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Testing Okun's Law in Spain and the G7 countries

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Abstract

This paper estimates Okun's law in the G7 countries and Spain between 1971 and 2020. It employs different specifications of the model, focusing on the dynamic part of the relationship. I find that the negative relationship between output and unemployment generally holds over the selected time period. Nevertheless, Okun's coefficient varies significantly across countries. The dynamic version can enlighten the different country estimations that can be found in the literature. Time lags are used to avoid any possible misspecification. Several regressions and tests are conducted to account for unit roots, auto-correlation and structural breaks.

Keywords: Okun's Law; Unemployment; Hamilton Filter
JEL classification: E24, C22, E47

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1 Introduction

Okun's law refers to a macroeconomic relationship that relates output and unemployment. Therefore, it connects the activity level in the labour market and the activity level in the goods market over a given business cycle. In his publication, Okun (1962) found that a three percent increase (decrease) in output is related to a one percent decrease (increase) in unemployment rate. Whereas this relationship was validated by early literature (see (Blackley, 1991)) more recent research has found this relationship to be closer to a two percent increase (decrease) in output resulting in a one percent decrease (increase) in unemployment rate, which means that variations in unemployment rate to real GDP growth rate are more responsive.

For a significant period of time, this relationship has attracted a good deal of attention, not only because of its empirical robustness, but because when combined with the Phillips curve, the aggregate supply curve is obtained. Furthermore, this relationship may be very useful for certain macroeconomic policies. For instance, in determining the optimal growth rate of an economy.

Two different dimensions are commonly discussed in the literature; the heterogeneity of Okun's law across several countries and the time-varying aspect. The literature focusing on the time-varying concept tests whether Okun's coefficient has decreased on increased over a certain period of time. Some examples can be found in Knotek (2007) or Lee and Sinclair (2000). In this context, Okun's law variations are usually tested over expansionary and recessionary periods. Knotek (2007) comes to the conclusion that Okun's coefficient is, on average, smaller in expansionary periods rather than in recessionary, which means that unemployment varies in a different way during different business cycles. As stated before, other studies focus on answering the question of whether all the economies react similarly to changes in output and unemployment (see Ball et al. (2019)). Usually, these studies focus on the characteristics of a country's labour market to understand the heterogeneous behaviour of Okun's coefficient.

A further distinction can be made of the literature that explains the relationship between output and unemployment rate. A first group estimates Okun's coefficient using the equations suggested by Okun (1962), the first difference model and the Gap model. A second group estimate and augmented production function in which, in the right hand side, a series of explanatory variables are included (capital productivity or labour, e.g).

This paper focuses on the G7 countries (Canada, France, Germany, Italy, Japan, USA and UK) and Spain during the last fifty years (1971-2020) using a time series approach. I decided to include Spain in this study due to its high levels of unemployment over the last four decades. I will estimate the model using the First difference and Gap model proposed by Okun (1962). However, I include several lags to both real GDP and unemployment. The reason for this is to take into account that firms do not react to changes in unemployment and output in an immediate way, as results are shown after some time. Furthermore, estimating different versions of Okun's coefficients may explain the reasons for the heterogeneous behaviour of Okun's coefficient.

Many studies focus primarily on the output-unemployment relationship in the short run. Very few papers opt for a dynamic version of the model, for e.g Ball et al. (2019), who include two lags of the output term in their model, and even fewer focus on the long run relationship. Furthermore, these studies focus mainly in the post-war period in the USA, without conducting any further investigation in the European context. On the other hand, Moosa (1999) shows that the long run effects that cyclical output has on cyclical unemployment differ from those in the short run.

Contrary to many other studies, this paper includes data from the financial crisis and the starting months of the COVID-19 crisis, which may have possibly distorted unemployment and output estimations made prior to these events.

The rest of the paper is organized as follows: Section 2 introduces the versions of Okun's law as first stated by Okun (1962) and discusses further extensions made by more recent literature. Section 3 focuses on the methodology, paying attention on explaining the differences between stationary and non-stationary variables, spurious regressions and several methods to de-trend data. Section 4 revises previous empirical literature. Section 5 explains the estimation methodology that is followed and describes the data used. Section 6 discusses the results that were obtained and Section 7 summarizes the main findings and concludes the paper.

2 Okun's Law: Model variations

Since the seminal paper of Okun (1962) and the well-known assumption that a three percent increase in output is correlated with a one percent decrease in unemployment rate, a substantial amount of literature has been devoted to the ”Okun‘s law”; how responsive unemployment rate is to variations in output. This rule has been broadly considered to be a fair representation of the negative relation between output and unemployment. To begin with, in his article in 1962, Okun presented two basic equations relating unemployment and output which, ever since, have been used as a rule of thumb for macroeconomic analysis. Since then, these equations have been modified and extended by numerous authors with the objective of improving the equation's statistical fit and the precision of their foundation.

The first specification suggested by Okun is the *First difference model*, which stipulates that the relationship between the observed unemployment rate (u_t) and the natural logarithm of the observed real output (y_t) is given by:

$$\Delta u_t = \beta_0 + \beta_1 \Delta y_t + \varepsilon_t \tag{1}$$

where β_0 represents the intercept, β_1 is Okun's coefficient, which measures by how much variations in output result in variations in unemployment rate, and ε represents the disturbance term.

Following the *First difference model*, Okun specified the *Gap model*, given by the following expression:

$$u_t - u_t^* = \beta_0 + \beta_1 (y_t - y_t^*) + \varepsilon_t \tag{2}$$

where y^* is the potential output, while u^* represents the natural rate of unemployment. As pointed out by Ball et al. (2019), the error term (ε_t) captures those factors that alter the output cyclical unemployment relationship, such as exceptional variations in labour force participation or productivity. On the other hand, the coefficient β_1 represents the level by which firms will adjust employment when there is a variation in output and the cyclical reaction of the labour force. This implies that:

$$e_t - e_t^* = \beta^e(y_t - y_t^*) + \varepsilon_{et} \tag{3}$$

$$l_t - l_t^* = \beta^l(y_t - y_t^*) + \varepsilon_{lt} \tag{4}$$

where e_t^* and l_t^* represent the trend values of, respectively, employment and the natural logarithm of labour force. Thus, the lesser the cyclical response of the labour force is, the greater the inverse correlation between β_1 (from eq (2)) and β^e will be.

However, the *Gap model* presents a considerable problem; no observable data can be obtained for u_t^* and y_t^*, which means these two terms need to be estimated. For instance, Okun suggested maintaining u_t^* constant at 4% as a rate of labour utilization and advocated for an uncomplicated time series trend to measure y_t^*. Yet, several authors support different techniques to estimate u_t^* and y_t^*. Most of these methods include, among others, deterministic techniques like the HP filter (such as Marinkov and Geldenhuys (2007)) or stochastic decomposition methods, as the Kalman filter (for instance, Silvapulle et al. (2004)). An in depth explanation of these procedures can be found later on in the paper.

One of the drawbacks that Okun remarked is that the unemployment rate can only be seen as a proxy variable for insignificant resources that affect losses in output. To settle this issue, a number of authors (see Gordon (1984)) suggested a new model relating the *Gap model* and the production function, considering that the equilibrium level of real output will be attained once each and every factor reaches its equilibrium level:

$$y_t - y_t^* = \beta_0 + \beta_1(u_t - u_t^*) + \beta_2(H_t - H_t^*) + \varepsilon_t \tag{5}$$

$(H_t - H_t^*)$ is a gaps vector between all observable inputs except labour and the equilibrium level. It is important to bear in mind that when estimating this model, instead of the unemployment rate, real output is used as the dependant variable.

Accounting for the existent time lags in the unemployment-output relationship, a number of studies (for instance, Weber (1995)) have advocated for the inclusion of dynamics in Okun's law and, therefore, rewrite eq. (1) as:

$$U_t = \beta_0 + \beta_1 Y_t + \beta_2 Y_{t-1} \varepsilon_t \tag{6}$$

Which is equal to:

$$U_t = \beta_0 + \beta_1 \Delta Y_t + Y_{t-1}(\beta_1 + \beta_2) + \varepsilon_t$$

Eq (6) makes it possible for the unemployment rate to match a delay in output. The term β_1 captures the effect in the short-run of output on unemployment, while the sum of β_1 and β_2 captures the total effect. Respectively, these two terms are known as the short-run and total Okun effect. Furthermore, when output growth is auto-correlated in a positive way, this model specification allows for a reduction in the possible bias of the simultaneous equation for the total effect that output has on unemployment (Sogner and Stiassny (2002)).

Even though specification 6 can be useful to judge whether Okun's law has suffered several variations over time, these may have been significantly slow. For instance, labour markets undergo structural changes that happen at a slow rate. The model assumes all parameters happen concurrently when affected by a discrete variation when, in practice, the parameters might vary in their own particular way. To account for this matter, Beaton (2010) suggests the TVP (Time Variation Parameter) adaptation from eq. (6):

$$u_t = \beta_0(t) + \beta_1(t)\Delta y_t + y_{t-1}(\beta_1(t) + \beta_2(t)) + \varepsilon_t$$

which can be written as:

$$u_t = \phi(t)X_t + \varepsilon_t \tag{7}$$

The TVPs are jointly accounted for by $\phi(t)$, while the remaining regressor will be captured by X_t. Moreover, and according to Sogner and Stiassny (2002), TVPs are expected to follow a random walk pattern, which can be expressed as:

$$\phi(t)_t = \phi(t)_{t-1} + w_t \tag{8}$$

where w_t is expected to be equal to 0.

The TVP specification applies a specific weight to each observation, with those closer to *t* having a greater effect than those further away in time. Moreover, it allows to interpret in an individual way the evolution of each and every term. Hence, it allows to recognize variations in Okun's law related with both kinds of changes; short-run and total effects.

Lastly, the *Asymmetric dynamic model* is an alternative proposed by Silvapulle et al. (2004). They base their model specification in the distributed time lag model of Weber (1995):

$$u_t^c = \sum_{j=1}^{p} \alpha_j u_{t-j}^c + \sum_{j=1}^{q} \beta_j y_{t-j}^c + \varepsilon_t \tag{9}$$

where the terms u_t and y_t refer, respectively, to the observable rate of unemployment and the natural logarithm of output, while u_t^n and y_t^n refer, respectively, to the natural employment rate and the natural logarithm of potential output. Thus, cyclical output (y_t^c) and cyclical unemployment (u_t^c) can be defined as $y_t^c = y_t - y_t^n$ and $u_t^c = u_t - u_t^n$, respectively.

A limitation of this model proposed by Weber (1995) is that it does not allow for any kind of asymmetry between cyclical unemployment and cyclical output. To tackle this matter, Silvapulle et al. (2004) suggest decomposing the term y_t^c into two different terms; $y_t^{c+} = y_t^c \geq 0$ and $y_t^{c+} = y_t^{c-} \leq 0$. In their article, they correlate positive and negative output gaps to upturns and downturns in an economy.

As a result, they suggest the following model with the goal of including the asymmetric relationship between cyclical unemployment and cyclical output:

$$u_t^c = \sum_{j=1}^{p} \alpha_j u_{t-j}^c + \sum_{j=1}^{q} [\phi_j y_{t-j}^{c+} + \lambda_j y_{t-j}^{c-}] + \varepsilon_t \tag{10}$$

In this case, ϕ refers to the relationship between unemployment and output which is expected to be negative.

3 Methodology

3.1 Stationary and non-stationary time series

As shown by Richard Harris (1995) models made up of non-stationary variables will most likely lead to a spurious regression issue. Hence, results obtained from the regression will suggest that statistically significant relationships can be found between certain variables when, truly, what the regression shows is a correlation over time rather than a substantial causal relationship.

For instance, suppose that variable y_t is obtained by a first-order autoregressive procedure:

$$y_t = \rho y_{t-1} + u_t \tag{11}$$

If the series is non-stationary, then the variance of y_t increases over time, without any proneness for the time series to return to any value close to the mean. This fact contrasts with the stationary-first difference model, $\Delta y_t = y_t - y_{t-1}$. The variance of stationary variables is finite and, the variables themselves, usually fluctuate around their mean value. A non-stationary variable can, however, become a stationary variable. To do so, it needs to be differenced.[1]

The fact that a variable is stationary or non-stationary depends on whether it has a unit

[1] Not just by first-differencing, it will depend on the number of unit roots it contains.

root. For instance, we can rewrite eq. (11) as:

$$(1 - \rho L)y_t = u_t \tag{12}$$

where y_{t-1} will be contained in L, the lag term. If the roots of eq. (12) are greater, in absolute terms, than 1, then then y_t is stationary. Hence, for the example shown previously, stationary requires ρ to be greater than 1 in absolute terms.

A second way to consider stationarity is to look at the different time trends found in variables. To do so, we add an intercept to eq. (11):

$$y_t = \beta + \rho y_{t-1} + u_t \tag{13}$$

Which, if $\rho = 1$, then it can be rearranged as:

$$y_t = y_0 + \beta_t + \sum_{j=1}^{T} u_j \tag{14}$$

It is observed that y_t is not returning a fixed trend ($y_0 + \beta t$), due to the accumulation of the random errors term. In fact, in eq.(13) if $\rho = 1$, then y_t will only depend on the sign of β, and therefore its value will drift downwards or upwards.

3.1.1 Spurious regressions

Spurious correlations show a relationship between variables in a regression, when what is actually correlated are different time trends. Thus, spurious regressions show a correlation between independent non-stationary variables.

Richard Harris (1995) explains this fact with a simple equation, where x and y refer to uncorrelated non-stationary variables:

$$y_t = \beta_0 + \beta_1 x_1 + \varepsilon_t \tag{15}$$

Generally, it should be possible to accept the null hypothesis of $\beta_1 = 0$. Nonetheless, due to the non-stationarity of the data, which implies that ε_t is non-stationary too, any time series growing tendency leads to the regression model to pick up a certain correlation level, even though the variables are not correlated.[2]

In sum, generally there is an issue of incorrectly assuming that there exists a relationship between unrelated non-stationary variables. This problem builds up when the sample size increases. We will also account for this later in the paper.

3.2 Alternatives to estimate trends of $u_t^*$and y_t^*

As stated previously, in order to estimate the *Gap model*, Okun made use of linear trends to calculate both the natural rate of unemployment and potential output. However, these two variables are considered to be non-stationary and, using deterministic trends techniques will most likely lead to misleading results. To avoid this we will test for cointegration.

Moreover, four alternative methods will be discussed next; the *HP filter*, the *BN filter*, the *Kalman filter* and the *Hamilton filter*.

3.2.1 HP filter

The *HP filter* is able to differentiate the cyclical component , α_t^c, and the stochastic trend ,α_t^*, from a given time series x_t. It can be expressed as:

$$\sum_{t=1}^{T}[(x_t - x_t^*)^2 + \lambda(\Delta x_{t+1}^* - \Delta x_t^*)^2] \tag{16}$$

[2]The issue of spurious correlation, resulting in a rejection of the null hypothesis, is based on the fact that the F and the t statistics' distributions generated by non-stationary time series differ from those generated by stationary series. With non-stationary series the propensity is to reject the null hypothesis. This tendency increases as the sample size grows. Richard Harris (1995) mentions the Monte Carlo experiment conducted by Banerjee et al. (1993), in which he regressed eq. (15) 10.000 times. The results showed that at a 5% significance level, the probability of rejecting the null hypothesis was of 75.3%

where the term λ controls for the smoothness of the time series. As a guideline, the author suggest λ should be equal to 1600 for quarterly data. (bear in mind that x_t^c, the cyclical component of the time trend, is equal to $x_t - x_t^*$).

Several authors have used this filter in their research papers, see King et al. (1995) or Giorno et al. (1995) where the *HP filter* was used to analyze the Philips curve which is the relationship between unemployment and inflation and to determine certain trends and cyclical characteristics from data, respectively.

Notwithstanding, this technique has recently received heavy criticism (see Hamilton (2017) for instance), as it can potentially result in false assumptions. Furthermore, Cogley and Nason (1995) concluded that the *HP filter* is able to create business cycles dynamics from data that does not present such cycles.

3.2.2 BN filter

Another alternative method to transform any given integrated time series - time ordered observations-into a stochastic trend - one that varies during each run as a result of the random nature of the process- is the *BN filter*, proposed by Beveridge and Nelson (1981). Assuming that x_t is an integrated time series and that Δx_t follows a stationary behaviour, the following model is suggested:

$$x_t^* = x_t + \sum_{i=1}^{\infty} E_t(\theta_0 + \Delta x_{t+i}) \tag{17}$$

where $E_t(x_t)$ is equal to x_t. Note that the cyclical factor x_t^c can be calculated as $x_t - x_t^*$. x_t^* can be described as the long-run estimation of x_t based on the information provided at period t. For instance, Attfield and Silverstone (1997) made use of the *BN filter* to estimate cyclical trends of unemployment an output for the UK.

3.2.3 Kalman filter

The *Kalman filter*, according to Lee and Sinclair (2000) is composed by two different equations:

$$x_k = Ax_{k-1} + Bu_k + w_{k-1} \tag{18}$$

and

$$z_k = Hx_k + v_k \tag{19}$$

This implies that each signal value, x_k, will be tested through a linear stochastic equation (eq. 18). x_k will be made up of a linear combination of its preceding value, adding a control signal, u_k, whereas w_{k-1} refers to a noise function.

Eq. (19) shows that z_k, a measurement value, can be expressed as a linear combination of x_k, a signal value and a measurement noise. Note that both terms are considered to follow a normal distribution. Terms H, B and A refer to general form matrices.

The next step is to determine the initial values and the needed parameters. To do so, there are two different set of equations; Time update and Measurement update.

"Time update" refers to predictions:

$$\widehat{x}_k^- = A\widehat{x}_{k-1} + Bu_k$$

$$P_k^- = AP_{k-1}A^T + Q$$

While "Measurement update" refers to corrections:

$$K_k = P_k^- H^T (HP_k^- H^T + R)^- 1$$

$$\widehat{x}_k = \widehat{x}_k^- + K_k(z_k - H\widehat{x}_k^-)$$

$$P_k = (I - K_k H)P_k^-$$

$\widehat{x}_k$ is the prior estimate, while P_k^- is to the prior error covariance. In the measurement set of equations, $\widehat{x}_k$ represents the estimation of x at moment k. P_k is, along with $\widehat{x}_k$, of much importance to estimate the future state $(k+1)$. Thus, as the measurement set of equations is known as the correction set, we can consider these the ”output” while the time update will be the ”input”. We will start estimating the time update equations considering $k = 0$. Next, we will use those results to estimate the measurement update equations, which outputs at k, will be the input used to estimate the time update equation for $k+1$, following a cycle.

3.2.4 Hamilton filter

The most up to date method discussed in this paper is the *Hamilton filter*. In his paper *Why you should never use the HP filter*, Hamilton (2017) proposed the following model:

$$y_{t+h} = \delta + \beta_0 y_t + \beta_1 y_{t-1} + ... + \beta_p y_{p-1} + v_{t+h} \tag{20}$$

where p is the amount of lags used in the model. The larger p is, the richer the model will be, although more parameters need to be estimated.

h refers to the lag that is most probable to be predicted incorrectly. Hamilton states that in financial and macro time series, the standard time horizon should be two years. Thus, for instance, if we want to estimate the quarterly business cycle, h should be equal to 8.

An important remark that Hamilton makes in his paper is that ”both h and p must be multiples of numbers of samples from a single year for a data with seasonality” Hamilton (2017). Thus, for instance, and following the author’s remarks, for quarterly data p should be equal to 4, while h should be equal to 8.

Hamilton (2017) highlights two main reasons why the HP filter should not be used:

- The HP filter creates time series with spurious relations that are not based on the data that is being processed.
- Values filtered at the end of the sample usually show spurious dynamics. In addition, they differ with those values obtained in the middle of the sample.

He continues stating that the main objective of the HP filter is detrending time series.

The main example of a time series process that is used to test the effectiveness of the HP filter is a random walk. Theory suggest that certain economic variables should all follow a random walk pattern. For instance, see Samuelson (2015) for future prices or Fama (1965) for stock prices. However, a number of studies have claimed deviations from a random walk pattern in certain time series. In addition, authors as Balcilar et al. (2015) have claimed that a random walk is extremely hard to beat and, therefore, for certain time series, there is no reason to use the HP filter.

Hamilton (2017) theoretically explains his critics, using the cyclical component formula for the HP filter:

$$c_t = \frac{\lambda(1-L)^3}{F(L)}\varepsilon_t \tag{21}$$

ε_t follows a random walk pattern, but c_t does not. This is, according to Hamilton (2017), highly predictable. Moreover, F(L) and L are solely dependent on λ, which means that the patterns of the cyclical component c_t rely on the HP filter's term that control for the smoothness, and it does not reflect any true dynamics of the data sample.

4 Empirical evidence

Since the publication of his seminal paper, Okun (1962), Okun's law has been tested in numerous occasions by a number of authors. As described previously, authors have modified and adapted Okun's original model to test for specific features of a country's economy. Next, I will review some examples.

Firstly, I will discuss the asymmetry in Okun's law. In 2001, Harris et al. (2001) suggested different reasons why testing for asymmetry would be of much interest: If asymmetry is not taken into account, it will certainly lead to forecasting inaccuracy. Furthermore, asymmetry hypothesis rejection would be helpful for stabilization and structural policies and ff the relationship between unemployment and output results asymmetric, it may imply that the Phillips curve may also be asymmetric.

To work with said asymmetry, Silvapulle et al. (2004), tested Okun's law using data from the USA for the post-war period and the distributed time lag model (previously discussed). In their paper they explain that they had two different estimation methods in mind; OLS in the case that no outliers where found in the response variable or the M-estimators[3] in case there were. Their results suggest that there were outliers in the response variable, which meant that the estimation methods chosen were M-estimations.

Their results suggest that Okun's law coefficients for the post-war period in the USA differ when cyclical output cycles are positive or negative. When the output gap is negative rather than positive, the return of unemployment to output is stronger, suggesting asymmetry is present. These results differ from those found in Weber (1995) and Moosa (1999), which were based in symmetric models.

Beaton (2010) investigated Okun's law cohesion for Canada and United States using the previously discussed time varying parameter (TVP) approach. For the United States, he used data on output and unemployment ranging from the first quarter of 1948 until the second quarter of 2009, while for Canada, a shorter period was used, from the first quarter of 1961 until the second quarter of 2009. In order to be able to compare the results obtained by the TVP model with those that already existed from previous literature, he estimated the model through the following methods. To begin, he first estimated the model using OLS estimators. To follow, he tested the stability through a Quandt likelihood ratio (QLR_T)[4] using rolling regressions and and a split-sample estimation. Finally, the model was estimated by TVP and all the results where compared. The results obtained by OLS suggest that Canada's short-run Okun coefficients is equal to -0.16, whilst in the long run he obtained a greater coefficient equal to -0.31. For the United States, results suggest a closer relationship between output and unemployment. According to Beaton (2010), one percentage point

[3]Robust estimation used in the presence of outliers, when the data does not follow a normal distributions or in the case of extreme observations

[4]Alternative version of the Chow test that uses the greatest F-statistic value that is obtained when conducting an ordinary Chow test on a certain data range

decrease in output will result in a 0.23% and in a 0.39% increase in the unemployment rate in the short-run and long-run respectively.

When testing for Okun's law stability, the author determines that in both Canada and the United States, Okun's law has suffered changes over time. The QLT_T suggests two structural breaks, taking place in 1979 and 1985 for Canada and in 1974 and 1983 in the case of the United States. The author suggests that the cause may well be the oil shocks from the 1970s. The rolling regression results indicate a similar conclusion. Specifically, in the case of Canada, from the late 70s until the early 80s,the absolute value of the short-run and total Okun's law coefficients increased. In the case of the United States, results point out a higher volatility in the relationship between output and unemployment rate. This relationship weakened between the late 60s and early 70s, and became stronger during the second part of the 70s, 80s and early 90s. Furthermore, referring to the United States, volatility derives into asymmetry in Okun's law. Specifically, the response of the unemployment rate to output variations tends to be greater over recessionary periods than over expansionary.TVP model results, however, suggest that Okun's law is more stable than the results suggested by rolling regressions. One of the reasons that the author brings up is the fact that rolling regressions estimations, as a result of the small sample size used, might overstate the effect caused by more recent observations. Following the TVP estimation, the author concludes that an increase in 1% in unemployment will result in a 2.6% and a 2% decrease in output in Canada and the United States, respectively.

Kim et al. (2014) tested Okun's law for several East Asian countries over the period 1986-2011. To do so, they used a time-varying version of the gap and first difference models, as these are able to capture the time variation parameter of Okun's coefficient. It is specified as follows:

$$y_t - y_t^* = \beta_0 + \beta_1(y_{t-1} - y_{t-1}^*) + d_t\Upsilon + \beta_2(t)(u_t - u_t^*) + \varepsilon_t \tag{22}$$

where terms u_t, y_t, u_t^* and y_t^* refer, respectively, to the real unemployment rate, output, natural rate of unemployment and potential level of output. $\beta_2(t)$ denotes Okun's time-varying parameter, the effect that unemployment variation rate has on the variation rate of real output over time. d_t refer to a $mx1$ vector of dummy variables at time t, while $(u_t - u_t^+)$ and $(y_t - y_t^*)$ refer to the unemployment gap captured by cyclical unemployment and output gap captured by cyclical level of output, respectively. In order to obtain variables u_t^* and y_t^*, the authors used the HP filter discussed in section 3.1.1. To estimate the model, authors used maximum likelihood estimators, as OLS estimates are inefficient of heteroscedasticity is present.

Other than Singapore, results show a very significant negative relationship between real GDP and unemployment rate over the whole time period. The authors suggest that this might be due to different economic situations between countries. In the case of Japan, when its economy was in an expansion state, small variations in unemployment rate had a greater impact on the economy in comparison to a recessionary period. In Korea's case, Okun's coefficients follow the same trend as its real GDP. The authors suggest that this is due to the stability of unemployment in the country. This effect implies that when the economy goes through a recession, the effect that unemployment has on output is greater than when it is expanding. On the other hand, Singapore's and Hong Kong's Okun's coefficients follow unemployment rate trend rather than GDP's.

Especially among young people, unemployment rates across several European countries have become a significant issue. Dunsch (2016) analyzes it in Germany and Poland. To do so, she uses two different estimation methods. Firstly, she regresses the *First Difference model* via OLS. Next, she constructs a balanced panel for both Germany and Poland in order to solve the problem of scarce observations available for OLS estimation. To solve this panel, least squares dummy variable model (LSDV) is used for Germany and Poland:

$$\Delta u_{jt} = \beta_{0j} A_j + \beta_j A_j GDPgrowth_t + \varepsilon_{jt} \tag{23}$$

where A_j is a dummy variable referring to different age groups. Results validate the negative relationship assumption between unemployment and output in both countries and among all age groups. Specifically, in the case of both Germany and Poland, it is found that Okun's coefficient is greater, in absolute terms, among young people than for any other age group, meaning that this age group's sensitivity to business cycles is larger.

A relevant point that must be discussed is whether Okun's law is validated for any economy, no matter whether advanced or developing. Ball et al. (2019) analyze this situation, using IMF's *World Economic Outlook* criteria to determine which economies are considered advanced and which developing. To check this hypothesis, the authors used both the *First difference model*, explained in eq. (1) and the variations of the *Gap model* explained in equations (3) and (4). To determine the trend of output, employment and unemployment rate and labour force, they used the *HP filter*. After estimating the models, they came to the following conclusions: Labour markets appear to be more responsive to fluctuations in output in developed economies rather than in developing economies. As an example, the variation of unemployment rate with respect to output is of about -0.2 for developing economies, while for developed economies is of about -0.4. Moreover, significant heterogeneity is found in Okun's coefficient for developing economies, similar to the results fromBall et al. (2013). To finish, they show that Okun's coefficients are, on average, approximately half as big in developing economies as in advanced economies.

To finish, we should ask ourselves if economists are consistently able to predict Okun's law coefficients correctly. Pierdzioch et al. (2011) tried to analyze this suggestion using various sets of data obtained from surveys carried out by the Consensus Economic Inc. These surveys are conducted in a monthly basis and, therefore, economists' predictions will be more uncertain in January and much more precise in December, at the end of the year. As a result, two estimations are carried out over two different months, January and April. Assuming that growth rate of real potential output and unemployment rate are constant, the authors used the following version of Okun's law, suggested by Moosa (1999):

$$E_{t,i}[u_{t+1}] - u_t = \alpha + \beta E_{t,i}[\Delta y_{t+1}] + \varepsilon_{t,i} \tag{24}$$

where $E_{t,i}$ and Δy_{t+1} refer to the prediction in time t of economist i and the growth rate in year t+1 of real output, respectively. α, the intercept term, is expected to be positive, while β, the slope coefficient, is expected to be negative. Eq. (24) is estimated by fixed effects, and via a F-test, it is tested whether the null hypothesis of identical constant for all economists participating in the survey can be rejected.

Results for the January estimation suggest a negative Okun coefficient, with a very similar magnitude to the one reported previously by various authors, excluding the United States coefficient, which is slightly smaller than those reported previously. However, comparisons of Okun's coefficients reveal a considerable variation between G7 countries. For instance, the largest coefficient estimated corresponds to the United Kingdom, -0.34, while the smallest is found in Japan, -0.11. The fact that the smallest coefficient is found in Japan goes in line with previous estimations.[5]. On the other hand, one explanation of the largest coefficient being found in the United Kingdom may lay on the flexibility of its labour market. According to Moosa (1999), the British labour market is the least regulated labour market of the European Economic Area (from which the UK was previously part). As a result, it is a very flexible labour market, which allows employers to reduce or expand their workforce during a recessionary or booming economic period.

Results from the forecast conducted over the month of April do not differ much. Okun's coefficient has, as expected, a negative sign and, with the exception of Germany, is statistically significant in every other country. However, United Kingdom's coefficient is no longer the largest. Canada's, France's and the United States' coefficients are bigger in absolute terms. Japan's coefficient, on the other hand, is still the smallest.
Figure (1) summarizes the bibliography review:

[5]See Moosa (1999) or Freeman (2001)

Figure 1: *Bibliography Summary*

Author & Year	Scope	Time period	Model used	Estimation Method	Results
Okun (1962)	USA	Post war period	First difference, trial gap and fitted trend	OLS	3% increase (decrease) in output would be associated with 1% decrease (increase) in unemployment.
Silvapulle et al. (2004)	USA	1945–1964	Distributed lag model	No outlier→OLS based method Outliers→M-estimators' methods	Response of unemployment to output is stronger when there is a negative output gap.
Pierdzioch et al. (2009)	G7 countries; the United States, Germany, Japan, the United Kingdom, France, Italy, and Canada	1986-2006	New version suggested by Moosa (97). Also used the Asymmetry model.	Time fixed effects model.	Significant negative relationship btw the expected change in unemployment rate and the expected growth rate real output in G7 countries.
Beaton (2010)	Canada and USA	1961-2009 for Canada 1948-2009 for the USA	TVP model explained in the previous section.	Comparison between OLS, Rolling Regression, QLR_T, M⁻estimators.	For USA: 2.6% decrease output -> 1% increase unemployment. Canada: 2% decrease output -> 1% increase unemployment.
Kim et al. (2014)	East Asian countries	1986-2011	Time varying version of first order diff and gap model Hp filter to obtain unobserved u* and y*	OLS problems with heteroscedasticity and inefficient, meaning maximum likelihood estimation method is used	Okun's coefficients are time-varying.
Dunsch (2016)	Germany and Poland	1992-2014	Own adaptation from 1st Diff model. Addition of new adaptation for fixed effects panel data.	OLS For panel data model→ Least squares dummy variable (LSDV)	Greater Okun's coefficient among the youth, which makes it a sensible group.
Ball et al. (2019)	Advanced and developing economies	1980-2015	Gap and first difference model. Hp filter for trend values of unemployment rate and output.	OLS and Fixed Effects	Weaker relationship btw jobs growth in developing economies.

Own elaboration

5 Data description

To test Okun's law, I have chosen data from the G7 group of countries, as well as Spain. I believe these 8 countries' output and labour market are remarkably different, hence it will be of much interest to test whether Okun's law holds. Data was extracted from the WorldBank database and ranges from 1971 to 2020, which will give an approximation of the way in which Okun's law has varied over time.

Unemployment data is calculated as the percentage share of total labour force that is without a job but available for and seeking employment, while output data will be proxied by the real GDP, in constant 2015 US dollars. Data for both output and unemployment was modified, and two new variables were obtained; *unemp.chg* and *y.pct.chg*, $u_t - u_{t-1}$ and $\frac{GDP_t - GDP_{t-1}}{GDP_{t-1}} * 100$ respectively.

Table (1) shows a summary of the main statistics of unemployment and the growth of real GDP from 1971 to 2020. The analyzed group of countries show an average GDP growth of 2.12%. On average, unemployment rates were 7.99%. However, the trends exhibited by the group of countries is significantly heterogeneous. For instance, Spain shows is one of the fastest growing economy on average, 2.3%, but it also shows the highest unemployment rate, 14.92%. The lowest unemployment rates on average are shown in Japan, 1.2%, while Spain shows the highest rates on average, 26.09%, peaking during the crisis.

In this paper, I employ time series methods. Okun's coefficient is estimated by a long-run relationship model and the gap model proposed by Okun (1962). The gap model is estimated by its dynamic versions. To do so, first I estimated the model including two lagged output variables in the right hand side of the model. Then, to the previous model, I also included two lagged unemployment variables, which account for possible delays int he response of cyclical unemployment to output. The GDP trend and natural rate of unemployment are not observable variables, hence I use the Hamilton filter in order to de-trend the data. Figure (2) and Figure (3) are two examples that show the real GDP trend and unemployment trend over the last 50 years, calculated using Hamilton filter.

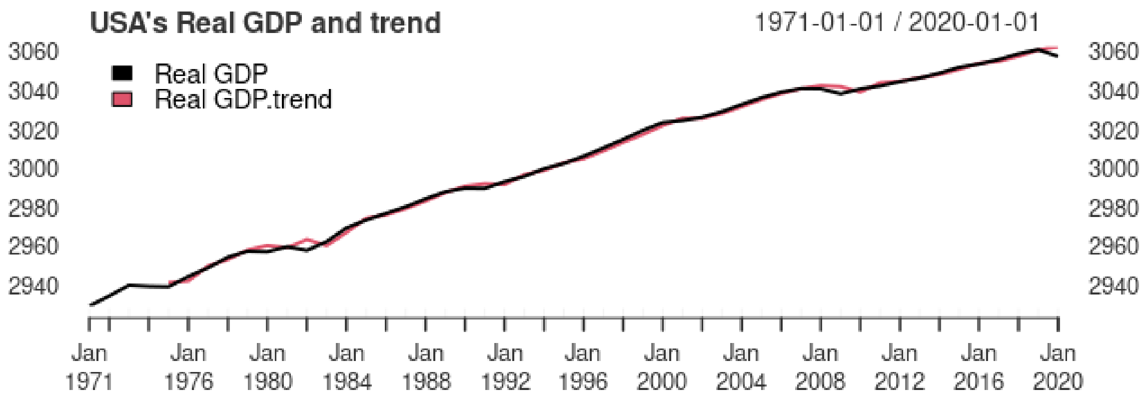
Table 1: *Main statistics summary*

Annual data (1971-2020)

	Uenmployment				**GDP growth**			
Country	**Mean**	**Std. Dev**	**Min**	**Max**	**Mean**	**Std. Dev**	**Min**	**Max**
Canada	8.00	1.71	5.4	12.02	2.42	2.53	-5.23	6.87
France	8.55	2.61	2.67	12.59	1.99	2.17	-7.86	6.34
Germany	6.98	2.14	3.14	11.17	1.83	2.14	-5.69	5.25
Italy	9.29	2.06	5.38	12.68	1.49	2.72	-8.93	7.13
Japan	3.13	1.17	1.2	5.39	2.26	2.75	-5.69	8.41
USA	6.27	1.58	3.67	9.7	2.66	2.12	-3.4	7.24
UK	6.8	2.52	2.6	11.51	1.99	2.63	-9.4	6.52
Spain	14.92	6.8	1.5	26.09	2.3	3	-10.82	8.15
Total	7.99	2.57	1.2	26.09	2.12	2.51	-10.82	8.41

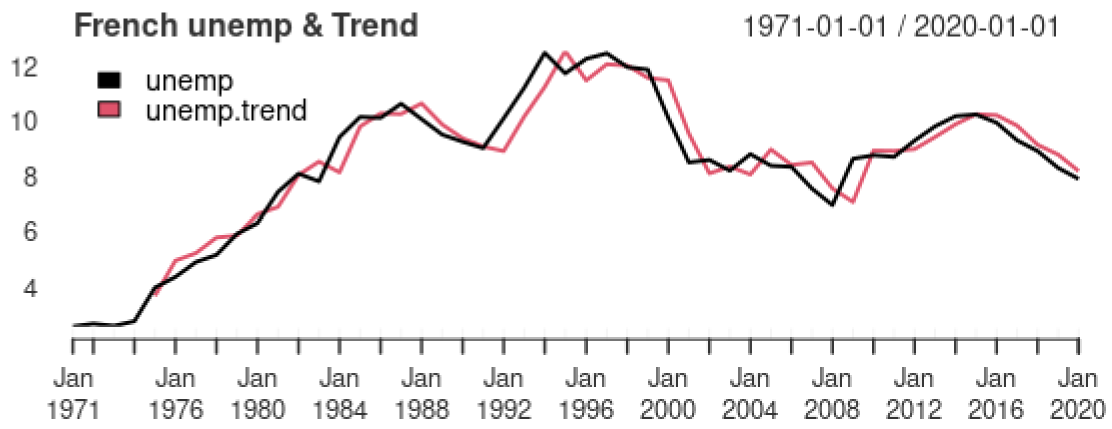
Note: Data based on annual real GDP growth and annual unemployment rates (in levels) downloaded from World Bank database. Own elaboration.

Figure 2: *USA's GDP Hamilton filer*



Own elaboration

Figure 3: *French unemployment Hamilton filter*



Own elaboration

Before starting with the regressions, I conduct a unit root test on the variables included in the model. To do so, I use the Augmented Dickey-Fuller Unit Root Test , the Elliot, Rothenberg and Stock Unit Root Test and the Phillips-Perron Unit Root Test. Table (2) shows the results obtained. According to the three tests, the null hypothesis can be rejected for both variables in most countries at a significance level of 5 %. Therefore, variables are stationary. The only exceptions are found in the Philips Perron test for Canada, Germany and Spain. This may be due to missing data.

Table 2: *Unit root tests*

	Augmented Dickey-Fuller test		Elliott, Rothenberg & Stock test		Phillips & Perron Test	
	Unemployment	**Output**	**Unemployment**	**Output**	**Unemployment**	**Output**
Canada	-4.22***	-3.67**	-3.99**	-3.84**	-3.93**	-3.26*
France	-4.36***	-4.39***	-4.38***	-3.58**	-5.78***	-4.39***
Germany	-3.68**	–5.68***	-3.85**	-3.84**	-3.22*	-5.52***
Italy	-3.83**	-5.19***	-3.52**	-4.49***	-4.85***	-5.77***
Japan	-4.23***	-5.03***	-4.76***	-3.93**	-4.67***	-5.69***
USA	-4.11***	-4.91***	-3.82**	-4.42***	-3.58**	-4.89***
UK	-6.06***	-3.92**	-6.27***	-3.46**	-4.71***	-3.74**
Spain	-4.22***	-3.16*	-3.55**	-3.61**	-6.30***	-3.27*

Note: Table shows values of tests statistics. "***","**" and "*" indicate rejection of the null (unitary root) at a significance level of 1%, 5% and 10%, respectively. Critical values: -4.04, -3.45 and -3.15 at 1%, 5% and 10% significance level. Own elaboration.

6 Empirical results

6.1 Long run relationship

To start, I estimate a long-run relationship between unemployment and output:

Table 3: *Long-run estimation*

	Okun Coeff.
Canada	-4.87***
France	-1.82*
Germany	-2.38***
Italy	-1.36**
Japan	-1.05**
USA	-4.33***
UK	-4.78***
Spain	-1.98*
Avg Adj. R2	0.401

Note: "***","**" and "*" indicate a 1%, 5% and 10% significance level. Own elaboration.

I find a great heterogeneity between countries, as stated previously, in line with the vast majority of literature. Coefficients vary from -1.05 in Japan to -4.87 in Canada. All of them are significant at, at least, 10%. The average R^2 is equal to 0.4, which is considerably good.

However, regressing the previous model with structural breaks will help us determine when and whether there has been an abrupt change in Okun's coefficient. The standard linear regression model assumes that variation does not exist among observations. However, in most cases, variation is present, and it plays a relevant role in time series analysis.

Bai and Perron (1998) suggest two different breakpoints estimation methods; Global maximization and Sequentially determined breakpoints. In this paper, I use the latter. The procedure is as follows:

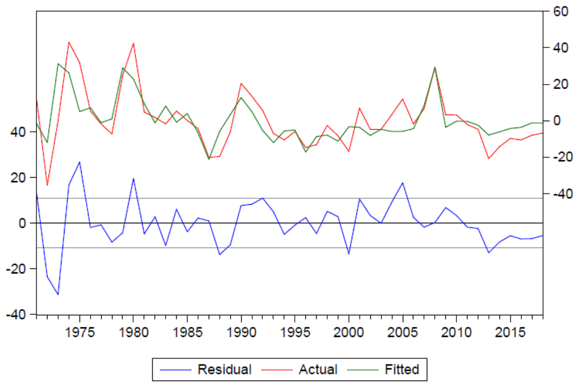
To start, I perform a test of parameter consistency on the full sample. I test for breakpoints in each sub-sample, and add a break-point whenever a sub-sample null is rejected. The procedure is repeated until the maximum specified number of breakpoints is reached. To finish, a refining is performed to re-estimate breakpoints that may be obtained from a sub-sample containing multiple breaks.

For each country, two structural breaks are found. Table (4) shows the years in which these happened and the Okun's coefficients estimated during these periods. Okun's coefficients are significantly close to those expected. Japan consistently shows the lowest coefficients, while the English speaking countries show the highest.

To further show the structural breaks, figure(4a) and figure(4b) show the residuals of the estimation for the UK and the USA, respectively.

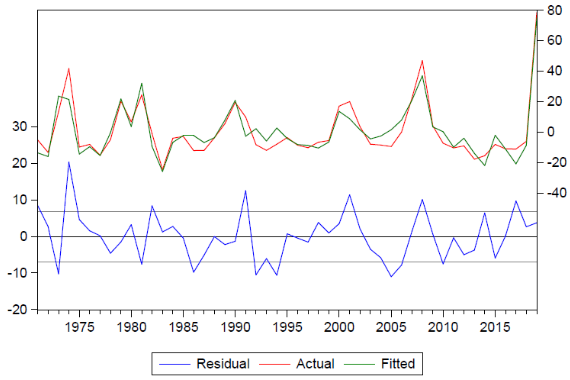
Figure 4: *Structural breaks*

(a) Residuals for the UK



Own elaboration.

(b) Residuals for the USA



Own elaboration.

Table 4: *Structural breaks estimation*

	Structural	**breaks**		**Okun coeff.**	
	T1	**T2**	α_1	α_2	α_3
Canada	1996	2006	-1.57**	-2.16**	-2.37*
France	1981	1997	-2.34*	-1.95*	-1.15**
Germany	1983	2005	-2.78***	-1.77*	-1.04*
Italy	1978	2001	-1.57**	-2.53*	-3.07
Japan	1981	2005	-1.02***	-1.27*	-1.57**
USA	1999	2010	-2.68**	-3.72*	-4.51**
UK	1986	1999	-2.88**	-3.59*	-4.31***
Spain	1983	2004	-1.51*	-2.25***	-3.07**

Note: Table shows values of Okun's coefficients and years in which there is a structural break. ”***”,”**” and ”*” indicate a significance level of 1%, 5% and 10%, respectively.Own elaboration.

The negative relationship between output and unemployment is present in every structural break. For every country, Okun's coefficient has increased after every structural break, with the exception of France and Germany. These countries undertook extensive labour market reforms from 1995 to 2005, which might explain the reason why the coefficient did not become larger. In fact, one structural break found for Germany happened in 2005 and for France in 1997, exactly when the reforms were undertaken. This may lead to believe that these two countries experience jobless growth, the fact that unemployment rate can be lowered without any substantial growth in GDP. Moreover, we can find a relationship between the average level of unemployment and the estimated Okun's coefficient: in countries with a higher unemployment rate, it fluctuates more with respect to variations in output, hence a higher Okun's coefficient is obtained, as it is the case of Spain or Italy. In the case of Spain, a growing Okun's coefficient may also be explained by the high incidence of temporary contracts. The labour market reforms undertaken in the 80s made it simpler for employers to employ workers on fixed-term contracts, without any employment protection, which is guaranteed for permanent workers. Over the last two decades, this type of contracts have accounted for about a third of Spanish employment. Temporary contracts reduce the difficulty for firms to adjust unemployment levels when output varies, increasing Okun's coefficient.

6.2 Gap & Auto-regressive distributed lag Gap model

Secondly, I estimated Okun's law using the original Gap version proposed by Okun (1962) in his seminal papers, in which he introduced two lagged output variables:

$$U_t - U_t^* = \alpha + \beta_0(Y_t - Y_t^*) + \beta_1(Y_{t-1} - Y_{t-1}^*) + \beta_2(Y_{t-2} - Y_{t-2}^*) \tag{25}$$

where Y* and U* are estimated using the Hamilton filter and the β coefficients refer to the lagged and current value of the output gaps.

Table (5) shows the results obtained by the regression. The fit is considerably good, with an R^2 over 0.35 for all countries except for Canada and the USA. Compared with the previous model, the average R^2 marginally increases to 0.44, while the average Okun's coefficient is -0.279.

The same heterogeneity between countries' Okun's coefficient is observed when estimating the Gap model. In this case, Okun's coefficient varies from -0.153 in Japan to -0.401 in the USA.

Table 5: *Gap version of Okun's law*

	(Okun´s coefficient) $\beta_0 + \beta_1 + \beta_2$	**Adj R2**
Canada	-0.307** (0.067)	0.296
France	-0.265** (0.203)	0.394
Germany	-0.168*** (0.076)	0.613
Italy	-0.296** (0.186)	0.357
Japan	-0.153*** (0.076)	0.421
USA	-0.401** (0.127)	0.395
UK	-0.374** (0.042)	0.491
Spain	-0.267*** (0.061)	0.563

Note: Table shows estimated coefficients. Standard error are shown in parenthesis. "***","**" and "*" indicate a significance level of 1%, 5% and 10%, respectively. Own elaboration.

Even though most literature supports the validity of Okun's law, Okun's coefficient' magnitude vary greatly depending on the model specification, and it seems to vary with time. Hence, and following Obst (2022), I estimate an auto-regressive lagged gap model, in which I include the lagged dependent variable as follows:

$$U_t - U_t^* = \alpha + \beta_0(Y_t - Y_t^*) + \beta_1(Y_{t-1} - Y_{t-1}^*) + \beta_2(Y_{t-2} - Y_{t-2}^*) + \rho_1(U_{t-1} - U_{t-1}^*) + \rho_2(U_{t-2} - U_{t-2}^*)$$

I tried to keep the model as simple as possible. Hence, I only included two lags of the dependent variable, the unemployment gap, as well as two lags of the output gap. As I included lags of the dependent variable, the long run effect will be given by the Long Run Multiplier (LRM) which can be defined as:

$$LRM = \frac{\sum_{i=0}^{n} \widehat{\beta}}{1 - \sum_{i=0}^{n} \widehat{\rho}}$$

This model has two main advantages with respect to the previous estimation methods (Obst (2022)). Firstly, it addresses the issue of serial correlation, unlike the Gap and long-run relationship estimations. Secondly, it controls, in an indirect way for any other explanatory variables that might affect unemployment in period *t* with a delay. The latter, is a widely discussed issue. Silvapulle et al. (2004), for instance, discussed this fact addressing that Okun's law simple version usually presents time variation as a result of various reasons, such as asymmetric adjustments over different business cycles and to the addition of explanatory variables to the regression analysis (e.g variations in the labour market). In my estimation of the model, these factors are indirectly controlled by the inclusion of the lagged effects of both output and unemployment gaps.

To test for serial correlation, I conducted the Breusch-Godfrey test, as the Durbin-Watson test is not applicable for this model, as I included lagged values of the dependent variable in the model. Table (6) shows the results obtained:

Table 6: *Auto-correlation test*

	Breusch-Godfrey test
Canada	0.3251
France	0.2694
Germany	0.5575
Italy	0.5589
Japan	0.2013
USA	0.6446
UK	0.5720
Spain	0.2013

Note: Own elaboration. Table shows p-values obtained by the test.

Breusch-Godfrey test uses the following hypotheses:

- H0: There is no auto-correlation at any order smaller than or equal to ρ.
- H1: There is auto-correlation at some order smaller or equal to ρ.

Therefore, from the output, we can conclude that auto-correlation does not exist among the residuals at any order smaller or equal to $\rho = 2$.

Once discussed the auto-correlation issue, I can continue describing the results obtained by the auto-regressive lagged Gap model. Table (7) summarizes the estimation results.The fit increases substantially compared to the previous model, with an average R^2 of 0.60. Hence, it appears to be a more fair representation of Okun's law than the basic model. In general, countries that showed a low R^2 in the Gap model, in the auto-regressive model show a significant higher value, as Canada (0.296 to 0.583) or Italy (0.357 to 0.636). Thus, an important remark to make is that, even though the Gap model model included two lagged variables of the output gap, it showed a poorer fit than the auto-regressive Gap model. The average Long Run Multiplier is barely the same as that obtained in table (5). The average Okun's coefficient is -0.258, which indicates a drop of about 0.3% in unemployment if output increases by 1%. Thus, results are close to the expected size of Okun's coefficient. On the other hand, and in line with Kim et al. (2014) and Pierdzioch et al. (2011), Japan shows the lowest coefficient (-0.08), while as expected, USA shows the largest coefficient (-0.472) with a R^2 equal to 0.77. These results are significantly similar to those obtained by Ball et al. (2013), who estimated an Okun's coefficient oscillating between -0.4 and 0.5, with an R^2 of 0.8.

Results indicate that English speaking countries' unemployment moves rapidly with any change in GDP. Hence, for policy implications, this group of countries are more likely to mitigate unemployment enhancing GDP growth through monetary easing or expansionary fiscal policies. On the other hand, the labour market of countries with a lower coefficient, especially Japan, are inert and not very respondent to changes in GDP. In the case of Japan, results lead to believe that Japanese workers have no difficulty in finding a new job when it is terminated. Therefore, the coefficient might not be as impacted with changes in output and unemployment. This might be one of the reasons to explain the abnormally low coefficient obtained. Another explanation, according to Ball et al. (2013), might be Japan's tradition of "lifetime employment", which makes employers hesitant to lay off workers.

Table 7: *Auto-regressive distributed lag Gap model*

	Canada	**France**	**Germany**	**Italy**	**Japan**	**USA**	**UK**	**Spain**
β_0	-0.466*** (0.064)	-0.153* (0.037)	-0.227*** (0.056)	-0.406* (0.049)	-0.034*** (0.042)	-0.747*** (0.068)	-0.413*** (0.076)	-0.332** (0.101)
β_1	0.017 (0.037)	0.013* (0.058)	0.010 (0.070)	0.307* (0.078)	-0.075 (0.052)	0.137 (0.140)	-0.157 (0.103)	0.268** (0.199)
β_2	0.022* (0.067)	0.031** (0.091)	-0.013 (0.170)	-0.084* (0.057)	0.027** (0.042)	0.206* (0.125)	0.220* (0.094)	-0.080 (0.166)
ρ_1	-0.176* (0.039)	0.024 (0.153)	0.072*** (0.127)	0.028 (0.182)	-0.064** (0.119)	0.057** (0.187)	0.057** (0.147)	0.014 (0.144)
ρ_2	0.087 (0.107)	0.059** (0.160)	-0.021* (0.017)	0.098 (0.171)	0.086 (0.112)	0.088 (0.175)	0.045 (0.119)	0.064* (0.136)
Adj R^2	0.583	0.467	0.577	0.636	0.723	0.771	0.683	0.398
Okun's coefficient	-0.392	-0.118	-0.242	-0.209	-0.08	-0.472	-0.389	-0.156

Note: Own elaboration. Standard errors in parenthesis; ***, ** and * indicate statistical significance at the 1%, 5% and 10% level, respectively.

7 Conclusions

Okun's law is a keystone in macroeconomics. Many studies focus on the output-unemployment relationship in the short run, while a very limited number of papers opt for a dynamic version of Okun's law model (see Ball et al. (2019)). However, most of these scarce number of papers focus on the USA. Hence, to study the dynamic version of Okun's law in European countries (as well as North American and Asian) is of much interest. Furthermore, contrary to most of the previous literature, this paper includes data from the global financial crisis and the COVID-19 crisis, two shocks affecting both output and unemployment. The empirical analysis conducted in this paper supports the negative relationship between output and unemployment.

Two of my model specifications use the GDP trend and the natural trend of unemployment.These are not observable variables, hence a filtering method needs to be used. Historically, the BN filter, the HP filter or the Kalman filter have been used to estimate these variables. However, and following the publication of Hamilton (2017), the Hamilton filter is the most appropriate filtering method. Thus, this is the method used in this paper to obtain the real GDP trend and the natural trend of unemployment over the last 50 years.

To address the issues that may arise when analysing time series, I conducted several tests on the variables included. First, in order to tests the stationarity of the variables, I conducted three different unit root tests; the Augmented Dickey-Fuller Unit Root Test , the Elliot, Rothenberg and Stock Unit Root Test and the Phillips-Perron Unit Root Test. After performing these tests, I concluded that the variables included in the models are stationary. Furthermore, variables were tested for auto-correlation too. As I included lagged values of the dependent variable in the auto regressive lag Gap model, I conducted the Breusch-Godfrey test. Results showed that auto-correlation does not exist among the residuals at any order smaller or equal to $\rho = 2$.

Analyzing the results obtained by the regressions, Spain and the G7 countries show that Okun's law is an undergoing regularity. Based on the long run estimation, coefficients lie between -1.05 in Japan to -4.87 in Canada. Nevertheless, the average Okun coefficient obtained across every country is -2.82, significantly close to the latest rule of thumb originally proposed by Okun (1962).

When regressing the previous model with structural breaks, I find that from 1971 to 2020 there have been two structural breaks for every country. Analyzing the evolution of the coefficient before and after the breaks helps me determine the way in which the labour market of each country behaves, and sheds some light on the reasons why we observe such a high heterogeneity between countries.

On average, following a 1% output increase, unemployment decreases by 0.279% based on the original Gap model, including two output lags, and 0.258% according to the auto-regressive lag Gap model. Therefore, results hold robust among the various models applied in this paper and are, quantitatively, notably similar to those obtained by most recent literature. Furthermore, when analyzing the fits of the models, I find that the auto-regressive lag Gap model is the most fair representation of Okun's law, with an average R^2 of 0.6. Once again, this is in line with recent literature, as Obst (2022) suggested. English speaking countries' (Canada, USA and UK) Okun's coefficient estimation is closer to the original finding of Okun, -0.392, -0.472 and 0.389 respectively while, on the other hand, estimations for Spain, Japan and France are slightly further from Okun's estimated values.

To finish, a possible fruitful avenue of future research to find an explanation for the heterogeneity of Okun's coefficient between countries is to refer to the main features of their labour market. It will be interesting to test whether, for instance, if the amount of temporary workers has an impact on the country's Okun coefficient, including variables to capture this effect. Countries with high Okun's coefficients (e.g USA) or lower values (e.g Japan) are specially interesting topics for future research. A further analysis may include different explanatory variables, as the educational features of the country, for instance, which might be able to capture the skilled and unskilled unemployment rate variations. Another possible area of interest for future research may include a sector-based analysis. This could include an examination of the structural change among different industrial sectors, as a reason that may explain the heterogeneous behaviour of Okun's coefficient among countries.

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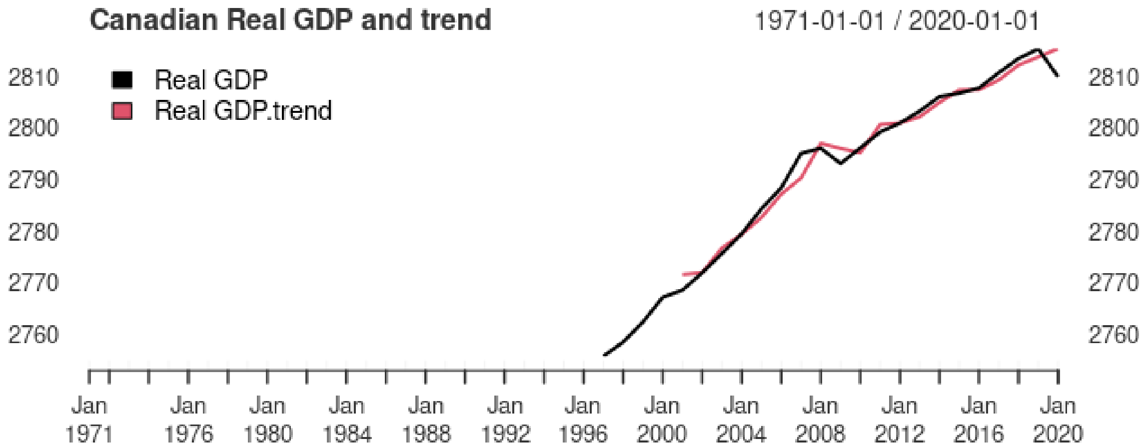
8 Appendix

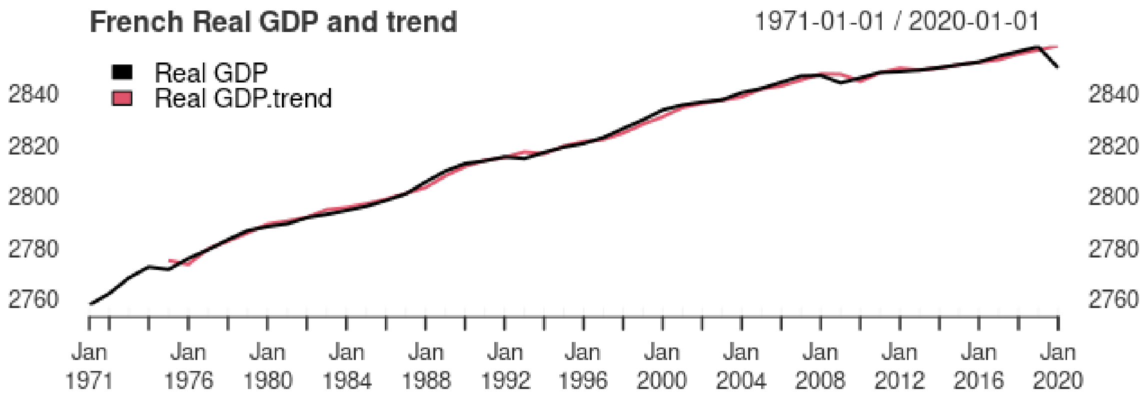
8.1 Appendix A: Hamilton filter GDP

Figure 5: *Canadian GDP Hamilton filer*



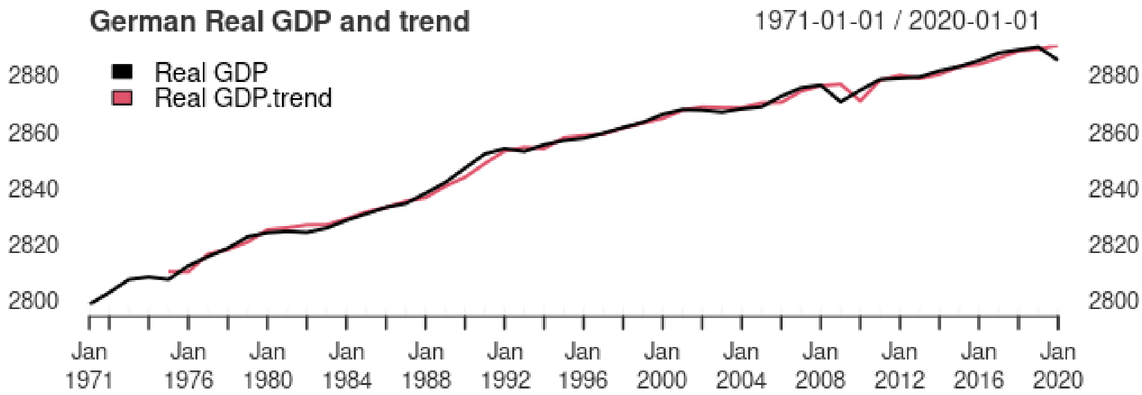
Note: No data available for the first 25 years. Own elaboration.

Figure 6: *French GDP Hamilton filer*



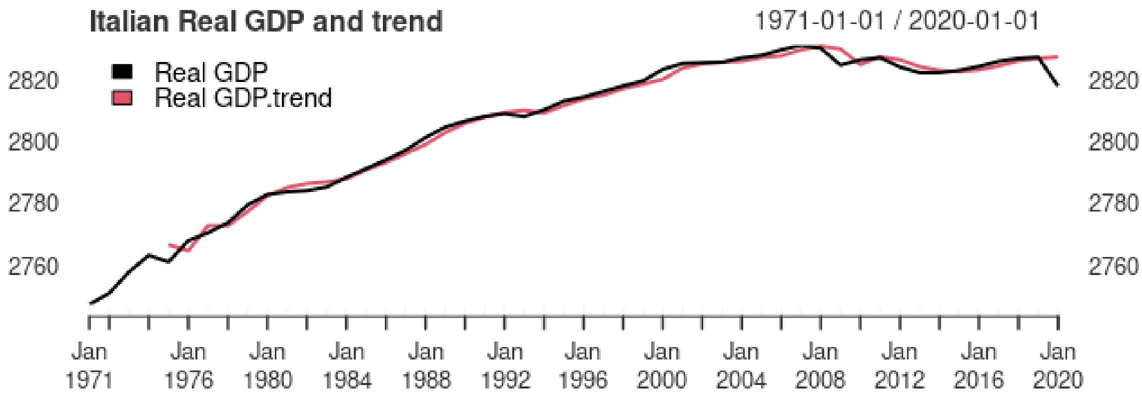
Own elaboration.

Figure 7: *German GDP Hamilton filter*



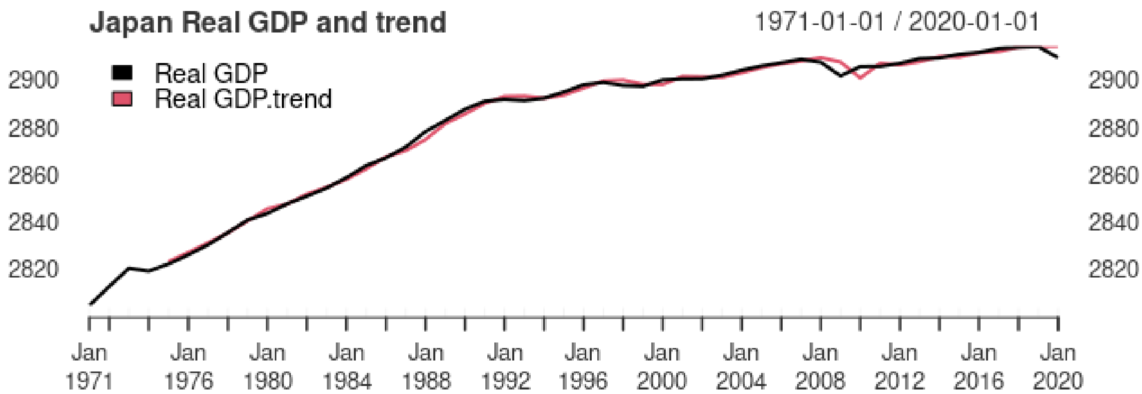
Own elaboration.

Figure 8: *Italian GDP Hamilton filter*



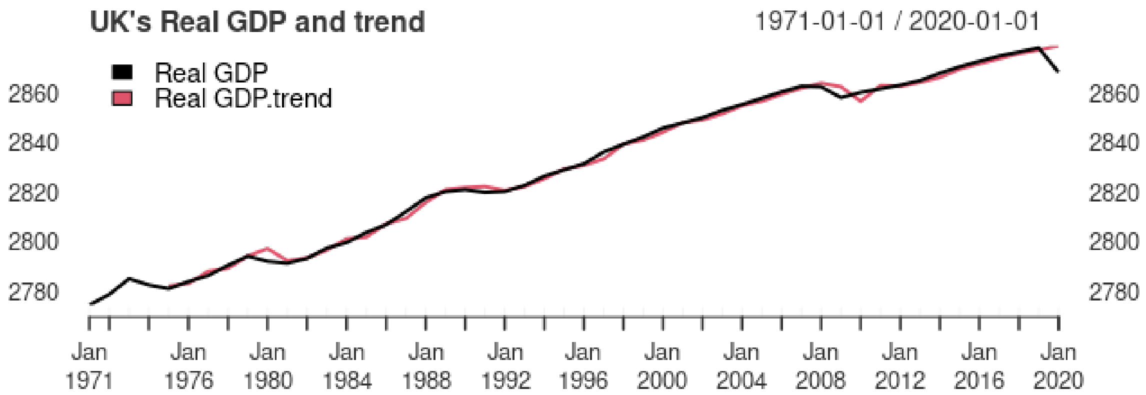
Own elaboration.

Figure 9: *Japanese GDP Hamilton filter*



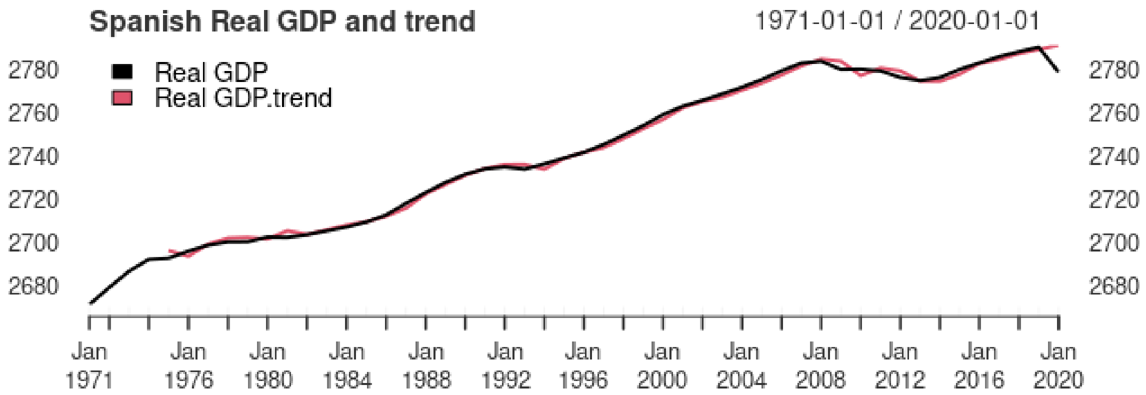
Own elaboration.

Figure 10: *UK's GDP Hamilton filter*



Own elaboration.

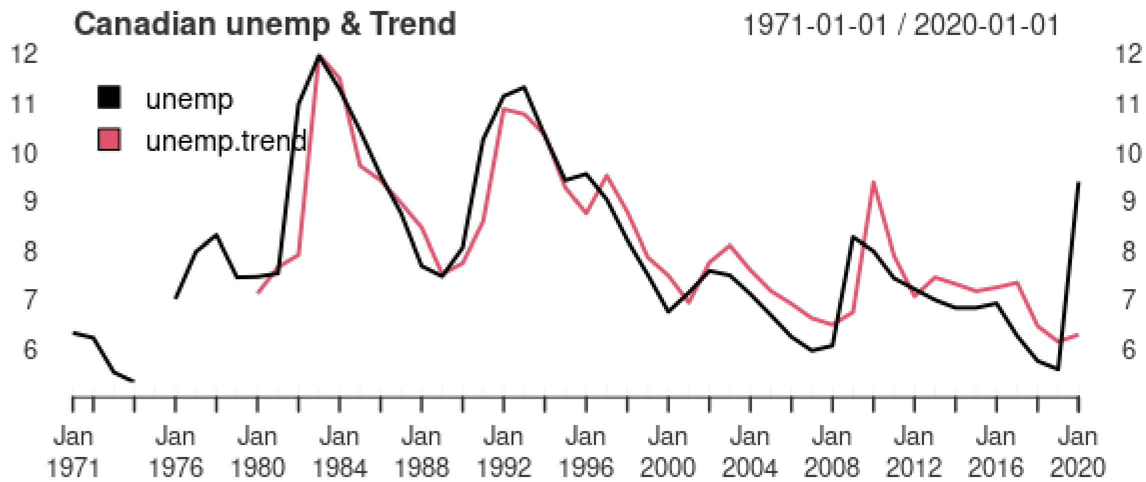
Figure 11: *Spanish GDP Hamilton filter*



Own elaboration.

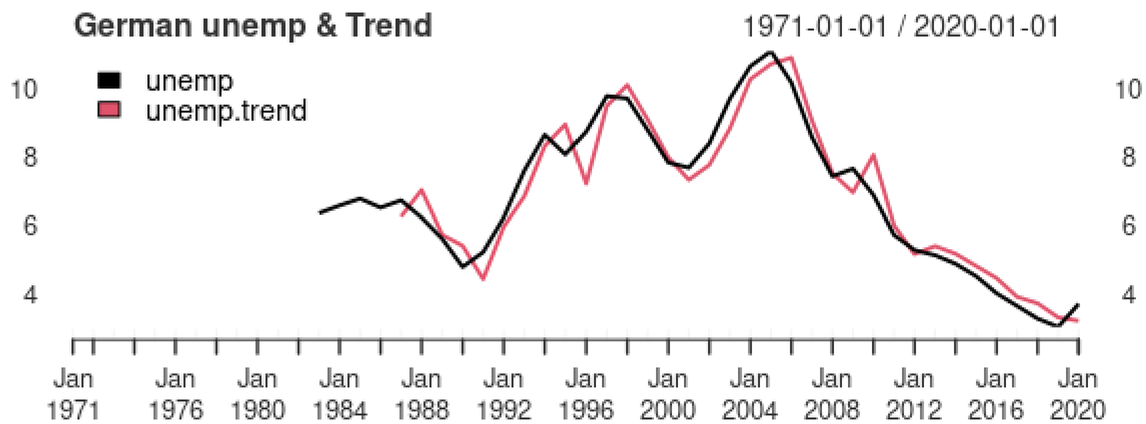
8.2 Appendix B: Hamilton filter Unemployment

Figure 12: *Canadian unemployment Hamilton filter*



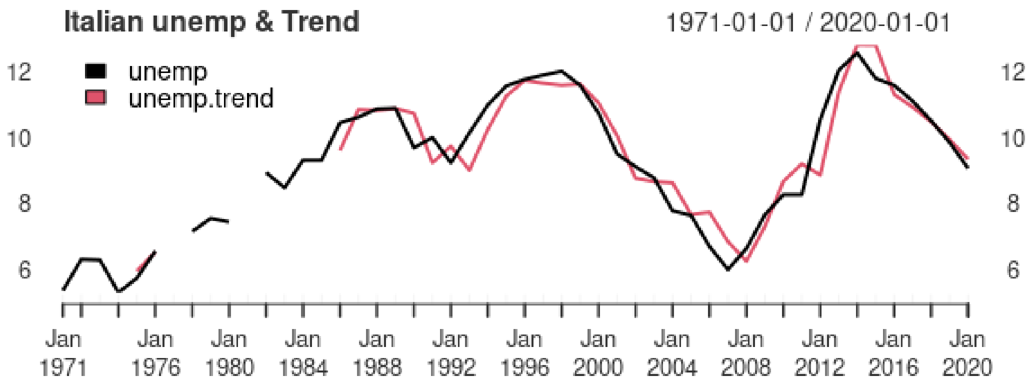
Own elaboration.

Figure 13: *German unemployment Hamilton filter*



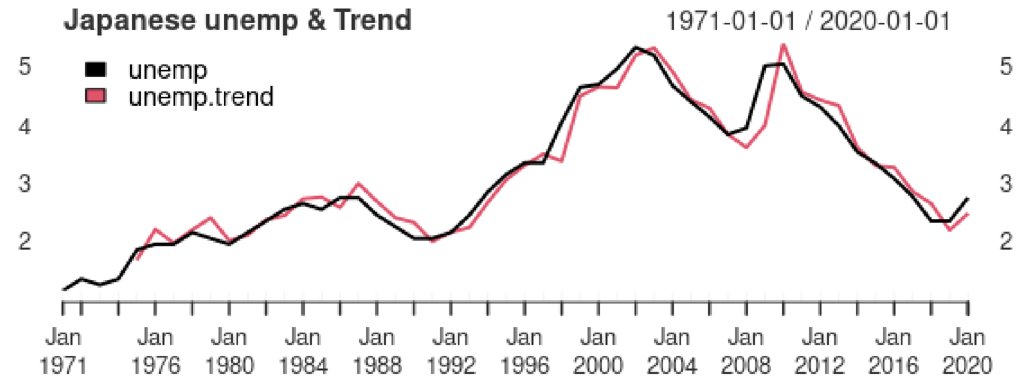
Own elaboration.

Figure 14: *Italian unemployment Hamilton filter*



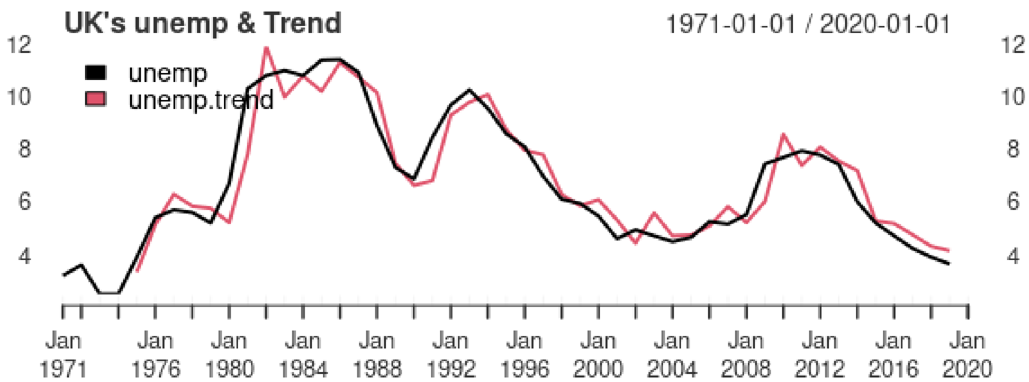
Own elaboration.

Figure 15: *Japanese unemployment Hamilton filter*



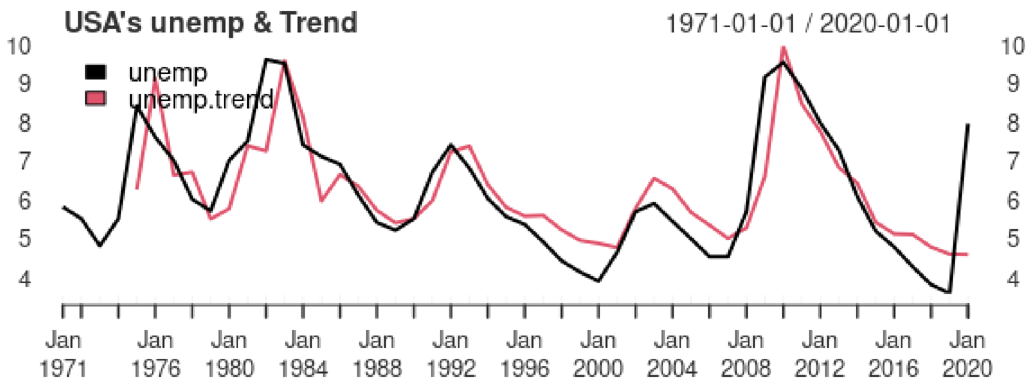
Own elaboration.

Figure 16: *UK's unemployment Hamilton filter*



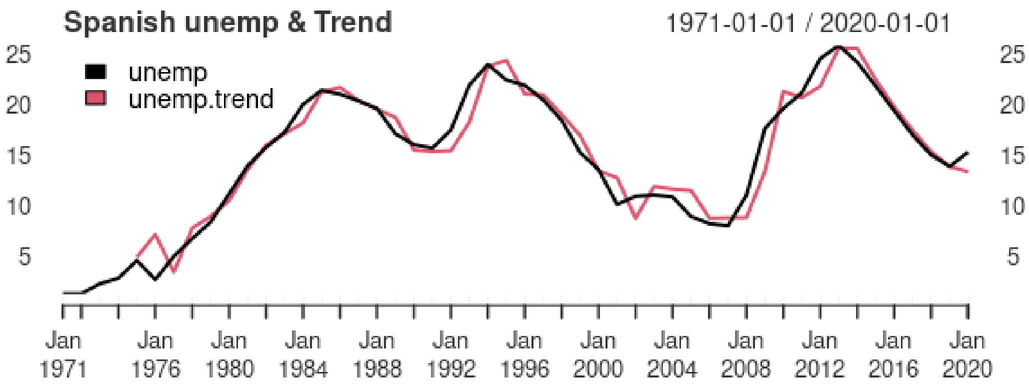
Own elaboration.

Figure 17: *USA's unemployment Hamilton filter*



Own elaboration.

Figure 18: *Spanish unemployment Hamilton filter*



Own elaboration.

8.3 Appendix C: First difference model

Given that the variables included are stationary, it does not make sense to estimate a first difference model, as it will be over-differentiated and its variance will become greater. Despite this fact, below I show the results I obtained:

Table 8: *Okun's first diff. model*

	Canada	**France**	**Germany**	**Italy**	**Japan**	**USA**	**UK**	**Spain**
$\Delta unemployment$	-2.118* (0.064)	-1.603 (0.042)	-1.300* (0.044)	-1.499 (0.055)	-2.724** (0.037)	-1.842.** (0.014)	-1.328* (0.024)	-1.773** (0.017)
Adj R^2	0.325	0.725	0.259	0.224	0.617	0.569	0.379	0.284

Note: Table shows estimated coefficients. Standard error are shown in parenthesis. "***","**" and "*" indicate a significance level of 1%, 5% and 10%, respectively. Own elaboration.