

Robomaths: Robotics to Learn Mathematics in an Architecture Degree

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The abstract part of mathematics is a difficult matter included in many subjects in university degrees. Specifically, in architecture degrees students lack interest in this topic if they don't experience its immediate application. In addition, technological skills are required at every educational level and the students of these degrees are usually more interested in art than in technology. With the aim of encouraging architecture students' interest in mathematics and technology, a methodology is presented here that includes the use of robotics in maths lectures. The key idea is to make the abstract part of mathematics understandable by means of robots.

Keywords: mobile robots, Mathematics, gamification, educational robotics

INTRODUCTION

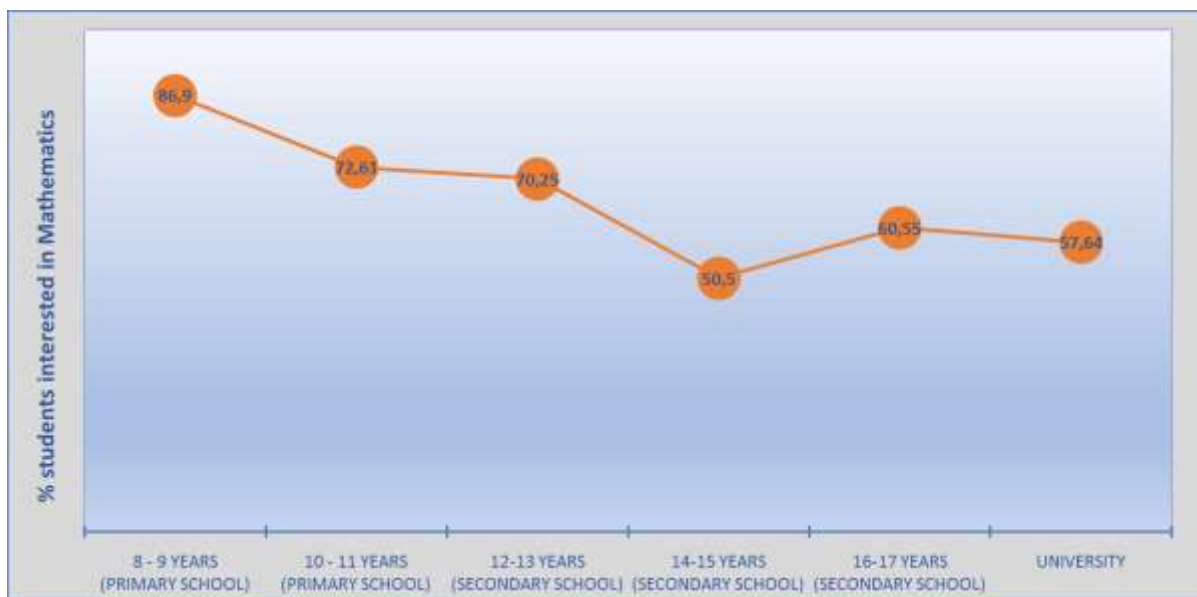
The rapid technological advances in our society introduce a high level of uncertainty in the new generations, since it is not possible to predict the world they will work in. Some works, such as (Gomez, et al., 2019), state that, in a few years, many different competences and technological skills will be required to young workers and, thus they will return to the figure of the Renaissance. Some of the necessary abilities will be directly related to technology, data analysis, business and management, design and innovation. In this sense, mathematical skills are currently being highly demanded by companies, as many of the technological advances in society rely on big data and artificial intelligence.

With this future prospect ahead, the role of university faculty has to be redrawn. A fundamental question arises: how could we provide our students with the best possible training for still non-existent jobs? A possible answer to this question is to provide them with as many competences as possible and to guide them in their learning process so that they can autonomously expand it when necessary. For this purpose, motivation is the key for them to become the true protagonists of the knowledge society.

In this context and regarding mathematical skills, the difficulty associated to the abstract part of mathematics is socially well-known, making it the least loved subject by students. As a matter of fact, some works (Weale, 2019) point out the anxiety suffered by some students when dealing with mathematical topics which leads to a loss of interest in the study of mathematics when children become teenagers, as remarked in (Muñoz, et al., 2015). In this work, different reasons responsible for this situation are analyzed and a proposal is described.

The main reason to be considered in the lack of students' interest in mathematics is the mechanical way of studying mathematical concepts through repetitive and unsubstantiated exercises, often without connection to real problems. A study carried out to 3187 Spanish students in different schools during three academic years (1999-2000, 2000-2001, 2001-2002) shows the progressive loss of interest in mathematics as they access higher educational levels (Hidalgo, et al., 2004) and the evolution of this interest is depicted in Figure 1.

FIGURE 1
EVOLUTION OF THE STUDENTS' INTEREST IN MATHEMATICS AS THEY ACCESS HIGHER EDUCATIONAL LEVELS



(Hidalgo, et al., 2004)

Despite this situation, the demand for math experts has grown exponentially in many jobs due to big data requirements. In fact, math professions are becoming increasingly attractive. If the students' interest does not increase, companies will need many more mathematicians than society is able to provide. Therefore, a solution must be found to reverse this situation and robotics may be the way.

The world is currently changing very fast, much faster than it used to. New technologies are already solving many problems in numerous companies. Some of these solutions involve the use of robots, already present in our daily life, also in the construction field (Davila Delgado et al., 2019). For that reason, the inclusion of educational robotics in official curricula is of main importance.

In this work, a methodology for introducing technological concepts in mathematical subjects of the architecture degree of University Cardenal Herrera-CEU (UCH-CEU, 2020) is proposed. In this sense, robotics has been selected as a means of connecting students with mathematics and technology in a motivating way.

Some works have been developed in elementary school, as in (Leoste & Heidmets, 2020a) where the effects of robot-supported math learning were evaluated when conducted by regular math teachers.

The paper is organized as follows. Firstly, the university context where the innovative approach has taken place is presented. Secondly, the particularities of the methodology and the developed implementations are explained. Finally, conclusions are drawn that can be of use in any other technical discipline.

BACKGROUND

In this section, the university context where the new methodology has been introduced is presented: the university, the degree and the students.

The CEU Group and the ESET

The CEU Group is the largest and most traditional private educational institution in Spain. It comprises 25 educational centres, which include a university, ten schools and other professional education centres. The teaching ranges from childhood education to undergraduate and postgraduate university studies, also including professional education. Annually, around 31,000 students attend classes in CEU classrooms and more than 100.000 professionals have studied at this institution in the past.

The University Cardenal Herrera-CEU (UCH-CEU) is located in Valencia and is made up of three different campuses, Castellon, Valencia and Elche, each located in one of the three provinces of the Valencian region. The objective of the university can be summarized in the following points:

- to guarantee the continuous training of graduates, postgraduates and doctors in the scientific, technological and economic fields, as well as in the social and human sciences,
- to lead fundamental and applied research activities in the scientific and technical fields,
- to contribute to the exploitation of results as well as to the dissemination of scientific and technical information,
- to maintain and extend international cooperation
- to ensure that their graduates are able to permanently adapt to the demands of scientific and industrial society.

UCH-CEU has five faculties or schools:

- Faculty of Law, Business and Political Science
- Faculty of the Humanities and Communication Sciences
- Faculty of Veterinary Medicine
- Faculty of Health Sciences
- Technical School of Design, Architecture and Engineering (ESET)

The ESET School occupies a former industrial building of 3000m². It was funded in 1987 by the CEU Foundation as a technical school for industry workers. The following studies are taught in the ESET:

- Bachelor's Degree in Industrial Design Engineering and Product Development
- Bachelor's Degree in Architecture
- Master's Degree in Interior Design
- Master's Degree in Design and Graphical Communication
- Master's Degree in Product Design

The Bachelor's Degree in Architecture

The most international degree offered by the ESET is the Bachelor's Degree in Architecture, with more than 90% of international students from more than 20 different countries among its freshmen: Norway, Sweden, USA, Zambia, Rwanda, Senegal, Romania, France, South Africa, etc ... This student profile motivates faculty to develop an innovative teaching seeking a learning process adapted to the new generations of students around the world, where the students have a key role in their learning process. The search for new learning methods has inspired the introduction of robotics into the architecture curriculum. The Bachelor's Degree in Architecture is a 5-year degree of 300 ECTS credits distributed in different types of subjects:

1. Core Modules (CM): Compulsory material present in all curricula leading to a specific official degree. These represent the 20% of the subject load.
2. Obligatory Subjects (OB): These are designated by the university as compulsory for the students within the corresponding curriculum. These represent the 76% of the subject load.
3. Optional Subjects (OP): The University establishes these subjects for students to choose from. These represent the 2% of the subject load.
4. Final Degree Project (FDP): It is a compulsory project that must be developed by every student in order demonstrate the knowledge acquired during the studies. It represents the 2% of subject load.

The Mathematics Major

The mathematics major belongs to the fundamental CM subjects and is organized in two different subjects: Mathematics I and Mathematics II. These subjects are taught in the first year of the degree, in the first and second semester, respectively, and their contents are described in Table 1. At the UCH-CEU, these subjects consist of 60 hours of classroom education, of which 30 are lectures and 30 seminars. Mathematics is obviously a fundamental discipline as it is the basis of many other majors taught in the following years of the degree.

TABLE 1
CONTENTS OF MATHEMATICS I AND II IN THE DEGREE IN
ARCHITECTURE AT UCH-CEU

MATHEMATICS I		
Unit	Content	Robotic Seminar?
Unit 1	Vector Spaces. Basis of Linear Algebra, Numerical Linear Algebra.	No
Unit 2	Numbers: Real, Complex, Irrational, Natural and Integer numbers (Golden Number and Fibonacci resonance). Complex numbers in rectangular form.	No
Unit 3	Trigonometry. Hyperbolic trigonometry. Catenary curve. Complex numbers in polar form.	Yes
Unit 4	Conics.	Yes
Unit 5	Parametric curves (Bézier, B-Splines, NURBS). Derivative rules.	Yes
Unit 6	Geometric Patterns. Fractal Geometry.	No
MATHEMATICS II		
Unit	Content	Robotic Seminar?
Unit 1	Techniques of Integration of one variable.	Yes
Unit 2	Differential Equations. Applications.	Yes
Unit 3	Functions of Several Variables: Quadrics. Parametric Surfaces.	Yes
Unit 4	Optimization and Optimization with Constraints.	Yes
Unit 5	Multiple Integration.	No

The students regard Mathematics as a difficult discipline, especially the students from the Degree in Architecture, as their main interests are focused on using their designs to improve people's lives through their projects. They usually work with real projects and objects built by them and, therefore, they value the ability to see and touch what they create. When these types of students face an abstract course like mathematics, they find it hard to understand the importance of the abstract formulation without a real application of each concept. Certainly, they need to find out the connection between the architecture field and mathematics and robotics may be the way.

ROBOMATHS: A PROPOSAL FOR USING ROBOTS IN MATH CLASSES

Considering the abovementioned situation, an excellent option is to use educational robotics to bring mathematics closer to reality. As a result, students will have the opportunity to manipulate and modify abstract concepts, such as algorithms and parametric curves, using robots. This has a two-fold objective: to introduce technology to architecture students and make mathematics more understandable in a fun way. In this sense, lecturers can benefit from the main features of the Z generation: open-minded, innovative, multitasking, impatient and willing to test new tools or methodologies.

The introduction of robotics in education also allows the gamification of some contents. Do not forget that, for the Z generation, the mathematical content should be sold as an attractive, fun product and, hence, the lecturer should stimulate the students' motivation and awaken their interest in learning (Hilario, 2019; Hilario, et al., 2019).

In the previous works (Hilario, 2019) and (Hilario, et al., 2020), the authors linked some of the contents of the Mathematics discipline with robotics concepts. For instance, **parametric curves** (Bézier, B-Splines, NURBS) are used in architecture to design buildings but also in mobile robotics to define smooth paths for the robot to navigate. Also, robot navigation algorithms can provide real explanations of abstract problems such as **optimization**. An in-depth study has now been carried out with the aim of determining the topics susceptible of implementation through practical work with robots and is summarized in

TABLE 1.

Let us describe the direct connection between Math's topics and robotics concepts and implementations. In Mathematics I, taught in the first Semester of the first year, the following units can be complemented with robotics implementations:

- Unit 3. Catenary curve
- Unit 4. Conics sections
- Unit 5. Parametric curves

Architects use catenary curves for designing arcs, conic sections for providing beauty and resistance and parametric curves, such as Bézier, B-Splines, NURBS, etc., are very common in Computer-Aided Geometric Design (CAGD). For instance, the use of circumference allows material savings in walls and enclosures, the parabola provides great resistance for buildings and is widely used in bridge design and the ellipse is common in amphitheaters and stadiums.

The aforementioned curves can be used as reference trajectories for mobile robots to navigate from a start to a goal location. This means that the robot can "draw" or follow these curves if the robot is programmed to do so. Among them, CAGD curves are the most frequently used to design smooth trajectories for mobile robots, as in (Hilario, et al., 2010, 2011).

Let us introduce an example of robot programming with Unit 4: Parametric curves. The idea is to first program the robot to follow these types of trajectories and then study their properties and how parametric curves work.

Let us focus on Bézier curves. In order to represent and graph a Bézier curve, some control points have to be computed. The students could "play" with the control points to experience how their position affects the shape of the curve, in this case being the trajectory of a mobile robot. Furthermore, this methodology would allow the visualization of the mathematical properties of the curves. In addition, Bézier curves present two very interesting properties for robot navigation:

1. They always pass through the first and last control points.
2. The Convex Hull property: A Bézier curve is always completely contained inside of the Convex Hull of the control points.

Both properties are remarkable because the first one assures that the robot begins and finishes at specific points of the path, therefore a start and goal positions can be established. Moreover, the second property controls the area of the plane where the mobile robot is going to follow the trajectory.

Regarding Mathematics II, taught in the second semester, the following units can be supplemented with robotics implementations:

- Unit 1: Techniques of Integration of one variable.
- Unit 2: Differential Equations. Applications.
- Unit 3: Functions of Several Variables: Quadrics. Parametric Surfaces.
- Unit 4: Optimization and Optimization with Constraints.

In these units, the students learn antiderivatives or integrals and optimization problems. Both math tools are useful in the implementation of safe robot path planning algorithms, which entails that the robot can navigate among obstacles without colliding.

Let us describe an example of robotic seminar for unit 3 that also uses the implementations developed in Mathematics I (parametric curves). In unit 3, students learn how to obtain critical points and classify them as functions of several variables. If a function has several stationary points, it might be possible to develop a game that designs a Bézier curve with these solutions in which, if the solutions are wrong, the mobile robot collides with an obstacle. Another possibility is to develop a collaborative game in which each group has to obtain a critical point related to a control point of a Bézier curve. If four groups are involved, a cubic Bézier curve could be designed and serve as a path for a mobile robot. In case any of the solutions is incorrect, the trajectory of the mobile robot will be wrong and the robot will collide. This approach foments collaborative work between classmates.

TOWARDS A USEFUL APP FOR ROBOMATH LECTURES

As indicated in the introduction, technology in the classroom is a milestone. Nevertheless, excessive technological requirements in mathematics classes can pose barriers such as the need for highly specialized and trained teachers or the additional training of students and even the acquisition of expensive technological devices. Therefore, it is essential to develop technological innovations with common products already existing in educational centers and easy to use or inexpensive in order to enable their rapid acquisition by universities and students.

In this work, a technological solution has been developed with the aim of improving the teaching of mathematics and, specifically, the learning of parametric curves mentioned in the preceding section. In the previous works (Hilario, et al., 2010), (Hilario, et al., 2011) and (Montés, et al., 2019) we developed robot navigation techniques based on parametric curves. This implies that the authors have experience in robot programming, very useful for the purpose of this work.

The initial experimental setup, used in (Montés, et al., 2019) and shown in Figure 2, was a LEGO Mindstorms EV3® robot (LEGO, 2020) without sensors, a 2x2 m² white square board located on the floor, a web-camera on the ceiling and connected to a computer via USB and a red disk indicating the target to be reached by the robot.

FIGURE 2
INITIAL EXPERIMENTAL SETUP



This setup, although very simple, has some economic and technological difficulties to be considered:

1. The robot programming was developed using Matlab® and, consequently, a license is required. This is not always a problem as many technical universities already have student licenses.
2. Wi-Fi communication with the robot is required to interact with it in real time. Therefore, a specific Wi-Fi network must be set up through a dedicated router.
3. A camera connected to a computer is required to provide the robot location in the scene, as the robot do not have specific sensors to measure its motion.
4. Specific lighting conditions are required for the application to work properly.

With the aim of avoiding some of these limitations, a redesign of the experimental setup has been performed in order to maintain only the features required by Math lessons.

First, the robot control has been developed in open loop, thus avoiding the use of a camera and the specific lighting conditions. This type of control would not be useful in a real navigation application because the robot would not be able to accurately follow the trajectory, but it is enough to demonstrate the possibility of a robot to follow a parametric curve in a time limited laboratory session.

Second, without a camera acquiring images, the need for a white floor is no longer a requirement, allowing the implementation of the application in any classroom with any type of floor.

Third, any cell phone can be used as a router because they have the ability to creating a Wi-Fi network to connect any device. Nowadays, every student has a cell phone and the acquisition of the device is also avoided.

Finally, the code developed in Matlab can be compiled into an executable program that can be run on any computer without the need for a license.

Therefore, any student can use the Robomath application with the sole need for a LEGO robot, a computer and a cell phone, as shown in Figure 3. In this sense, new educational activities can be introduced into Math instruction that would be impossible to carry out with the initial setup, such as taking the robot home for individual hands-on exercises or group assignments.

Additionally, a website can be developed with the course details combining mathematics and robotics education. This type of website will allow students to simulate the robot's path for students to test it in class and at home.

FIGURE 3
NEW SETUP: ROBOT, COMPUTER AND CELL PHONE



DISCUSSION

The proposed implementations allow the use of robotics technology in the classroom with minimal students and teacher training, offering lecturers the easy integration of robotics into Math classes. Moreover, the new teaching strategy gives new Z-generation students the opportunity to interact with new and abstract knowledge by introducing gamification into the classroom, making learning mathematics more attractive and motivating. The preliminary introduction of this proposal has yielded encouraging results among students.

The presented approach has been possible to develop because the authors belong to a multidisciplinary research group, including mathematicians, engineers and computer scientists. Their main research field is robot navigation and they have developed algorithms for mobile robot navigation based on parametric curves of Computer Aided Geometric Design (CAGD), such as Bézier, B-Splines or NURBS in (Hilario, et al., 2010, 2011). These algorithms allow the avoidance of dynamic obstacles and are computed using optimization problems with constraints. Therefore, the connection of the authors' research field with the math teaching has been straightforward.

As a result, an issue arises: innovative and technological teaching is only possible if mathematics lecturers have technological training? Do lecturers have to acquire advanced technological skills or should teaching methodologies be developed by lecturers in multidisciplinary teams? Is collaborative teaching a real possibility? The answer to these questions is yet to come but initial studies have already been done in elementary school, as in (Leoste & Heidmets, 2020b), where it was found that math teachers should be accompanied by an educational technologist with basic knowledge regarding the topic taught.

In the end, robotics education becomes a powerful tool to develop technological and mathematical abilities. However, difficulties exist as an open-minded vision is required as well as the will to change the teaching methodologies, since it is not easy to get out of the comfort zone. In spite of that, if lecturers observe and question their own methodologies, they will discover a new range of possibilities to make teaching more attractive and understandable.

With this type of applications, options are opened to use mobile robots in other kind of courses or subjects. For instance, in the architecture degree, subjects like physics, descriptive geometry and drawing could carry out similar developments. A thorough analysis and classification of the contents should be performed in order to develop proper robotic applications.

CONCLUSIONS AND FUTURE WORK

This work presents a change of teaching methodologies in Mathematics education at university. Considering that education for the XXI century has to provide technological skills for the new generations

of students, the introduction of technology in abstract subjects, such as Mathematics, is key for this competence acquisition.

In this sense, a new approach is proposed here for experiencing mathematical concepts through robotics applications in the Degree in Architecture at University Cardenal-Herrera CEU. An application has been developed in Matlab for making a mobile robot follow a parametric curve.

The future work includes an extensive use in mathematics subjects and the evaluation of the students' motivation and the learning before and after the introduction of robotics in teaching in order to validate the proposal. This would likely result in an increase in the students' motivation and grading regarding difficult and technical contents. Also there exist the possibility of extending the approach to other subjects. Finally, new implementations could be also developed in open-source programs, such as Octave instead of Matlab.

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