

$$Minimize: \quad \sigma_P^2 = x' \cdot V \cdot x \quad (1)$$

$$Maximize: \; E_P = \sum_{k=1}^{n} x_k \cdot E_k \quad (2)$$

$Subject\ to\ the\ following\ restrictions$:

The sum of all the weights of each security in the portfolio cannot be greater than 1, i.e., the sum of the values must equal 1.

$$\sum_{k=1}^{n} x_k = 1 \quad (3)$$

In addition, Markowitz sets a condition of non-negativity, which means that the portfolio weights cannot be negative, i.e., they must be equal to or greater than 0.

$$\forall k \in \{1,2,\dots,n\}, x_k = 0 \quad (4)$$

Being:

$\sigma_P^2 = Portfolio\ variance\ p$

$x' = Column\ vector\ of\ the\ proportion\ of\ each\ asset\ x_k$

$x = Row\ vector\ of\ the\ proportion\ of\ each\ asset\ x_k$

$V = Variance - covariance\ matrix\ of\ the\ annualised\ returns$

$E_P = Expected\ portfolio\ return\ p$

$E_k = Expected\ return\ on\ each\ security$

$x_k = Proportion\ of\ the\ investor's\ budget\ allocated\ to\ asset\ k$

2.5 Modes of model resolution:

As mentioned above, the model has two objectives: to obtain the maximum return for a given level of risk or to have the minimum possible risk for a given level of return.

These objectives can be achieved in two diverse ways:

- Graphically.
- With mathematical optimisation methods.

The use of each will depend on the number of assets selected.

If the number of assets to be worked with is small, two or three, the graphical representation can be used to solve the model, since the calculations are not overly complex.

For a large number of assets, as is the case in this work, where it will be seen later that 12 assets are selected, mathematical optimisation methods must necessarily be used.

One of the mathematical methods that we can use to solve the Markowitz model is the so-called critical line method, which is a quadratic programming algorithm that aims to detect the corner portfolio. To perform the calculations of this method we will use the Microsoft Excel Solver tool for its simple and fast use.

A corner portfolio is one in which an asset that was not previously in the portfolio is added to the portfolio, or an asset that was previously in the portfolio disappears from the portfolio, i.e., the corner portfolio is one that quantitatively changes the composition of the portfolio. In any table of corner portfolios, the first one is the one that offers the highest expected return and the last one is the one that offers the lowest risk according to the measure selected in each model.

Finally, once the efficient frontier has been determined, each investor will choose their optimal portfolio, i.e., the one they are most satisfied with, which will depend on the investor's degree of risk aversion. This selection will be subjective, as it will depend on the qualitative and quantitative variables of the investor, such as their wealth, family and work situation, age, etc... Therefore, the investor will choose the portfolios closer to maximum profitability the riskier they are and the portfolios closer to minimum risk the more risk-averse they are.

2.6 Effect of asset correlation on the variance of the portfolio:

Firstly, as the model seeks the lowest volatility (risk) for a given level of return and to develop the model, correlations between assets are key. It is of significant importance to highlight the fundamental effect that correlation and diversification have on the variance (risk) of the portfolio.

It is worth noting that the covariance depends on the correlation between assets.

$$\sigma_{ij} = \rho_{ij} \cdot \sigma_i \cdot \sigma_j \quad (5)$$

And that in turn, the variance depends on the covariance.

$$\sigma^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} X_i \cdot X_j \cdot \sigma_{ij} \quad (6)$$

Being:

$\sigma_{ij} = Covariance$

$\rho_{ij} = Correlation\ coefficient$

$\sigma_i = Standard\ deviation\ of\ i$

$\sigma_j = Standard\ deviation\ of\ j$

$\sigma^2 = Variance$

$X_i = Weight\ of\ asset\ i$

$X_j = Weight\ of\ asset\ j$

If we examine the two equations above we can see that if the correlation between the assets is positive, the variance (risk) will increase, and conversely, if the assets are negatively correlated, the variance, i.e. risk, will decrease, which will be essential for our model since, as mentioned above, its objective is to minimise risk as much as possible for a given level of return.

Next, we will demonstrate with an example (see annexe 1) that by increasing the number of assets, we will still obtain the same return (profitability), but by having a greater diversification, the variance (risk) will decrease.

Finally, it should be noted that if $\sigma_{ij} > 0$ the risk will decrease more slowly than if $\sigma_{ij} < 0$.

2.7 Building an efficient portfolio:

As mentioned above, to find the corner portfolios we will use the Critical Line Method, in which a corner portfolio is defined as efficient if and only if it is a convex linear combination between two consecutive corner portfolios. In our case, a convex linear combination of two points, in this case, two consecutive corner portfolios, is a straight line.

Assume that by applying the algorithm to n assets we will obtain m corner portfolios, knowing the following data:

Corner Portfolio	Expected Return	Risk (Standard Deviation)	Asset Ratios
1	E_1	σ_1^2	$X_1 = (X_{1.1} \quad X_{1.2} \quad ... \quad X_{1,n})$
…	…	…	…
h	E_h	σ_h^2	$X_1 = (X_{h,1} \quad X_{h,2} \quad ... \quad X_{h,n})$
h+1	E_{h+1}	σ_{h+1}^2	$X_1 = (X_{h+1.1} \quad X_{h+1.2} \quad ... \quad X_{h+1,n})$
…	…	…	…
m	E_m	σ_m^2	$X_1 = (X_{m,1} \quad X_{m,2} \quad ... \quad X_{m,n})$

Table 1: Expected return, risk, and asset weighting of each corner portfolio (theoretical).

Source: own elaboration.

Let E^* be the desired expected return satisfying the condition $E_h \geq E\ E^* \geq_{h+1:}$

$$E^* = \lambda \cdot E_h + (1-\lambda) \cdot E_{h+1} \ (7) \ \ con \ \lambda \in [0.1], where \ \lambda = \frac{E^* - E_{h+1}}{E_h - E_{h+1}} \quad 8$$

Starting from the value ofλ, the efficient portfolio gives an expected return of E^*will be obtained by applying:

$$X^* = \lambda \cdot X_h + (1-\lambda) \cdot X_{h+1} \quad (9), i.e.:$$

$$\begin{bmatrix} X_1^* \\ X_2^* \\ ... \\ X_n^* \end{bmatrix} = \lambda \cdot \begin{bmatrix} X_{h,1} \\ X_{h,2} \\ ... \\ X_{h,n} \end{bmatrix} + (1-\lambda) \cdot \begin{bmatrix} X_{h+1.1} \\ X_{h+1.2} \\ ... \\ X_{h+1,n} \end{bmatrix} \quad (10)$$

In conclusion, once all the corner portfolios are available, to obtain any efficient portfolio it is not necessary to use any mathematical programme, it is only necessary to find the value of λ and look for the desired return that is within the efficient frontier, i.e., that is between the portfolio of maximum return and the efficient portfolio of minimum volatility.

2.8 Criticisms of the Markowitz model:

The Markowitz model is a model that has been remarkably successful at a theoretical level due to its great usefulness, which is why it has given rise to multiple subsequent developments and derivations based on it, such as the Sharpe or Black-Litterman models. However, in its early days, its practical use was not as successful among investment analysts and portfolio managers as might have been expected. The reasons why the model was more successful in theory than in practice were: the mathematical complexity and the restrictive assumptions of the model. The first of these is since, being a parametric quadratic programme, the resolution algorithm is complex. This problem ceased to exist with the creation of software and hardware tools such as Microsoft Excel's Solver, which performs the calculations immediately. This tool did not exist and therefore could not be used when the model was published, being of great difficulty to obtain the simple calculation of the variance of a portfolio with several assets.

Some of these restrictive assumptions are that the model considers neither taxes nor transaction costs, that it considers the selected securities to be perfectly divisible and that the model does not provide any tool for the investor to assess his attitude to risk and to derive his utility function.

Despite the above, Michaud (1989) points out several advantages of using an optimisation technique such as the Markowitz model, such as satisfying investors' objectives and constraints, controlling the portfolio's exposure to risk, establishing an investment style, efficient use of information, etc.

II. DATA:

To develop the practical application of the model, we define the time horizon over which we will conduct the study, the stock market we will focus on and the number of assets with which we will work.

The first aspect to consider when constructing the optimal portfolio is the time horizon we are going to use. In this case, we have defined a two-year timeline, specifically, from 1 January 2018 to 31 December 2019. On the other hand, one of the reasons why we have chosen this time horizon is to simplify the search for the dividends distributed by the assets used, necessary for the calculation of the yields, since the corporate websites of the companies only publish the dividends distributed in recent years.

Concerning the selection of the stock market from which we are going to extract the assets, in this case, the equity market we have chosen is the IBEX-35, as it is the benchmark stock market index of the Spanish stock exchange, made up of the 35 companies listed on the Spanish Stock Exchange Interconnection System (SIBE) with the highest liquidity, which will facilitate access to the data necessary to implement the study, allowing us to have greater knowledge of the assets and the market.

Table 9 in annexe 2 shows the 35 companies that make up the IBEX-35 divided by sector.

Once the above distribution had been made, we decided to select two assets per sector, except for real estate services, since we only had two assets and selecting them would not have been objective. Despite this, we selected the assets based on their correlations, looking for the pair of assets with the most negative correlation or, failing that, the lowest correlation, which we will explain in detail later on. This method was chosen because it is the closest to the Markowitz method, seeking to reduce risk as much as possible.

Finally, once all the above aspects have been defined, we have created a database in Microsoft Excel, formed by the daily closing prices from 1 January 2018 to 31 December 2019 of the 35 companies that make up the IBEX-35[1], extracting several 486 closing prices per asset, also adding the dividends[2] contributed by each of the companies during the selected period and the splits[3] produced.

The data obtained in this database have been used to calculate the monthly returns of each of the assets, which is why we have introduced dividends and splits since they influence the returns of the assets, something that we will explain in greater depth in the methodology of this work.

[1] Data on the daily closing prices of the 35 IBEX-35 companies for the chosen time horizon have been obtained from the website www.elespanol.com/invertia/.

[2] The dividend data have been obtained from the following websites: https://es.investing.com/, http://www.infobolsa.es/ and the corporate websites of each of the entities.

[3] The data on splits have been obtained from https://www.bolsamadrid.es/, checking the information obtained on them in the 2018 audited consolidated annual accounts of the MásMóvil group, https://www.grupomasmovil.com/informacion-economica-y-financiera/memorias-anuales/.

III. METHODOLOGY:

Once the necessary aspects have been defined and the appropriate database has been obtained for this model, in this section we will develop the methodology used to find the optimal investment portfolio, i.e., the set of procedures that we have conducted to achieve the objective of this work.

Calculation of monthly returns for each asset:

To calculate the monthly returns, we have first defined the type of return we are going to use, being the simple return, which is characterised by the assumption that the reinvestment rate is 0, i.e. it is the most conservative hypothesis when reinvesting, based basically on the difference between the capital obtained and that invested on the capital invested.

In this case, when calculating monthly returns, in addition to the gain or loss in share values, we have considered whether companies have distributed dividends during the month and whether there have been splits.

The expressions we have used to calculate the returns are as follows:

$$RS = \frac{P_f + div_{share} - P_i}{P_i} \quad 11$$

$$RS = \frac{P_f + dif_{share} - P_i}{P_i} \quad (12)$$

To calculate dif_{share}, i.e., the difference that occurs when there is a split between the price of the old and the new share, we have used the following expressions, explained with an example below.

$$dif_{share} = P_v - P_n \qquad (13)$$

$$n^{o}_{old\ shares} * P_v = n^{o}_{new\ shares} * P_n \quad (14)$$

Where: RS = Simple Return

P_f = Final month's share price

P_i = Initial month's share price

div_{share} = dividends distributed during the month

$diff_{share}$ = difference between old and new share price

P_v = Old share price

P_n = New share price

$n^{o}_{old\ shares}$ = number of old shares

$n^{o}_{new\ shares}$ = number of new shares

The expression used for the calculation of simple returns is the first expression (11), as long as there are no splits, in which case, we will use the second expression (12), except when a company, in addition to having splits, has distributed dividends, in which case, we will use both expressions, which is not the case here, since, during the period we have used for the study, there is no company in which there has been a split and at the same time distribution of dividends.

The first expression (11) consists of adding the dividend distributed during that month, if any, to the closing price of that month minus the closing price of the previous month and dividing by the closing price of the previous month.

The second (12) is the same as the previous one, but instead of adding the distributed dividend, we will add the difference between the price of the old share and the new one.

Once these calculations have been made, we have obtained a total of 24 simple monthly returns for each asset, increased in the months in which dividends or splits have been distributed.

As an example, we are going to calculate the first simple monthly yield of Amadeus, an entity that pays dividends every six months, to observe the variation in the yield caused by the distribution of dividends.

$$RS_{12\ AMADEUS\ (\ January\ 2018\ )} = \frac{62{,}48 + 0{,}48 - 60{,}5}{60{,}5} * 100 = 4{,}07\%$$

If the dividends paid out had not been considered, the simple return for the same month would have been as follows:

$$RS_{12\ AMADEUS\ (\ January\ 2018\ without\ dividends)} = \frac{62{,}48 - 60{,}5}{60{,}5} * 100 = 3{,}3\%$$

As can be seen in the calculations made above, considering the dividend pay-out when calculating returns increase them, in this case, the return has increased by approximately 0,77% thanks to the dividend pay-out.

In the case of calculating the simple return with a split, we will use the company MásMóvil as an example, as it is the only company that has conducted a split during the period selected for the model.

This Split took place on 13 December 2018 and consists of dividing the company's share capital in the ratio of five new shares for each old share (5:1), converting the 24.042.100 old shares into 120.210.500 newly issued shares without changing the company's share capital.

To be able to calculate the returns, we have first calculated what we call the difference between the old and the new share price (13) by making the following calculations:

Split 5:1 $$P_{n\ (\ 13\ December\ 2018\ )} = \frac{1*101}{5} = 20{,}2$$

$$dif_{share} = P_v - P_n = 101 - 20{,}2 = 80{,}8$$

First, we have calculated the price of the new shares, i.e., the price that the shares will have when the Split takes place. This calculation has been made using the formula discussed above (14), which is formed by the number of new and old shares and their price, the unknown being the price of the new shares; in this case, as we have explained, having a ratio of 5:1, the number of new shares will be 5, while the number of old shares will be 1, and the price of the old shares is 101€/share since this is the closing price of MásMóvil shares the day before the Split (11 December 2018). With all this, we can now solve the equation, and clearing it we have obtained the price of the new share; $P_{n\ (13\ December\ 2018)}$ = 20,2 €/share.

Then, having both the old share price (Pv = 101 €/share) and the new share price (Pn = 20,2 €/share), we can calculate the difference between them, resulting in a difference between the share prices of 80,8 €/share.

Once this difference has been obtained, we can calculate the returns as follows:

$$RS_{\ 12\ MÁSMÓVIL\ (\ December\ 2018\ )} = \frac{19{,}5 + 80{,}8 - 107{,}6}{107{,}6} * 100 = \ -6{,}78\%$$

The calculation of simple monthly returns for the remaining assets follows the same procedure as the one used above for the Amadeus entity.

In the specific case of the Amadeus entity, the data and results obtained are as follows:

DATE	MONTHLY RETURN
Jan 3, 2018	-
Jan 31, 2018	4,07%
Feb 28, 2018	-2,35%
Mar 29, 2018	5,41%
April 30, 2018	2,36%
May 31, 2018	8,22%
Jun 29, 2018	-0,33%
Jul 31, 2018	8,79%
Aug 31, 2018	10,42%

Sep 28, 2018	-0,84%
Oct 31, 2018	-13,26%
Nov 30, 2018	-10,90%
Dec 31, 2018	-6,11%
Jan 31, 2019	4,15%
Feb 28, 2019	3,44%
Mar 29, 2019	7,53%
Apr 30, 2019	-1,09%
May 31, 2019	0,62%
Jun 28, 2019	2,20%
Jul 31, 2019	0,82%
Aug 30, 2019	-5,97%
Sep 30, 2019	-2,17%
Oct 31, 2019	2,50%
Nov 29, 2019	7,62%
Dec 31, 2019	1,93%

Table 2 Amadeus monthly returns.

Source: own elaboration.

One aspect to bear in mind is that, as we can see, the dates do not always coincide with the end of the month, this is because the last day of the month fell on a weekend or holiday, so the stock market would be closed. For this reason, we have taken the last day of the month when the Spanish stock market was open.

Finally, the tables with the monthly returns of the other IBEX-35 companies used in the study can be found in appendix 3 (tables 10 to 16).

Calculation of volatility:

The second step to be conducted to reach our portfolio is the calculation of the variance since together with the profitability they are the two variables that the Markowitz model considers. In this case, we have calculated the standard deviation to measure the risk, which is the root of the variance, to eliminate quadratic values and facilitate the handling of the data.

This is the mathematical expression by which we have calculated the variance and to which we have applied a square root to finally arrive at the standard deviation.

$$\sigma^2 (R) = E\,[R - E\,(R)]^2 = E\,(R)^2 - E^2(R) \qquad (15)$$

$$\sigma\,(R) = \sqrt{\sigma^2(R)} \qquad 16$$

Where: σ^2 (R) = variance of the asset

σ (R) = standard deviation of the asset

R = Return on assets

Annualization of securities returns and volatilities:

Having calculated the standard deviation on the monthly return of each asset, we have obtained its average monthly volatility. As we need the annual variables to be able to run the model correctly, we are going to annualise the average monthly returns and volatilities of each of these assets.

To annualise the average returns, all we have done is apply the following expression:

$$Average\ Annual\ Return = Average\ Monthly\ Return * 12 \qquad (17)$$

Regarding the annualization of volatility, we have used the following expression.

$$\sigma_{Annual} = \sqrt{12} * \sigma_{monthly} \qquad (18)$$

Continuing with the example used above of the Amadeus entity, we observe that the average monthly return is 1,13% and its average monthly volatility is 5,82%. If we apply the above expressions to annualise these values, we obtain the following data:

$$Average\ Annual\ Return = 1.13\% * 12 \simeq 13{,}53\%.$$

$$\sigma_{Annual} = \sqrt{12} * 5{,}82\% \simeq 20{,}17\%$$

The annual conversions of the remaining assets follow the same procedure as previously used for the Amadeus entity.

Table 17 in annexe four shows the average monthly and annual returns and volatilities of the 35 assets that make up the IBEX-35.

Calculation of correlations:

Once the returns and volatilities of the thirty-five securities that make up the IBEX-35 have been annualised, the next step is to calculate the correlations between them to be able to select the assets that will make up our portfolio.

Correlation is the reciprocal or corresponding link that exists between two or more elements, specifically, the linear relationship that exists between them, in this case, the correlation relates the behaviour that two assets have with each other, these correlations will be between the following interval [-1,1], being negative correlations (< 0) when the two assets are correlated in opposite sense, that is to say, when the value of an asset increases, the value of the other decreases and vice versa, becoming completely opposite in the value -1, in the case that the correlation was equal to 0 the variables will be incorrectly related, not having the assets no relation between them, when one of the assets suffers a variation in its value (rise or fall) we do not know how the other one will vary, and finally, positive correlation will exist (> 0) when the two assets are directly correlated, with both assets making the same movement, either upwards or downwards, and this movement will be in the same proportion when the correlation is equal to 1.

The mathematical formula that allows us to calculate the correlation coefficient is as follows:

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x * \sigma_y} \qquad (19)$$

Where: $\sigma_{xy} = covariance\ of\ the\ monthly\ returns\ of\ the\ two\ assets$

$\sigma_x = standard\ deviation\ of\ monthly\ returns\ of\ one\ of\ the\ assets$

$\sigma_y = standard\ deviation\ of\ the\ monthly\ returns\ of\ the\ other\ asset$

In our case, to facilitate the calculation of the correlations of all the assets comprising the IBEX-35, we have used the Excel tool, specifically the data analysis function, and within this the correlation coefficient, for which we have had to previously make a table with all the monthly returns of all the assets ordered by dates (see tables 18 and 19 in annexe 5), once all this has been done, we obtain the correlation matrix (see tables 20 and 21 in annexe 6), in which we find all the existing relationships between the 35 assets.

If we look at this matrix (tables 20 and 21 in annexe 6), we can see that there are correlations of all kinds, incredibly positive, positive, negative, and null. For example, in the case of Banco Santander (SAN) and Banco Bilbao Vizcaya Argentaria (BBVA), there is an exceedingly high degree of association, with a correlation coefficient of 0,89, while Acciona (ANA) and Actividades de Construcción y Servicios (ACS) still have a positive correlation coefficient of 0,39, but the degree of association is not as high as in the previous case. On the other hand, Iberdrola (IBE) and MásMóvil (MAS) have a negative correlation coefficient of -0,30, i.e., there is an inverse relationship between them. And finally, between Grifols (GRF) and Mapfre (MAP), we correlate zero, so there is no linear association between them.

Assets selected for the portfolio

Once the correlations between the thirty-five assets have been calculated, we have selected the 12 stocks that are going to form our investment portfolio, specifically two companies per sector, selecting those stocks with the most negative correlation or those closest to zero, thus reducing the risk to obtain the most optimal portfolio possible.

The assets making up this portfolio are as follows:

- Oil and energy:
  - Naturgy Energy Group (NTGY)
  - Red Eléctrica Corporación (REC)

- Basic materials, industry, and construction:
  - Acerinox (ACX)
  - Ferrovial (FER)

- Consumer goods:
  - Ence Energía y Celulosa (ENC)
  - Grifols (GRF)

- Consumer services:
  - Melia Hotels International (MEL)
  - International Airlines Group (IAG)

- Financial services:
  - Banco Sabadell (SAB)
  - Bankinter (BKT)

- Technology and telecommunications:
  - Cellnex (CLNX)
  - MásMóvil (MAS)

Portfolio securities data:

Once we have selected the twelve assets that are going to form our investment portfolio, we are going to extract, among the 35 assets calculated, the data of the 12 selected, to be able to correctly set out the Markowitz model.

- Average returns and volatilities

First, we have gathered the monthly and annual average returns and volatilities of the twelve selected assets, which can be seen in the first and second column, respectively, of table 3:

STOCKS	AVERAGE MONTHLY RETURN	AVERAGE MONTHLY VOLATILITY	AVERAGE ANNUAL RETURN	AVERAGE ANNUAL VOLATILITY
ACERINOX	-0,43%	8,54%	-5,12%	29,59%
BANCO SABADELL	-1,05%	8,77%	-12,56%	30,39%
BANKINTER	-0,32%	6,77%	-3,78%	23,44%
CELLNEX	2,51%	5,99%	30,08%	20,73%
ENCE	-0,43%	12,16%	-5,22%	42,13%
FERROVIAL	2,10%	3,79%	25,22%	13,12%
GRIFOLS	1,10%	5,91%	13,17%	20,47%
IAG	1,08%	8,35%	12,90%	28,92%
MASMOVIL	-2,85%	17,84%	-34,24%	61,81%
MELIA HOTELS	-1,35%	5,29%	-16,19%	18,32%
NATURGY	0,83%	4,56%	9,99%	15,80%
R.E. C	0,69%	3,88%	8,27%	13,43%

Table 3: Average monthly and annual returns and volatilities of the twelve selected assets.

Source: own elaboration.

As we can see, six of the selected assets have a negative return, however, this does not imply that these assets will be discarded when applying the Markowitz model since what this model considers are the return and the risk. of the diversified portfolio, and some of these assets with a negative return may be found in one of the corner portfolios.

- Correlations:

In this case, we have assembled the correlations that exist between the 12 selected assets in a correlation matrix, in which we can see that there are different degrees of correlation so that we can later demonstrate in the practical application of the Markowitz model, the influence that correlations can have on the creation of the optimal investment portfolio generated through the application of the Markowitz model.

Table 22 in annexe seven shows the full correlation matrix for the 12 selected assets to 6 decimal places.

Matrix of variances and covariances:

Firstly, it is worth highlighting the importance of calculating this matrix, as it will be necessary to conduct the Markowitz model, specifically, we will need the variances and covariances matrix of annual returns, in this case, we are going to calculate the matrix using monthly returns, but we will subsequently annualise them.

Let's recall that the variance is a measure of dispersion that represents, in this case, the variability of the average monthly returns of the selected assets, concerning their arithmetic mean; on the other hand, the covariance is a value that indicates the degree of joint variation that exists between two random variables, in this case, two assets of the 12 selected, concerning their means.

To perform the calculations that make up this matrix, we first grouped the previously calculated monthly returns of the 12 selected assets in a table (to facilitate the calculation), and then, using this data and the Excel VARP formula, we obtained the variance and covariance matrix (see table 23 in annexe 8), in our case 12 by 12, in which we can see all the existing covariances and variances, the latter being found on the diagonal of the matrix since the covariance of an asset with itself is its variance.

Finally, name the variance and covariance formula used by Excel to obtain the values of this matrix:

$$VAR_X = \frac{\sum_{j=1}^{n} (x_j - \bar{x})^2}{(n-1)} \qquad (20)$$

$$COV_{x,y} = \frac{1}{n}\sum_{j=1}^{n}(x_j - \bar{x})(y_j - \bar{y}) \qquad (21)$$

Annualised variance and covariance matrix:

As mentioned above, to perform the Markowitz model we need to have the annual variances and covariances, therefore, as we have done previously with the returns, we are going to annualise them by multiplying by twelve each monthly variance and covariance, obtaining, as a result, the matrix of annualised variances and covariances of the 12 selected assets, which we are going to have to complete symmetrically, as the order of the securities does not alter the result of the covariance, $COVx,y = COVy,x$ i.e. the order of the securities does not alter the result of the covariance.

As an example, in the case of Acerinox, to obtain the annualised variance we have looked for its variance in the monthly variance and covariance matrix and multiplied it by twelve, obtaining the following result:

$$\sigma^2{}_{Acerinox\ Annual} = \sigma^2{}_{Acerinox\ monthly} * 12 = 0{,}007295936 * 12 = 0{,}0875512323$$

For the covariance of Acerinox with Banco Sabadell, or with any other asset, the procedure we have followed is the same as in the previous case, i.e., multiply the monthly covariance by twelve, obtaining the following result:

$$COV_{ACX,SAB\ annual} = COV_{ACX,SAB\ monthly} * 12 = 0{,}003189221 * 12 = 0{,}0382706515$$

Finally, to annualise the variances and covariances of the rest of the selected securities, we followed the same procedure as in the case of Acerinox. As a result, we obtained the annual variance and covariance matrix for the twelve selected securities (see table 24 in appendix 9), which we will subsequently use to calculate the risk of our portfolio.

Calculation of portfolio risk:

As mentioned above, we will use the annualised variance and covariance matrix to calculate the variance of the portfolio (σ_p^2).

Two of them will be formed by the weights of the assets that we are going to use, one in the form of a row and the other in the form of a column, the row matrix will be 1x12, and the column matrix will be the same but transposed, that is to say, 12x1, where the weights will change depending on the profitability required of the portfolio; the last matrix will be formed by the annualised variances and covariances of the assets, which we have calculated in the previous section and will not modify, remaining fixed.

Once these matrices are set up, the row matrix of the weights (w) is multiplied by the annualised variance and covariance matrix, and by the column matrix of the weights (w^T) will give us the variance of the portfolio, i.e:

$$\sigma_p^2 = (\,w_1 \quad w_2 \; \dots \; w_n) \begin{pmatrix} \sigma_{11}^2 & \sigma_{12} & \sigma_{1n} \\ \sigma_{21} & \sigma_{22}^2 & \sigma_{2n} \\ \sigma_{n1} & \sigma_{n2} & \sigma_{nn}^2 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ w_n \end{pmatrix} = (w * V * w)^T \qquad (22)$$

With the variance of the portfolio calculated (σ_p^2), we can obtain the risk of the portfolio by calculating the square root of the portfolio.

$$\sigma_p\,(R) = \sqrt{\sigma_p^2\,(R)} \qquad (23)$$

IV. PRACTICAL APPLICATION:

Once the theory of the Markowitz model has been explained and the data obtained have been analysed, all that remains is to conduct its practical application. To do so, before we start, we will collect the previous data to obtain a better understanding.

First of all, remember that the aim is to create an efficient portfolio of 12 IBEX-35 assets, for which we have used a timeline of returns from 1 January 2018 to 31 December 2019. This portfolio is made up of two companies from each sector, selected using the correlations between the thirty-five assets that make up the IBEX-35.

Before the selection of the assets that make up the portfolio, both the returns and their monthly average standard deviations (or volatilities) of the thirty-five assets were calculated and then annualised. Subsequently, once the twelve assets had been selected, their 12 annual returns and standard deviations were grouped in the following table:

STOCKS	AVERAGE ANNUAL RETURN	AVERAGE ANNUAL VOLATILITY
ACERINOX	-5,12%	29,59%
BANCO SABADELL	-12,56%	30,39%
BANKINTER	-3,78%	23,44%
CELLNEX	30,08%	20,73%
ENCE	-5,22%	42,13%
FERROVIAL	25,22%	13,12%
GRIFOLS	13,17%	20,47%
IAG	12,90%	28,92%
MASMOVIL	-34,24%	61,81%
MELIA HOTELS	-16,19%	18,32%
NATURGY	9,99%	15,80%

R.E. C	8,27%	13,43%

Table 4: Annual returns and volatilities of the twelve selected assets.
Source: own elaboration.

Next, we have chosen the **IBEX-35** as the stock market to extract the assets as it is the benchmark stock market index of the Spanish stock market.

In table 5 below, we have grouped the return and volatility of this index for 2018, and 2019 and the average of both periods.

	Profitability	**Volatility**
2018	-14,97%	15,04%
2019	11,88%	13,70%
Average 2018 and 2019	-1,55%	14,37%

Table 5: IBEX-35 returns and volatilities in 2018, 2019 and the average of both periods.
[4] *Source:* https://www.bolsasymercados.es/esp/Home

As we can see in the table above, the IBEX-35 ended 2018 with a fall of 14,97%, which it managed to recover in 2019, reaching a return of 11,88%. If we look at the two periods together, we see that the average for both years is -1,55%, this result is because the profitability obtained in 2019 has not been able to completely compensate for the fall in 2018. In contrast, the volatility of the index has remained fairly constant over the two years, averaging 14,37%.

Once the above data have been collected, we proceed to find the **efficient frontier** of the selected portfolio of twelve securities. This process has been conducted by solving the Markowitz optimisation model by complying with the boundary conditions mentioned above in formulas 1,2,3 and 4.

[4] The data has been obtained from the website https://www.bolsasymercados.es/esp/Home, from the 2018 and 2019 market reports published by the website.
https://www.bolsasymercados.es/docs/infmercado/2018/esp/IM2018.pdf
https://www.bolsasymercados.es/docs/infmercado/2019/esp/IM2019.pdf

In our case, i.e., for twelve assets, solving the optimisation problem with these conditions is complex, so we have used the Microsoft Excel *Solver* tool, as it allows us to establish a series of boundary conditions on the decision variables.

Next, to minimise the variance of the portfolio we have created a matrix or row vector of twelve-by-one and its transpose. In this **matrix** what we will find is the **weight**, which is to say, the weighting of each asset k in the portfolio, this is dynamic since its values vary depending on the profitability that we give to the portfolio. To make the calculation correctly, before executing the programme we assign the same weight to all the assets, which is to say, as there are twelve assets, we distribute the total of the portfolio among them equally, each one weighting 8,3333...%, thus fulfilling the first of the restrictions of the model. $\sum_{k=1}^{n} x_k = 1$

Before calculating the volatility of the portfolio, using the above data we have calculated the **expected return of the portfolio** by default, using the Excel formula SUMAPRODUCT, which multiplies the annual return of each of the twelve assets by the weight that we have initially given by default $(8,\hat{3}\%)$, and subsequently adds them all together, thus obtaining an expected return of **1,876%**. Then, as mentioned above, we calculate the variance of the portfolio and with its square root we obtain the standard deviation or **volatility** of the portfolio, the latter being **26,51%.**

Continuing with the practical application of the model, we will apply the **critical line method.**

The procedure to follow to obtain the corner portfolios in *Solver* is as follows: first, we must locate the target cell, which is the value we want to optimise, i.e., minimise or maximise, in our case, we seek to minimise the standard deviation (the risk). Next, we introduce the restrictions that must be met when minimising risk, which is: that the sum of the weights is equal to 1, that they are positive or zero and that all this is done while maintaining the required level of profitability.

To obtain the minimum variance subject to the above conditions, there must be a matrix with dynamic values, i.e., that change as the profitability we require in the system varies. In this case, this dynamic matrix will be the row matrix of weights, and as the row matrix varies, the column matrix will also vary, since I have defined both to be equal.

The weights that result after applying the *Solver* are the percentages of shares of each asset that the investor must purchase about their total budget.

With all the above mentioned we proceed to run *Solver*, obtaining the minimum risk portfolio, which we will name later as portfolio ten. This portfolio has a profitability of 7,58%, a risk of 7,89% and is formed by the weights of table 5. Once obtained this portfolio corner we proceed to find the following ones, for it, I have been increasing the desired profitability in percentages of 2,5% being able to observe how the composition of the portfolio is changing with the entrance of new titles or the exit of titles that already were present.

	ACX	SAB	BKT	CLNX	ENC	FER	GRF	IAG	MAS	MEL	NTGY	REC
Portfolio ten	0%	0%	15,17%	1,20%	0%	16,68%	14,86%	0%	0,66%	9,81%	0%	41,61%

Table 6: Weights corner portfolio ten
Source: own elaboration.

In this way, we have obtained a set of ten portfolios that offer us the minimum risk for a given return while complying with the boundary conditions, known as the Markowitz efficient frontier.

Table 7 below shows the weights for each of the ten corner portfolios obtained.

	ACX	*SAB*	*BKT*	*CLNX*	*ENC*	*FER*	*GRF*	*IAG*	*MAS*	*MEL*	*NTGY*	*REC*
Portfolio 1	0%	0%	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%
Portfolio 2	0%	0%	0%	48,57%	0%	51,43%	0%	0%	0%	0%	0%	0%
Portfolio 3	0%	0%	2,62%	25,90%	0%	66,19%	5,29%	0%	0%	0%	0%	0%
Portfolio 4	0%	0%	7,23%	22,01%	0%	59,01%	7,77%	0%	0%	0%	0%	3,98%
Portfolio 5	0%	0%	9,68%	17,54%	0%	51,22%	9,62%	0%	0%	0%	0%	11,95%
Portfolio 6	0%	0%	12,12%	13,07%	0%	43,43%	11,47%	0%	0%	0%	0%	19,92%
Portfolio 7	0%	0%	14,57%	8,60%	0%	35,63%	13,32%	0%	0%	0%	0%	27,89%
Portfolio 8	0%	0%	16,17%	4,98%	0%	28,27%	14,71%	0%	0%	1,38%	0%	34,50%
Portfolio 9	0%	0%	15,68%	3,39%	0%	22,05%	14,99%	0%	0%	6,17%	0%	37,72%
Portfolio 10	0%	0%	15,17%	1,20%	0%	16,68%	14,86%	0%	0,66%	9,81%	0%	41,61%

Table 7: Weights of the ten corner portfolios obtained.
Source: own elaboration.

Table 8 below shows the returns and standard deviations (risk) of the above portfolios:

	Return	***Standard Deviation***
Portfolio 1	30,08%	20,73%
Portfolio 2	27,58%	13,80%
Portfolio 3	25,08%	11,76%
Portfolio 4	22,58%	10,65%
Portfolio 5	20,08%	9,70%
Portfolio 6	17,58%	8,93%
Portfolio 7	15,08%	8,38%
Portfolio 8	12,58%	8,08%
Portfolio 9	10,08%	7,94%
Portfolio 10	7,58%	7,89%

Table 8: Returns and standard deviations of the 10 corner portfolios obtained.
Source: own elaboration.

Analysing the above data, we can observe the following properties in the formation of the portfolios:

Firstly, if we analyse the 10 corner portfolios and the annual returns of the assets that form them, we can see that there are assets such as Bankinter, MasMovil and Melia Hotels that, despite having negative returns, form part of some of the portfolios. This is because when calculating the corner portfolios using the critical line method, the Markowitz model does not consider whether the assets have positive or negative returns; its only objective is to minimise the risk for a given level of return or to maximise the expected return for a given level of risk.

Next, we have distinguished five assets that do not participate in any of the portfolios, these are Acerinox (ACX), Banco Sabadell (SAB), Ence Energía y Celulosa (ENC), International Airlines Group (IAG) and Naturgy Energy Group (NTGY), this is because the correlations that exist between all of them are considerable (on average approximately 0,40) which hinders the entry of the corner portfolios because as correlations increase it becomes more difficult to reduce volatility.

Likewise, we have distinguished three assets that carry a lot of weight in these portfolios, which are: Cellnex (CLNX), Ferrovial (FER) and Red Eléctrica Corporación (REC). Portfolio 1, which is the portfolio with the highest profitability, is made up of 100% of Cellnex. This asset will remain present in all the diversified portfolios but will gradually lose weight until the last portfolio in which it will weigh 1,20%, therefore, as the profitability required of the portfolio decreases, the weight represented by the asset in this portfolio will also decrease. On the contrary, Red Eléctrica Corporación has an opposite development, it enters from the fourth portfolio, but its weight increases as the volatility of the portfolio decreases, reaching a weight of 41,61% in the portfolio with the lowest volatility (portfolio 10). This same development also occurs with the assets Grifols (GRF) and Bankinter (BKT) but to a lesser extent, which begin to participate in the third portfolio, from which they gradually increase their weight during the following portfolios until the last one in which they suffer a minuscule decrease, this participation in most of the portfolios with not excessively large weights compared to the three assets mentioned above, is since the assets have low and negative correlations with most of the assets in the portfolio.

As mentioned above, Ferrovial (FER) is an important asset because it is in all the corner portfolios except the one with the highest profitability and in all of them with a significant weight, specifically, it has approximately 50% in five of the portfolios in which it participates and ends with a weight of 16,68%.

MásMóvil (MAS) and Melia Hotels International (MEL) are shares that participate little in the portfolios, specifically, MásMóvil only forms part of the last portfolio and with an insignificant weight of 0,66%, while Melia Hotels International appears in the last three portfolios, reaching a weight of 9,81% also in the last portfolio.

Once the properties of the portfolios have been analysed, an investor's decision will depend on what he wants, if he is looking for a high return without caring about the risk that this may entail, the investor will choose among the first portfolios assuming a low diversification, on the other hand, In my case, I would choose portfolio 5 or 6 since both are diversified with 5 securities and from the last portfolio to these, there is not a big increase in risk but there is a big increase in profitability.

Finally, the correlations between the assets that make up the portfolios I have mentioned can be seen in the correlation matrix of the 12 assets in table 22 of annexe 7.

Once the 10 corner portfolios have been obtained, we transfer the results to a scatter plot in which we place the volatility on the abscissa axis and the expected return of the portfolio on the ordinate axis. If we then join the points belonging to each portfolio, we get the following **Markowitz efficient frontier**.

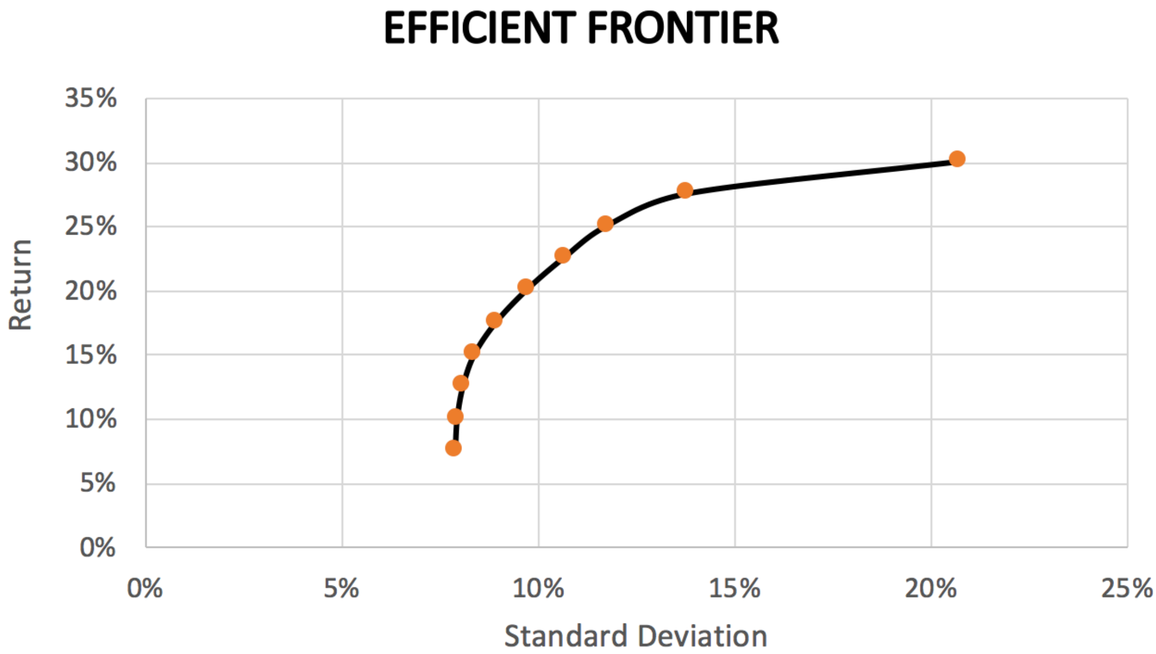

Figure 2: Markowitz Efficient Frontier.
Source: own elaboration.

As we can see in Figure 2, the efficient frontier has the shape of a parabola and contains infinitely many efficient portfolios. The parabolic shape of the curve indicates that portfolio returns decrease faster than volatility (risk) until a point is reached where risk will increase despite decreasing returns, the latter point is the one with the lowest volatility of all efficient portfolios.

Next, in order to compare the Markowitz efficient frontier with the IBEX-35, we have compiled the data on its annual average return and volatility for the time horizon used. As we can see in Table 5, the first of these was - 1.55%, which is due to the large fall suffered in 2018 by this stock market. In terms of volatility, the Spanish stock market has a volatility of 14.37%.

Next, for the same purpose, we have gathered the data obtained by applying the **naïve strategy 1/N**. Therefore, if we assign each asset a weighting of $8,\hat{3}\%$ $\left(\frac{1}{12}\right)$ the expected return of the portfolio will be 1,876% and the volatility (risk) of the portfolio will be 26,51%.

With the above data we are going to **compare graphically** the Markowitz Efficient Frontier with the IBEX-35, and with the 1/N strategy. This comparison is conducted to check if the objective of the work is fulfilled, that is if active management could beat passive management, specifically if the portfolios constructed by applying the Markowitz model can beat the Spanish stock market index (IBEX-35) and/or the naïve 1/N strategy.

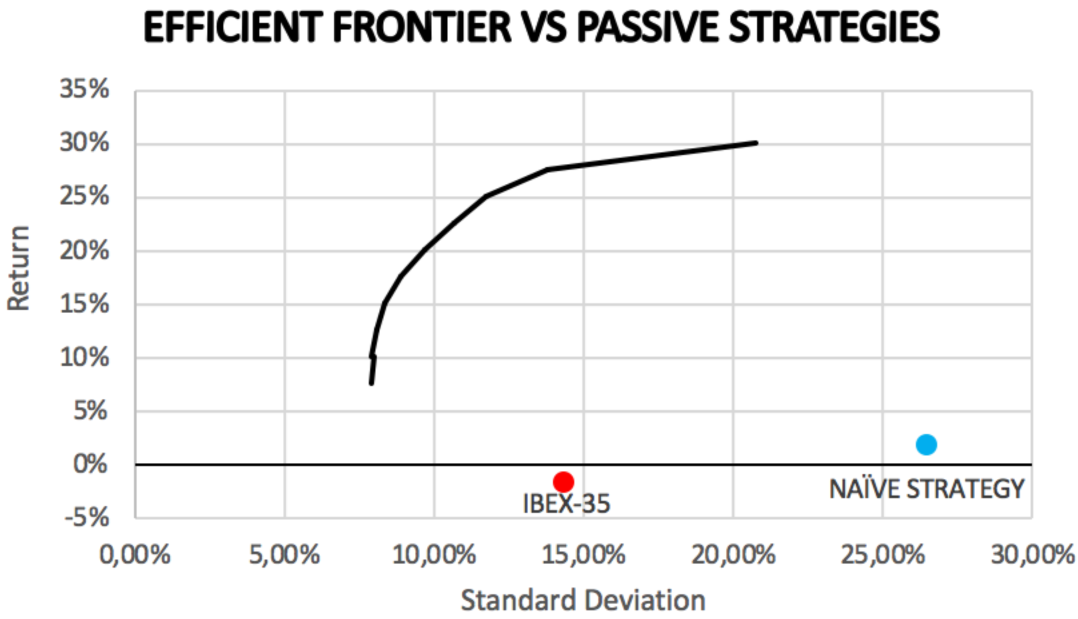

Figure 3: Comparison of the Markowitz efficient frontier with the IBEX-35 and the naïve 1/N strategy.

Source: own elaboration.

If we look at the previous graph (figure 3) we can see that all the portfolios that form the efficient frontier beat the naïve strategy 1/N (blue dot), since all of them offer a higher return and a lower risk, even the riskiest portfolio of the efficient frontier has less risk than the naïve strategy 1/N. On the other hand, if we compare it with the IBEX-35 (red point) we also observe that all the diversified portfolios of the efficient frontier offer much more profitability than this point, since this one has a negative profitability of -1,55%, if we look at the volatility, the great majority of the portfolios have a lower volatility than the IBEX-35 (14,37%) and offer profitability much higher than the index, only two of the portfolios have more risk than the index, but their profitability is abysmally higher. The corner portfolio that offers a higher return has a difference in volatility from that of the index of 6,36% since this portfolio has a volatility of 20,73% and the IBEX has a volatility of 14,37%; but the difference in returns between them is much greater, specifically 31,63%, since the return of the riskiest portfolio is 30,08% and that of the Spanish market is -1,55%.

V. CONCLUSION:

In this paper, we have studied the model proposed by Markowitz in 1952 in his Portfolio Selection Theory together with its practical application. This model is a theoretical reference in the field of portfolio selection theory. It marked a before and after in the history of investment, since investors before the creation of this model did not take risk into account, but simply sought to maximise profitability, which changed with Markowitz's model. The model stated the importance of considering both return and risk and highlighted the risk-reducing impact that diversification could have on risk. In terms of its practical application, it is currently a model that is easy to apply, which was not the case in its early days. This difference is because technological developments have contributed to the availability of the software and hardware necessary for its use.

Let us recall that the main objective of this paper is to evaluate whether active management can outperform passive management, in particular, whether the portfolios selected by applying the Markowitz model can outperform the Spanish stock market index and/or the naïve strategy 1/N.

Based on the daily closing price data from 1 January 2018 to 31 December 2019 of the 35 companies that make up the IBEX-35 and considering both the dividends paid and the splits produced, we have applied the portfolio selection model to obtain the following conclusions.

First, by comparing the efficient portfolios obtained through the empirical study of the Markowitz model with the naïve strategy 1/N, we have clearly observed that any portfolio in the Markowitz efficient frontier beats the Markowitz strategy, as all of them provide higher returns at lower risk.

On the other hand, if we compare the efficient portfolios obtained using the Markowitz model with the IBEX-35. Firstly, we can observe that the minimum risk portfolio (portfolio 10) outperforms the Spanish market index, since this portfolio offers higher profitability and less risk, specifically it provides an increase in profitability of 9,13% (7,58% of the minimum risk portfolio + 1,55% of the IBEX-35) and a decrease in risk of 6,48% (7,89% of the minimum risk portfolio – 14,37% of the IBEX-35). Next, if we compare with the entire Markowitz efficient frontier, we reach the same conclusion, i.e., that the corner portfolios obtained by applying the Markowitz model outperform the index. This is because all the portfolios, except the two with the highest risk, provide a much higher return than the index with the lowest risk. In this case, the investor's decision will depend on the investor's risk aversion, which may depend on the type of investment, the investor's age, education, and time horizon, among other things.

With the above, we can affirm that the objective of the study is met, i.e., that active management using the Markowitz model beats both the IBEX-35 and the naïve strategy 1/N (passive management).

It is worth mentioning that this model does not allow predicting the future behaviour of the selected assets or the index. The results obtained are calculations made a priori with historical data that show past performance and serve as a reference for the future creation of investment portfolios, therefore, these calculations need not be repeated in future periods, as stock markets may vary due to any national or international factor. For all these reasons, it is advisable to recalculate the composition of the portfolio over time, i.e., to actively manage it. It should also be borne in mind that the results have been obtained using data from two years and that perhaps if the sample were extended the results might be different, which we cannot say for certain.

Finally, although the model has its critics, we believe that it is a valuable tool to have as a reference when creating an investment portfolio.

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ANNEXES

ANNEXE 1

Example demonstrating that diversification leads to lower risk

In the following, we will demonstrate with an example that with $\sigma_{ij} = 0$ and applying the same weights (w), returns (r) and standard deviation (σ for all chosen assets, diversification will cause that as the number of chosen assets increases, the risk decreases.

We first choose two assets and calculate their return and variance under the following conditions:

$\sigma_{ij} = 0$

$r_i = 0{,}05$

$w_i = \frac{1}{2} = 0{,}5$

$\sigma_i = 0{,}05$

$$r_p = w_1 r_1 + w_2 r_2 = 2 \cdot (0{,}05 \cdot 0{,}5) = \mathbf{0,05}$$

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12} = 0{,}5^2 \cdot 0{,}05^2 + 0{,}5^2 \cdot 0{,}05^2 + 2 \cdot 0{,}5 \cdot 0{,}5 \cdot 0 = 1{,}25 \cdot 10^{-3} = \mathbf{0,00125}$$

Next, we will recalculate the same thing under the same conditions, but in doing so for three assets the $w_i = \frac{1}{3} = 0,\hat{3}$

$$r_p = w_1 r_1 + w_2 r_2 + w_3 r_3 = 3 \cdot \left(0{,}05 \cdot \frac{1}{3}\right) = \mathbf{0,05}$$

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + 2(w_1w_2\sigma_{12} + w_1w_3\sigma_{13} + w_2w_3\sigma_{23}) = 0,\hat{3}^{\ 2} \cdot 0{,}05^2 + 0,\hat{3}^{\ 2} \cdot 0{,}05^2 + 0,\hat{3}^{\ 2} \cdot 0{,}05^2 + 2 \cdot \left(0,\hat{3} \cdot 0,\hat{3} \cdot 0 + 0,\hat{3} \cdot 0,\hat{3} \cdot 0 + 0,\hat{3} \cdot 0,\hat{3} \cdot 0\right) = 8{,}33 \cdot 10^{-4} = \mathbf{0,000833}$$

With this example, we can conclude that by increasing the number of assets, we will still obtain the same return (profitability), but by having more diversification the variance (risk) will decrease.

Finally, it should be noted that if $\sigma_{ij} > 0$ the risk will decrease more slowly than if $\sigma_{ij} < 0$.

ANNEXE 2

The 35 IBEX-35 companies by sector

SECTOR	COMPANY	ACRONYMS
Oil and energy:	Repsol	REP
	Enagas	ENG
	Endesa	ELE
	Iberdrola	IBE
	Naturgy Energy Group	NTGY
	Red Eléctrica Corporación	REC
Basic materials, industry, and construction:	ACERINOX	ACX
	ArcelorMittal	MTS
	Cie Automotive	CIE
	Siemens Gamesa Renewable Energy	SGRE
	Acciona	ANA
	Construction and Service Activities	ACS
	Ferrovial	FER
Consumer goods:	Viscofan	VIS
	Inditex	ITX
	Ence Energía y Celulosa	ENC
	Grifols	GRF

Consumer services:	Melia Hotels International	MEL
	Mediaset España Comunicación	TL5
	Aena	AENA
	International Airlines Group	IAG
Financial services:	Banco Bilbao Vizcaya Argentaria	BBVA
	BANCO SABADELL	SAB
	Banco Santander	SAN
	Bankia	BKIA
	BANKINTER	BKT
	Caixabank	CABK
	Mapfre	MAP
Technology and telecommunications:	Cellnex	CLNX
	MásMóvil	MAS
	Telefónica	TEF
	Amadeus	AMS
	Indra Systems	DR
Real estate services	Inmobiliaria Colonial	COL
	Merlin Properties	MRL

Table 9: The 35 IBEX-35 companies by sector.

Source: own elaboration.

ANNEXE 3

Monthly performance of the 35 IBEX-35 assets

DATE	ACCIONA	ACERINOX	ACS	AENA	AMADEUS
	MONTHLY RETURN	MONTHLY RETURN	MONTHLY RETURN	MONTHLY RETURN	MONTHLY RETURN
3-ene.-2018	—	—	—	—	—
31-ene.-2018	6,02%	-3,5%	-0,65%	1,77%	4,07%
28-feb.-2018	-3,86%	4,5%	-11,07%	-4,20%	-2,35%
29-mar.-2018	-6,20%	-5,2%	14,92%	-2,04%	5,41%
30-abr.-2018	11,00%	2,8%	13,00%	7,70%	2,36%
31-may.-2018	-6,59%	-3,7%	0,03%	-5,52%	8,22%
29-jun.-2018	6,78%	-5,1%	-4,77%	-4,31%	-0,33%
31-jul.-2018	8,32%	14,7%	12,44%	1,94%	8,79%
31-ago.-2018	3,21%	-6,4%	-2,34%	-1,04%	10,42%
28-sep.-2018	3,25%	6,4%	2,69%	-1,35%	-0,84%
31-oct.-2018	-4,26%	-20,7%	-10,00%	-4,11%	-13,26%
30-nov.-2018	7,79%	-10,4%	2,39%	0,50%	-10,90%
31-dic.-2018	-9,99%	-4,3%	-1,05%	-3,24%	-6,11%
31-ene.-2019	11,10%	10,7%	8,60%	10,07%	4,15%
28-feb.-2019	2,40%	-2,5%	8,62%	3,87%	3,44%
29-mar.-2019	10,83%	-4,8%	-1,46%	2,92%	7,53%
30-abr.-2019	4,03%	1,3%	4,41%	7,21%	-1,09%
31-may.-2019	-4,53%	-6,8%	-8,73%	-0,51%	0,62%
28-jun.-2019	-3,58%	11,7%	-4,51%	4,40%	2,20%
31-jul.-2019	4,42%	-16,0%	10,74%	-6,01%	0,82%
30-ago.-2019	0,72%	0,8%	-6,88%	-2,06%	-5,97%
30-sep.-2019	0,15%	2,5%	6,08%	1,54%	-2,17%
31-oct.-2019	-2,20%	7,3%	0,55%	-2,86%	2,50%
29-nov.-2019	0,97%	10,4%	-4,12%	-0,42%	7,62%
31-dic.-2019	0,86%	6,2%	3,03%	4,38%	1,93%

Table 10: Monthly returns of Acciona, Acerinox, Actividades de Construcción y Servicios (ACS), Aena and Amadeus.
Source: own elaboration.

DATE	ARCELORMITTAL	BANCO SABADELL	BANCO SANTANDER	BANKIA	BANKINTER
	MONTHLY RETURN	MONTHLY RETURN	MONTHLY RETURN	MONTHLY RETURN	MONTHLY RETURN
3-ene.-2018	–	–	–	–	–
31-ene.-2018	1,65%	12,46%	9,68%	0,72%	16,30%
28-feb.-2018	-2,16%	-9,16%	-4,61%	-3,20%	-2,93%
29-mar.-2018	-3,65%	0,09%	-3,45%	-4,16%	-4,17%
30-abr.-2018	9,72%	1,67%	2,73%	4,66%	4,70%
31-may.-2018	-3,13%	-11,40%	-14,78%	-11,07%	-5,48%
29-jun.-2018	-9,96%	-3,27%	-3,38%	-4,04%	0,42%
31-jul.-2018	11,63%	1,03%	5,63%	5,81%	-0,24%
31-ago.-2018	-7,83%	-7,58%	-9,78%	-4,60%	-5,55%
28-sep.-2018	3,20%	1,44%	1,62%	3,21%	5,12%
31-oct.-2018	-17,28%	-12,50%	-2,25%	-17,22%	-10,48%
30-nov.-2018	-9,86%	-3,76%	-0,31%	4,55%	0,54%
31-dic.-2018	-12,87%	-9,17%	-7,00%	-12,33%	-5,71%
31-ene.-2019	14,22%	-0,16%	3,88%	0,40%	-2,46%
28-feb.-2019	-3,27%	10,22%	5,57%	8,27%	7,25%
29-mar.-2019	-11,05%	-11,31%	-2,59%	-14,00%	-3,78%
30-abr.-2019	1,44%	13,13%	5,61%	8,39%	1,54%
31-may.-2019	-28,76%	-6,83%	-10,88%	-8,12%	-8,73%
28-jun.-2019	19,49%	-8,28%	3,57%	-5,97%	-3,97%
31-jul.-2019	-8,87%	-14,42%	-6,22%	-14,89%	-5,72%
30-ago.-2019	-8,37%	-1,32%	-11,24%	-8,48%	-10,60%
30-sep.-2019	0,26%	17,44%	8,79%	9,69%	12,17%
31-oct.-2019	3,75%	11,06%	-2,19%	-0,64%	8,58%
29-nov.-2019	10,77%	1,37%	-2,02%	1,91%	0,89%
31-dic.-2019	2,12%	4,12%	7,12%	8,40%	4,74%

Table 11: Monthly returns of Arcelormittal, Banco Sabadell, Banco Santander, Bankia and Bankinter. Source: own elaboration.

DATE	BBVA	CAIXABANK	CELLNEX	CIE AUTOMOTIVE	ENAGAS
	MONTHLY RETURN	MONTHLY RETURN	MONTHLY RETURN	MONTHLY RETURN	MONTHLY RETURN
3-ene.-2018	–	–	–	–	–
31-ene.-2018	6,68%	9,22%	0,32%	15,95%	-7,42%
28-feb.-2018	-7,62%	-8,09%	-2,76%	0,29%	-3,30%
29-mar.-2018	-3,21%	0,62%	4,78%	11,11%	6,06%
30-abr.-2018	10,43%	7,97%	3,25%	4,30%	9,53%
31-may.-2018	-13,69%	-12,76%	-2,65%	3,17%	-5,51%
29-jun.-2018	0,93%	-0,70%	-1,55%	-24,72%	8,82%
31-jul.-2018	5,72%	7,88%	5,67%	3,75%	-0,54%
31-ago.-2018	-13,72%	-2,55%	-2,68%	3,20%	1,14%
28-sep.-2018	2,54%	1,52%	2,44%	5,15%	-3,41%
31-oct.-2018	-9,61%	-7,49%	-3,89%	-13,97%	0,00%
30-nov.-2018	-0,28%	5,47%	13,63%	-5,29%	4,88%
31-dic.-2018	-8,95%	-12,79%	-7,75%	-6,38%	0,42%
31-ene.-2019	10,86%	5,40%	8,81%	16,48%	5,83%
28-feb.-2019	5,33%	2,48%	-7,68%	2,38%	-1,38%
29-mar.-2019	-5,32%	-10,31%	8,28%	-5,07%	4,01%
30-abr.-2019	5,13%	0,89%	3,47%	3,15%	-0,78%
31-may.-2019	-9,00%	-3,77%	19,22%	-13,01%	-1,21%
28-jun.-2019	1,13%	-7,05%	4,26%	15,83%	-4,32%
31-jul.-2019	-7,52%	-12,04%	3,60%	-7,44%	-9,48%
30-ago.-2019	-7,04%	-8,97%	4,24%	-6,39%	0,97%
30-sep.-2019	11,76%	19,60%	3,55%	10,68%	7,35%
31-oct.-2019	2,61%	8,14%	1,20%	-0,98%	3,16%
29-nov.-2019	1,02%	1,13%	2,03%	1,58%	3,10%
31-dic.-2019	6,17%	5,03%	0,34%	-1,77%	5,98%

Table 12: Monthly returns of BBVA, Caixabank, Cellnex, Cie Automotive and Enagas.
Source: own elaboration.

DATE	ENCE	ENDESA	FERROVIAL	GRIFOLS	IAG
	MONTHLY RETURN	MONTHLY RETURN	MONTHLY RETURN	MONTHLY RETURN	MONTHLY RETURN
3-ene.-2018	–	–	–	–	–
31-ene.-2018	-4,43%	3,95%	-2,97%	3,60%	-2,63%
28-feb.-2018	2,67%	-3,73%	-2,87%	-11,28%	-6,05%
29-mar.-2018	18,49%	3,71%	1,34%	5,50%	1,92%
30-abr.-2018	4,95%	8,49%	5,57%	5,13%	2,78%
31-may.-2018	1,20%	-3,97%	-1,44%	6,96%	5,26%
29-jun.-2018	10,14%	-0,58%	1,76%	1,62%	-4,08%
31-jul.-2018	7,60%	7,27%	1,81%	-3,42%	9,63%
31-ago.-2018	-1,16%	-1,98%	6,34%	1,00%	-1,15%
28-sep.-2018	9,44%	-3,05%	-2,40%	-4,41%	-4,83%
31-oct.-2018	-15,24%	-0,11%	-3,28%	1,90%	-5,85%
30-nov.-2018	-16,74%	6,52%	5,34%	-2,71%	0,68%
31-dic.-2018	-1,93%	1,33%	-3,46%	-8,77%	0,36%
31-ene.-2019	24,08%	12,21%	10,50%	0,09%	10,47%
28-feb.-2019	-6,22%	1,89%	3,18%	-0,57%	-6,53%
29-mar.-2019	-23,75%	3,74%	0,53%	5,54%	-15,26%
30-abr.-2019	-4,89%	-2,12%	4,62%	-1,47%	7,07%
31-may.-2019	-22,52%	1,41%	-0,41%	-5,16%	-18,29%
28-jun.-2019	8,00%	0,89%	6,38%	11,64%	5,10%
31-jul.-2019	-19,95%	1,35%	4,07%	12,00%	-1,90%
30-ago.-2019	7,41%	3,04%	8,74%	-2,64%	2,24%
30-sep.-2019	4,39%	2,20%	0,99%	-6,08%	14,33%
31-oct.-2019	6,39%	2,13%	0,88%	8,37%	15,36%
29-nov.-2019	-6,25%	1,77%	1,29%	5,48%	3,34%
31-dic.-2019	7,88%	-0,59%	3,93%	4,01%	13,84%

Table 13: Monthly returns of Ence, Endesa, Ferrovial, Grifols and International Airlines Group (IAG). Source: own elaboration.

DATE	IBERDROLA	INDITEX	INDRA	INMOBILIARIA COLONIAL	MAPFRE
	MONTHLY RETURN	MONTHLY RETURN	MONTHLY RETURN	MONTHLY RETURN	MONTHLY RETURN
3-ene.-2018	–	–	–	–	–
31-ene.-2018	3,97%	0,00%	1,63%	7,27%	5,88%
28-feb.-2018	-6,61%	-11,77%	-6,27%	-3,50%	-3,79%
29-mar.-2018	0,74%	5,96%	2,56%	7,92%	2,04%
30-abr.-2018	7,72%	0,35%	4,84%	0,05%	6,81%
31-may.-2018	-6,96%	3,85%	-8,67%	-7,29%	-10,90%
29-jun.-2018	7,36%	6,63%	-7,24%	4,38%	0,25%
31-jul.-2018	3,23%	-4,17%	1,37%	-2,12%	4,67%
31-ago.-2018	-2,46%	-6,90%	1,34%	3,32%	-5,12%
28-sep.-2018	-1,03%	2,39%	-5,23%	-4,07%	6,55%
31-oct.-2018	-1,60%	-5,75%	-12,21%	-1,72%	-1,34%
30-nov.-2018	4,31%	7,75%	6,97%	0,28%	-3,63%
31-dic.-2018	6,04%	-19,37%	-11,64%	-6,66%	-6,02%
31-ene.-2019	2,89%	8,56%	10,52%	11,35%	5,80%
28-feb.-2019	4,73%	8,28%	6,91%	2,20%	0,69%
29-mar.-2019	7,32%	-2,06%	0,30%	0,94%	-1,56%
30-abr.-2019	4,22%	3,37%	5,79%	4,75%	7,09%
31-may.-2019	4,63%	-6,46%	-8,89%	2,59%	-2,38%
28-jun.-2019	4,04%	8,27%	-4,41%	-1,06%	1,87%
31-jul.-2019	-1,95%	2,19%	-13,28%	4,30%	-4,45%
30-ago.-2019	10,84%	2,03%	-8,09%	2,04%	-4,78%
30-sep.-2019	1,00%	1,97%	10,51%	5,63%	3,30%
31-oct.-2019	-2,25%	-0,78%	9,55%	3,49%	1,58%
29-nov.-2019	-2,70%	1,63%	10,37%	0,17%	1,11%
31-dic.-2019	6,82%	13,66%	8,88%	0,89%	-4,70%

Table 14: Monthly returns of Iberdrola, Inditex, Indra, Inmobiliaria Colonial and Mapfre. Source: own elaboration.

DATE	MASMOVIL	MEDIASET	MELIA HOTELS	MERLIN	NATURGY
	MONTHLY RETURN	MONTHLY RETURN	MONTHLY RETURN	MONTHLY RETURN	MONTHLY RETURN
3-ene.-2018	–	–	–	–	–
31-ene.-2018	10,82%	-3,44%	-2,31%	2,07%	-8,30%
28-feb.-2018	15,61%	4,50%	6,96%	1,60%	-1,44%
29-mar.-2018	0,50%	-10,35%	-3,85%	6,01%	5,99%
30-abr.-2018	-2,76%	-3,28%	9,79%	2,40%	7,34%
31-may.-2018	-1,84%	2,35%	-6,61%	-8,54%	-1,22%
29-jun.-2018	-14,29%	-8,37%	-1,92%	4,97%	8,21%
31-jul.-2018	0,73%	-4,89%	-3,64%	2,06%	5,83%
31-ago.-2018	-0,71%	-5,24%	-5,01%	-2,99%	0,78%
28-sep.-2018	1,84%	2,44%	-10,49%	-1,43%	-0,80%
31-oct.-2018	14,83%	-4,30%	-5,61%	-3,67%	-7,42%
30-nov.-2018	-7,03%	1,69%	-3,34%	1,04%	2,72%
31-dic.-2018	-6,78%	-10,96%	-6,49%	-2,88%	0,32%
31-ene.-2019	-1,65%	10,06%	5,41%	8,73%	8,08%
28-feb.-2019	-1,08%	5,74%	-5,36%	-1,21%	-1,32%
29-mar.-2019	8,03%	-2,72%	-2,65%	3,37%	7,23%
30-abr.-2019	0,31%	7,99%	6,97%	3,67%	1,28%
31-may.-2019	2,57%	-7,10%	-7,43%	0,69%	0,31%
28-jun.-2019	-1,31%	1,72%	4,48%	2,87%	-5,87%
31-jul.-2019	3,74%	-17,51%	-4,02%	0,90%	-4,39%
30-ago.-2019	-10,27%	9,18%	-4,21%	-2,17%	2,37%
30-sep.-2019	-0,85%	-0,87%	-2,33%	5,96%	1,84%
31-oct.-2019	-8,32%	-8,88%	2,03%	4,04%	0,33%
29-nov.-2019	0,59%	8,50%	1,28%	-1,22%	0,26%
31-dic.-2019	3,93%	-1,63%	6,00%	0,00%	-2,14%

Table 15: Monthly returns of Masmovil, Mediaset, Melia Hotels, Merlin and Naturgy.

Source: own elaboration.

DATE	R.E.C	REPSOL	SIEMENS GAMESA	TELEFONICA	VISCOFAN
	MONTHLY RETURN	MONTHLY RETURN	MONTHLY RETURN	MONTHLY RETURN	MONTHLY RETURN
3-ene.-2018	–	–	–	–	–
31-ene.-2018	-4,81%	-1,11%	7,71%	1,07%	2,36%
28-feb.-2018	-7,25%	-3,89%	5,82%	-2,49%	-7,02%
29-mar.-2018	6,39%	2,96%	5,55%	2,50%	8,55%
30-abr.-2018	4,54%	9,31%	8,03%	5,50%	-0,81%
31-may.-2018	-3,90%	2,74%	-6,83%	-10,37%	3,17%
29-jun.-2018	4,97%	1,58%	-14,05%	-1,91%	3,53%
31-jul.-2018	7,21%	4,68%	7,33%	6,13%	1,03%
31-ago.-2018	0,47%	-2,01%	7,39%	-8,80%	4,76%
28-sep.-2018	-0,06%	2,88%	-14,74%	-2,67%	-0,55%
31-oct.-2018	1,10%	-8,18%	-6,12%	6,00%	-17,69%
30-nov.-2018	4,11%	-1,99%	22,96%	7,91%	-6,67%
31-dic.-2018	2,47%	-7,97%	-17,49%	-5,28%	-3,35%
31-ene.-2019	3,18%	10,58%	12,90%	1,20%	4,07%
28-feb.-2019	-4,26%	-0,95%	2,48%	0,84%	4,50%
29-mar.-2019	1,41%	0,96%	6,25%	-0,13%	2,86%
30-abr.-2019	-1,60%	-1,63%	11,37%	-1,05%	-4,03%
31-may.-2019	4,27%	-2,46%	-9,94%	-1,95%	-11,85%
28-jun.-2019	-4,29%	-4,83%	2,70%	3,14%	3,09%
31-jul.-2019	-2,89%	5,64%	-13,83%	-4,84%	-4,77%
30-ago.-2019	3,57%	-7,36%	-4,18%	-8,36%	-10,67%
30-sep.-2019	2,53%	7,26%	1,01%	10,15%	-1,10%
31-oct.-2019	-2,49%	2,76%	0,90%	-1,02%	14,74%
29-nov.-2019	-1,72%	-5,65%	14,60%	0,55%	-1,10%
31-dic.-2019	3,55%	-1,52%	8,39%	-5,43%	0,95%

Table 16: Monthly returns of Red Eléctrica Corporación (R.E.C), Repsol, Siemens Gamesa, Telefónica and Viscofan.

Source: own elaboration.

ANNEXE 4

Monthly and annual returns and volatilities of the 35 IBEX-35 assets

STOCKS	AVERAGE MONTHLY PROFIT	AVERAGE MONTHLY VOLATILITY	AVERAGE ANNUAL PROFIT	AVERAGE ANNUAL VOLATILITY
ACCIONA	1,69%	5,85%	20,32%	20,27%
ACERINOX	-0,43%	8,54%	-5,12%	29,59%
ACS	1,33%	7,25%	15,96%	25,12%
AENA	0,36%	4,24%	4,31%	14,69%
AMADEUS	1,13%	5,82%	13,53%	20,17%
ARCELORMITTAL	-2,03%	10,65%	-24,40%	36,88%
BANCO SABADELL	-1,05%	8,77%	-12,56%	30,39%
BANCO SANTANDER	-1,10%	6,57%	-13,24%	22,76%
BANKIA	-2,20%	7,92%	-26,36%	27,44%
BANKINTER	-0,32%	6,77%	-3,78%	23,44%
BBVA	-0,65%	7,56%	-7,82%	26,20%
CAIXABANK	-0,46%	8,17%	-5,58%	28,30%
CELLNEX	2,51%	5,99%	30,08%	20,73%
CIE AUTOMOTIVE	0,50%	9,74%	6,00%	33,73%
ENAGAS	1,00%	4,95%	11,95%	17,15%
ENCE	-0,43%	12,16%	-5,22%	42,13%

ENDESA	1,91%	3,83%	22,90%	13,27%
FERROVIAL	2,10%	3,79%	25,22%	13,12%
GRIFOLS	1,10%	5,91%	13,17%	20,47%
IAG	1,08%	8,35%	12,90%	28,92%
IBERDROLA	2,26%	4,54%	27,15%	15,74%
INDITEX	0,82%	7,15%	9,83%	24,76%
INDRA	-0,18%	7,86%	-2,19%	27,23%
INMOBILIARIA COLONIAL	1,47%	4,38%	17,58%	15,17%
MAPFRE	-0,04%	4,72%	-0,51%	16,37%
MÁSMÓVIL	-2,85%	17,84%	-34,24%	61,81%
MEDIASET	-1,47%	7,00%	-17,69%	24,25%
MELIA HOTELS	-1,35%	5,29%	-16,19%	18,32%
MERLIN	1,09%	3,68%	13,12%	12,76%
NATURGY	0,83%	4,56%	9,99%	15,80%
R.E. C	0,69%	3,88%	8,27%	13,43%
REPSOL	0,07%	5,07%	0,89%	17,56%
SIEMENS GAMESA	1,59%	10,23%	19,11%	35,42%
TELEFÓNICA	-0,39%	5,18%	-4,66%	17,95%
VISCOFAN	-0,67%	6,77%	-8,01%	23,47%

Table 17: Monthly and annual returns and volatilities of the 35 assets that make up the IBEX-35. Source: own elaboration.

ANNEXE 5

Monthly performance by date of the 35 IBEX-35 assets (1/2)

DATE	ACCIONA	ACERINOX	ACS	AENA	AMADEUS	ARCELORMITTAL	BANCO SABADELL	BANCO SANTANDER	BANKIA	BANKINTER	BBVA	CAIXABANK	CELLNEX	CIE AUTOMOTIVE	ENAGAS	ENCE	ENDESA
31-dic-2019	0,86%	6,23%	3,03%	4,38%	1,93%	2,12%	4,12%	7,12%	8,40%	4,74%	6,17%	5,03%	0,34%	-1,77%	5,98%	7,88%	-0,59%
29-nov-2019	0,97%	10,40%	-4,12%	-0,42%	7,62%	10,77%	1,37%	-2,02%	1,91%	0,89%	1,02%	1,13%	2,03%	1,58%	3,10%	-6,25%	1,77%
31-oct-2019	-2,20%	7,30%	0,55%	-2,86%	2,50%	3,75%	11,06%	-2,19%	-0,64%	8,58%	2,61%	8,14%	1,20%	-0,98%	3,16%	6,39%	2,13%
30-sep-2019	0,15%	2,51%	6,08%	1,54%	-2,17%	0,26%	17,44%	8,79%	9,69%	12,17%	11,76%	19,60%	3,55%	10,68%	7,35%	4,39%	2,20%
30-ago-2019	0,72%	0,80%	-6,88%	-2,06%	-5,97%	-8,37%	-1,32%	-11,24%	-8,48%	-10,60%	-7,04%	-8,97%	4,24%	-6,39%	0,97%	7,41%	3,04%
31-jul-2019	4,42%	-16,04%	10,74%	-6,01%	0,82%	-8,87%	-14,42%	-6,22%	-14,89%	-5,72%	-7,52%	-12,04%	3,60%	-7,44%	-9,48%	-19,95%	1,35%
28-jun-2019	-3,58%	11,72%	-4,51%	4,40%	2,20%	19,49%	-8,28%	3,57%	-5,97%	-3,97%	1,13%	-7,05%	4,26%	15,83%	-4,32%	8,00%	0,89%
31-may-2019	-4,53%	-6,78%	-8,73%	-0,51%	0,62%	-28,76%	-6,83%	-10,88%	-8,12%	-8,73%	-9,00%	-3,77%	19,22%	-13,01%	-1,21%	-22,52%	1,41%
30-abr-2019	4,03%	1,29%	4,41%	7,21%	-1,09%	1,44%	13,13%	5,61%	8,39%	1,54%	5,13%	0,89%	3,47%	3,15%	-0,78%	-4,89%	-2,12%
29-mar-2019	10,83%	-4,81%	-1,46%	2,92%	7,53%	-11,05%	-11,31%	-2,59%	-14,00%	-3,78%	-5,32%	-10,31%	8,28%	-5,07%	4,01%	-23,75%	3,74%
28-feb-2019	2,40%	-2,53%	8,62%	3,87%	3,44%	-3,27%	10,22%	5,57%	8,27%	7,25%	5,33%	2,48%	-7,68%	2,38%	-1,38%	-6,22%	1,89%
31-ene-2019	11,10%	10,74%	8,60%	10,07%	4,15%	14,22%	-0,16%	3,88%	0,40%	-2,46%	10,86%	5,40%	8,81%	16,48%	5,83%	24,08%	12,21%
31-dic-2018	-9,99%	-4,29%	-1,05%	-3,24%	-6,11%	-12,87%	-9,17%	-7,00%	-12,33%	-5,71%	-8,95%	-12,79%	-7,75%	-6,38%	0,42%	-1,93%	1,33%
30-nov-2018	7,79%	-10,45%	2,39%	0,50%	-10,90%	-9,86%	-3,76%	-0,31%	4,55%	0,54%	-0,28%	5,47%	13,63%	-5,29%	4,88%	-16,74%	6,52%
31-oct-2018	-4,26%	-20,68%	-10,00%	-4,11%	-13,26%	-17,28%	-12,50%	-2,25%	-17,22%	-10,48%	-9,61%	-7,49%	-3,89%	-13,97%	0,00%	-15,24%	-0,11%
28-sep-2018	3,25%	6,39%	2,69%	-1,35%	-0,84%	3,20%	1,44%	1,62%	3,21%	5,12%	2,54%	1,52%	2,44%	5,15%	-3,41%	9,44%	-3,05%
31-ago-2018	3,21%	-6,42%	-2,34%	-1,04%	10,42%	-7,83%	-7,58%	-9,78%	-4,60%	-5,55%	-13,72%	-2,55%	-2,68%	3,20%	1,14%	-1,16%	-1,98%
31-jul-2018	8,32%	14,71%	12,44%	1,94%	8,79%	11,63%	1,03%	5,63%	5,81%	-0,24%	5,72%	7,88%	5,67%	3,75%	-0,54%	7,60%	7,27%
29-jun-2018	6,78%	-5,14%	-4,77%	-4,31%	-0,33%	-9,96%	-3,27%	-3,38%	-4,04%	0,42%	0,93%	-0,70%	-1,55%	-24,72%	8,82%	10,14%	-0,58%
31-may-2018	-6,59%	-3,72%	0,03%	-5,52%	8,22%	-3,13%	-11,40%	-14,78%	-11,07%	-5,48%	-13,69%	-12,76%	-2,65%	3,17%	-5,51%	1,20%	-3,97%
30-abr-2018	11,00%	2,78%	13,00%	7,70%	2,36%	9,72%	1,67%	2,73%	4,66%	4,70%	10,43%	7,97%	3,25%	4,30%	9,53%	4,95%	8,49%
29-mar-2018	-6,20%	-5,18%	14,92%	-2,04%	5,41%	-3,65%	0,09%	-3,45%	-4,16%	-4,17%	-3,21%	0,62%	4,78%	11,11%	6,06%	18,49%	3,71%
28-feb-2018	-3,86%	4,46%	-11,07%	-4,20%	-2,35%	-2,16%	-9,16%	-4,61%	-3,20%	-2,93%	-7,62%	-8,09%	-2,76%	0,29%	-3,30%	2,67%	-3,73%
31-ene-2018	6,02%	-3,54%	-0,65%	1,77%	4,07%	1,65%	12,46%	9,68%	0,72%	16,30%	6,68%	9,22%	0,32%	15,95%	-7,42%	-4,43%	3,95%

Table 18: Monthly returns sorted by dates of the 35 assets that make up the IBEX-35 (1/2).

Source: own elaboration.

Monthly performance by date of the 35 IBEX-35 assets (2/2)

DATE	FERROVIAL	GRIFOLS	IAG	IBERDROLA	INDITEX	INDRA	INMOBILIARIA COLONIAL	MAPFRE	MASMOVIL	MEDIASET	MELIA HOTELS	MERLIN	NATURGY	R.E.C	REPSOL	SIEMENS GAMESA	TELEFONICA	VISCOFAN
31-dic-2019	3,93%	4,01%	13,84%	6,82%	13,66%	8,88%	0,89%	-4,70%	3,93%	-1,63%	6,00%	0,00%	-2,14%	3,55%	-1,52%	8,39%	-5,43%	0,95%
29-nov-2019	1,29%	5,48%	3,34%	-2,70%	1,63%	10,37%	0,17%	1,11%	0,59%	8,50%	1,28%	-1,22%	0,26%	-1,72%	-5,65%	14,60%	0,55%	-1,10%
31-oct-2019	0,88%	8,37%	15,36%	-2,25%	-0,78%	9,55%	3,49%	1,58%	-8,32%	-8,88%	2,03%	4,04%	0,33%	-2,49%	2,76%	0,90%	-1,02%	14,74%
30-sep-2019	0,99%	-6,08%	14,33%	1,00%	1,97%	10,51%	5,63%	3,30%	-0,85%	-0,87%	-2,33%	5,96%	1,84%	2,53%	7,26%	1,01%	10,15%	-1,10%
30-ago-2019	8,74%	-2,64%	2,24%	10,84%	2,03%	-8,09%	2,04%	-4,78%	-10,27%	9,18%	-4,21%	-2,17%	2,37%	3,57%	-7,36%	-4,18%	-8,36%	-10,67%
31-jul-2019	4,07%	12,00%	-1,90%	-1,95%	2,19%	-13,28%	4,30%	-4,45%	3,74%	-17,51%	-4,02%	0,90%	-4,39%	-2,89%	5,64%	-13,83%	-4,84%	-4,77%
28-jun-2019	6,38%	11,64%	5,10%	4,04%	8,27%	-4,41%	-1,06%	1,87%	-1,31%	1,72%	4,48%	2,87%	-5,87%	-4,29%	-4,83%	2,70%	3,14%	3,09%
31-may-2019	-0,41%	-5,16%	-18,29%	4,63%	-6,46%	-8,89%	2,59%	-2,38%	2,57%	-7,10%	-7,43%	0,69%	0,31%	4,27%	-2,46%	-9,94%	-1,95%	-11,85%
30-abr-2019	4,62%	-1,47%	7,07%	4,22%	3,37%	5,79%	4,75%	7,09%	0,31%	7,99%	6,97%	3,67%	1,28%	-1,60%	-1,63%	11,37%	-1,05%	-4,03%
29-mar-2019	0,53%	5,54%	-15,26%	7,32%	-2,06%	0,30%	0,94%	-1,56%	8,03%	-2,72%	-2,65%	3,37%	7,23%	1,41%	0,96%	6,25%	-0,13%	2,86%
28-feb-2019	3,18%	-0,57%	-6,53%	4,73%	8,28%	6,91%	2,20%	0,69%	-1,08%	5,74%	-5,36%	-1,21%	-1,32%	-4,26%	-0,95%	2,48%	0,84%	4,50%
31-ene-2019	10,50%	0,09%	10,47%	2,89%	8,56%	10,52%	11,35%	5,80%	-1,65%	10,06%	5,41%	8,73%	8,08%	3,18%	10,58%	12,90%	1,20%	4,07%
31-dic-2018	-3,46%	-8,77%	0,36%	6,04%	-19,37%	-11,64%	-6,66%	-6,02%	-81,88%	-10,96%	-6,49%	-2,88%	0,32%	2,47%	-7,97%	-17,49%	-5,28%	-3,35%
30-nov-2018	5,34%	-2,71%	0,68%	4,31%	7,75%	6,97%	0,28%	-3,63%	-7,03%	1,69%	-3,34%	1,04%	2,72%	4,11%	-1,99%	22,96%	7,91%	-6,67%
31-oct-2018	-3,28%	1,90%	-5,85%	-1,60%	-5,75%	-12,21%	-1,72%	-1,34%	14,83%	-4,30%	-5,61%	-3,67%	-7,42%	1,10%	-8,18%	-6,12%	6,00%	-17,69%
28-sep-2018	-2,40%	-4,41%	-4,83%	-1,03%	2,39%	-5,23%	-4,07%	6,55%	1,84%	2,44%	-10,49%	-1,43%	-0,80%	-0,06%	2,88%	-14,74%	-2,67%	-0,55%
31-ago-2018	6,34%	1,00%	-1,15%	-2,46%	-6,90%	1,34%	3,32%	-5,12%	-0,71%	-5,24%	-5,01%	-2,99%	0,78%	0,47%	-2,01%	7,39%	-8,80%	4,76%
31-jul-2018	1,81%	-3,42%	9,63%	3,23%	-4,17%	1,37%	-2,12%	4,67%	0,73%	-4,89%	-3,64%	2,06%	5,83%	7,21%	4,68%	7,33%	6,13%	1,03%
29-jun-2018	1,76%	1,62%	-4,08%	7,36%	6,63%	-7,24%	4,38%	0,25%	-14,29%	-8,37%	-1,92%	4,97%	8,21%	4,97%	1,58%	-14,05%	-1,91%	3,53%
31-may-2018	-1,44%	6,96%	5,26%	-6,96%	3,85%	-8,67%	-7,29%	-10,90%	-1,84%	2,35%	-6,61%	-8,54%	-1,22%	-3,90%	2,74%	-6,83%	-10,37%	3,17%
30-abr-2018	5,57%	5,13%	2,78%	7,72%	0,35%	4,84%	0,05%	6,81%	-2,76%	-3,28%	9,79%	2,40%	7,34%	4,54%	9,31%	8,03%	5,50%	-0,81%
29-mar-2018	1,34%	5,50%	1,92%	0,74%	5,96%	2,56%	7,92%	2,04%	0,50%	-10,35%	-3,85%	6,01%	5,99%	6,39%	2,96%	5,55%	2,50%	8,55%
28-feb-2018	-2,87%	-11,28%	-6,05%	-6,61%	-11,77%	-6,27%	-3,50%	-3,79%	15,61%	4,50%	6,96%	1,60%	-1,44%	-7,25%	-3,89%	5,82%	-2,49%	-7,02%
31-ene-2018	-2,97%	3,60%	-2,63%	3,97%	0,00%	1,63%	7,27%	5,88%	10,82%	-3,44%	-2,31%	2,07%	-8,30%	-4,81%	-1,11%	7,71%	1,07%	2,36%

Table 19: Monthly returns sorted by dates of the 35 assets that make up the IBEX-35 (2/2).

Source: own elaboration.

ANNEXE 6

Correlation matrix of the 35 IBEX-35 assets (1/2)

	ACCIONA	ACERINOX	ACS	AENA	AMADEUS	ARCELORMITTAL	BANCO SABADELL	BANCO SANTANDER	BANKIA	BANKINTER	BBVA	CAIXABANK	CELLNEX	CIE AUTOMOTIVE	ENAGAS	ENCE
ACCIONA	1															
ACERINOX	0,156710	1														
ACS	0,386726	0,173983	1													
AENA	0,552639	0,486606	0,402851	1												
AMADEUS	0,224364	0,430219	0,338614	0,215117	1											
ARCELORMITTAL	0,298760	0,805572	0,404410	0,535758	0,407696	1										
BANCO SABADELL	0,211354	0,425670	0,364601	0,465172	0,098473	0,389133	1									
BANCO SANTANDER	0,441035	0,408562	0,426611	0,670191	0,017553	0,573542	0,693050	1								
BANKIA	0,367825	0,569167	0,446040	0,588717	0,150905	0,526312	0,821238	0,722537	1							
BANKINTER	0,321693	0,347761	0,350960	0,346272	0,178311	0,434672	0,825259	0,748957	0,734670	1						
BBVA	0,531197	0,566279	0,529479	0,722586	0,093382	0,649339	0,775160	0,886830	0,802050	0,757805	1					
CAIXABANK	0,404720	0,402504	0,451528	0,492427	0,105240	0,423458	0,834386	0,735312	0,832025	0,793527	0,832551	1				
CELLNEX	0,321252	0,070624	0,080997	0,287048	0,001747	-0,098904	-0,032624	-0,008051	0,063450	-0,140576	0,134802	0,188080	1			
CIE AUTOMOTIVE	0,087655	0,536623	0,436317	0,527740	0,423009	0,723376	0,422315	0,492635	0,424571	0,425599	0,465519	0,410331	0,019187	1		
ENAGAS	0,282654	0,166952	0,204908	0,342047	-0,049520	0,072821	0,266552	0,182512	0,342393	0,130592	0,398383	0,465826	0,162243	-0,131023	1	
ENCE	-0,012295	0,625339	0,310957	0,234008	0,218365	0,622296	0,331077	0,236756	0,371330	0,199261	0,421306	0,341341	-0,206897	0,466928	0,334436	1
ENDESA	0,562724	0,207464	0,506175	0,556226	0,065781	0,300876	0,179228	0,347623	0,203919	0,129808	0,520134	0,426930	0,455446	0,283475	0,414792	0,173043
FERROVIAL	0,478621	0,252240	0,318370	0,548203	0,136189	0,356238	0,094805	0,099380	0,251541	-0,132756	0,288682	0,154172	0,319739	0,213865	0,312584	0,290928
GRIFOLS	0,123762	-0,066207	0,227361	0,030062	0,393986	0,297075	-0,127315	-0,002837	-0,231577	0,018184	0,014196	-0,111927	0,009871	0,139189	-0,098995	-0,036195
IAG	0,011154	0,537509	0,413096	0,270518	0,106023	0,656371	0,496532	0,385604	0,494566	0,364080	0,505812	0,506752	-0,140643	0,446825	0,273317	0,617267
IBERDROLA	0,377620	0,053463	0,118584	0,480640	-0,183879	-0,074851	0,198851	0,265565	0,155888	0,049430	0,348957	0,132251	0,255372	-0,176859	0,401889	0,013693
INDITEX	0,358832	0,198197	0,378599	0,402424	0,149130	0,400146	0,340532	0,400104	0,438788	0,302209	0,503155	0,352466	0,213582	0,244115	0,225542	0,300091
INDRA	0,414702	0,507433	0,438134	0,652725	0,321282	0,538352	0,726085	0,620258	0,789459	0,637481	0,722155	0,785371	0,157909	0,502996	0,530254	0,292068
INMOBILIARIA COLONIAL	0,406025	-0,014232	0,332312	0,389293	0,173426	0,097613	0,451857	0,343376	0,251721	0,275808	0,438998	0,474515	0,329081	0,257084	0,290572	0,204538
MAPFRE	0,484617	0,443034	0,419349	0,615597	0,118747	0,545652	0,629237	0,741250	0,564861	0,556381	0,792613	0,657334	0,193868	0,462838	0,207406	0,334018
MASMOVIL	0,320659	0,008362	-0,016767	0,145476	0,222045	0,148978	0,092921	0,229132	0,158066	0,135848	0,138533	0,214348	0,276406	0,196949	-0,144124	-0,112922
MEDIASET	0,181688	0,471183	-0,174797	0,473804	-0,016784	0,403376	0,283681	0,213303	0,411645	0,081627	0,284074	0,129617	0,037906	0,350439	0,058625	0,229615
MELIA HOTELS	0,273763	0,474458	0,113873	0,576262	0,093898	0,577253	0,254953	0,415017	0,386688	0,233091	0,495676	0,268198	0,029872	0,292624	0,325610	0,313687
MERLIN	0,428043	0,318245	0,372780	0,486771	0,089890	0,330422	0,412679	0,533754	0,358991	0,344067	0,643103	0,533375	0,413189	0,277927	0,456439	0,346175
NATURGY	0,463458	0,253941	0,386106	0,294953	0,246753	0,081551	0,065846	-0,049744	0,202781	-0,076320	0,246120	0,215490	0,326985	-0,096133	0,727421	0,331122
R.E.C	0,241600	-0,018827	0,320607	0,166013	-0,098133	-0,177362	0,003831	-0,000438	0,097118	-0,198954	0,159397	0,262127	0,423993	-0,254540	0,674304	0,197634
REPSOL	0,531026	0,237749	0,735005	0,343992	0,410110	0,374987	0,276307	0,304993	0,339542	0,380297	0,529213	0,499335	0,256854	0,351080	0,272046	0,353115
SIEMENS GAMESA	0,440513	0,325216	0,226257	0,598064	0,210708	0,451223	0,341847	0,443305	0,540848	0,287512	0,417908	0,472664	0,283901	0,468976	0,321063	0,069406
TELEFONICA	0,294772	0,092578	0,304974	0,384366	-0,245532	0,256642	0,349525	0,644943	0,391967	0,369354	0,594199	0,618777	0,301588	0,230894	0,362139	-0,018693
VISCOFAN	0,147276	0,435596	0,453677	0,188357	0,687003	0,504453	0,360670	0,222780	0,301325	0,476055	0,331291	0,330466	-0,178650	0,445783	0,199866	0,493342

Table 20: Correlation matrix of the 35 IBEX-35 assets (1/2).

Source: own elaboration.

Correlation matrix of the 35 IBEX-35 assets (2/2)

	ENDESA	FERROVIAL	GRIFOLS	IAG	IBERDROLA	INDITEX	INDRA	INMOBILIARIA COLONIAL	MAPFRE	MASMOVIL	MEDIASET	MELIA HOTELS	MERLIN	NATURGY	R.E.C	REPSOL	SIEMENS GAMESA	TELEFONICA	VISCOFAN
ACCIONA																			
ACERINOX																			
ACS																			
AENA																			
AMADEUS																			
ARCELORMITTAL																			
BANCO SABADELL																			
BANCO SANTANDER																			
BANKIA																			
BANKINTER																			
BBVA																			
CAIXABANK																			
CELLNEX																			
CIE AUTOMOTIVE																			
ENAGAS																			
ENCE																			
ENDESA	1																		
FERROVIAL	0,476202	1																	
GRIFOLS	0,083525	0,254985	1																
IAG	0,203787	0,355444	0,174309	1															
IBERDROLA	0,449873	0,391892	-0,076761	-0,079967	1														
INDITEX	0,188678	0,536785	0,486699	0,353519	0,212680	1													
INDRA	0,428768	0,379072	0,074408	0,541681	0,093652	0,465810	1												
INMOBILIARIA COLONIAL	0,459294	0,475644	0,206178	0,160817	0,229194	0,420687	0,469464	1											
MAPFRE	0,425566	0,126061	0,000746	0,197292	0,213667	0,227093	0,479163	0,452937	1										
MASMOVIL	-0,051104	0,079335	0,271471	-0,119503	-0,300065	0,394055	0,200901	0,291104	0,240878	1									
MEDIASET	0,043724	0,334154	-0,248950	0,158277	0,040196	0,318393	0,384250	-0,015771	0,192599	0,243100	1								
MELIA HOTELS	0,232357	0,403489	0,175556	0,438405	0,114695	0,242596	0,475192	0,236786	0,336388	0,196796	0,305744	1							
MERLIN	0,535191	0,327161	0,035122	0,235434	0,286043	0,290600	0,483833	0,729927	0,645253	0,150742	-0,045011	0,478584	1						
NATURGY	0,488132	0,378158	-0,162089	0,085582	0,357442	0,103925	0,318606	0,244300	0,209621	-0,173374	0,057522	0,163309	0,473499	1					
R.E.C	0,495645	0,255830	-0,188146	0,093320	0,530579	0,071859	0,103930	0,186131	0,119507	-0,235105	-0,230564	-0,134143	0,272806	0,666532	1				
REPSOL	0,463337	0,269475	0,205093	0,326809	-0,058227	0,350038	0,378385	0,411546	0,433837	0,223568	-0,132910	0,201199	0,522400	0,501291	0,236120	1			
SIEMENS GAMESA	0,440555	0,427825	0,074931	0,315425	0,040917	0,339479	0,773392	0,324469	0,285532	0,377877	0,476390	0,549165	0,328982	0,200401	0,022898	0,110535	1		
TELEFONICA	0,533178	-0,002794	-0,069151	0,141165	0,140212	0,147862	0,417705	0,231390	0,599283	0,217732	0,027526	0,195879	0,528527	0,141023	0,268641	0,262798	0,414683	1	
VISCOFAN	0,130085	0,144081	0,425456	0,407918	-0,093280	0,325732	0,506921	0,282314	0,230183	-0,060571	-0,123062	0,165049	0,344373	0,287382	-0,090319	0,470377	0,188574	-0,069672	1

Table 21: Correlation matrix of the 35 IBEX-35 assets (2/2).

Source: own elaboration.

ANNEXE 7

Completed correlation matrix of the 12 selected titles

	ACERINOX	BANCO SABADELL	BANKINTER	CELLNEX	ENCE	FERROVIAL	GRIFOLS	IAG	MASMOVIL	MELIA HOTELS	NATURGY	REC
ACERINOX	1	0,425670	0,347761	0,070624	0,625339	0,252240	-0,066207	0,537509	0,008362	0,474458	0,253941	-0,018827
BANCO SABADELL	0,425670	1	0,825259	-0,032624	0,331077	0,094805	-0,127315	0,496532	0,092921	0,254953	0,065846	0,003831
BANKINTER	0,347761	0,825259	1	-0,140576	0,199261	-0,132756	0,018184	0,364080	0,135848	0,233091	-0,076320	-0,198954
CELLNEX	0,070624	-0,032624	-0,140576	1	-0,206897	0,319739	0,009871	-0,140643	0,276406	0,029872	0,326985	0,423993
ENCE	0,625339	0,331077	0,199261	-0,206897	1	0,290928	-0,036195	0,617267	-0,112922	0,313687	0,331122	0,197634
FERROVIAL	0,252240	0,094805	-0,132756	0,319739	0,290928	1	0,254985	0,355444	0,079335	0,403489	0,378158	0,255830
GRIFOLS	-0,066207	-0,127315	0,018184	0,009871	-0,036195	0,254985	1	0,174309	0,271471	0,175556	-0,162089	-0,188146
IAG	0,537509	0,496532	0,364080	-0,140643	0,617267	0,355444	0,174309	1	-0,119503	0,438405	0,085582	0,093320
MASMOVIL	0,008362	0,092921	0,135848	0,276406	-0,112922	0,079335	0,271471	-0,119503	1	0,196796	-0,173374	-0,235105
MELIA HOTELS	0,474458	0,254953	0,233091	0,029872	0,313687	0,403489	0,175556	0,438405	0,196796	1	0,163309	-0,134143
NATURGY	0,253941	0,065846	-0,076320	0,326985	0,331122	0,378158	-0,162089	0,085582	-0,173374	0,163309	1	0,666532
REC	-0,018827	0,003831	-0,198954	0,423993	0,197634	0,255830	-0,188146	0,093320	-0,235105	-0,134143	0,666532	1

Table 22: Correlation matrix completed for the 12 selected titles.

Source: own elaboration.

ANNEXE 8

Monthly variance and covariance matrix of the 12 selected securities

	ACERINOX	BANCO SABADELL	BANKINTER	CELLNEX	ENCE	FERROVIAL	GRIFOLS	IAG	MASMOVIL	MELIA HOTELS	NATURGY	REC
ACERINOX	0,007295936	0,003189221	0,002010025	0,000361068	0,006495910	0,000816029	-0,000334202	0,003832548	0,000127442	0,002143259	0,000989318	-0,000062326
BANCO SABADELL	0,003189221	0,007693821	0,004898249	-0,000171281	0,003531696	0,000314957	-0,000659960	0,003635633	0,001454258	0,001182682	0,000263430	0,000013025
BANKINTER	0,002010025	0,004898249	0,004578876	-0,000569358	0,001639779	-0,000340240	0,000072717	0,002056542	0,001640162	0,000834144	-0,000235549	-0,000521767
CELLNEX	0,000361068	-0,000171281	-0,000569358	0,003582532	-0,001506031	0,000724841	0,000034915	-0,000702706	0,002951864	0,000094556	0,000892658	0,000983553
ENCE	0,006495910	0,003531696	0,001639779	-0,001506031	0,014789980	0,001340049	-0,000260137	0,006266402	-0,002450284	0,002017516	0,001836684	0,000931515
FERROVIAL	0,000816029	0,000314957	-0,000340240	0,000724841	0,001340049	0,001434510	0,000570734	0,001123789	0,000536129	0,000808202	0,000653262	0,000375533
GRIFOLS	-0,000334202	-0,000659960	0,000072717	0,000034915	-0,000260137	0,000570734	0,003492478	0,000859900	0,002862489	0,000548680	-0,000436900	-0,000430929
IAG	0,003832548	0,003635633	0,002056542	-0,000702706	0,006266402	0,001123789	0,000859900	0,006968245	-0,001779890	0,001935415	0,000325842	0,000301911
MASMOVIL	0,000127442	0,001454258	0,001640162	0,002951864	-0,002450284	0,000536129	0,002862489	-0,001779890	0,031835272	0,001856979	-0,001410915	-0,001625772
MELIA HOTELS	0,002143259	0,001182682	0,000834144	0,000094556	0,002017516	0,000808202	0,000548680	0,001935415	0,001856979	0,002796876	0,000393920	-0,000274947
NATURGY	0,000989318	0,000263430	-0,000235549	0,000892658	0,001836684	0,000653262	-0,000436900	0,000325842	-0,001410915	0,000393920	0,002080297	0,001178224
REC	-0,000062326	0,000013025	-0,000521767	0,000983553	0,000931515	0,000375533	-0,000430929	0,000301911	-0,001625772	-0,000274947	0,001178224	0,001502065

Table 23: Monthly variance and covariance matrix of the 12 selected titles.

Source: own elaboration.

ANNEXE 9

Annual variance and covariance matrix of the 12 selected securities

	ACERINOX	*BANCO SABADELL*	*BANKINTER*	*CELLNEX*	*ENCE*	*FERROVIAL*	*GRIFOLS*	*IAG*	*MASMOVIL*	*MELIA HOTELS*	*NATURGY*	*REC*
ACERINOX	0,087551232	0,038270651	0,024120299	0,004332819	0,077950919	0,009792348	-0,004010428	0,045990577	0,001529303	0,025719109	0,011871812	-0,000747907
BANCO SABADELL	0,038270651	0,092325849	0,058778991	-0,002055369	0,042380350	0,003779489	-0,007919516	0,043627602	0,017451093	0,014192180	0,003161162	0,000156302
BANKINTER	0,024120299	0,058778991	0,054946508	-0,006832293	0,019677344	-0,004082882	0,000872601	0,024678501	0,019681943	0,010009728	-0,002826592	-0,006261207
CELLNEX	0,004332819	-0,002055369	-0,006832293	0,042990384	-0,018072373	0,008698097	0,000418977	-0,008432478	0,035422363	0,001134675	0,010711902	0,011802640
ENCE	0,077950919	0,042380350	0,019677344	-0,018072373	0,177479756	0,016080588	-0,003121640	0,075196827	-0,029403404	0,024210195	0,022040207	0,011178184
FERROVIAL	0,009792348	0,003779489	-0,004082882	0,008698097	0,016080588	0,017214117	0,006848807	0,013485467	0,006433549	0,009698423	0,007839141	0,004506395
GRIFOLS	-0,004010428	-0,007919516	0,000872601	0,000418977	-0,003121640	0,006848807	0,041909740	0,010318798	0,034349867	0,006584155	-0,005242801	-0,005171146
IAG	0,045990577	0,043627602	0,024678501	-0,008432478	0,075196827	0,013485467	0,010318798	0,083618940	-0,021358682	0,023224985	0,003910108	0,003622934
MASMOVIL	0,001529303	0,017451093	0,019681943	0,035422363	-0,029403404	0,006433549	0,034349867	-0,021358682	0,382023259	0,022283747	-0,016930977	-0,019509266
MELIA HOTELS	0,025719109	0,014192180	0,010009728	0,001134675	0,024210195	0,009698423	0,006584155	0,023224985	0,022283747	0,033562513	0,004727044	-0,003299363
NATURGY	0,011871812	0,003161162	-0,002826592	0,010711902	0,022040207	0,007839141	-0,005242801	0,003910108	-0,016930977	0,004727044	0,024963568	0,014138693
REC	-0,000747907	0,000156302	-0,006261207	0,011802640	0,011178184	0,004506395	-0,005171146	0,003622934	-0,019509266	-0,003299363	0,014138693	0,018024785

Table 24: Matrix of annual variances and covariances for the 12 selected titles.

Source: own elaboration.