SELECTION OF PORTFOLIOS THAT MINIMISE THE VARIANCE OF THE ESTIMATED BETA FOR EFFICIENT PORTFOLIOS IN MARKOWITZ'S SENSE.

UNIVERSITAT JAUME I

DEGREE IN FINANCE AND ACCOUNTING
2018 / 2019

MARTA FERRANDO TRAVER

al285144@uji.es

TUTOR: Alejandro José Barrachina Monfort

The work that we present below is the practical application of the model proposed by McInish et al. (1984), where the main objective is to find a portfolio of risky assets – formed by individual asset weights – that minimizes the variance of its estimated beta with respect to market performance. In this paper we have chosen, as target betas, the estimated betas of twelve efficient portfolios in the sense of Markowitz formed by seven actions of the IBEX 35, which has allowed us to compare the results provided by both models. The main conclusion of the work is that, for the considered data, both models coincide in their results, that is to say that the efficient portfolios in the sense of Markowitz minimize the variance of their estimated beta with the market, starting from a beta around 1.3.

Keywords:

- CAPM.
- Mean-variance approach.
- Portfolio optimization.
- Variance of the Beta.

JEL Codes: G11

INDEX

Tables, figures and graphs.

1. INTRODUCTION.

Throughout history several models have been define don the Theory of Portfolios, but the approach that Harry Markowitz provided in an article called Portfolio Selection and published in 1952, revolutionized the world of finances and is still used today.

This model considers the performance variance of an asset as a measure of risk (volatility), in such a way that the risk of a portfolio will be determined by the variances and covariances of the financial yields of the assets that form it.

According to Markowitz, the optimal portfolio of a rational and risk-averse investor must be efficient, that is, it must minimize the risk given the expected profitability. The set of efficient portfolios will form what is known as the Markowitz Efficient Frontier and this will be made up of those portfolios that minimize the risk for each possible level of profitability expected.

Following de Markowitz model line, the Capital Asset Pricing Model (CAPM) was introduced by Sharpe (1964), Lintner (1965) and Mossin (1966), and it is based on the work of Harry Markowitz on diversification and the Theory of Portfolios. Sharpe was awarded, along with Harry Markowitz and Merton Miller, with the Nobel Prize in Economics in the year 1990.

This model also assumes that investors involved in a competitive stock market are averse to risk and behave in a rational way. In the same way as the Markowitz model, the CAPM also considers that we can measure the risk of assets through the variance of their financial yields. Another assumption of the model is that investors involved in the stock market are able to minimize the specific risk of their investments (non-systematic risk) by creating a sufficiently diversified portfolio.

The CAPM model considers that, in equilibrium, the expected performance of any asset or portfolio of assets quoted on the market, must meet the following relationship:

$$E(R_i) = R_f + \beta_i(E(R_M) - R_f)$$

Where:

- $E(R_i)$: Expected profitability of a particular asset.
- R_f : Profitability of the risk-free asset.
- β_i : Measurement of the sensitivity of the asset to systematic risk (non-diversifying)
- $E(R_M)$: Expected profitability on the market in which the asset is listed.

The above expression is equivalent to:

$$E(R_i) - R_f = \beta_i(E(R_M) - R_f)$$

where

- $[E(R_i) - R_f]$: Risk premium associated with the asset.
- $[E(R_M) - R_f]$: Risk premium associated with the market in which the asset is quoted.

In other words, according to the CAPM, in equilibrium, the risk associated with the asset is directly related to the average market risk, weighted by the degree of sensitivity of the asset with respect to the systematic risk (non-diversifying), which is measured by the parameter beta.

If beta were zero, the expected profitability of the asset would be equal to the profitability of the risk-free asset. If beta were equal to 1. The expected profitability of the asset would be equal to the expected profitability of the market. If beta is less than 1 would mean that the systematic risk of the asset is greater than the average risk of the market, and on conversely, if beta is greater than 1 would indicate that the risk of the asset would be less than the risk of the market.

In other words, the greater the degree to which the asset is affected by the systematic risk, the investors of this asset are compensated with a higher expected profitability. However, according to the CAPM, investors are not compensated for the non-systematic risk of the asset because the model assumes that these investors have been able to eliminate this risk through diversification.

According to the CAPM, the equilibrium expression of the beta of an asset is:

$$\beta_i = \frac{Cov\ (R_i, R_M)}{Var\ (R_M)}$$

The previous expression tells us that the beta of an asset or a portfolio is the estimation of the slope of a regression line between the financial yield of the market and the performance of the asset or portfolio. Therefore, as an estimator of the slope of a regression line, the variance can be calculated and this would be the origin of the proposal of McInish et al. (1984), (Portfolio selection to achieve a target beta); to form portfolios that minimize the variance of your beta with the market.

Therefore, the objective of this paper is to compare the efficient portfolios in the sense of Markowitz, which will have a certain estimated beta with the market, with the portfolios that will minimize the variance of the mentioned estimated beta.

2. DATA.

There are different types of assets according to the different types of companies: on the one hand we have the *blue chips*, which refer to actions in large-capitalization companies and leaders in their sector. On the other hand, the actions called *penny stocks* belong to small companies, with a high risk type and very volatile. We also have the *utilities*, these are shares in service companies within very regulated sectors, are counter-cyclical companies and their income tends to be quite stable. Finally, we could also distinguish between *sustained-growth* actions and *high-growth* actions.

In our case we have come to the historical data of the actions provided by website *invertia* and we have chosen seven companies belonging to the IBEX 35. In addition, as an indicator of the evolution of the market, we have chosen the data of the IBEX 35. The period of time that we will analyze are the contributions that there have been for two years, from January 2nd, 2014 until December 31^{rt}, 2015 (data obtained from the website *invertia*). The analysis will be carried out with daily data, that is, a total of 512 data for each of the selected companies and for the market evolution index. Below we will detail the seven selected companies:

- **Acciona S.A.** is a Spanish company dedicated to the promotion and management of infrastructure and renewable energies. This company operates within a fairly regulated sector and could report dividends on a regular basis.

- **Gas Natural SDG, S.A.** is a company dedicated to the generation, marketing and distribution of natural gas and electricity. It is a public services company and we believe that its actions are part of the group of *utilities*, it is consolidated within a very regulated sector and the income that could be provided could be quite stable and with a high probability of receiving dividends.

- **Iberdrola S.A.** is a company dedicated to energy production, distribution and commercialization. Just as the previous case, this company operates in a very regulated sector and the benefits of its shares could be quite stable.

- **Mediaset España Comunicación S.A.** is a communication group dedicated to the production and display of television content. It is the only company that we have of this sector, in this way we can diversify in several sectors and not focus our attention just in one.

- **Grupo ACS S.A.** is a company dedicated to the construction industry and the development of infrastructures, both civil and industrial. This is a diversified company with participations in other companies of world importance as Hochtief.

- **Melia Hotels International S.A.** is a Spanish hotel company and we have chosen these actions because, as it has a different activity from the previous ones, we can diversify in different sectors.

- **Gamesa Corporación Tecnológica S.A.** is a company dedicated to the manufacture, sale and installation wind turbines. It is one of the leading manufacturers of wind turbines in the world, and manufacturer of new technologies applied to emerging activities such as robotics, that is why we believe that we can report benefits.

With the historical quotations of the assets explained above, we calculate the logarithmic yields for the period considered by this expression:

$$Performance\ (X_i) = ln\left(\frac{quotation_1}{quotation_{n-1}}\right)$$

Subsequently, we calculate the average of the yields calculated above and we obtain the variance of each asset with respect to the IBEX 35. The results obtained are as follows:

	ACCIONA	GAS NATURAL	IBERDROLA	MEDIASET	ACS	MELIA HOTELS	SIEMENS
E(Yi)	0,124932%	0,001252%	0,067677%	0,034963%	0,015013%	0,052060%	0,143985%
Variance	0,037235%	0,017662%	0,011479%	0,032136%	0,028221%	0,023638%	0,065928%

Table 1: Average expected yield and variance of individual assets.

At the same time, as an indicator of the evolution of the market we have chosen the stock index of IBEX 35 and we have chosen the same period of time to study as for stocks. In this case we have also calculated the logarithmic performance of the daily variations for the period considered. Subsequently, with these data and with the daily logarithmic yields of the quotes of the assets calculated above, we have calculated the slope of the regression line between each asset with respect to the market, and in this way we have obtained the estimated beta of each asset:

β	ACCIONA	GAS NATURAL FENOSA	IBERDROLA	MEDIASET ESPAÑA	ACS	MELIA HOTELS	SIEMENS GAMESA
	1,125352	0,766820	0,685511	0,915614	1,045371	0,637989	1,375386

Table 2: Estimated beta of each asset with respect to the market.

On the other hand, as a risk-free asset we have chosen a Spanish bond at 20 years. With daily quotes from August 1st, 2015 to December 31st, 2015 (data obtained from the website *investing*) we have calculated the average daily logarithmic yield and is of 0.07%.

3. METHODOLOGY.

To carry out the comparison between the efficient portfolios in the sense of Markowitz, each one of them with a certain beta with the market, and the portfolios that would minimize the variance of this estimated beta. We have followed a process formed by four stages that we are going to develop below:

In the first stage we will focus in calculating the efficient portfolios in the sense of Markowitz, with the seven assets considered in the previous section. These efficient portfolios will be those that give us a higher expected profitability with a known level of risk, measured by the variance of portfolio yields.

In the second stage we will estimate the betas of each of the previous portfolios with the market. These betas will result from the sum of the optimal weights of the assets in each portfolio weighted by the beta of each asset, remember that the beta of each asset was the slope of the regression line between the daily yields of each asset and the daily market yields. The results of the beta of each asset can be observed at the end of point 2 in Table 2.

In the third stage we will calculate the portfolios that minimize the variance of the estimated betas in the previous section and, to do this, we will apply the model proposed by McInish et al. (1984).

Finally, in the fourth stage we will compare the weights obtained in the efficient portfolios in the sense of Markowitz, calculated in the first stage, with the weights obtained in the portfolios that minimize the variance of the corresponding estimated beta, calculated in the thirds stage.

In the next three subsections we will detail the first stages of the process. We will develop the fourth stage in point number 4 in a more extensive way.

3.1. Efficient portfolios in the sense of Markowitz.

This first stage consists in obtaining the efficient portfolios in the sense of Markowitz. Each of these portfolios consists of the seven assets described in point 2. We will calculate the optimal weights of each asset for a given risk that maximize the expected performance and in total we will obtain twelve efficient portfolios, having studied a risk that varies between a range of [0.0105%, 0.065%]. In Table 3 we can observe the optimal weights of each asset in each of the twelve efficient portfolios calculated:

	PORTFOLIO 1	PORTFOLIO 2	PORTFOLIO 3	PORTFOLIO 4	PORTFOLIO 5	PORTFOLIO 6	PORTFOLIO 7	PORTFOLIO 8	PORTFOLIO 9	PORTFOLIO 10	PORTFOLIO 11	PORTFOLIO 12
ACCIONA	0,02%	23,43%	37,21%	47,58%	56,24%	63,80%	50,56%	36,16%	25,34%	12,70%	8,35%	1,20%
GAS NATURAL	10,32%	0,01%	0,01%	0,01%	0,01%	0,01%	0,01%	0,01%	0,01%	0,01%	0,01%	0,01%
IBERDROLA	68,61%	64,84%	44,93%	28,80%	15,41%	3,72%	0,01%	0,01%	0,01%	0,01%	0,01%	0,01%
MEDIASET	2,60%	0,01%	0,01%	0,01%	0,01%	0,01%	0,01%	0,01%	0,01%	0,01%	0,01%	0,01%
ACS	0,01%	0,01%	0,01%	0,01%	0,01%	0,01%	0,01%	0,01%	0,01%	0,01%	0,01%	0,01%
MELIA HOTELS	18,43%	1,65%	0,01%	0,01%	0,01%	0,01%	0,01%	0,01%	0,01%	0,01%	0,01%	0,01%
SIEMENS	0,02%	10,05%	17,82%	23,59%	28,31%	32,44%	49,39%	63,79%	74,61%	87,25%	91,60%	98,75%
σ^2_p	0,0105%	0,015%	0,020%	0,025%	0,030%	0,035%	0,040%	0,045%	0,050%	0,055%	0,060%	0,065%

Table 3: Efficient portfolios in the sense of Markowitz.

To obtain these efficient portfolios in the sense of Markowitz, the Excel application spreadsheets and its Solver tool, which allows to solve complex optimization problems. The details of the procedure to obtain the efficient portfolios in the sense of Markowitz are generally known and this is not a relevant aspect of this work. A detail that is important, in order to understand the results obtained throughout this work, is that one of the restrictions established when calculating the efficient portfolios is that the weightings of the assets in each portfolio are, at least 0.01%, to avoid possible errors in the calculations.

As we can see in table number 3, in ACS we would only invest the minimum set as a restriction, regardless of the risk we face. In Gas Natural and Mediaset is the same as in ACS, except that when the risk is minimal (0.0105%) we would dedicate some of the budget to these two assets, without exceeding 20%. In Melia Hotels the same thing happens as in the previous case, except that in this asset we would stop investing when the risk exceeds 0.015%, from portfolio 2.

On the contrary, in Siemens when the risk is the minimum studied, its weighting is also the minimum but is increasing as the risk is rising until it reaches portfolio 12 with a risk equal to 0.065% and a weighting of 98.75%, almost the entire budget. With respect to Iberdrola, the opposite is the case in Siemens, the weighting decreases as the risk increases, starting from portfolio 7, with a risk equal to 0.04%, the weighting of this asset becomes the minimum. Finally, Acciona behaves in a striking way because its weighting is increasing until reaching its maximum in portfolio number 6, with a risk equal to 0.035%, where it begins to decrease until reaching 1.2% in the portfolio 12, that is, its evolution has a concave shape.

By way of summary, we can emphasize that, according to the Markowitz model, when the risk exceeds 0.04%, we should only dedicate our budget to investing in Acciona and in Siemens Gamesa.

3.2. Estimation of the beta of the efficient portfolios in the sense of Markowitz.

This stage consists on estimating the beta of each of the previous portfolios and, for that, we have calculated, first of all, the beta of each asset, whose results can be seen in Table 2.

The beta of each asset (β_i) is the slope of the regression line between the average of the daily performance of the asset, R_i, and the average of the daily performance series of the market, R_M. As mentioned above, as an indicator of the evolution of market performance we have chosen the series of daily variations of the IBEX 35.

The beta of an asset portfolio (β_C) will be calculated as the weighted sum of the beta of each asset by its optimal weighting within each portfolio. In our case, we have calculated the betas of each asset and the efficient portfolios in the sense of Markowitz, formed by seven assets, therefore, the beta of each of these portfolios will be defined by:

$$\beta_c = w_1\beta_1 + w_2\beta_2 + w_3\beta_3 + \cdots + w_k\beta_k$$

Where:

- β_i: Is the beta of each asset in the portfolio.
- w_i: Is the asset weighting within the portfolio.

Complying:

$$\sum_{i=1}^{n} w_i = 1$$

$$w_i \geq 0{,}0001 \;\; ; \;\; i = 1,2,\dots,n$$

In other words, the beta of each efficient portfolio that we have obtained in the previous section results from the product between the optimal weighting of each asset in each of the portfolios and the slope that exists between: the series of daily performances of the asset and the series of daily yields of the market, the latter is represented by the IBEX 35.

Below we present a summary table and a graph- represented by Figure 1- in which the beta of each portfolio obtained in this section is related to the expected performance.

SUMMARY TABLE		
	β Markowitz	Expected yield
P_1	0,6913618	0,0571%
P_2	0,8571554	0,0885%
P_3	0,9721753	0,1026%
P_4	1,0575478	0,1129%
P_5	1,1282470	0,1215%
P_6	1,1899778	0,1289%
P_7	1,2486902	0,1343%
P_8	1,2847025	0,1370%
P_9	1,3117565	0,1391%
P_{10}	1,3433399	0,1415%
P_{11}	1,3542205	0,1423%
P_{12}	1,3721103	0,1437%

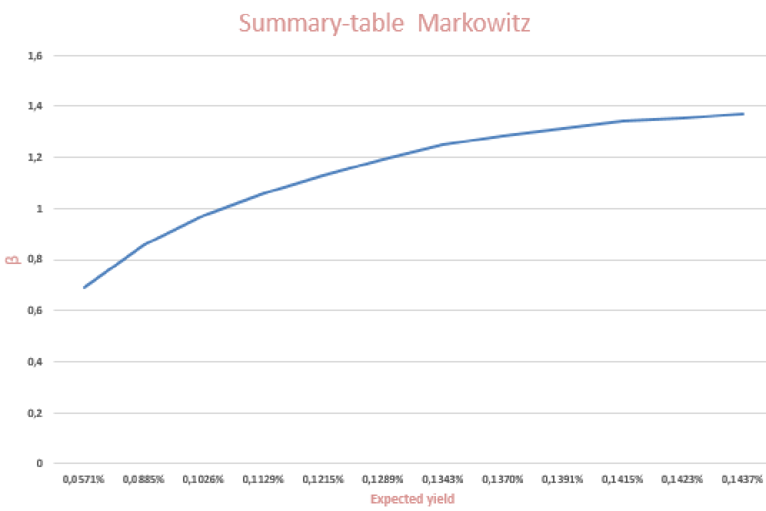


Figure 1: Relationship between the beta of each portfolio and the expected yield.

As can see in the Figure 1, the twelve efficient portfolios in the sense of Markowitz obtained in the previous section fulfil the intuitive idea posed by, for example, the CAPM, as we have told in the introduction. In other words, the idea that the grater the beta of an asset or a portfolio, the greater the expected performance of that asset or portfolio. Since the target beta measures the degree to which the systematic risk affects the asset, the higher this beta, the higher the expected performance of the asset to compensate for the increased risk supported.

3.3. Portfolios that minimize the variance of the estimated beta.

The main objective of this stage, where we develop the basic idea of this work, is to obtain the portfolios that minimize the variance of the estimated betas calculated in the previous section and that we can see in Figure 1. In other words, we seek to minimize the variance of the estimator of the linear regression slope, which is a random variable.

To do this, we will base on the model posed by McInish et al. (1984). First, we will explain the theoretical basis of the model and, then, we will detail all the calculations that we have carries out to apply the model.

3.3.1. Theoretical explanation of the model.

The main objective of the model posed by McInish et al. (1984) is, for a given beta, find the portfolio (defined by the weights of the different assets that form it) that minimizes the variance of this beta. Since the beta of an asset is the slope of the regression line between asset yields and market yields, it is to find the portfolio that minimizes the estimator of the slope of this linear regression, which is a random variable.

Therefore, we are operating in the environment of the Simple Linear Regression Econometric Model, whose parameters (constant term and pending) can be estimated by Ordinary Least Squares (OLS), estimation method that is welcome to the Gauss-Markov Theorem.

This theorem is based on four assumptions which ensure the unbiasedness of the estimators by OLS of the parameters (constant term and pending) of the Simple Linear Regression Model. In particular, the OLS estimators are unbiased estimators of the population parameters, that is, the mean of the density distribution of the estimator coincides with the true value of the parameter.

These four assumptions that ensure the unbiasedness of the estimators by OLS are the following:

(1) Linearity in the parameters: the population regression function is linear in the population parameters ($\beta_0, \beta_1, \ldots, \beta_k$).

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + u$$

(2) Random sampling: at the time of obtaining the population we should select only a random sample of *n* observations.

$$\{(x_{1i}, x_{2i}, \ldots, x_{nj,} y_i) : i = 1,2,\ldots,n\}$$

For a randomly extracted observation of the population we have to:

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_n x_{in} + u_i$$

(3) Null conditioned mean: the mean value of *u* does not depend on the value that the explanatory variables take.

$$E\ (u|x_1, x_2, \dots, x_n) = 0$$

Caution must be taken with this assumption as it may fail in case the functional relationship between the explained variable and the explanatory variable is not properly specified and/or if we omit relevant explanatory factors (included in *u*) that are correlated with the explanatory variable.

(4) Sample variation in the independent variable: in the sample, the explanatory variable should take different values, it should not be equal to a constant.

In summary, and under the assumptions 1 to 4, it can be demonstrated the unbiasedness of the estimators by OLS of the parameters of the Simple Linear Regression Model. That is, it can be shown that:

Unbiasedness: $E\left(\widehat{\beta_j}\right) = \beta_j \quad ; \quad j = 0, 1, \dots, k$

So far, we have focused on the hope of estimators by OLS of the parameters of the Simple Linear Regression Model. The fifth of the Gauss-Markov Theorem is important with respect to the variance or standard deviation of these estimators by OLS, that is, to the extent of the dispersion of its sample distribution. The variance or standard deviation of an estimator measures its accuracy.

(5) Homoscedasticity: The conditional variance of the error term (*u*) does not depend on the values that the explanatory variable takes. That is, σ^2 is an unknown constant.

$$var\ (u|x_1, \dots, x_n) = \sigma^2$$
$$var\ (y|x_1, \dots, x_n) = \sigma^2$$

For all possible combinations of values of the explanatory variable, the variance of the error term is the same.

In the present study, what interests us is the variance of the OLS estimator of the slope of the Simple Linear Regression Model, $var\ (\widehat{\beta_1})$. Under the assumptions 1 to 5 (Gauss-Markov) it can be shown that:

$$var\ (\widehat{\beta_1}) = \frac{\sigma^2}{\sum(x_{1i} - \overline{x})^2}$$

- σ^2: is the variance of the error term.
- $\bar{x}$: is the average of the series of the variable x_1.

3.3.2. Calculations.

In this stage, we will explain the steps followed to elaborate in Excel the template where we will calculate the portfolios that minimize the variance of the estimated beta for the optimal portfolios in the sense of Markowitz obtained in the previous section, all using de Solver tool.

(1) We reserve a column for the daily performance series of the IBEX 35 that we call R_M. The formula that we will follow is the following:

$$Performance\ (IBEX) = \ln\left(\frac{quotation_1}{quotation_{n-1}}\right)$$

(2) Another of the columns will be destined to the daily performance of each portfolio formed by the seven assets considered.

$$Performance\ (X_1) = \ln\left(\frac{quotation_1}{quotation_{n-1}}\right)$$

Having the series of yields for each of the individual assets {n = 1,2,…,7} and, fixing in Excel the cells that are going to define the weights of each of these assets (W_i) in the different portfolios, we can calculate the series of daily returns of each possible portfolio (R_c) by adding the daily returns of the assets weighted by their weight in each possible portfolio.

$$R_{ci} = w_1 R_{1i} + w_2 R_{2i} + w_3 R_{3i} + w_4 R_{4i} + w_5 R_{5i} + w_6 R_{6i} + w_7 R_{7i}$$

Therefore, the expected performance of the portfolio will vary as the asset weights are changed in each possible portfolio.

(3) For each daily performance of the IBEX 35, and each daily performance of the portfolio, we subtract the yield of the risk-free asset (R_f), in such a way that we will already have the two series on which we must carry out the linear regression:

$$X = R_M - R_f$$

$$Y = R_c - R_f$$

Here you have to take into account that the estimator of the slope of the linear regression between two variables will not vary if these two variables are subtracted a constant (which in our case will no vary over time). This is why, so far, we were referring to the beta of an asset or portfolio as the slope of linear regression between its yields and market yields. However, at this point in the work exhibition, we must specify that, as we will discuss later, the variance of the estimator of this slope depends on the errors of the estimated regression line. For the calculation of which it is necessary the estimator of the constant term of linear regression, which does vary when the variables are subtracted a constant like this. Therefore, it is necessary to estimate between the series of daily yields of the portfolio and the market, subtracting the return on risk-free assets, which are the two variables related to the CAPM model.

(4) Knowing the values that take *X* and *Y*, we can define in a cell of our template the Excel formula that calculates the constant term (*alpha*) of the linear regression between *X* and *Y*:

$$\alpha_{portfolio} = INTERCEPT\ (X_1, X_2, \dots, X_n; Y_1, Y_{2,} \dots, Y_n)$$

And in another cell define the Excel formula that calculates the slope (*beta*) of the linear regression between these two variables:

$$\beta_{portfolio} = SLOPE\ (X_1, X_2, \dots, X_n; Y_1, Y_{2,} \dots, Y_n)$$

(5) Then, in another cell, we define the variance of the estimated beta as:

$$var\ (\widehat{\beta_c}) = \frac{\sigma^2}{\sum(x_i - \overline{x})^2}$$

Where σ^2 is the variance of the error term *u* and $\bar{x}$ is the mean of the series of the variable *X*.

To estimate the numerator of the previous formula, which is the variance of the error term (σ^2), we use its unbiased estimator:

$$\widehat{\sigma^2} = \frac{\sum_{i=1}^{n} \widehat{u_i^2}}{n-2}$$

$$u = y - \hat{y}$$

Where Y is the real value that the variables takes, and $\hat{Y}$ is the value estimated by linear regression, that is, $u = y - (\alpha + \beta * X_i)$. Finally, *n* is the number of observations.

Therefore, in a column of the Excel template, we calculate the daily error of the estimate squared. To know the value it takes $\widehat{\sigma^2}$ we will add the whole series of squared errors and divide it between $n - 2$.

On the other hand, to calculate the denominator of the formula of $var\ (\widehat{\beta_c})$ we create a column in which we calculate $(x - \bar{x})^2$ for each daily observation of the variable X.

(6) We already have well defined, on the one hand the cell with the estimated beta of linear regression between X and Y, and on the other hand, the cell with the variance of this estimated beta. Therefore, we can proceed to work with the Excel Solver tool the problem of minimizing the variance of the estimated beta of a portfolio regarding the weightings of the assets that will define each portfolio.

We program Solver to minimize the variance of the beta (calculated in the corresponding Excel cell) specifying that the variables to change are the weights of the individual assets. The operation must be subject to several restrictions. First, the weight of each asset always has to be greater than zero (we have to invest something, even at least, in each of the assets). More specifically, it has been considered the same restriction that was taken into account when obtaining the optimal portfolios in the sense of Markowitz, that the weightings of the assets in each portfolio was, at least, 0.01% to avoid possible errors in calculations with Solver. Another restriction of the minimization problem is that the total sum of the weights should be 100% of our budget, that is, we have to invest all the money we have available.

The minimization problem is resolved by specifying and setting the estimated beta value from which we seek to minimize its variance. In other words, this minimization problem is solved for each of the estimated betas for the twelve optimal portfolios in the sense of Markowitz, obtained in the first stage of the methodology followed in the present study.

After solving the twelve minimization problems through the Solver tool, as we explained above, we obtain twelve portfolios, each with the same estimated beta as the optimal portfolio in the sense of corresponding Markowitz, but defined by weights of the seven assets that minimize the variance of this estimated beta. The composition of each of these portfolios, as well as their estimated alpha, estimated beta and the variance of this beta, are shown below in Table 4.

	ACCIONA	GAS NATURAL	IBERDROLA	MEDIASET	ACS	MELIA	SIEMENS			
	W_1	W_2	W_3	W_4	W_5	W_6	W_7	variance (β)	β portfolio	α portfolio
PORTFOLIO 1	0,0099%	13,2386%	65,7038%	1,7759%	0,0100%	19,2518%	0,0100%	0,00033895	0,6913608	0,0004464
PORTFOLIO 2	9,1970%	13,8562%	35,7635%	10,1312%	17,6621%	8,0443%	5,3457%	0,00023932	0,8571563	0,0005304
PORTFOLIO 3	16,5975%	10,6417%	21,6869%	10,2470%	26,2447%	1,8460%	12,7362%	0,00037189	0,9721753	0,0006822
PORTFOLIO 4	22,3189%	8,2870%	8,3007%	9,8647%	33,0012%	0,0100%	18,2176%	0,00056697	1,0575477	0,0007912
PORTFOLIO 5	27,0972%	2,2192%	0,0100%	8,2445%	38,7355%	0,0100%	23,6836%	0,00079910	1,1282469	0,0009173
PORTFOLIO 6	29,9480%	0,0100%	0,0100%	0,0100%	33,4159%	0,0100%	36,5961%	0,00111817	1,1899788	0,0011381
PORTFOLIO 7	29,9778%	0,0100%	0,0100%	0,0100%	15,6028%	0,0100%	54,3794%	0,00170688	1,2486902	0,0014023
PORTFOLIO 8	29,9961%	0,0100%	0,0100%	0,0100%	4,6763%	0,0100%	65,2876%	0,00223087	1,2847035	0,0015643
PORTFOLIO 9	25,3354%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	74,6146%	0,00271129	1,3117564	0,0016493
PORTFOLIO 10	12,7038%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	87,2462%	0,00342220	1,3433398	0,0016920
PORTFOLIO 11	8,3517%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	91,5983%	0,00370727	1,3542215	0,0017067
PORTFOLIO 12	1,1967%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	98,7533%	0,00422064	1,3721113	0,0017308

Table 4: portfolios that minimise the estimated beta variance.

As a summary of the results obtained according to this model, we must emphasize that when β is equal or greater than 1.1, Gas Natural, Iberdrola, Mediaset and Melia Hotels are no longer profitable for our investment. On the other hand, the evolution of ACS has concave shape, reaching its maximum in Portfolio 5, but never surpassing 40% of the budget. Finally, the two companies in which in most cases studied invest would be Acciona and Siemens Gamesa.

However, we will be able to see in detail the evolution of each portfolio and of each asset below in point 4.

4. COMPARISON OF THE MODELS.

Next, we will compare the results we have obtained according to the efficient portfolios in the sense of Markowitz and according to the model that minimizes the variance of the estimated beta, comparing the results reported by both models. This comparison will allow us to:

1) Analyse if the weights that define the twelve efficient portfolios in the Markowitz sense calculated (that minimize the risk of the portfolio measured through the variance of its yields) and the optimal weights that would minimize the variance of their estimated betas follow some trend or pattern.
2) To see to what extent these optimal weights in the sense of Markowitz are or are not similar to the weights that would minimize the variance of the estimated betas for the portfolios.
3) Check if the weights come to converge at some point, in such a way that the efficient portfolios in the sense of Markowitz would also be minimizing the variance of their estimated beta.

First of all we will compare the results by portfolios and then we will do it for the assets.

4.1. Comparison by portfolios.

At this point we proceed to compare the two models studied and detail the behaviour that we observe portfolio to portfolio. In this way, we can know the weighting of each asset and the differences that we find in the results reported by both models.

We start from **portfolio 1**, with β = 0.6913618. According to the Markowitz model, the efficient portfolio with this estimated beta has a variance of its yields equal to 0.0105%, and this is the efficient portfolio with the minimum variance of its yields that the data, on which this study is based, allows calculating. For this estimated beta we can see that the results are very similar, both according to the

Markowitz model and the model that minimizes the variance of the estimated beta; in Acciona, ACS and Siemens Gamesa we would invest the minimum allowed, followed by Mediaset where we would invest around 2% of our budget, that is, with this beta and according to these models, it would not be advisable to invest in the four assets mentioned above, nor to minimize the volatility of the portfolio's yields or to minimize the variance of its estimated beta. Next, we would invest in Gas Natural and Melia Hotels, but without exceeding 20% of our budget, and finally both models agree that the asset to which we should devote a greater part of our capital would be Iberdrola, which we should dedicate between a 65% and 68% of the total.

In summary, with a beta equal to 0.6913618, both models are very similar to the weighting of the assets and dedicate a very similar part of the total budget to each individual asset.

		PORTFOLIO 1	
		β_{C1}	0,6913618
		MARKOWITZ	MINVARBETA
ACCIONA	W_1	0,02%	0,01%
GAS NATURAL	W_2	10,32%	13,24%
IBERDROLA	W_3	68,61%	65,70%
MEDIASET	W_4	2,60%	1,78%
ACS	W_5	0,01%	0,01%
MELIA HOTELS	W_6	18,43%	19,25%
SIEMENS	W_7	0,02%	0,01%

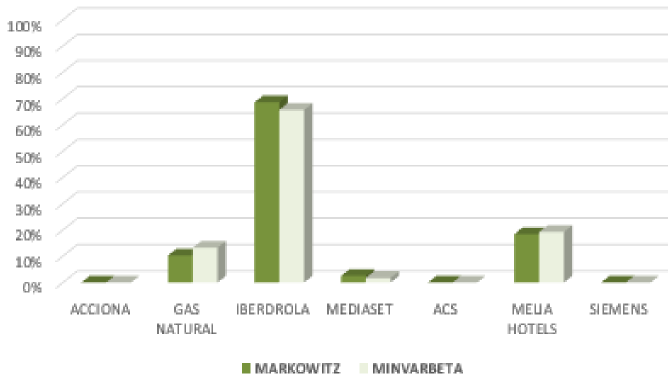


Figure 2: Table of results portfolio 1, with β = 0.6913618.

In **portfolio 2** with β = 0.8571553 we can see that the results reported by both models are quite different. The assets in which, according to the Markowitz approach to portfolio optimization, the minimum allowed would be Gas Natural, Mediaset and ACS, followed by Melia Hotels. On the contrary, the model that minimizes the variance of beta would invest less budget in Siemens Gamesa, Melia Hotels, Acciona and Mediaset. Although the two models agree that Iberdrola should have the highest weighting, it should be noted that Markowitz's model dedicates almost 65% and the model that minimizes variance dedicates almost half that the Markowitz model, 35%.

		PORTFOLIO 2	
		β_{C2}	0,8571553
		MARKOWITZ	MINVARBETA
ACCIONA	W_1	23,43%	9,20%
GAS NATURAL	W_2	0,01%	13,86%
IBERDROLA	W_3	64,84%	35,76%
MEDIASET	W_4	0,01%	10,13%
ACS	W_5	0,01%	17,66%
MELIA HOTELS	W_6	1,65%	8,04%
SIEMENS	W_7	10,05%	5,35%

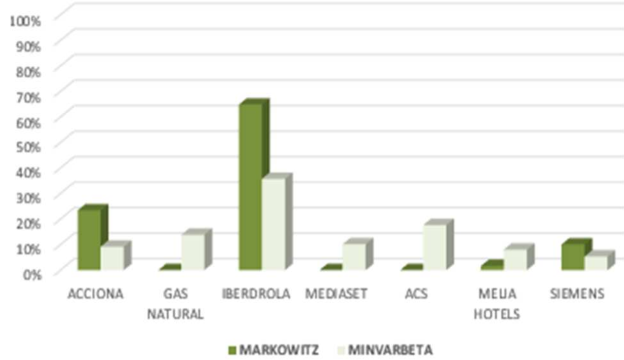


Figure 3: Table of results portfolio 2, with β = 0.8571553.

Regarding **portfolio 3** with β = 0.9721753 no weight is matched, each model reports different results. The Markowitz model would dedicate almost half of the budget to Iberdrola, secondly it would invest 23.43% in Acciona, and in third place it would invest 17.82% in Siemens Gamesa, in the rest of the assets it would invest the minimum budget allowed because, according to this model, Gas Natural, Mediaset and ACS are not convenient to maximize the expected return at this level of risk. On the other hand, the model that minimizes the variance of the beta dictates that the asset to which most of the budget is to be devoted is ACS (according to the Markowitz model in this asset it would invest 0.01%, the minimum weight required when solving the optimization problem) followed by Iberdrola.

The difference between both models is that the model that minimizes the variance of the beta would distribute the budget among six of seven assets and the Markowitz model would focus on three main assets, while the remaining four would invest the least. Therefore, the differences of both models in this portfolio are quite significant.

		PORTFOLIO 3	
		β_{C3}	0,9721753
		MARKOWITZ	MINVARBETA
ACCIONA	W_1	37,21%	16,60%
GAS NATURAL	W_2	0,01%	10,64%
IBERDROLA	W_3	44,93%	21,69%
MEDIASET	W_4	0,01%	10,25%
ACS	W_5	0,01%	26,24%
MELIA HOTELS	W_6	0,01%	1,85%
SIEMENS	W_7	17,82%	12,74%

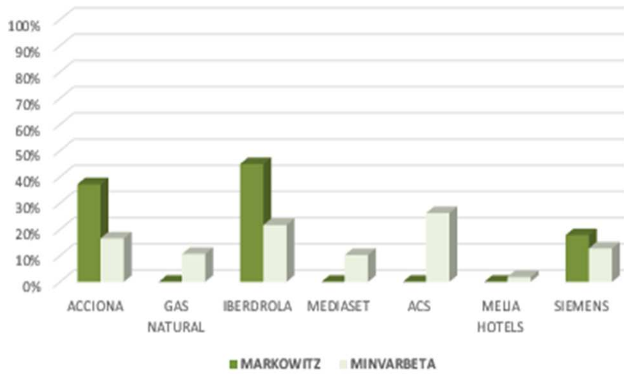


Figure 4: Table of results portfolio 3, with β = 0.9721753.

In **portfolio 4** with β = 1.0575477 the results of the models only coincide in that in the Melia Hotels asset we should invest 0.01% of our budget, that is, the minimum.

On the other hand, the Markowitz model dictates that the asset in which we should invest almost half of our budget would be Acciona, followed by Iberdrola and Siemens Gamesa. On the other hand, the model that minimizes the variance does not say the same thing and concludes that the asset to which we should devote the biggest part of the budget should be ACS, the Acciona and thirdly Siemens Gamesa.

According to this model, Gas Natural, Iberdrola and Mediaset should dedicate more or less the same capital, around 9%.

Finally, according to Markowitz's model, Gas Natural, Mediaset, ACS and Melia Hotels should dedicate the minimum investment. The model that minimizes the variance of the estimated beta only matches the Markowitz model in which only the minimum allowed in Melia Hotels would be invested.

Therefore, the differences for this estimated beta between the two models are important, although both agree that you have to invest very little in Melia Hotels.

		PORTFOLIO 4	
		β_{C4}	1,0575477
		MARKOWITZ	MINVARBETA
ACCIONA	W_1	47,58%	22,32%
GAS NATURAL	W_2	0,01%	8,29%
IBERDROLA	W_3	28,80%	8,30%
MEDIASET	W_4	0,01%	9,86%
ACS	W_5	0,01%	33,00%
MELIA HOTELS	W_6	0,01%	0,01%
SIEMENS	W_7	23,59%	18,22%

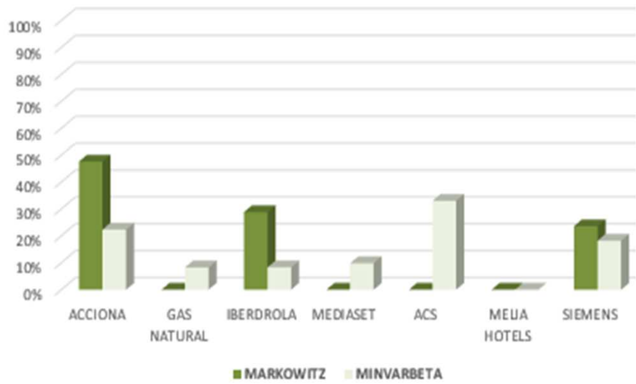


Figure 5: Table of results portfolio 4, with β = 1.0575477.

In the **portfolio 5** with β = 1.1282469, according to Markowitz's model, it resolves that the only assets in which we must invest are Acciona (more than half of the available budget), Siemens Gamesa and Iberdrola, in this order. In the rest of assets we would have to invest the minimum allowed. On the contrary, the model that minimizes the variance of the beta resolves us that we would have to invest in all assets except Iberdrola and Melia Hotels, where we would have to invest 0.01% only. According to this last model, we would have to dedicate 38.74% of our budget in ACS, 27.1% in Acciona, 23.68% in Siemens Gamesa, 8.24% in Mediaset and 2.22% in Gas Natural.

Therefore, the two models agree that the asset in which we should invest less is Melia Hotels and that in Acciona we would have to invest an important part, according to the Markowitz model 56.24% and a half part according to the model that minimizes the beta variance, 27.1%. But, for example, in ACS, according to the Markowitz model, the minimum required would be inverted when solving the optimization problem, while for the model that minimizes the variance, this would be the asset in which more money would have to be invested. Therefore, for this estimated beta there are significant differences between the portfolios obtained by each model.

		PORTFOLIO 5	
		β_{C5}	1,1282469
		MARKOWITZ	MINVARBETA
ACCIONA	W_1	56,24%	27,10%
GAS NATURAL	W_2	0,01%	2,22%
IBERDROLA	W_3	15,41%	0,01%
MEDIASET	W_4	0,01%	8,24%
ACS	W_5	0,01%	38,74%
MELIA HOTELS	W_6	0,01%	0,01%
SIEMENS	W_7	28,31%	23,68%

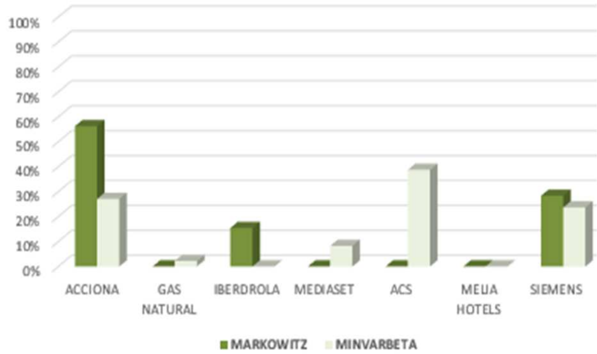


Figure 6: Table of results portfolio 5, with β = 1.1282469.

In **portfolio 6** with β = 1.1899778, the result obtained according to the Markowitz model is similar to that obtained for the estimated beta of the previous portfolio, only the budget would be dedicated to Acciona, Siemens Gamesa and Iberdrola, in this order of importance. It should be noted that almost two thirds of the total budget would be devoted to Acciona and almost one third would be devoted to Siemens Gamesa.

On the other hand, the model that minimizes the estimated beta variance reaches results similar to those of the Markowitz model but referring to different assets. This model would distribute its budget between Siemens Gamesa, ACS and Acciona, in this order, it would dedicate approximately one third of the budget to each one. In Gas Natural, Iberdrola, Mediaset and Melia Hotels it would invest the minimum allowed.

In summary, for this estimated beta the two models coincide in investing very little in Gas Natural, Iberdrola, Mediaset and Melia Hotels, the difference lies mainly in ACS, because according to the Markowitz model, the minimum investment should be made and, according to the model that minimizes the variance would have to invest a third of our budget. It also highlights the result of Acciona, since the model that minimizes the variance of the beta resolves that should be invested half of the budget that would be spent. According to the Markowitz model it should be invested in this asset.

		PORTFOLIO 6	
		β_{C6}	1,1899778
		MARKOWITZ	MINVARBETA
ACCIONA	W_1	63,80%	29,95%
GAS NATURAL	W_2	0,01%	0,01%
IBERDROLA	W_3	3,72%	0,01%
MEDIASET	W_4	0,01%	0,01%
ACS	W_5	0,01%	33,42%
MELIA HOTELS	W_6	0,01%	0,01%
SIEMENS	W_7	32,44%	36,60%

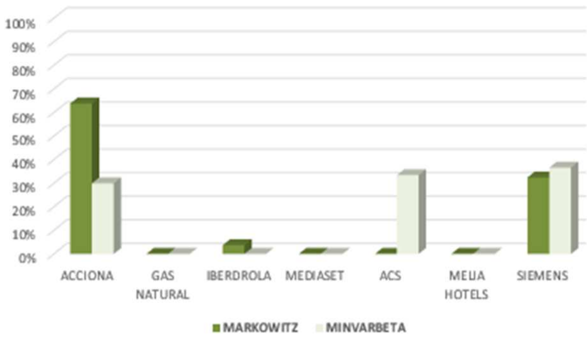


Figure 7: Table of results portfolio 6, with β = 1.1899778.

In **portfolio 7** with β = 1.2486902 the results reported by both models are similar: investing in Acciona and Siemens Gamesa would be the best option, although the Markowitz model would invest almost the same proportion of the budget in both companies and the model that minimizes the variance of beta would invest 54.38% in Siemens Gamesa, 29.98% in Acciona and 15.6% in ACS. That is, the latter model also considers it advisable to invest in the ACS company, in contrast to the Markowitz model. In addition, according to both models, the minimum allowed in the remaining companies would be invested. Therefore, for this estimated β, the weights for the assets that we report both models are quite similar.

		PORTFOLIO 7	
		β_{C7}	1,2486902
		MARKOWITZ	MINVARBETA
ACCIONA	W_1	50,56%	29,98%
GAS NATURAL	W_2	0,01%	0,01%
IBERDROLA	W_3	0,01%	0,01%
MEDIASET	W_4	0,01%	0,01%
ACS	W_5	0,01%	15,60%
MELIA HOTELS	W_6	0,01%	0,01%
SIEMENS	W_7	49,39%	54,38%

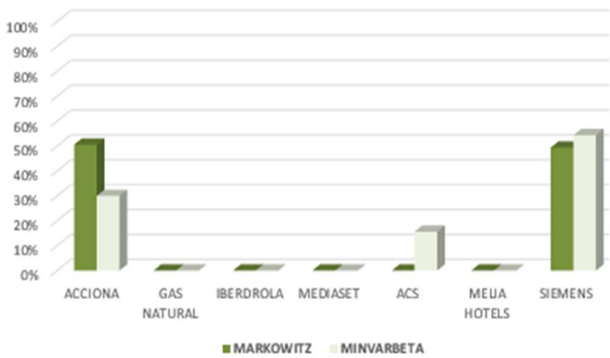


Figure 8: Table of results portfolio 7, with β = 1.2486902.

In **portfolio 8** with β = 1.2847025 we can see that the results reported by both models are also quite similar; both agree to invest the minimum budget allowed in Gas Natural, Iberdrola, Mediaset and Melia Hotels. In ACS, according to the Markowitz model, the minimum allowed for the optimization problem would also be inverted, but according to the model that minimizes the estimated beta variance, a 4.68% of the total would be invested. Therefore, according to both models, the majority of their budget would be invested in the remaining two assets, in Siemens Gamesa they would invest almost two thirds of the total and in Acciona they would invest a third of the budget.

		PORTFOLIO 8	
		β_{C8}	1,2847025
		MARKOWITZ	MINVARBETA
ACCIONA	W_1	36,16%	30,00%
GAS NATURAL	W_2	0,01%	0,01%
IBERDROLA	W_3	0,01%	0,01%
MEDIASET	W_4	0,01%	0,01%
ACS	W_5	0,01%	4,68%
MELIA HOTELS	W_6	0,01%	0,01%
SIEMENS	W_7	63,79%	65,29%

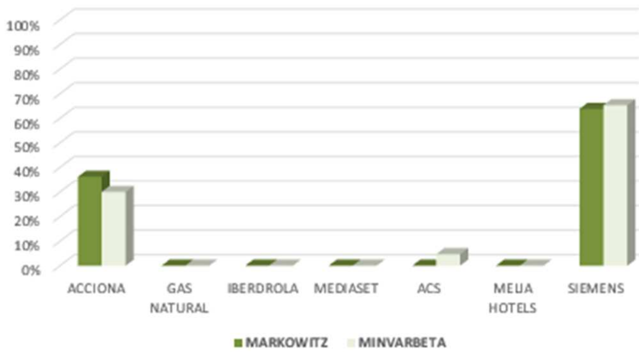


Figure 9: Table of results portfolio 8, with β = 1.2847025.

In **portfolio 9** with β = 1.3117564 and following the same pattern as the two previous portfolios, we would only invest in Acciona and in Siemens Gamesa. In this portfolio and with this beta = 1.311756, it should be noted that the results reported by both models are the same as in Siemens Gamesa we would invest 74.61% and in Acciona 25.34%. In this way we can conclude that with a β = 1.311756, the Markowitz model has defined an optimal portfolio that minimizes the variance of its yields while minimizing the variance of its estimated beta with the market.

		PORTFOLIO 9	
		β_{C9}	1,3117564
		MARKOWITZ	MINVARBETA
ACCIONA	W_1	25,34%	25,34%
GAS NATURAL	W_2	0,01%	0,01%
IBERDROLA	W_3	0,01%	0,01%
MEDIASET	W_4	0,01%	0,01%
ACS	W_5	0,01%	0,01%
MELIA HOTELS	W_6	0,01%	0,01%
SIEMENS	W_7	74,61%	74,61%

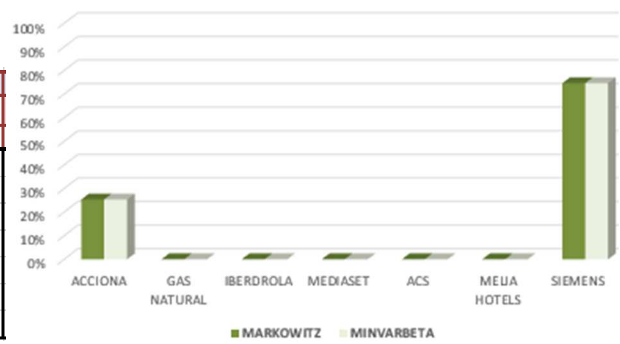


Figure 10: Table of results portfolio 9, with β = 1.3117564.

In **portfolio 10** with β = 1.3433398 the results of both models coincide and we follow the pattern of only investing money in Acciona and in Siemens Gamesa. Compared with portfolio 9, it should be noted that we will now invest 12.64% more in Siemens Gamesa, and that same percentage we subtract it from the investment in Acciona.

		PORTFOLIO 10	
		β_{C10}	1,3433398
		MARKOWITZ	MINVARBETA
ACCIONA	W_1	12,70%	12,70%
GAS NATURAL	W_2	0,01%	0,01%
IBERDROLA	W_3	0,01%	0,01%
MEDIASET	W_4	0,01%	0,01%
ACS	W_5	0,01%	0,01%
MELIA HOTELS	W_6	0,01%	0,01%
SIEMENS	W_7	87,25%	87,25%

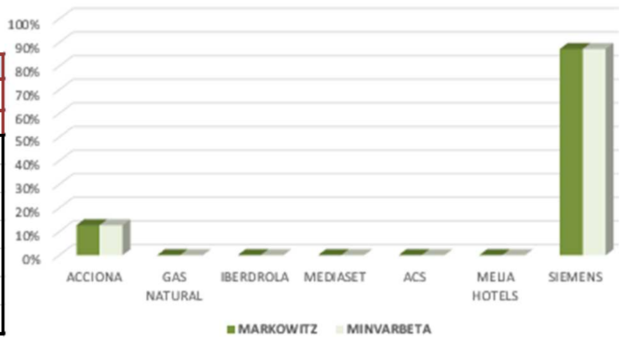


Figure 11: Table of results portfolio 10, with β = 1.3433398.

In **portfolio 11** with β = 1.3542205 both models report the same results and we also follow the same evolution as in the two previous portfolios; we continue investing only in Acciona and in Siemens Gamesa and, the percentage that we increase in the investment of Siemens Gamesa we subtract it from the investment in Acciona. In this portfolio, investment in Siemens Gamesa already exceeds 91% of the total budget and investment in Acciona decreases, compared to the previous portfolio, up to 8.35%.

		PORTFOLIO 11	
		β_{C10}	1,3542205
		MARKOWITZ	MINVARBETA
ACCIONA	W_1	8,35%	8,35%
GAS NATURAL	W_2	0,01%	0,01%
IBERDROLA	W_3	0,01%	0,01%
MEDIASET	W_4	0,01%	0,01%
ACS	W_5	0,01%	0,01%
MELIA HOTELS	W_6	0,01%	0,01%
SIEMENS	W_7	91,60%	91,60%

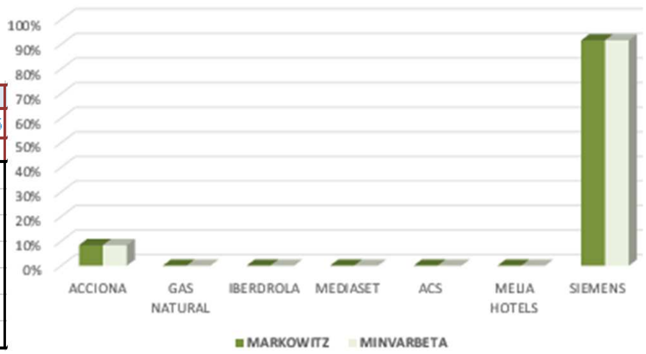


Figure 12: Table of results portfolio 11, with β = 1.3542205.

Finally, in **portfolio 12** with β = 1.3721103 both models continue to coincide in the results of the weights, we would continue investing only in Acciona and in Siemens Gamesa and the investment in the latter would continue to increase, in this portfolio it would reach 98.75% of the total budget, this is almost the total money that we have to carry out this investment.

		PORTFOLIO 12	
		β_{C11}	1,3721103
		MARKOWITZ	MINVARBETA
ACCIONA	W_1	1,20%	1,20%
GAS NATURAL	W_2	0,01%	0,01%
IBERDROLA	W_3	0,01%	0,01%
MEDIASET	W_4	0,01%	0,01%
ACS	W_5	0,01%	0,01%
MELIA HOTELS	W_6	0,01%	0,01%
SIEMENS	W_7	98,75%	98,75%

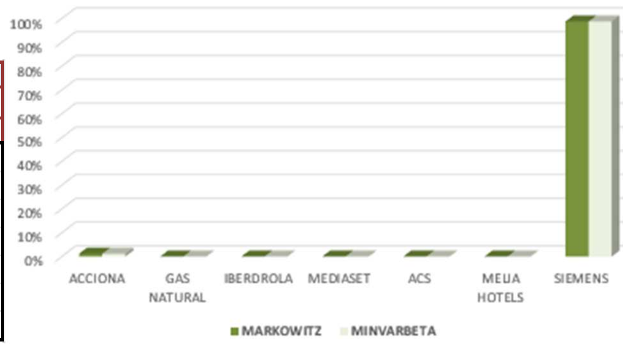


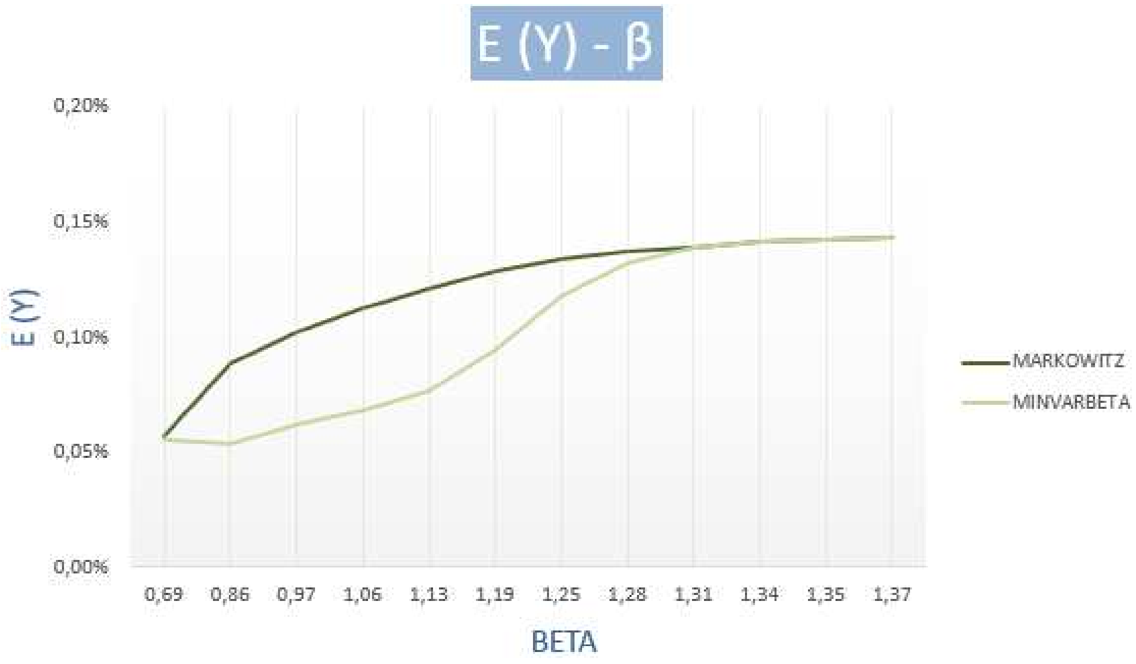
Figure 13: Table of results portfolio 12, with β = 1.3721103.

This is the highest estimated beta on which the present study is based because the data on which it is based does not allow calculating efficient portfolios in the sense of Markowitz for higher levels of risk. From the analysis performed, two interesting results can easily be observed:

I. The optimal portfolios, according to the perspective of each model, converge when the estimated beta of the portfolio is approximately 1.3.

II. It also seems that both models reach the same optimal portfolio for an estimated beta around 0.69. However, the data on which this study has been based does not allow us to obtain more evidence in this regard given that they do not allow obtaining efficient portfolios in the sense of Markowitz for lower levels of risk.

These two results are quite well reflected in the following graph that represents the relationship between the expected performance of each portfolio (according to the Markowitz model and according to the model that minimizes the variance of the estimated beta) regarding each beta.



Graph 1: Beta – Expected yield

For a beta around 0.69, both models reach an optimal portfolio with the same expected performance. That is to say, the portfolio that minimizes the variance of its yields for that expected performance is the same as minimizing the variance of its estimated beta. For higher estimated betas, the optimal portfolio is different for each model, but reconverge for estimated betas around 1.3 and above.

Graph 1 also allows us to observe that for estimated betas in which each model arrives to different portfolios (betas between 0.69 and 1.3), the optimal portfolios according to the Markowitz model always provides higher expected performance than optimal portfolios depending on the model that minimizes the estimated beta variance. As beta is increasing, the expected performance of the portfolios obtained by both models is also increasing, that is, a clear upward trend is seen.

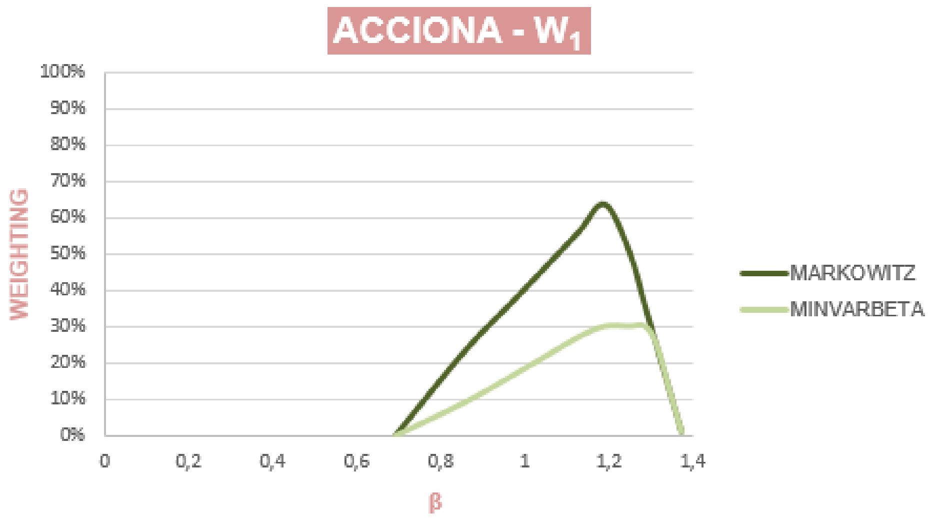
III. There are certain assets whose weights converge on both models in estimated betas less than 1.3. This is the case, for example, of Gas Natural and Mediaset, which begins to converge from $\beta = 1.18$, or from Melia Hotels that converge from $\beta = 1.05$.

We can observe this last conclusion better in the following subsection, where we compare the weights of the individual assets according to each model.

4.2. Comparison by weightings of individual assets.

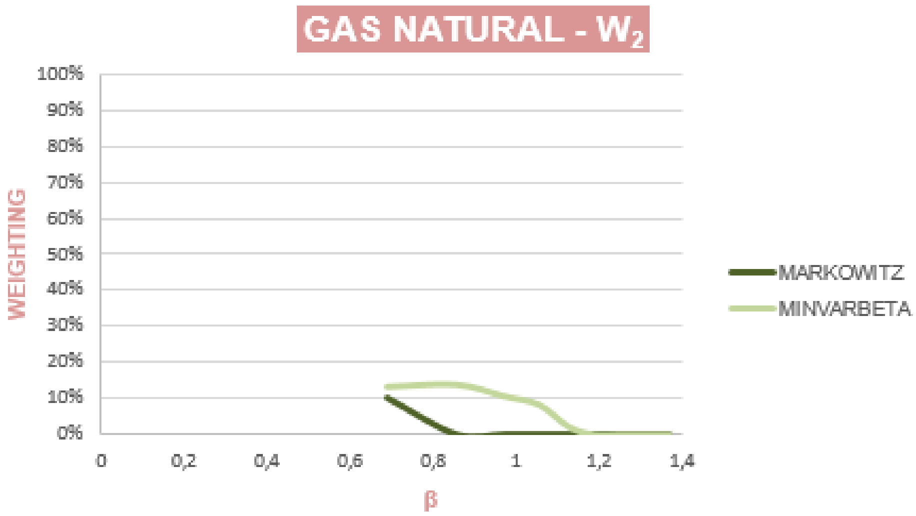
During the development of this point we will obtain an overview of what happens to the weights of each individual asset in the optimal portfolios, according to the Markowitz model and those that minimize the variance of their estimated beta, throughout our study. There are some assets whose behaviours are very similar in both models and other assets with quite different behaviours. Next, we will explain in detail the behaviour of each one of the assets that make up the portfolios and we will attach graphs to explain it in a more visual way.

Acciona (W_1) follows the same trend in both models, obtaining a concave curve, although the Markowitz model gives a greater weight and has a steeper slope. This asset reaches the maximum weight in portfolio 6 with β = 1.18. On the contrary, in the model that minimizes the variance, it reaches the maximum in portfolio 8 with β = 1.28. Both models manage to converge from β = 1.3, point from which the optimal portfolios according to the Markowitz model manage to minimize the variance of their estimated beta with the market.



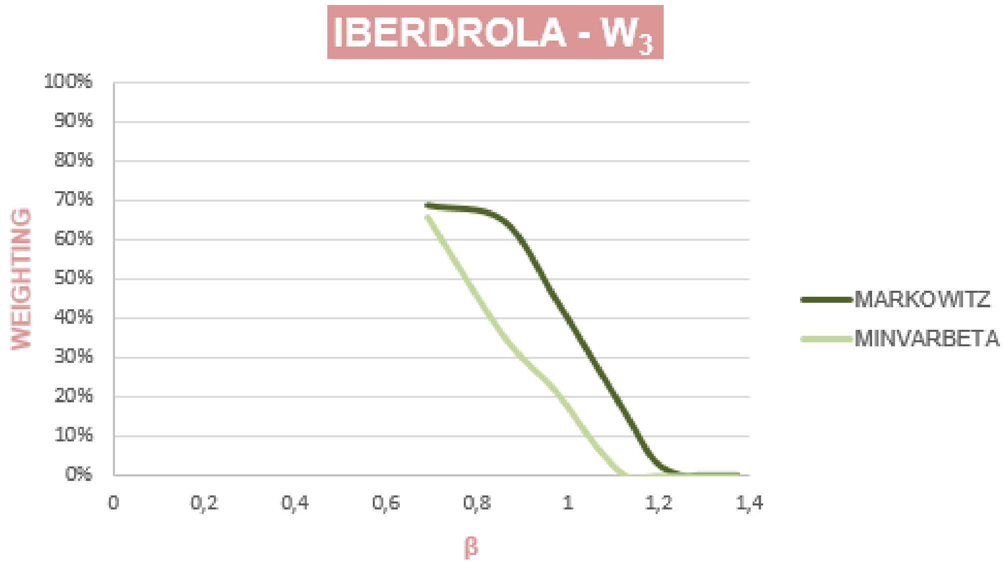
Graph 2: Evolution of Acciona (W_1).

Gas Natural (W_2) has a series of weights with descending slope. The Markowitz model resolves that this asset is no longer suitable for minimizing the variance portfolio yields from β = 0.85 and the model that minimizes the variance of the estimated beta dictates that this occurs from β = 1.18, but both agree that, when the risk and the target beta begin to increase, in this asset we will have to invest the minimum allowed. The two models studied manage to converge from β = 1.18 because it is at this point when this asset is no longer convenient to minimize the variance of beta.



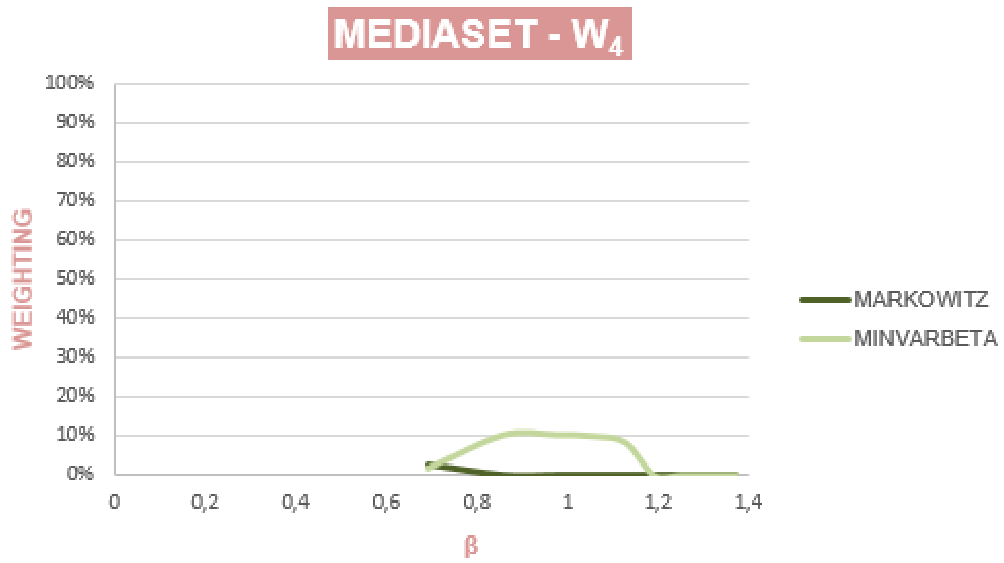
Graph 3: Evolution of Gas Natural (W_2).

Iberdrola (W_3) has a downward slope; in portfolio 1 with β = 0.69 it is considered that this asset is profitable for the two models studied (according to Markowitz's model 68.61% of the budget would have to be dedicated and according to the model that minimizes the variance of the beta 65.7%) and as the risk and the target beta are increasing, the weighting of this asset within the portfolios is declining. In the Markowitz model, the minimum is reached in β = 1.24 and in the model that minimizes the variance, it is reached in β = 1.12. Both models converge from β = 1.24.



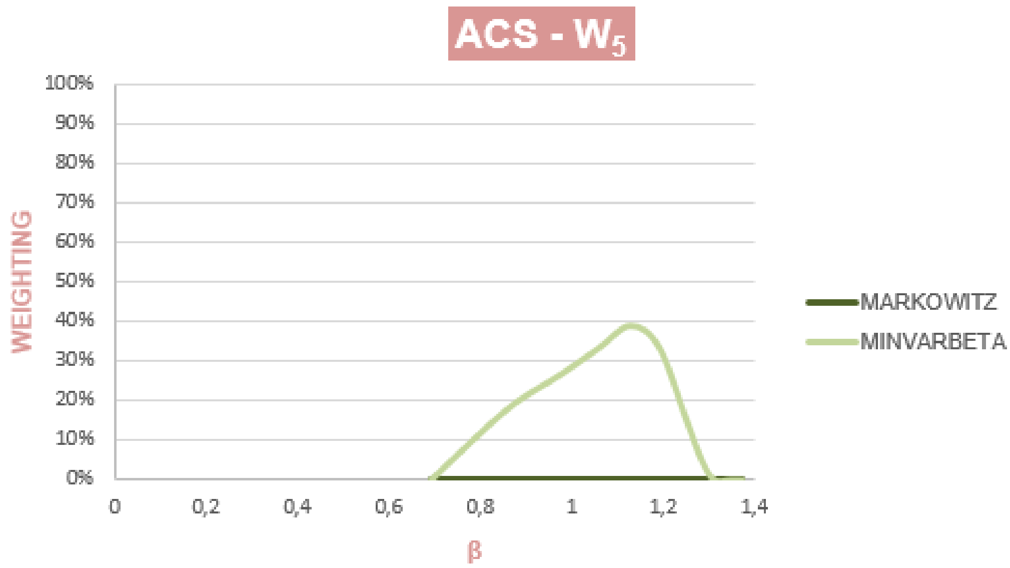
Graph 4: Evolution of Iberdrola (W_3).

Mediaset ($\boldsymbol{W_4}$) is an asset that, as we can see in Graph 5, is very inconvenient, both to minimize the variance of portfolio's yields and to minimize the variance of its estimated beta. According to the Markowitz model, we would only have to invest in this asset in Portfolio 1 and only 2.6% of the budget. On the other hand, the model that minimizes variance would invest in this asset a maximum of 10.24% in Portfolio 3 but, would stop investing in it from Portfolio 6 with β = 1.18. Therefore, from Portfolio 6 we would invest the minimum allowed in this asset, either according to the Markowitz model or according to the model that minimizes the variance of beta, that is, the models converge in β = 1.18 as we have highlighted above.



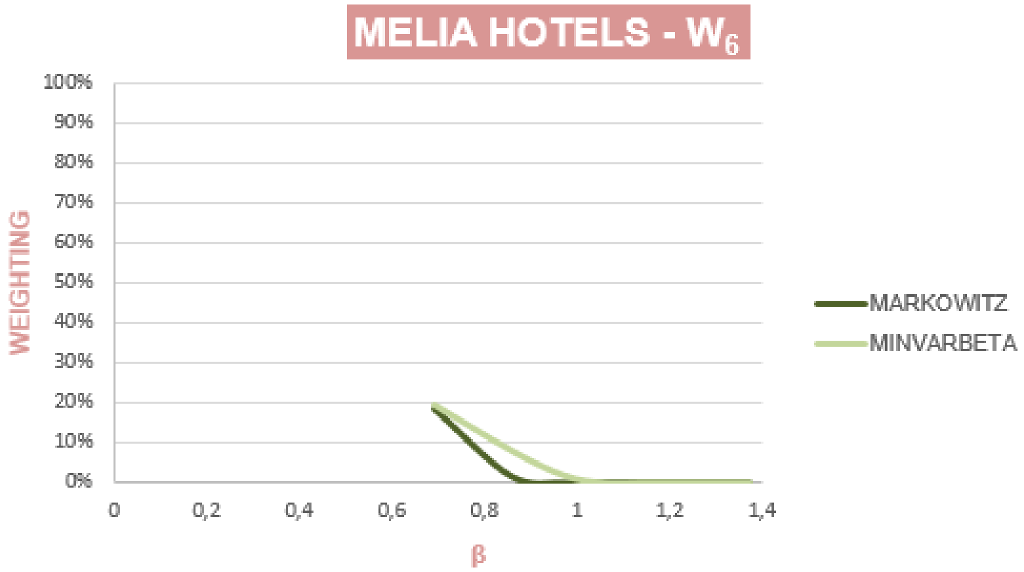
Graph 5: Evolution of Mediaset (W_4).

ACS (W_5) for the Markowitz model is an asset in which we should invest only the minimum allowed by the optimization problem, whatever the portfolio we are studying, and on the contrary for the model that minimizes the variance of beta is an asset to which we should pay close attention until reaching β = 1.31. According to the model that minimizes the variance of beta, the weighting of this asset is increasing until reaching its maximum in β = 1.12; where we should invest in this asset more than a third of our budget, and on the contrary the Markowitz model resolves that this asset is not profitable for our investment. In this active, both models coincide from β = 1.3.



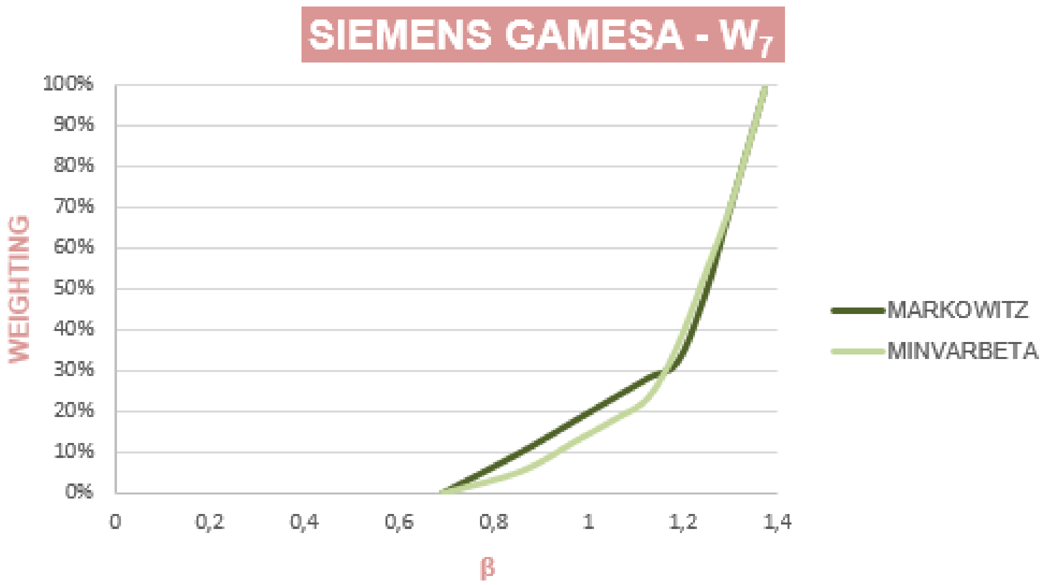
Graph 6: Evolution of ACS (W_5).

Melia Hotels (W_6) behaves in a similar way to Gas Natural and Mediaset since the two models studied resolve that this asset is not suitable either to minimize the variance of portfolio yields or to minimize the variance of its estimated beta. In β = 0.69 both models would invest in it – never exceeding 20% of the budget – but as the risk increases and the target beta increases, its weighting is becoming smaller, until reaching β = 1.05, where none of the two models would invest money in it. Both models converge from β = 1.05 as we have indicated in the previous point.



Graph 7: Evolution of Melia Hotels (W_6).

Siemens Gamesa ($\boldsymbol{W_7}$) behaves in a very different way than the rest of the assets. This asset has an upward slope throughout the studied range. In β = 0.69 none of the two models studied considers this asset to be convenient or to minimize the variance of the portfolio yields or to minimize the variance of its estimated beta. However, as the level of risk increases and the target beta is increasing in value, the weighting of Siemens Gamesa is growing until it reaches β = 1.37 where both models would invest 98.75% of their available budget in this asset, almost the whole. In this case, the two models studied converge from β = 1.3.



Graph 8: Evolution of Siemens Gamesa (W_7).

To conclude with this point we must emphasize that, although each asset behaves in a very different way from the rest, finally the two models studied coincide at some point and converge in such a way that the optimal portfolios according to the Markowitz model manage to minimize the variance of your estimated beta with the market. This occurs in all the studied assets from β = 1.3 approximately, although in some the models converge with β less than 1.3.

5. CONCLUSION.

The objective of this work was to apply the model proposed by McInish et al. (1984), that is, to form portfolios that minimized the variance of their estimated beta with the market. In our case, we have used as estimated beta the beta of twelve optimal portfolios in the sense of Markowitz formed by seven shares of the IBEX 35. In other words, we were looking for the portfolios that minimize the variance of the estimator of the slope of a linear regression between the performance of the portfolio and the performance of the market. Taking as an objective value of this outstanding, the value of each one of the betas of the twelve efficient portfolios in the sense of Markowitz considered.

After solving the twelve optimization problems, we have obtained twelve portfolios, each of them with the same estimated beta that the corresponding optimal portfolio in the sense of Markowitz, but defined by weights of the seven individual assets that manage to minimize the variance of this estimated beta.

After comparing both models and carrying out an Excel study, we have come to the conclusion that there is a point at which the optimal weights of individual assets coincide in the two models and this occurs when beta is approximately equal to 1.3. From this moment, both models converge and this means that the Markowitz model defines optimal portfolios that minimize the variance of their yields, while minimizing the variance of their estimated beta with the market.

6. BIBLIOGRAPHY.

Invertia. *Acciones Ibex*. Available on: <https://www.invertia.com/es/mercados/bolsa/indices/acciones/-/indice/ibex/IB011IBEX35>

[retrieved April 2018]

Invertia. *Cotizaciones históricas Ibex 35*. Available on: <https://www.invertia.com/es/mercados/bolsa/indices/historico/-/indice/ibex/IB011IBEX35>

[retrieved April 2018]

Investing. *Rentabilidad del bono español a 20 años*. <https://es.investing.com/rates-bonds/spain-20-year-bond-yield-historical-data> [retrieved April 2018]

Lintner, J. (1965). "The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets". Review of Economics and Statistics. 47 (1)

McInish, T. H., Morse, J. N., Saniga, E. M. (1984). “Portfolio selection to achieve a target beta”. RAIRO, Recherche opérationnelle 18 (2): 131-145.

Mossin, J. (1966). "Equilibrium in a Capital Asset Market". Econometrica. 34 (4): 768–783

Sharpe, W. F. (1964). "Capital asset prices: A theory of market equilibrium under conditions of risk". Journal of Finance. 19 (3): 425–442.

Wahlen, J., Baginski, S. and Bradshaw, M. (2011) *Financial Reporting, Financial Statement Analysis and Valuation*. South-Western College Pub, 7th edition