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Analysis of investment in financial markets:

Markowitz against Value at Risk historical approach

Final Degree Dissertation in Finance and Accounting

Author: Alejandro Ramón Camáñez Mont

E-mail: al285274@uji.es

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Tutor: Alejandro José Barrachina Monfort

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ABSTRACT

This study compares three approaches to portfolio optimization, the approach suggested by Markowitz (1952), and the approach based on employing the historical approach to Value at Risk (VaR), at both the 90% and 95% levels of confidence, as risk measure. To fulfill this purpose, real data of stock prices for seven different companies that have been listed on the Ibex 35 were used to empirically obtain optimal portfolios according to these three approaches. To do it, the program used was Excel, with special relevance to the tool Solver, obtaining optimal portfolios for eight different levels of expected returns. Although the behaviour of the asset's weights in the different portfolios that minimize risk measured by VaR is quite erratic, in general portfolios that minimize risk measured by 95% VaR are more similar to the ones obtained under Markowitz's (1952) approach than portfolios that minimize risk measured by 90% VaR.

Keywords: Portfolio Optimization, Value at Risk, Volatility, Expected Return.

INDEX

LIST OF TABLES

LIST OF FIGURES

Analysis of investment in financial markets: Markowitz against Value at Risk

1. INTRODUCTION

This final degree dissertation pretends to compare the optimal portfolios obtained applying two different approaches. On the one hand, the approach suggested by Markowitz (1952), according to which the risk measure considered to obtain optimal portfolios is the volatility of their returns. On the other hand, the approach that considers the historical approach to Value at Risk (VaR) as the risk measure. To fulfill this purpose, real data of stock prices for seven different companies that have been listed on the Ibex 35 were obtained and used to obtain optimal portfolios according to both approaches. The main reason because I chose Markowitz's (1952) approach is that it is the classical one to modern portfolio optimization and I had worked with it in the subject FC1029 – Markets and Financial Institutions. On the other side, I chose VaR because it is generally accepted in the international markets as a risk measure since the Basle Committee (1996) published its regulation.

Previous studies, such as Campbell, Huisman, and Koedijk (2001), Benati and Rizzi (2007) and Yoshida (2009) have all dealt with portfolio optimization under VaR as risk measure, but only Campbell, Huisman, and Koedijk (2001) has used the historical approach to VaR as in this final degree dissertation. Nevertheless, they focus on obtaining the efficient frontier of portfolios (namely, portfolios that for a given expected return minimize the risk) under the historical approach to VaR as risk measure, rather than comparing the weights of the different assets in those optimal portfolios with the weights those assets would have if the volatility of portfolio's returns were considered as risk measure, which is the objective of the present study.

The structure that follows this document is presented below. Section 2 discusses the theorical foundation of the two approaches considered to obtain optimal portfolios, including their history and abridgement. Sections 3 and 4 describe, respectively, the data used and the methodology followed in the present study. Section 5 exposes the optimal portfolios with both methods. Section 6, discusses the results obtained in this final degree dissertation as well as other previous studies obtained by other authors. A conclusion based on the analysis of the results and recommendations are include in section 6. Finally, the bibliography to consult every academic source that has been used in this final degree dissertation.

2. THEORETICAL FOUNDATION

2.1. RISK MEASURES

When the Markowitz mean-variance optimization problem is used, the form of measure the risk is obtaining the variance of the future portfolio return. The problem with the variance is that is defined as the expected squared deviation from the mean value, thus it does not measure if the deviations are positive or negative. Moreover, standard deviation could only be considered accurate if the future value of the portfolio is approximately normal distributed, and that condition is too restrictive to do a correct analysis of the financial markets.

There are some mathematical properties that are considerate interesting to measures of risk, let *p*(X) be a function measuring the risk of a stochastic variable X: (Isaksson, 2016)

- Translation invariance. **$p(cR0 + X) = -c + p(X)$ for $c \in R$.** As the c is added with a risk-free interest the risk of the portfolio will be reduced its risk by the same amount.
- Monotonicity. **If $X2 < X1$, then $p(X1) \leq p(X2)$.** If it could be known that the X1 portfolio will be larger in the future than X2, it is considered that this first portfolio is less risky.
- Convexity**. $p(\lambda X1+(1-\lambda)X2) \leq \lambda p(X1)+(1-\lambda)\, p(X2)$, for any real $\lambda \in [0,1]$** the risk measure prefers diversification, it is preferable more assets in one portfolio than less assets.
- Normalization. **$p(0) = 0$.** It is considerate the case of one empty portfolio, in consequence it has not risk.
- Positive homogeneity. **$p(\lambda X) = \lambda p(X)$ for $\lambda \geq 0$** For instance, if you double or triple a position in the portfolio, then you are doubling or tripling the risk assumed with that position.
- Subadditivity. **$p(X1 + X2) \leq p(X1) + p(X2)$** This property indicates that the risk measure rewards diversification. A company that keep two business units is interpreted as less risky compared to the two business units if they were separate companies.

With respect to the risk measures considered in the present study, volatility of the returns (measured by their variance) and VaR, it is generally accepted that none of them respects all the above properties. In particular, variance does not respect translation and monotonicity properties, while VaR does not respect subadditivity property. In this sense, the objective of the present study is to analyse the effects of considering risk measures that satisfy different properties over portfolio optimization.

2.2. MARKOWITZ, HISTORY AND APPROXIMATION TO THIS ANALYTICAL METHOD

The following theory that I am going to explain was developed by Markowitz, originally in his doctoral dissertation. This first work (Markowitz,1950-51 cited in Markowitz, 1959, p.viii) was supported by the Social Science Research Council and the Cowles Commission for Research in Economics.

The hypothesis on which Markowitz's previous investors were based is that a good investor should maximize the expected returns. According to Markowitz (1952) this investment rule must be rejected because it is not useful enough as a theory to explain and neither like a correct way to follow for the investors. In contrast, a good investor should consider expected return as a positive event and the variance of return an adverse event. Markowitz (1952, p. 77) named this new theory as the rule of "Expected Returns-Variance of returns"

One of the reasons why the original hypothesis that only maximizes the expected returns matters should be abandoned, is because if it is supposed that the market does not have imperfections, the portfolio does not necessity to be diversified. For Markowitz (1952), a diversified portfolio is always preferable to a portfolio no diversified. For him, a rule that does not implicate the superiority of a diversified portfolio over another portfolio no diversified must be rejected in any case.

"The law of high numbers", this rule implies that the investor should maximize the expected return and also reduce the variance of them trough diversification. It is done investing in many securities which the maximum expected return, therefore the real performance of the portfolio should have similar results to the expected portfolio. In this theory, it is supposed the existence of one optimal portfolio, and it is commended to the investors.

Markowitz (1952) said that the assumption in "The law of high numbers" cannot be accepted because the diversification does not depend on the number of securities which one investor has in his portfolio. The elimination of the variance depends on how much they are intercorrelated. Thus, the portfolio that has the maximum expected return should be the one which has the minimum variance, but it is also possible that it has not the minimum variance. The most common situation is that the investors could choose between a higher expected return, with the assumption of a higher variance, or the reserve situation, a lowest expected return but a lowest variance too.

The theory of "Expected Returns-Variance of returns" (E-V) not only implies diversification, it also implies the correct diversification. For instance, following "The law of high numbers" previous, an investor could build a portfolio with one hundred of companies specialized in petroleum ten years ago from now, 2019. If one investor would have done it, he would have had severe losses between the years 2014-2016. Following the law previously mentioned, it must not happen, because the investor has an elevated number of securities in his portfolio. In spite of this, it would happen. Instead, if the investor would have built a portfolio with petroleum, food, banking, and transportation companies, such as railroad or airlines, and they were companies of different countries or continents, when one sector falls, it only affects to the weight that this sector has in his portfolio. Consequently, not all the portfolio falls, and maybe it could be compensated by the rise of other sector present in the portfolio. Therefore, the looking for low covariances between companies with a highest expected return should be the prime objective for investors.

In the more extensive publication of Markowitz (1959), he developed the E-V theory and wrote about other concepts that might illustrate better this theory. To him, the purpose of the analysis is not to find the best portfolio, it is to find the best portfolio to satisfy the objectives of every investor.

In that point, Markowitz (1959, p.4) announced that "Uncertainly is a salient feature of security investment" with this sentence, it is possible to see about the division of the total risk, in diversifiable risk and non-diversifiable risk. The diversifiable risk is the risk that, through the diversification of the investment, taking into account the covariances between them, it might be eliminated partially. The non-diversifiable risk is the risk that might not be eliminated, even one investor knows the market perfectly, and he has the most privileged information, such as the decisions of the executives of the companies in which he might invest, unexpected events might change the course of the economy. For example, the change of a government after elections, a new war between two countries, changes in weather conditions, a new regulatory law, a commercial agreement and other events could affect to the results of most of the companies in that economy.

In spite of the existence of non-diversifiable risk, it does not mean that security analysis is useless, only that in every investment is necessary to assume a minimum risk and that the objective of the diversified portfolio should be found this minimum.

To determinate the objectives of a portfolio it has to take account some considerations. For investors exist two commonly objectives that are the same for all of them. First, they want a high return and secondly, the investors want that their return is stable. In other words, they prefer most return than less return, and they want less uncertainty than more uncertainty. It could be said that the investors are rational.

Other important point is the distinction between an "efficient" portfolio and an "inefficient" portfolio. As Markowitz (1959, p.6) said "If portfolio A has both a higher likely return and a lower uncertainty of return than portfolio B and meets the other requirements of the investor, it is clearly better than portfolio B. Portfolio B may be eliminated from consideration, since it yields less return with greater uncertainty than does another available portfolio. We refer to portfolio B as "inefficient." After eliminating all such inefficient portfolios – all such portfolios which are clearly inferior to other available portfolios – we are left with portfolios which we shall refer to as "efficient." These consist of: the portfolio with less uncertainty than any other with a 6% likely return, the portfolio with less uncertainty than any other with a 7% likely return, and so on**."**

Analysing all of the above, some conclusions could be drawn: efficient portfolios must be separated of inefficient portfolios, find out the risk that the investor is disposed to assume, and finally, to determinate the portfolio which provides the better combination of risk and expected return (Markowitz, 1959).

2.3. VALUE AT RISK, HISTORY AND APROXIMATION TO THIS ANALYTICAL METHOD

The successive crisis and the computational advances applied to the financial industry made necessary the establishment of a statistical measure that could be used by economist, investors and regulators to estimate the level of risks. The Value at Risk (VaR) was selected to do that function. As Vasileiou (2017, p.952) said **"**VaR is a statistical measure which assumes that if the market conditions are normal over a given period of time, a portfolio's (or a financial instrument's) maximum losses will not be above the VaR estimation and this is statistically confident at a certain confidence level (usually 95% or 99%)**"**

The first appearance of VaR was in 1952, shortly after of the Markowitz publication about the E-V. Both of them works were looking for a system to optimize the reward for a given level of risk. Other of the similitudes was the estimate of the covariances between the risk factors to show the effects of headings and diversification. However, they have several differences, such as Holton (2002, p.3) wrote **"**Markowitz used a variance of simple return metric. Roy used a metric of shortfall risk that represents an upper bound on the probability of the portfolio's gross return being less than some specified "catastrophic return."**"**

Lietaer (1971) explained how use VaR measure for foreign exchange risk. Later of the World War II, most currencies had devaluated in some moment of their history and the governments keep these devaluations on secret. For this reason, many corporations maintained ongoing hedges. To carry out, it was necessary to consider two assumptions: first, the devaluations occurred randomly, and second, the conditional magnitude of a devaluation is normally distributed.

The previous paragraphs are one example of what had supposed the technological and market changes for VaR in 1970s and 1980s. It could be summarized in three points (Holton, 2002).

- VaR increased the number of assets in which might be applied.
- The system of companies to take risk changed.
- New means to apply VaR in that new environment.

By the 1980's, it became necessary that the financial institutions had developed VaR measures more sophisticated. Principally by two motives, the volatile in financial markets was increasing and sources of market risk be multiplied, especially with the apparition of leverage. Several financial institutions implemented sophisticated variants of VaR in this decade, but it was not regulated and this practice depended on every institution. During the 1990's the value of proprietary VaR measures was recognized by the Basle Committee, which authorized their use by banks for performing regulatory capital calculations.

The document of Basle Committee (1996) on Banking Supervision is a guide for the official institutions of investment due to the necessity to measure risk. VaR acquires special relevance in this document on the Quantitative Standards. It established that banks could have flexibility in design their owns methods of risk measure, nevertheless, they have to obligation to respect minimum parameters. Without limiting the foregoing, the authorities responsible of their supervision could apply more stricter standards. The minimum parameters founded in Basle Committee (1996, pp.44-45) are the following:

"(a) "Value-at-risk" must be computed on a daily basis.

(b) In calculating the value-at-risk, a 99th percentile, one-tailed confidence interval is to be used.

(c) In calculating value-at-risk, an instantaneous price shock equivalent to a 10 day movement in prices is to be used, i.e., the minimum "holding period" will be ten trading days. Banks may use value-at-risk numbers calculated according to shorter holding periods scaled up to ten days by the square root of time (for the treatment of options, also see (h) below).

(d) The choice of historical observation period (sample period) for calculating value at-risk will be constrained to a minimum length of one year. For banks that use a weighting scheme or other methods for the historical observation period, the "effective" observation period must be at least one year (that is, the weighted average time lag of the individual observations cannot be less than 6 months).

(e) Banks should update their data sets no less frequently than once every three months and should also reassess them whenever market prices are subject to material changes. The supervisory authority may also require a bank to calculate its value-at-risk using a shorter observation period if, in the supervisor's judgement, this is justified by a significant upsurge in price volatility.

(f) No particular type of model is prescribed. So long as each model used captures all the material risks run by the bank, as set out in B.3, banks will be free to use models based, for example, on variance-covariance matrices, historical simulations, or Monte Carlo simulations.

(g) Banks will have discretion to recognise empirical correlations within broad risk categories (e.g., interest rates, exchange rates, equity prices and commodity prices, including related options volatilities in each risk factor category). The supervisory authority may also recognise empirical correlations across broad risk factor categories, provided that the supervisory authority is satisfied that the bank's system for measuring correlations is sound and implemented with integrity.

(h) Banks' models must accurately capture the unique risks associated with options within each of the broad risk categories. The following criteria apply to the measurement of options risk:

- banks' models must capture the non-linear price characteristics of options positions;
- banks are expected to ultimately move towards the application of a full 10 day price shock to options positions or positions that display option-like characteristics. In the interim, national authorities may require banks to adjust their capital measure for options risk through other methods, e.g., periodic simulations or stress testing;
- each bank's risk measurement system must have a set of risk factors that captures the volatilities of the rates and prices underlying option positions, i.e., vega risk. Banks with relatively large and/or complex options portfolios should have detailed specifications of the relevant volatilities. This means that banks should measure the volatilities of options positions broken down by different maturities.

(i) Each bank must meet, on a daily basis, a capital requirement expressed as the higher of (i) its previous day's value-at-risk number measured according to the parameters specified in this section and (ii) an average of the daily value-at-risk measures on each of the preceding sixty business days, multiplied by a multiplication factor.

(j) The multiplication factor will be set by individual supervisory authorities on the basis of their assessment of the quality of the bank's risk management system, subject to an absolute minimum of 3. Banks will be required to add to this factor a "plus" directly related to the ex-post performance of the model, thereby introducing a built-in positive incentive to maintain the predictive quality of the model. The plus will range from 0 to 1 based on the outcome of so-called "backtesting." If the backtesting results are satisfactory and the bank meets all of the qualitative standards set out in B.2 above, the plus factor could be zero. The accompanying document, Supervisory framework for the use of backtesting in conjunction with the internal models approach to market risk capital requirements, presents in detail the approach to be applied for backtesting and the plus factor.

(k) Banks using models will be subject to a separate capital charge to cover the specific risk of interest rate related instruments and equity securities51 as defined in the standardised approach to the extent that this risk is not incorporated into their models. However, for banks using models, the total specific risk charge applied to interest rate related instruments or to equities should in no case be less than half the specific risk charges calculated according to the standardised methodology.**"**

It is generally accepted that, volatility is a basic risk measure in finance studies. It is also accepted that the volatility grows during crises periods. More concretely, the “volatility feedback hypothesis” describe how the price of assets should fall when volatility are increasing in the financial markets. Thus, if the financial crisis is related with volatility rises, and it is commonly accepted to be a risk measure, it might seem that VaR has not place in the study of risk in financial markets (Vasileiou, 2017).

If the reader takes into account the previous paragraph, without financial knowledge, he could ask himself how to measure the risk in investments. Probably his answer could be to measure the volatility in percentage, and this answer would be incorrect. For this reason, the VaR is so important as a statistical measure, because it is not only able to estimate the potential losses and it is also able to do it in monetary terms.

It may be assumed that the main methods of VaR are three: the historical simulation, the Delta Normal and the Monte Carlo.

All these methods use the last *X* observations and get the VaR estimations, but for Vasileiou (2017, pp.955) they use different assumptions.

“1) The historical approach uses real historical data and recalculates the portfolio’s returns for the last × observations, assuming that the following days will be similar to the previous x days, and from these x returns the VaR is the 1% or 5% of the lowest returns

2) Delta Normal calculates the variance-covariance matrix, portfolios sigma and under the normal distribution assumption estimates the VaR

3) Monte Carlo has in most cases similar procedures to Delta Normal, but additionally generates several random scenarios using the same data set.**”**

The VaR method that has been used in this final degree dissertation is the first one, the historical VaR. Some of the main advantages of this method are that the estimation is simple and that the results are clearly communicable. Nevertheless, it has other disadvantages. If the parameters such as the duration of the historical data selected and the confidence level are inadequate, probably the VaR results will be inadequate too.

3. EXTRACTION OF REAL DATA

As it is exposed previously, this final degree dissertation is based on the academic work of subject Markets and Financial Institutions, for this reason, the data that has been worked is the data that I have used in that academic work about Markowitz. These data were obtained from the webpage *Investing* (https://www.investing.com/).

Despite the listed companies that could be used were limited to companies of Ibex 35, it was possible to apply for diversification, every corporation that I chose is dedicated to one different sector of the others companies in the portfolio. The ones selected were the following:

- ABERTIS: Corporation dedicated at the construction of highway
- AENA: Corporation focused on the airport management
- BANKINTER: Bank company
- GRIFOLS: Company of investigation, concretely on biomedical research
- SIEMENS GAMESA: Renewable technology
- INMOBILIARIA COLONIAL: Corporation dedicated to real estate management
- AMADEUS: Technology applied to travel

The data selected are the final quote of the day for these seven corporations, every day they quote during three years, since 25/03/2015 to 26/03/2018. Before continuing with the explanation, two more conditions must be taken into account. In order to obtain a correct result, all days that for any reason did not have quotation in some of the companies of the portfolio were eliminated from the calculation. With these data, the mathematical expectation was calculated and also it was necessary that the expected return of every active was positive.

Each formula that has been described in the following paragraphs has been applied in Excel. To calculate the daily returns of every company, a concrete method was used. Instead of divide the final price of one day by the previous day, it was substituted by the neperian logarithm (LN). The reason behind this, is that not only is taken into account the differences between prices, it is also important take into account the price volatility.

Afterwards, the variance of the daily returns was calculated. To do this, the mathematical formula of VAR.P in Excel was used in the set of results previously used to calculate the daily returns.

Finally, the standard deviation that indicates how the profitability varies with respect to the mean was calculated, using the formula RAIZ, which is the square root of the variance.

Table 1: Results obtained analysing the real data

	EXPECTED RETURN	VARIANCE	STANDAR DEVIATION
ABERTIS	0,000104225	0,000154545	0,012431593
AENA	0,000922619	0,000204293	0,014293118
BANKINTER	0,000199839	0,000216427	0,014711459
GRIFOLS	0,000113067	0,000208082	0,014425051
SIEMENS GAMESA	0,000170151	0,000618809	0,024875872
INMOBILIARIA COLONIAL	0,00046642	0,000221171	0,014871812
AMADEUS	0,000544385	0,000183323	0,01353969

Source: Own development.

After obtaining these data, the study continued with the collecting the covariance matrix of the returns. Covariance teaches how much relation has a title respect to another. This has been calculated using COVARIANCE.P taking the average of the Neperian Logarithms of each company for it.

Table 2: Covariance Matrix

	ABERTIS	AENA	BANKINTER	GRIFOLS	SIEMENS GAMESA	INMOBILIARIA COLONIAL	AMADEUS
ABERTIS	0,000154545	0,000080870394	0,000089935698	0,000063941220	0,000102261705	0,000063121419	0,000067713042
AENA	0,000080870394	0,000204293	0,000080405060	0,000084159376	0,000127773653	0,000083536054	0,000087991949
BANKINTER	0,000089935698	0,000080405060	0,000216427	0,000082330554	0,000128944049	0,000086638570	0,000077306361
GRIFOLS	0,000063941220	0,000084159376	0,000082330554	0,000208082	0,000109487154	0,000073961538	0,000097910671
SIEMENS GAMESA	0,000102261705	0,000127773653	0,000128944049	0,000109487154	0,000618809	0,000106210084	0,000127534034
INMOBILIARIA COLONIAL	0,000063121419	0,000083536054	0,000086638570	0,000073961538	0,000106210084	0,000221171	0,000070674440
AMADEUS	0,000067713042	0,000087991949	0,000077306361	0,000097910671	0,000127534034	0,000070674440	0,000183323

Source: Own development.

As it can be observed, the diagonal of the covariance is the variance of the coincident asset. Also, the two parts in which the covariance matrix is divided are equivalents.

4. METHODOLOGY OF THE STUDY

As stated in the Introduction, the objective of the present study is to compare the optimal portfolios obtained applying two different approaches. On the one hand, the classical approach in the Modern Portfolio Theory suggested by Markowitz (1952), according to which the risk of portfolios is measured by the volatility of their returns. On the other hand, the approach that measures the risk of portfolios through the historical approach to VaR.

In this sense, the study applies a methodology in three stages to achieve this objective using the data of the seven stocks described in the previous section. In the first stage, several optimal portfolios are obtained applying the Markowitz's (1952) approach. In the second stage, for each expected return of the optimal portfolios obtained in the previous stage, the portfolios that minimize risk measured by the historical approach to VaR at two levels of confidence (90% and 95%) are obtained. Finally, the third and last stage compares the optimal portfolios from both perspectives. Namely, the optimal weights of each stock in each pair of portfolios with the same expected return obtained in the two previous stages are compared in order to identify similarities and differences between optimal portfolios according to both risk measures, the volatility of returns and the historical approach to VaR. The next two subsections lead with the first two stages of the methodology, while the third one is discussed in the next section of the study.

4.1. OPTIMAL PORTFOLIOS UNDER MARKOWITZ'S (1952) APPROACH: VOLATILITY OF RETURNS AS RISK MEASURE

This subsection deals with the first stage of the methodology discussed above. Namely, it deals with obtaining several optimal portfolios according to the Markowitz's (1952) approach, this is, considering the volatility of returns as the risk measure of portfolios.

Although some more were calculated, finally only eight were selected due to the fact that later they have been compared with VaR and these portfolios collect the most interesting data for it.

Therefore, one of the first steps to obtain these optimal portfolios is to define the total risk (volatility) of a portfolio relating the cells of systematic risk and no systematic risk. The process to calculate the systematic risk was add the total weight of each asset to the square, multiplied by its own variance.

Table 3: Estimate of systematic risk

SUMA =O808*O808*D796+O809*O809*E797+O810*O810*F798+O811*O811*G799+O812*O812*H800+O813*O813*I801+O814*O814*J802

	H	I	J	K	L	M	N	O	P	
793										
794										
795	SIEMENS GAMESA	INMOBILIARIA COLONIAL	AMADEUS							
796	0,000102261705	0,000063121419	0,000067713042							
797	0,000127773653	0,000083536054	0,000087991949							
798	0,000128944049	0,000086638570	0,000077306361							
799	0,000109487154	0,000073961538	0,000097910671							
800	0,000618809	0,000106210084	0,000127534034							
801	0,000106210084	0,000221171	0,000070674440							
802	0,000127534034	0,000070674440	0,000183323							
803										
804										
805						Portfolio				
806										
807	Portfolio 7	Portfolio 8							SYSTEMATIC RISK	NO SYS
808	0,01760%	0,01908%					ABERTIS	0,0001000000	=O808*O808*D796+O809*(	
809	0,08698%	0,08993%					AENA	0,9103362283		
810							BANKINTER	0,0001000000		
811							GRIFOLS	0,0001000000		
812							SIEMENS GAMESA	0,0001000000		
813							INMOBILIARIA COLONIAL	0,0001000000		
814							AMADEUS	0,0891637717		
815								1,00		
816										

Source: Own development.

Subsequently, to obtain the non-systematic risk is necessary to extract the following data table. For a better compression, if the reader sees the first line, *Xi* is the equivalent to the weight in the portfolio of the asset ABERTIS and *Xj* is the equivalent of the weight in the portfolio of the asset AENA. When they are multiplied, and multiply that result by two and their covariance, the last calculation is to add all these relations of the assets between them and this number is the no systematic risk. In other words, the diversifiable risk.

Table 4: Estimate of non-systematic risk

=SUMA(R820:R840)

	Xi*Xj	2*Xi*Xj	COVARIANZA	2*Xi*Xj*COVAR
ABERTIS - AENA	0,00910%	0,000182067	0,000080870394	0,0000000147238
ABERTIS - BANKINTER	1E-08	2E-08	0,000089935698	0,0000000000018
ABERTIS - GRIFOLS	0,00000%	2E-08	0,000063941220	0,0000000000013
ABERTIS - SIEMENS GAMESA	1E-08	2E-08	0,000102261705	0,0000000000020
ABERTIS - INMOBILIARIA COLONIAL	1E-08	2E-08	0,000063121419	0,0000000000013
ABERTIS - AMADEUS	0,00089%	1,78328E-05	0,000067713042	0,0000000012075
AENA -BANKINTER	9,10336E-05	0,000182067	0,000080405060	0,0000000146391
AENA - GRIFOLS	9,10336E-05	0,000182067	0,000084159376	0,0000000153227
AENA - SIEMENS GAMESA	9,10336E-05	0,000182067	0,000127773653	0,0000000232634
AENA - INMOBILIARIA COLONIAL	9,10336E-05	0,000182067	0,000083536054	0,0000000152092
AENA - AMADEUS	0,081169012	0,162338023	0,000087991949	0,0000142844391
BANKINTER - GRIFOLS	1E-08	2E-08	0,000082330554	0,0000000000016
BANKINTER - SIEMENS GAMESA	1E-08	2E-08	0,000128944049	0,0000000000026
BANKINTER - INMOBILIARIA COLONIAL	1E-08	2E-08	0,000086638570	0,0000000000017
BANKINTER - AMADEUS	8,91638E-06	1,78328E-05	0,000077306361	0,0000000013786
GRIFOLS - SIEMENS GAMESA	1E-08	2E-08	0,000109487154	0,0000000000022
GRIFOLS - INMOBILIARIA COLONIAL	1E-08	2E-08	0,000073961538	0,0000000000015
GRIFOLS - AMADEUS	8,91638E-06	1,78328E-05	0,000097910671	0,0000000017460
SIEMENS GAMESA - INMOBILIARIA COLONIAL	1E-08	2E-08	0,000106210084	0,0000000000021
SIEMENS GAMESA -AMADEUS	8,91638E-06	1,78328E-05	0,000127534034	0,0000000022743
INMOBILIARIA COLONIAL - AMADEUS	8,91638E-06	1,78328E-05	0,000070674440	0,0000000012603
				=SUMA(R820:R840)

Source: Own development.

Completed these steps, the total risk is the add of these two risks. Finally, the expected return was calculated as the weight of every asset multiplied by its matemathical hope.

Table 5: Estimate of Expected Return

=O808*D770+O809*D773+O810*D776+O811*D779+O812*D782+O813*D785+O814*D788

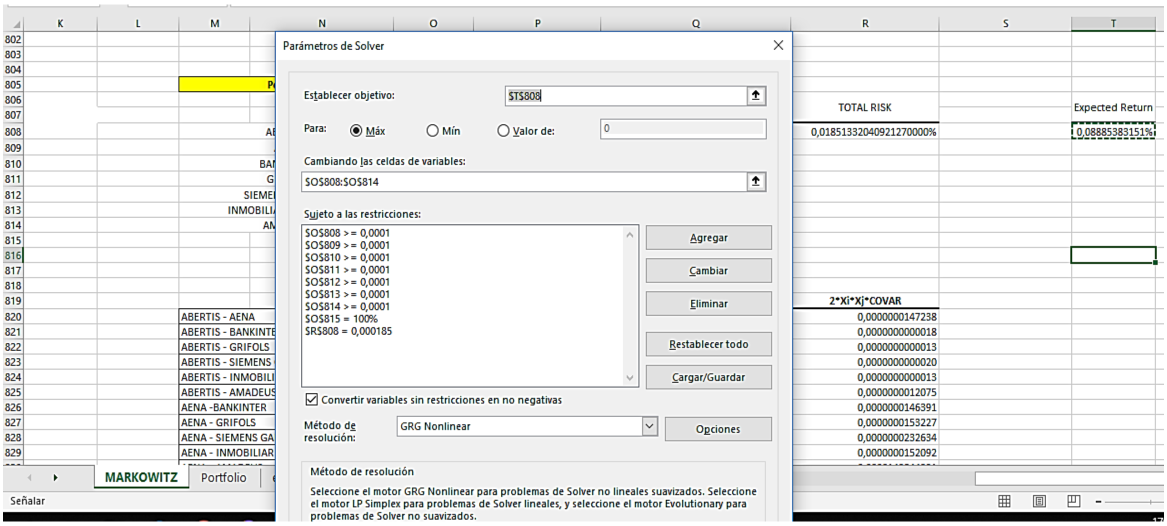
Portfolio		SYSTEMATIC RISK	NO SYSTEMATIC RISK	TOTAL RISK	Expected Return
ABERTIS	14,2857142857	36,870405078%	77,253348048%	114,1237531263360000000%	i+O814*D788
AENA	14,2857142857				
BANKINTER	14,2857142857				
GRIFOLS	14,2857142857				
SIEMENS GAMESA	14,2857142857				
INMOBILIARIA COLONIAL	14,2857142857				
AMADEUS	14,2857142857				
	100,00				

Source: Own development.

It is important to say that before doing the next step, the cells of the weight of assets are equals and all of them togheter add up to one hundred. Such as the portfolio is composed by seven assets the calculation “=100/7” was used in every cell.

With all these steps, the portfolio could be calculated. As a result of this, it is necessary to activate the function Solver and to apply some restrictions. First, it must be found the maximum expect value of the portfolio. In addittion, all the assets at least must have a value of one part per thousand, and all the assets must add up one hundred per cent. Lastly, one objective of assumed risk was selected, and in every portfolio it has been increasing. On account of this process, it is possible to obtain an efficient portfolios, as it has been explained above, it consist in selecting a level of risk, and selecting the portfolio with maximum expected return. The function Solver, with these restrictions, solutionates the problem of doing complex calculations and of finding the most optimized portfolio.

Table 6: Restrictions of Solver



Source: Own development.

In order to see the evolution of the portfolios as the risk increases a graph was drawn. The date of risk and expect return of the portfolios selected are the following:

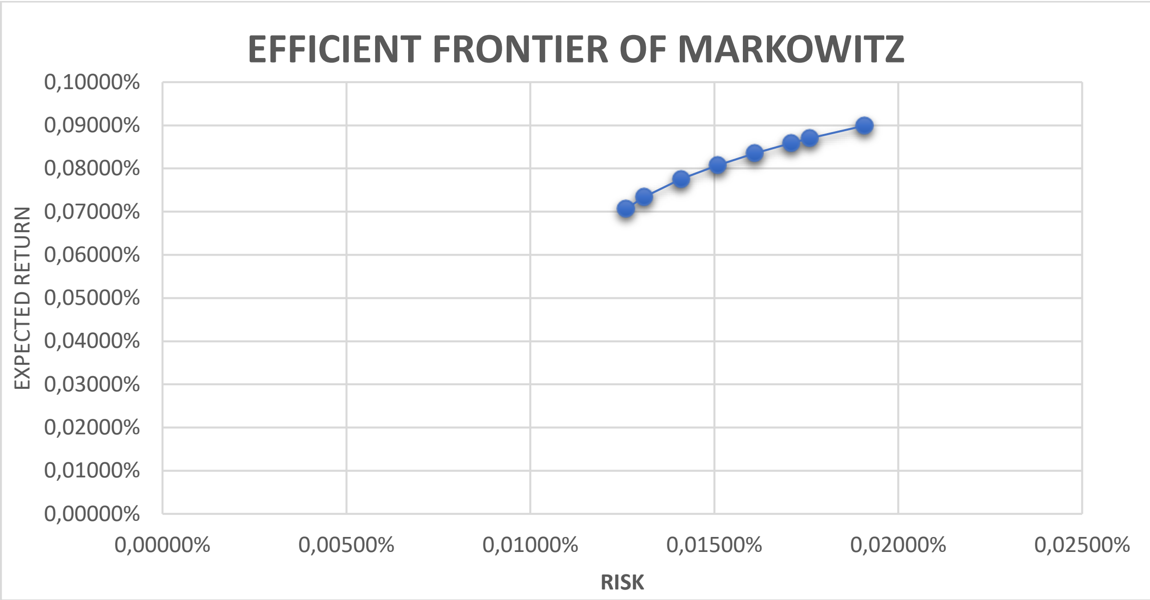
Table 7: Risk and Expected Return of final portfolios selected

	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5	Portfolio 6	Portfolio 7	Portfolio 8
Risk	0,01260%	0,01310%	0,01410%	0,01510%	0,01610%	0,01710%	0,01760%	0,01908%
Expected Return	0,07065%	0,07341%	0,07757%	0,08078%	0,08349%	0,08587%	0,08698%	0,08993%

Source: Own development.

And the graph was the next:

Figure 1: Efficient Frontier of Markowitz



Source: Own development.

If this graph is compared with the theorical model it could be observed that both of them have similar characteristics. In the horizontal axis it is posible to identify the level of risk, and in the vertical axis the expected return. The points on the graph represent the maximum expected value for every level of risk. When these points are united it is formed a line dished. With all of these elements it is formed the efficient frontier of Markowitz.

Figure 2: Theorical Efficient Frontier of Markowitz

Source: Marín, Rubio, and Mas-Colell, 2001. p.243.

Some criticisms (Michaud, 1989) have been made to Markowitz's model, but they are discussed in the discussion section.

4.2. OPTIMAL PORTFOLIOS UNDER THE HISTORICAL APPROACH TO VaR AS RISK MEASURE

This subsection discusses the process followed to obtain optimal portfolios under the historical approach to VaR (at both confidence levels, 90% and 95%) as risk measure. As when obtaining the optimal portfolios according to Markowitz's (1952) approach in the previous subsection, the spreadsheet Excel together with its tool Solver have been used to calculate optimal portfolios under historical VaR.

In particular, for comparison reason, the study is focus on obtaining the portfolios that minimize risk measured by the historical approach to VaR for each expected return of the eight optimal portfolios according to Markowitz's approach obtained in the previous stage. Therefore, the perspective considered to obtain these optimal portfolios has been to minimize portfolio's risk measured by historical VaR for each expected return of the portfolios obtained in the previous subsection. The following discussion details the steps followed to prepare the spreadsheet of Excel and its tool Solver to make the calculations.

First of all, the expected return of the portfolio is calculated multiplying the weight of every asset in the portfolio by its individual expected return and adding all of them. To clarify, the expected return of the stocks of each company was specified in the section extraction of data.

Table 8: Expected return of the portfolio

SUMA =(C765*I770)+(F765*I771)+(I765*I772)+(L765*I773)+(O765*I774)+(R765*I775)+(U765*I776)

	A	B	C	D	E	F	G	H	I	J	K	L	M
753		12-mar-2018	18,58	-0,0029558	12-mar-2018	170,45	-0,00058651	12-mar-2018	8,7	0,00207	12-mar-2018	23,68	0,0033841
754		13-mar-2018	18,58	0	13-mar-2018	171,35	0,00526625	13-mar-2018	8,696	-0,00046	13-mar-2018	23,32	-0,0153194
755		14-mar-2018	18,61	0,0016133	14-mar-2018	170,85	-0,00292227	14-mar-2018	8,654	-0,00484	14-mar-2018	22,87	-0,0194854
756		15-mar-2018	18,58	-0,0016133	15-mar-2018	170,45	-0,00234398	15-mar-2018	8,694	0,00461	15-mar-2018	23,33	0,0199141
757		16-mar-2018	18,165	-0,0225891	16-mar-2018	171	0,003221558	16-mar-2018	8,698	0,00046	16-mar-2018	23,34	0,0004285
758		19-mar-2018	18,21	0,0024742	19-mar-2018	169,95	-0,00615928	19-mar-2018	8,69	-0,00092	19-mar-2018	22,45	-0,038878
759		20-mar-2018	18,22	0,000549	20-mar-2018	169,75	-0,00117751	20-mar-2018	8,66	-0,00346	20-mar-2018	22,27	-0,0080501
760		21-mar-2018	18,18	-0,0021978	21-mar-2018	167,45	-0,01364197	21-mar-2018	8,55	-0,01278	21-mar-2018	22,44	0,0076046
761		23-mar-2018	18,195	0,0008247	23-mar-2018	162,35	-0,0309303	23-mar-2018	8,236	-0,03742	23-mar-2018	21,98	-0,0207121
762		26-mar-2018	18,19	-0,0002748	26-mar-2018	161,55	-0,00493981	26-mar-2018	8,284	0,00581	26-mar-2018	21,76	-0,0100595
763													
764													
765		EXP.RETURN	0,000104225		EXP.RETURN	0,000922619		EXP.RETURN	0,000199839		EXP.RETURN	0,000113067	
766													
767													
768													
769													
770							ABERTIS		14,2857142857				
771							AENA		14,2857142857				
772							BANKINTER		14,2857142857				
773							GRIFOLS		14,2857142857				
774							SIEMENS GAMESA		14,2857142857				
775							INMOBILIARIA COLONIAL		14,2857142857				
776							AMADEUS		14,2857142857				
777									100				
778													
779													
780							E_r portfolio	)+(U765*I776)					

Source: Own development.

In addition, it is necessary to know the expected daily returns for all the assets used in the portfolio. To find out, in every single day, the weight of every company in the portfolio was multiplied by the result obtained using the neperian logarithm with the division of one day and its previous day. Then, all of them were added, and this process is done with every day of the data.

Table 9: Estimate of the expected daily returns

SUMA | =(I770*D6)+(I771*G6)+(I772*J6)+(I773*M6)+(I774*P6)+(I775*S6)+(I776*V6)

	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
753	12-mar-2018	18,58	-0,0029558	12-mar-2018	170,45	-0,00058651	12-mar-2018	8,7	0,00207	12-mar-2018	23,68	0,0033841	12-mar-2018	12,645	0,001582905	12-mar-2018
754	13-mar-2018	18,58	0	13-mar-2018	171,35	0,00526625	13-mar-2018	8,696	-0,00046	13-mar-2018	23,32	-0,0153194	13-mar-2018	12,42	-0,0179538	13-mar-2018
755	14-mar-2018	18,61	0,0016133	14-mar-2018	170,85	-0,00292227	14-mar-2018	8,654	-0,00484	14-mar-2018	22,87	-0,0194854	14-mar-2018	12,88	0,036367644	14-mar-2018
756	15-mar-2018	18,58	-0,0016133	15-mar-2018	170,45	-0,00234398	15-mar-2018	8,694	0,00461	15-mar-2018	23,33	0,0199141	15-mar-2018	12,91	0,002326484	15-mar-2018
757	16-mar-2018	18,165	-0,0225891	16-mar-2018	171	0,003221558	16-mar-2018	8,698	0,00046	16-mar-2018	23,34	0,0004285	16-mar-2018	12,905	-0,00038737	16-mar-2018
758	19-mar-2018	18,21	0,0024742	19-mar-2018	169,95	-0,00615928	19-mar-2018	8,69	-0,00092	19-mar-2018	22,45	-0,038878	19-mar-2018	13,03	0,009639558	19-mar-2018
759	20-mar-2018	18,22	0,000549	20-mar-2018	169,75	-0,00117751	20-mar-2018	8,66	-0,00346	20-mar-2018	22,27	-0,0080501	20-mar-2018	13,29	0,019757482	20-mar-2018
760	21-mar-2018	18,18	-0,0021978	21-mar-2018	167,45	-0,01364197	21-mar-2018	8,55	-0,01278	21-mar-2018	22,44	0,0076046	21-mar-2018	13,625	0,024894468	21-mar-2018
761	23-mar-2018	18,195	0,0008247	23-mar-2018	162,35	-0,0309303	23-mar-2018	8,236	-0,03742	23-mar-2018	21,98	-0,0207121	23-mar-2018	13,27	-0,02640049	23-mar-2018
762	26-mar-2018	18,19	-0,0002748	26-mar-2018	161,55	-0,00493981	26-mar-2018	8,284	0,00581	26-mar-2018	21,76	-0,0100595	26-mar-2018	13,115	-0,01174924	26-mar-2018
763																
764																
765	EXP.RETURN	0,000104225		EXP.RETURN	0,000922619		EXP.RETURN	0,000199839		EXP.RETURN	0,000113067		EXP.RETURN	0,000170151		EXP.RETURN
766																
767																
768																
769																
770						ABERTIS		14,2857142857					Portfolio Expected Returns			
771						AENA		14,2857142857							26-mar-2015	=(I770*D6)+
772						BANKINTER		14,2857142857							27-mar-2015	0,578802275
773						GRIFOLS		14,2857142857							30-mar-2015	1,845558012
774						SIEMENS GAMESA		14,2857142857							31-mar-2015	0,408003713
775						INMOBILIARIA COLONIAL		14,2857142857							01-abr-2015	1,032918163
776						AMADEUS		14,2857142857							02-abr-2015	0,692704558
777								100							07-abr-2015	0,430628907

Source: Own development.

After obtaining the result, the following is to observe which is the number of observations in the data. According to the data specified in the corresponding section of the study, there are 757 observations (n=757). In order to calculate the historical VaR at the 90% of confidence of the daily returns of a portfolio, the number of observations must be multiplied by 10% in order to find the position of the lowest return of the portfolio such that the probability of obtaining a lower or equal return in the historical distribution is 10%. Moreover, in the case of 95% VaR, all the steps had been equal but multiplying the number of observations by 5%. To illustrate and to aboid redundancy, only the 95% VaR had been adjusted in this final degree dissertation.

As can be observed in the image below, the 5% of n, being n 757 is 37’85. That position does not exist, because for every day, only exist one return in the form of entire unity. Therefore, the solution is interpolate between the thirty-seventh lowest regular return and the thirty-eighth lowest regular return. First, these positions were looking for using the formula in Excel “K.ESIMO.MENOR”, and with the two positions obtained, the interpolation was calculated.

Table 10: Research of the thirty-seventh lowest regular return

SUMA =K.ESIMO.MENOR(Q771:Q1527;37)

	G	H	I	N	P	Q
767						
768						
769						
770	ABERTIS		14,2857142857	Portfolio Expected Returns		
771	AENA		14,2857142857		26-mar-2015	-0,025286239
772	BANKINTER		14,2857142857		27-mar-2015	0,578802275
773	GRIFOLS		14,2857142857		30-mar-2015	1,845558012
774	SIEMENS GAMESA		14,2857142857		31-mar-2015	0,408003713
775	INMOBILIARIA COLONIAL		14,2857142857		01-abr-2015	1,032918163
776	AMADEUS		14,2857142857		02-abr-2015	0,692704558
777			100		07-abr-2015	0,430628907
778					08-abr-2015	-0,630354558
779					09-abr-2015	1,060597255
780	E_r portfolio	0,036010079			10-abr-2015	0,848831546
781					13-abr-2015	-0,210296943
782	n	757			14-abr-2015	-1,471052577
783	5%	37,85			15-abr-2015	-0,125482505
784					16-abr-2015	-1,241328487
785	37	771:Q1527;37)			17-abr-2015	-1,862337561
786	38	-1,61398167			20-abr-2015	-0,223885154
787					21-abr-2015	0,969734124
788	Var 5%	-1,617446996			22-abr-2015	-0,212357346
789					23-abr-2015	-1,297454797

Source: Own development.

Table 11: Estimate of interpolation

SUMA =H785+(H786-H785)*0,85

	G	H
778		
779		
780	E_r portfolio	0,036010079
781		
782	n	757
783	5%	37,85
784		
785	37	-1,637083842
786	38	-1,61398167
787		
788	Var 5%	I786-H785)*0,85

Source: Own development.

Once the historical VaR of the portfolio's returns is defined in an Excell cell as the interpolation between these two positions, that cell will be established as the objective of the optimization problem specified in Solver. In particular, given that the historical VaR at the 95% of confidence is defined as the lowest return of the historical distribution is 5% (so it is sittuated to the left of queued in historical distribution) and, therefore the historical VaR will be defined by a negative return, the risk measured by historical VaR is minimized when historical VaR is maximized. Namely, the objective of the optimization problem specified in Solver is to find the stock weights in the portfolio that maximize the historical VaR of the historical distribution of the returns.

In order to compare the portfolios effectively, this optimization problem is solved given the expected return of each one of the eight optimal portfolios according to Markowitz's

(1952) approach obtained in the previous stage of the study. Furthermore, all the assets at least must have a value of one part per thousand, and all the assets must add up one hundred per cent. In other words, the Markowitz conditions are mainteined.

The same procedure is followed for the case of the historical VaR at the 90% of confidence. At the end, eight set of portfolios, in which each set consist in three portfolios, are obtained. Each portfolio in each one of these sets has the same expected return, but, one is optimal according to Markowit's (1952) approach, namely under volatility as risk measure (obtained in the previous stage of the study), another is optimal under historical VaR at the 95% of confidence, and the last one is optimal under historical VaR at the 90% of confidence. The following section of the study compares the optimal weights of the stocks according to the different risk measures in portfolios with the same expected return.

Some criticisms Čorkalo (2011) have been made to the VaR model, but they are discussed in the discussion section.

5. RESULTS OF THE ANALYSIS

5.1. Markowitz – 90% Value at Risk

In this section Markowitz and VaR at the 90% of confidence have been compared asset by asset of the portfolio. This section has been elaborated in an objective way, without any type of subjectivity. The finality is to see how change the elaboration of the portfolio using a complex method as Markowitz or one more simple such as VaR with a 90% confidence level.

5.1.1. ABERTIS

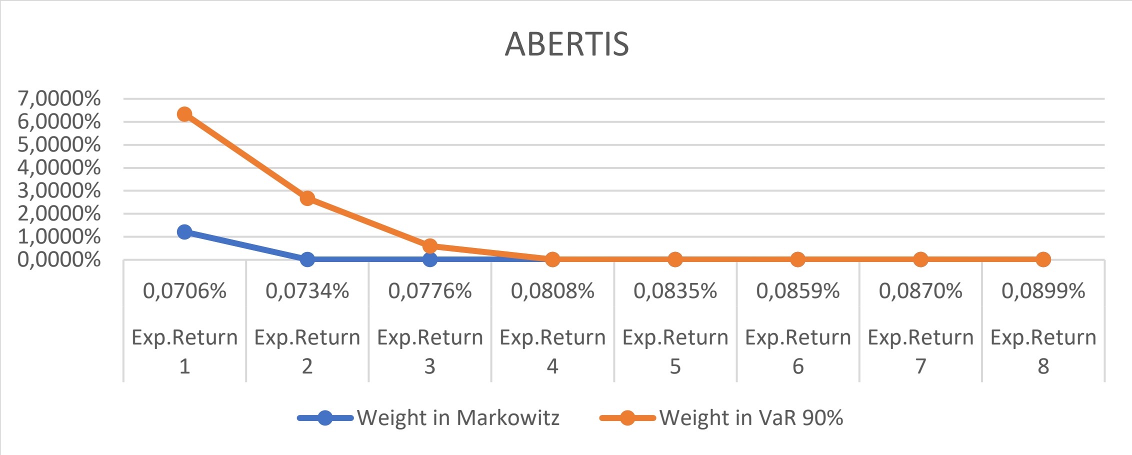
Table 12: Weights in the portfolios for Abertis, Markowitz and VaR 90%

	Exp.Return 1	Exp.Return 2	Exp.Return 3	Exp.Return 4	Exp.Return 5	Exp.Return 6	Exp.Return 7	Exp.Return 8
	0,0706%	0,0734%	0,0776%	0,0808%	0,0835%	0,0859%	0,0870%	0,0899%
Weight in Markowitz	1,2125%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	0,0099%
Weight in VaR 90%	6,3376%	2,6698%	0,5982%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%

Source: Own development.

In Abertis, the main difference resides in the first expected return of 0,0706%. While Markowitz's (1952) approach barely invest 1%, the approach that employs VaR as risk measure was investing more than 6% of the portfolio in this company. In the second expected return of 0,0734%, being this higher of the first level (to clarify, the lowest level of expected return will always be the first level, and in every level the expected return will be increasing since achieve the last level with the higher expected return) it could be observed, how in both cases the weight of Abertis in the portfolio is decreasing. Most acutely, in the case of VaR as risk measure with a 90% confidence level, which loses approximately 4% against a little more than 1% in Markowitz's (1952) case. In the third expected return 0,0776%, the difference is barely significant, and they are finally coincident in the fourth expected return of 0,0808% and the following until the last expected return of 0,0899%. Moreover, these similar expected returns are coincident with the condition of that every asset must have at least one weight in the portfolio of one part per thousand. Taking it into account, and observing that the maximum weight of Abertis looking at two approaches is around 6%, it could be said that none of both approaches considers Abertis such an interesting asset to minimize risk.

Figure 3: Graph of weights in the portfolios for Abertis, Markowitz and VaR 90%



Source: Own development.

5.1.2. AENA

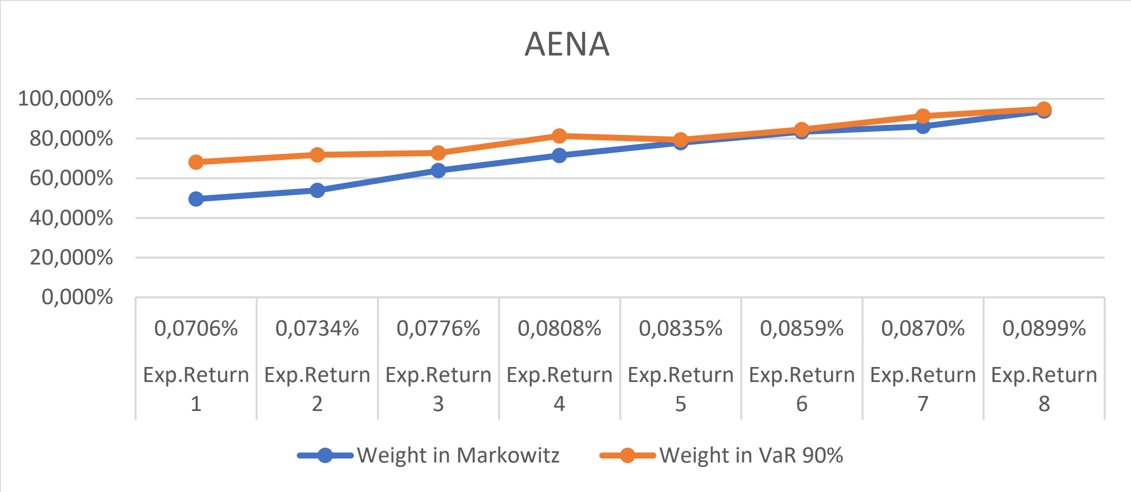
Table 13: Weights in the portfolios for AENA, Markowitz and VaR 90%

	Exp.Return 1	Exp.Return 2	Exp.Return 3	Exp.Return 4	Exp.Return 5	Exp.Return 6	Exp.Return 7	Exp.Return 8
	0,0706%	0,0734%	0,0776%	0,0808%	0,0835%	0,0859%	0,0870%	0,0899%
Weight in Markowitz	49,501%	53,816%	63,837%	71,409%	77,923%	83,411%	86,201%	93,879%
Weight in VaR 90%	68,085%	71,814%	72,802%	81,328%	79,380%	84,489%	91,357%	94,915%

Source: Own development.

This case is significatively different of Abertis. As could be observed in the graphic, the weight of the asset in the portfolio is increasing in the expected return according to both approaches to portfolio optimization. The weight in the first expected return of 0,0706% is high, according to Markowitz's (1952) approach is nearly of 50% and according to that employs VaR as risk measure with a 90% confidence level is nearly of 70%. In spite of this difference between them of almost 20% in the first expected return and the second expected return of 0,0734%, this distance is reduced until being practically coincident since the fifth expected return of 0,0835% until the last expected return of 0,0899%, when they are virtually the same. The two approaches are constant in their increase, except in the case of the approach VaR with 90% of confidence level at fourth expected return 0,0808%, being in this case higher than the fifth expected return. The both approaches considers Aena such an interesting asset in order to minimize risk.

Figure 4: Graph of weights in the portfolios for AENA, Markowitz and VaR 90%



Source: Own development

5.1.3. BANKINTER

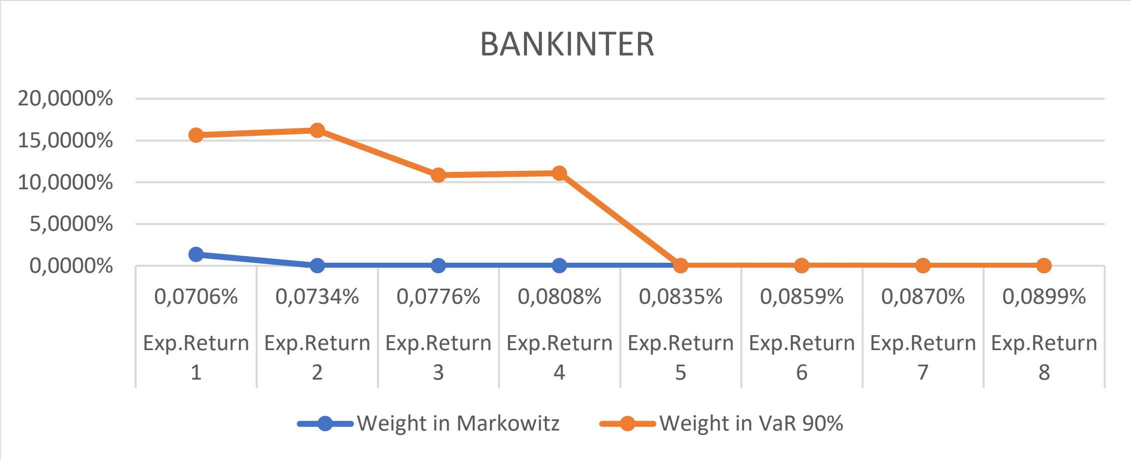
Table 14: Weights in the portfolios for Bankinter, Markowitz and VaR 90%

	Exp.Return 1	Exp.Return 2	Exp.Return 3	Exp.Return 4	Exp.Return 5	Exp.Return 6	Exp.Return 7	Exp.Return 8
	0,0706%	0,0734%	0,0776%	0,0808%	0,0835%	0,0859%	0,0870%	0,0899%
Weight in Markowitz	1,3508%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%
Weight in VaR 90%	15,6498%	16,2129%	10,8408%	11,0838%	0,0100%	0,0231%	0,0100%	0,0100%

Source: Own development.

The case of Bankinter has similitudes with the case of Abertis. However, the distance between the two approaches until the fifth expected return of 0,0835% is bulkier. In the Markowitz's (1952) approach the weight of Bankinter is firstly around 1% and then it falls to the minimum that it could be, one part per thousand. In VaR approach at the 90% of confidence level, the invest until the fourth expected return of 0,0808% is moderated, between 15% and 10%, but when the expected return increases more than those levels, it falls to the minimum condition too. Bankinter is complicated to perform from Solver point of view. On one side, both methods are coincident from an expected return of 0,0835%, point at which this asset becomes also not convenient to minimize risk measured according to the approach that employs VaR as risk measure at the 90% of confidence level. On the other side, it seems that Markowitz's (1952) approach discards Bankinter since the beginning (at least for these levels of risk) but, VaR approach at 90% of confidence level invest one significatively part in their portfolios until the fifth expected return of 0,0835%. Maybe the results of VaR approach at 90% of confidence level does not invest a large proportion of this company in their portfolios, but the results suggest that this method considers Bankinter a good asset to diversify and reduce the risk levels.

Figure 5: Graph of weights in the portfolios for Bankinter, Markowitz and VaR 90%



Source: Own development.

5.1.4. GRIFOLS

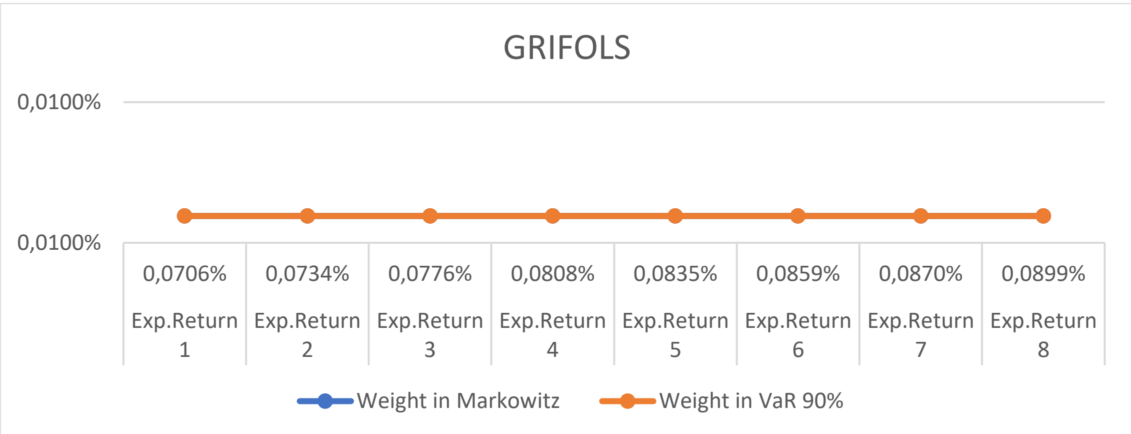
Table 15: Weights in the portfolios for Grifols, Markowitz and VaR 90%

	Exp.Return 1	Exp.Return 2	Exp.Return 3	Exp.Return 4	Exp.Return 5	Exp.Return 6	Exp.Return 7	Exp.Return 8
	0,0706%	0,0734%	0,0776%	0,0808%	0,0835%	0,0859%	0,0870%	0,0899%
Weight in Markowitz	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%
Weight in VaR 90%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%

Source: Own development.

Until now, the case of Grifols is the unique case in which both approaches have done the same investments in all the expected returns. The totally of the investments are coincident with the minimum condition of their weights at their portfolios. It is very clear that none of both approaches considers this company as the less engaging and it makes sense. If the reader setback to the Table 1 he will observe that Grifols is the second company with less expected return, and its variance is as high as the other companies' variance with higher expected return.

Figure 6: Graph of weights in the portfolios for Grifols, Markowitz and VaR 90%



Source: Own development.

5.1.5. SIEMENS GAMESA

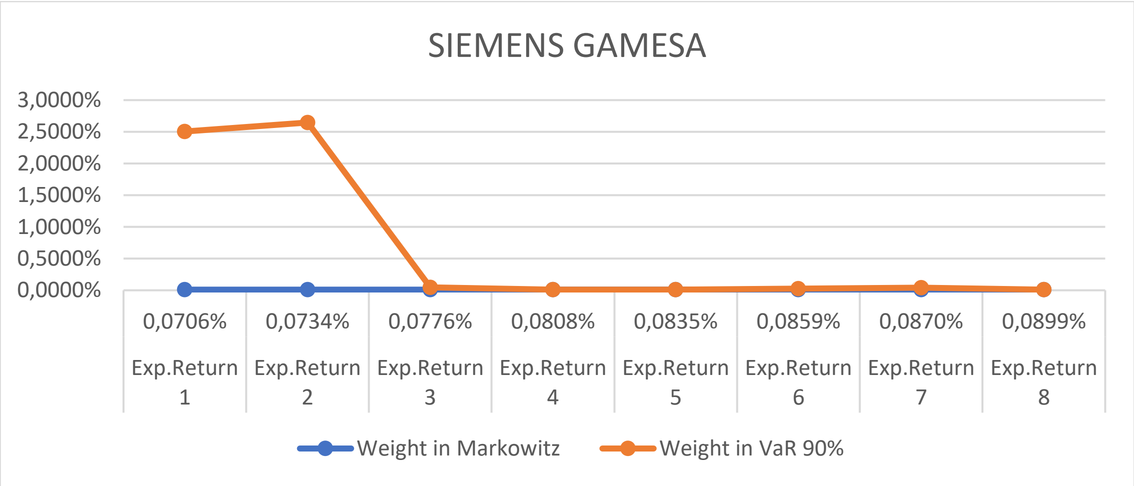
Table 16: Weights in the portfolios for Siemens Gamesa, Markowitz and VaR 90%

	Exp.Return 1	Exp.Return 2	Exp.Return 3	Exp.Return 4	Exp.Return 5	Exp.Return 6	Exp.Return 7	Exp.Return 8
	0,0706%	0,0734%	0,0776%	0,0808%	0,0835%	0,0859%	0,0870%	0,0899%
Weight in Markowitz	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%
Weight in VaR 90%	2,5040%	2,6463%	0,0475%	0,0100%	0,0100%	0,0237%	0,0400%	0,0100%

Source: Own development.

In the case of Siemens Gamesa, it is possible to see the resembling with Bankinter. But in this case, the distance between the two approaches until the third expected return of 0,0776% is not significant. In Markowitz's (1952) approach the weight of Siemens Gamesa is directly the minimum that it could be, one part per thousand. In VaR approach at the 90% of confidence level, the invest until the second expected return of 0,0734% is low, around 2,5%, but when the expected return increases as third expected return of 0,0776%, the VaR approach at 90% of confidence level falls to the minimum condition too. The analysis looking at Siemens Gamesa is very closely to Bankinter analysis. On one side, both approaches are coincident since the third expected return. On the other side, it seems that Markowitz's (1952) approach discards Siemens Gamesa directly but, invest a small proportion of this company in their expected returns until the third expected return of 0,0776%. It might be said that neither of the two approaches consider Siemens Gamesa as an interesting asset to invest.

Figure 7: Graph of weights in the portfolios for Siemens Gamesa, Markowitz and VaR 90%



Source: Own development.

5.1.6. INMOBILIARIA COLONIAL

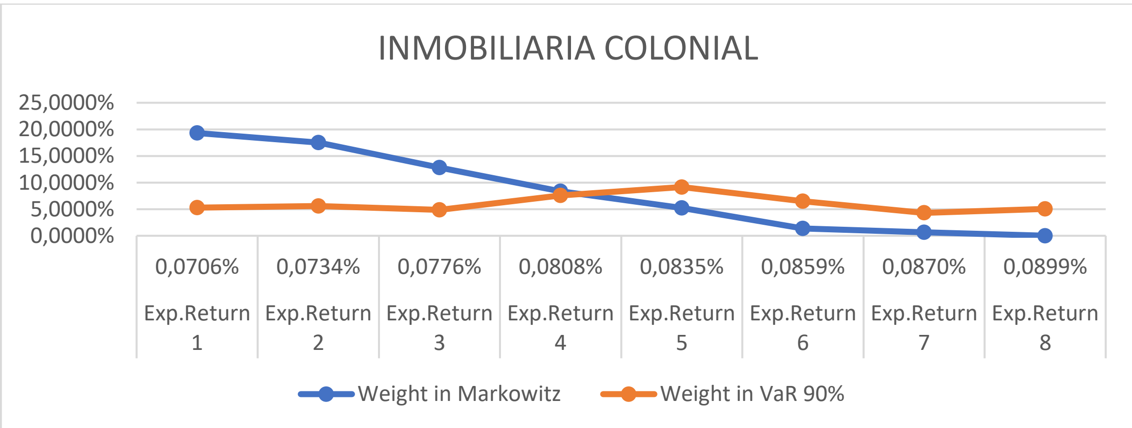
Table 17: Weights in the portfolios for Inmobiliaria Colonial, Markowitz and VaR 90%

	Exp.Return 1	Exp.Return 2	Exp.Return 3	Exp.Return 4	Exp.Return 5	Exp.Return 6	Exp.Return 7	Exp.Return 8
	0,0706%	0,0734%	0,0776%	0,0808%	0,0835%	0,0859%	0,0870%	0,0899%
Weight in Markowitz	19,3107%	17,5337%	12,8044%	8,3640%	5,2464%	1,3627%	0,6350%	0,0100%
Weight in VaR 90%	5,3000%	5,5741%	4,8592%	7,5483%	9,1599%	6,4699%	4,2981%	5,0348%

Source: Own development.

Inmobiliaria Colonial is unique in this analysis. Paying attention in Markowitz's (1952) approach it is clearly observable that it is decreasing as the expected return is increasing. Additionally, the levels of investment are significative, especially at the early portfolios, and the decrease is very clean. In other words, when the expected return increases, the weight in this company always is lower than the previous. Now, looking at VaR approach at 90% of confidence level, it begins with a lower weight in the portfolio than Markowitz's (1952) approach, around a 15%. Moreover, it has a lineal evolution being practically coincident with Markowitz in the fourth expected return of 0,0808% and then, overcoming it since the last expected return of 0,0899%, where it has a weight very similar to the first of them. The conclusions in this case are not coincident for both approaches. If Markowitz's (1952) is used, Inmobiliaria Colonial is an interesting company to invest, in order to minimize risk but only for small levels of expected return. For VaR approach at 90% of confidence level, it looks like a company to do a little diversification but being this constant for all of its portfolios.

Figure 8: Graph of weights in the portfolios for Inmobiliaria Colonial, Markowitz and VaR 90%



Source: Own development.

5.1.7. AMADEUS

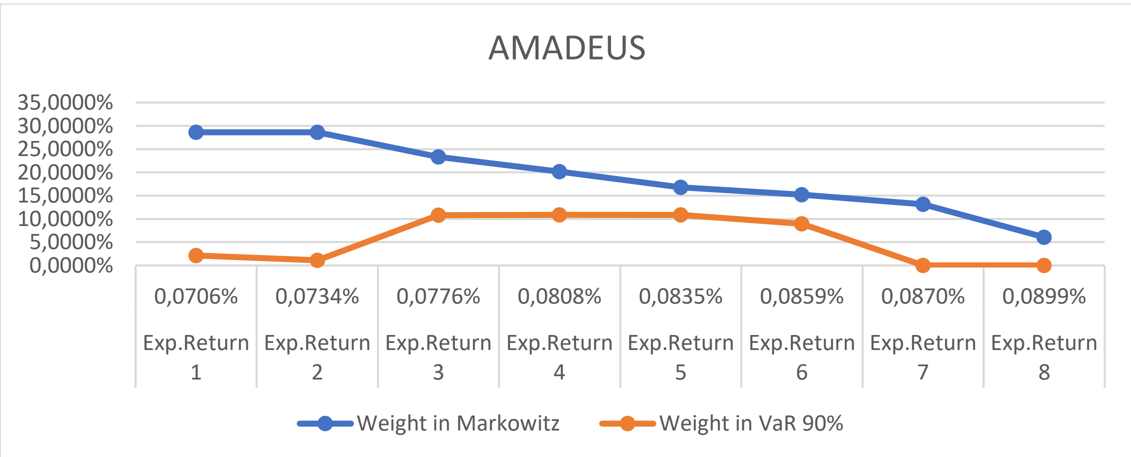
Table 18: Weights in the portfolios for Amadeus, Markowitz and VaR 90%

	Exp.Return 1	Exp.Return 2	Exp.Return 3	Exp.Return 4	Exp.Return 5	Exp.Return 6	Exp.Return 7	Exp.Return 8
	0,0706%	0,0734%	0,0776%	0,0808%	0,0835%	0,0859%	0,0870%	0,0899%
Weight in Markowitz	28,6047%	28,6104%	23,3184%	20,1874%	16,7903%	15,1866%	13,1239%	6,0707%
Weight in VaR 90 %	2,1036%	1,0728%	10,8328%	10,8428%	10,8498%	8,9744%	0,0100%	0,0100%

Source: Own development.

In Amadeus case, the two approaches have several differences. Markowitz's (1952) approach begins with an important investment in Amadeus, then it keeps this weight until the second expected return of 0,0734%, in which it begins to fall, but in this case, the minimum to these levels of expected returns is 6 %, not the minimum condition of one part per thousand. The VaR approach at 90% of confidence level is single, because it begins with a level barely significative, between 2% and 1%, at the third expected return of 0,0776% it increases to levels around the 10% of the weight of the expected returns until the seventh expected return of 0,0870% in which the weight of Amadeus is the same that minimum condition applied. The optimal weight for this asset according to Markowitz's (1952) approach is always higher than its optimal weight considering VaR at the 90% of confidence as risk measure. Although the differences between the two approaches are higher at early expected returns, but as the expected return is growing the distance between them is lower. Nevertheless, the asset always is relevant to minimize risk only according to Markowitz's (1952) approach. The results of Markowitz's (1952) approach indicates that is an important asset to minimize risk for low levels of expected return but its weight in the portfolio should be reduced for high levels of expected return. The results of VaR approach at 90% of confidence level are not conclusive.

Figure 9: Graph of weights in the portfolios for Amadeus, Markowitz and VaR 90%



Source: Own development.

5.2. Markowitz – Value at Risk 95%

5.2.1. ABERTIS

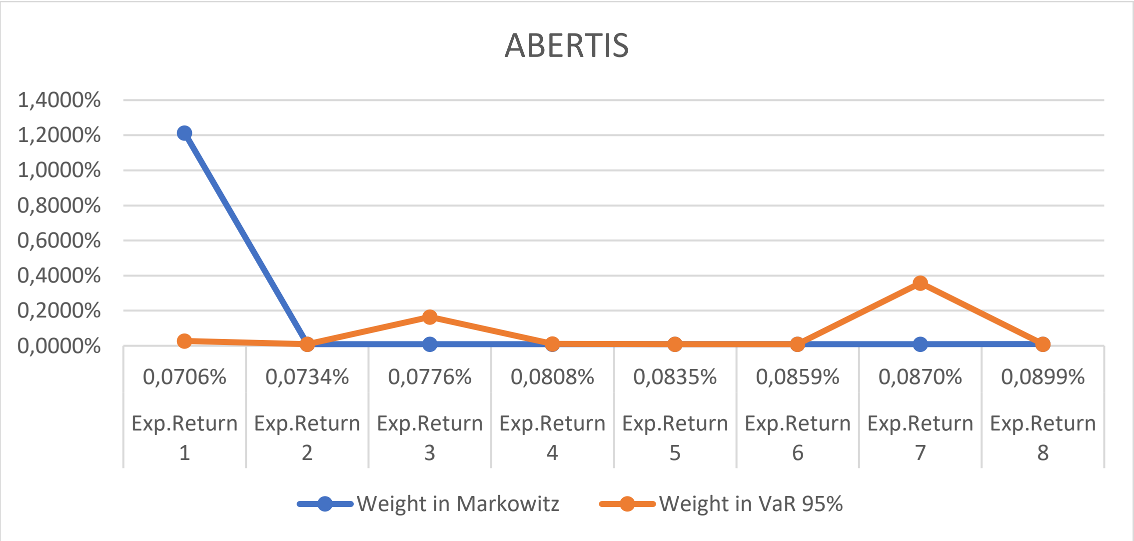
Table 19: Weights in the portfolios for Abertis, Markowitz and VaR 95%

	Exp.Return 1	Exp.Return 2	Exp.Return 3	Exp.Return 4	Exp.Return 5	Exp.Return 6	Exp.Return 7	Exp.Return 8
	0,0706%	0,0734%	0,0776%	0,0808%	0,0835%	0,0859%	0,0870%	0,0899%
Weight in Markowitz	1,2125%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	0,0099%
Weight in VaR 95%	0,0277%	0,0100%	0,1635%	0,0103%	0,0101%	0,0100%	0,3577%	0,0100%

Source: Own development.

In Abertis, it is possible to observe some changes with the new confidence level of VaR. It could be observed that the VaR approach at 95% of confidence level is not regular, it has peaks in the expected returns three of 0,0776% and seven of 0,0870%. In spite of this, paying attention to the levels of investment, they are under 0'5% thus these weights in the expected returns are not representative. Analysing both methods, Abertis seems useless. In the major part of the expected returns analysed for the two approaches only invest the minimum condition of one part per thousand, and in which that they invest one part the percentage is very low, which implies that the results under VaR at the 95% of confidence are closer to the Markowitz's (1952) approach than the results under 90% VaR.

Figure 10: Graph of weights in the portfolios for Abertis, Markowitz and VaR 95%



Source: Own development.

5.2.2. AENA

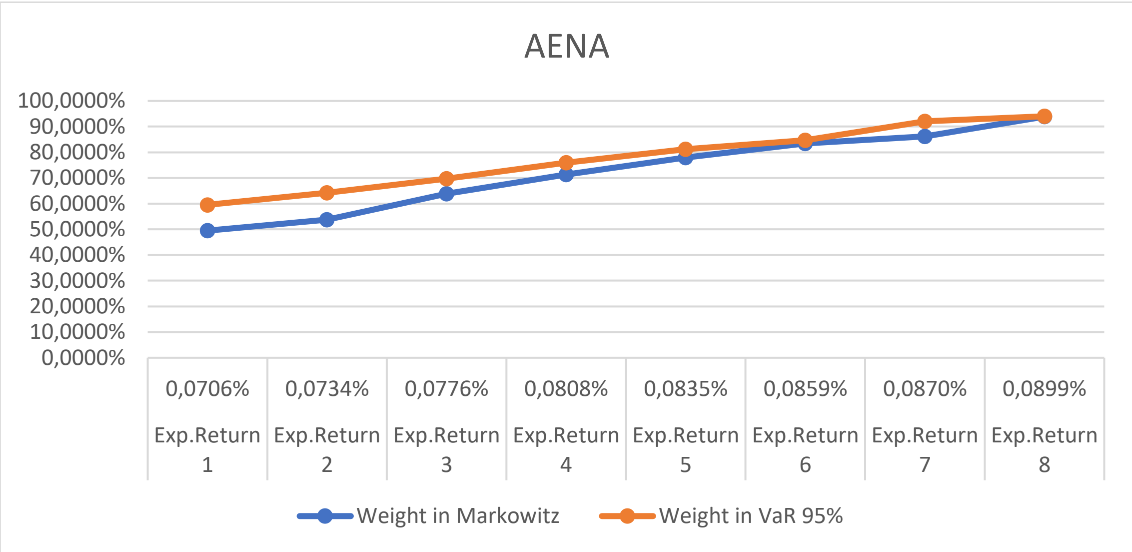
Table 20: Weights in the portfolios for Aena, Markowitz and VaR 95%

	Exp.Return 1	Exp.Return 2	Exp.Return 3	Exp.Return 4	Exp.Return 5	Exp.Return 6	Exp.Return 7	Exp.Return 8
	0,0706%	0,0734%	0,0776%	0,0808%	0,0835%	0,0859%	0,0870%	0,0899%
Weight in Markowitz	49,5013%	53,8159%	63,8372%	71,4086%	77,9233%	83,4107%	86,2011%	93,8794%
Weight in VaR 95%	59,5559%	64,2663%	69,7183%	75,9757%	81,1573%	84,7231%	92,0268%	94,0212%

Source: Own development.

Aena with the three types of analysis has obtained very similar results. The distinction between VaR at the 90% of confidence level and VaR at 95% of confidence level is visible in the three earlier expected returns, but since that moment they are practically equal. The evolution is the same that could be seen in 5.1.2, they are increasing their weights as it is growing the levels of expected return and the difference is reduced until being virtually coincident. However, this initial difference between Markowitz's (1952) approach and VaR approach at 90% of confidence level was reduced with the confidence level of 95% in VaR, and they are generally more similar during all the graphic. The two approaches have a constant and clean increase in their weights. The conclusion extracted is the same for that in 5.1.2., using the two methods Aena is a useful asset.

Figure 11: Graph of weights in the portfolios for AENA, Markowitz and VaR 95%



Source: Own development.

5.2.3. BANKINTER

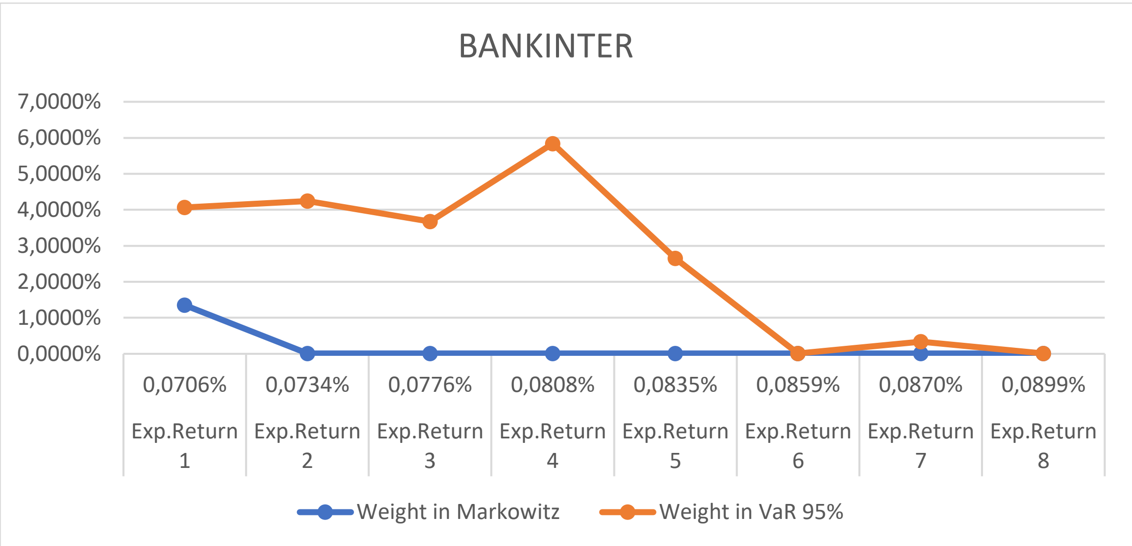
Table 21: Weights in the portfolios for Bankinter, Markowitz and VaR 95%

	Exp.Return 1	Exp.Return 2	Exp.Return 3	Exp.Return 4	Exp.Return 5	Exp.Return 6	Exp.Return 7	Exp.Return 8
	0,0706%	0,0734%	0,0776%	0,0808%	0,0835%	0,0859%	0,0870%	0,0899%
Weight in Markowitz	1,3508%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%
Weight in VaR 95%	4,0652%	4,2438%	3,6732%	5,8394%	2,6470%	0,0100%	0,3327%	0,0100%

Source: Own development.

Bankinter is the first asset in which could be appreciated several changes between the two VaR approaches applied. They are coincident at the end of the graph, but, in the earlier while the VaR at the 90% of confidence level invest a proportion of their weight's superior of 10%, with the VaR at the 95% of confidence level the maximum proportion invested is less than 6%. In this sense, the optimal weights for this asset employing VaR at 95% of confidence level as risk measure are more similar to the one obtained under the Markowitz's (1952) approach than when employing VaR at 90% of confidence. It is observable too that this VaR presents peaks, the most evident is in the fourth expected return of 0,0808%, but the tendency is clearly negative, as when the VaR at the 90% of confidence is considered as risk measure. However, the optimal weight of this asset under 95% VaR as risk measure becomes equal to the one obtained according to the Markowitz's (1952) approach from an expected return of 0,859%, higher than when 90% VaR is considered as risk measure. The results of both methods are not conflicting this time, so Bankinter seems an active that should not have presence in a portfolio that has the objective to obtained an elevated expected return, and neither seems indicate that it should has a relative presence in a portfolio which less risk assumed.

Figure 12: Graph of weights in the portfolios for Bankinter, Markowitz and VaR 95%



Source: Own development.

5.2.4. GRIFOLS

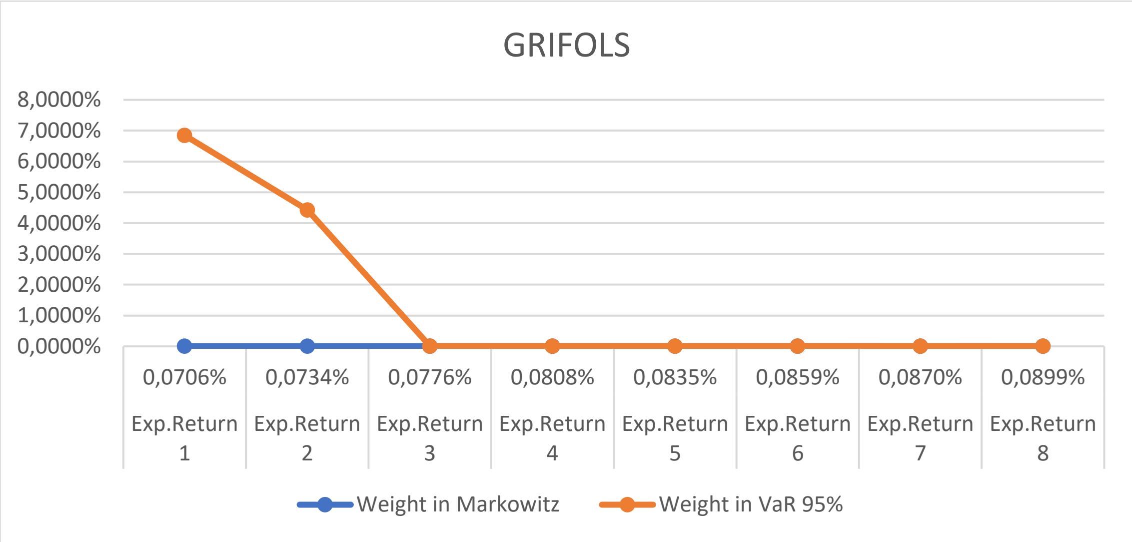
Table 22: Weights in the portfolios for Grifols, Markowitz and VaR 95%

	Exp.Return 1	Exp.Return 2	Exp.Return 3	Exp.Return 4	Exp.Return 5	Exp.Return 6	Exp.Return 7	Exp.Return 8
	0,0706%	0,0734%	0,0776%	0,0808%	0,0835%	0,0859%	0,0870%	0,0899%
Weight in Markowitz	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%
Weight in VaR 95%	6,8455%	4,4233%	0,0100%	0,0100%	0,0100%	0,0100%	0,0101%	0,0100%

Source: Own development.

The company Grifols was exclude of the investment until the minimum condition by Markowitz's (1952) approach and the VaR at the 90% of confidence level, but with VaR at the 95% of confidance level it has a change in the two earlier expected returns. Grifols begins with a weight of approximately a 7% and then it falls to around 4'5% to finally bein reduced to the minimum condition.

Figure 13: Graph of weights in the portfolios for Grifols, Markowitz and VaR 95%



Source: Own development.

5.2.5. SIEMENS GAMESA

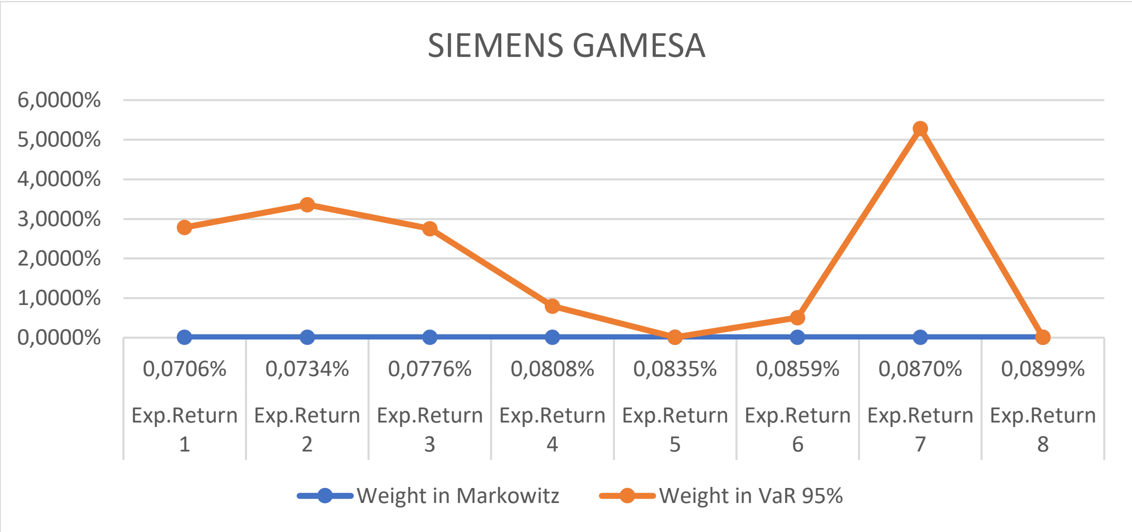
Table 23: Weights in the portfolios for Siemens Gamesa, Markowitz and VaR 95%

	Exp.Return 1	Exp.Return 2	Exp.Return 3	Exp.Return 4	Exp.Return 5	Exp.Return 6	Exp.Return 7	Exp.Return 8
	0,0706%	0,0734%	0,0776%	0,0808%	0,0835%	0,0859%	0,0870%	0,0899%
Weight in Markowitz	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%	0,0100%
Weight in VaR 95%	2,7821%	3,3565%	2,7523%	0,7932%	0,0100%	0,5069%	5,2813%	0,0100%

Source: Own development.

The optimal weights of Siemens Gamesa with VaR at the 95% of confidence level as risk measure shows an erratic behaviour. The unique point in common with Markowitz's (1952) approach and VaR approach at the 90% of confidence level is that it has a lower weight in all the portfolios. Siemens Gamesa has three peaks, one more remarkable in the seventh expected return in which achieves its maximum weight with a 5'28%, while Markowitz's (1952) approach only invest the minimum condition and VaR approach at the 90% of confidence level follows that way when the expected return grows. The results for VaR approach at the 95% of confidence level just make it clear that this company should not have a considerable weight in any portfolio.

Figure 14: Graph of weights in the portfolios for Siemens Gamesa, Markowitz and VaR 95%



Source: Own development.

5.2.6. INMOBILIARIA COLONIAL

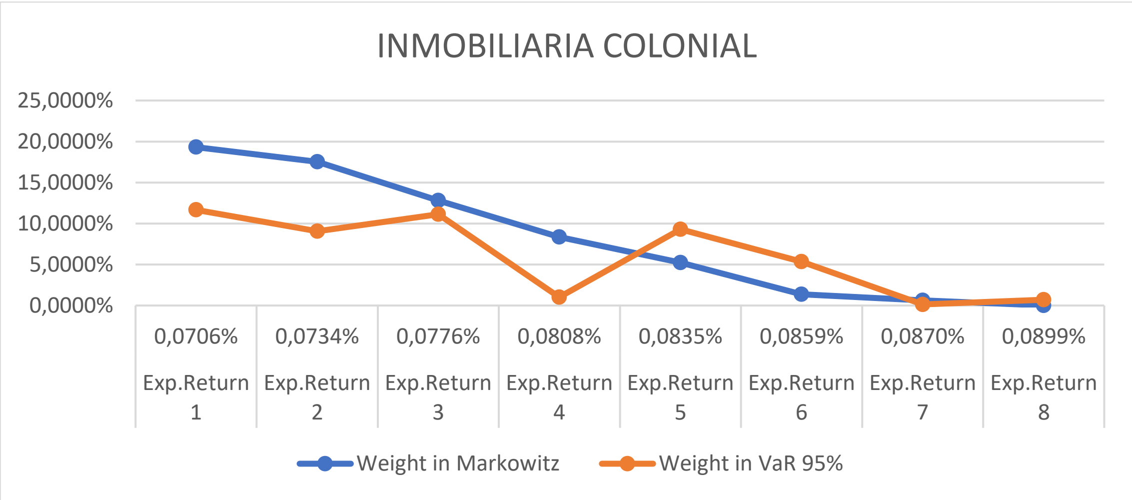
Table 24: Weights in the portfolios for Inmobiliaria Colonial, Markowitz and VaR 95%

	Exp.Return 1	Exp.Return 2	Exp.Return 3	Exp.Return 4	Exp.Return 5	Exp.Return 6	Exp.Return 7	Exp.Return 8
	0,0706%	0,0734%	0,0776%	0,0808%	0,0835%	0,0859%	0,0870%	0,0899%
Weight in Markowitz	19,3107%	17,5337%	12,8044%	8,3640%	5,2464%	1,3627%	0,6350%	0,0100%
Weight in VaR 95%	11,6612%	9,0422%	11,1178%	0,9985%	9,2817%	5,3443%	0,1358%	0,6974%

Source: Own development.

The case of Inmobiliaria Colonial with VaR at the 95% of confidence level is different of VaR at the 90% of confidence level. It begins with a weight more elevated in the expected returns, almost the weight of Markowitz. Moreover, it shows an erratic behaviour, increasing at third expected return of 0,0776% and then, it forms a peak downward at fourth expected return of 0,0808% and later it continues decreasing until approximate to the minimum condition of investment. In general, the optimal weights of this asset when employing VaR at the 95% confidence as risk measure are more similar to the ones according to the Markowitz's (1952) approach than when 90% VaR is employed.

Figure 15: Graph of weights in the portfolios for Inmobiliaria Colonial, Markowitz and VaR 95%



Source: Own development.

5.2.7. AMADEUS

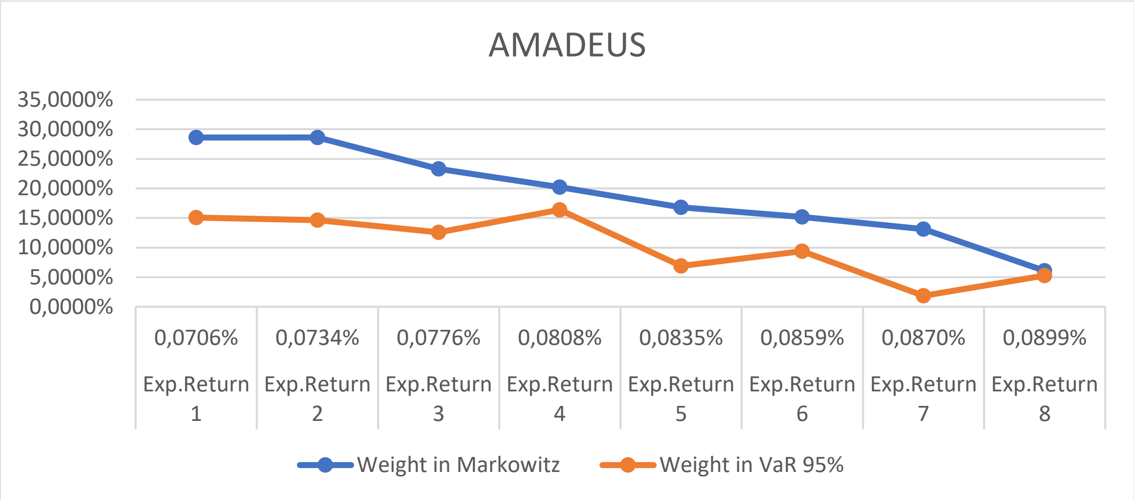
Table 25: Weights in the portfolios for Amadeus, Markowitz and VaR 95%

	Exp.Return 1	Exp.Return 2	Exp.Return 3	Exp.Return 4	Exp.Return 5	Exp.Return 6	Exp.Return 7	Exp.Return 8
	0,0706%	0,0734%	0,0776%	0,0808%	0,0835%	0,0859%	0,0870%	0,0899%
Weight in Markowitz	28,6047%	28,6104%	23,3184%	20,1874%	16,7903%	15,1866%	13,1239%	6,0707%
Weight in VaR 95%	15,0624%	14,6579%	12,5650%	16,3730%	6,8840%	9,3958%	1,8456%	5,2414%

Source: Own development.

The case of Amadeus with VaR at the 95% of confidence level is similar to the Inmobiliaria Colonial. It does not have a linear evolution, it has three peaks, in the fourth, sixth and eight expected returns. Similarly, to the approach that employs VaR at the 90% of confidence as risk measure, the weights of this asset that contribute to minimize risk measured by VaR at the 90% of confidence are always smaller than its optimal weights according to Markowitz's (1952) approach. However, this asset is always considered relevant to minimize risk measured by VaR at the 95% of confidence level and its optimal weights under this risk measure are closer to Markowitz's (1952) approach in almost all the portfolios than VaR at 90% of confidence level. In addition, despite the peaks and the no lineal tendency, it has on balance a negative tendency. It could be interpreted the same than Markowitz, it is positive to invest in Amadeus when the priority is less risk, and when the priority become in increase the expected return, this company should lose weight in the portfolios.

Figure 16: Graph of weights in the portfolios for Amadeus, Markowitz and VaR 95%

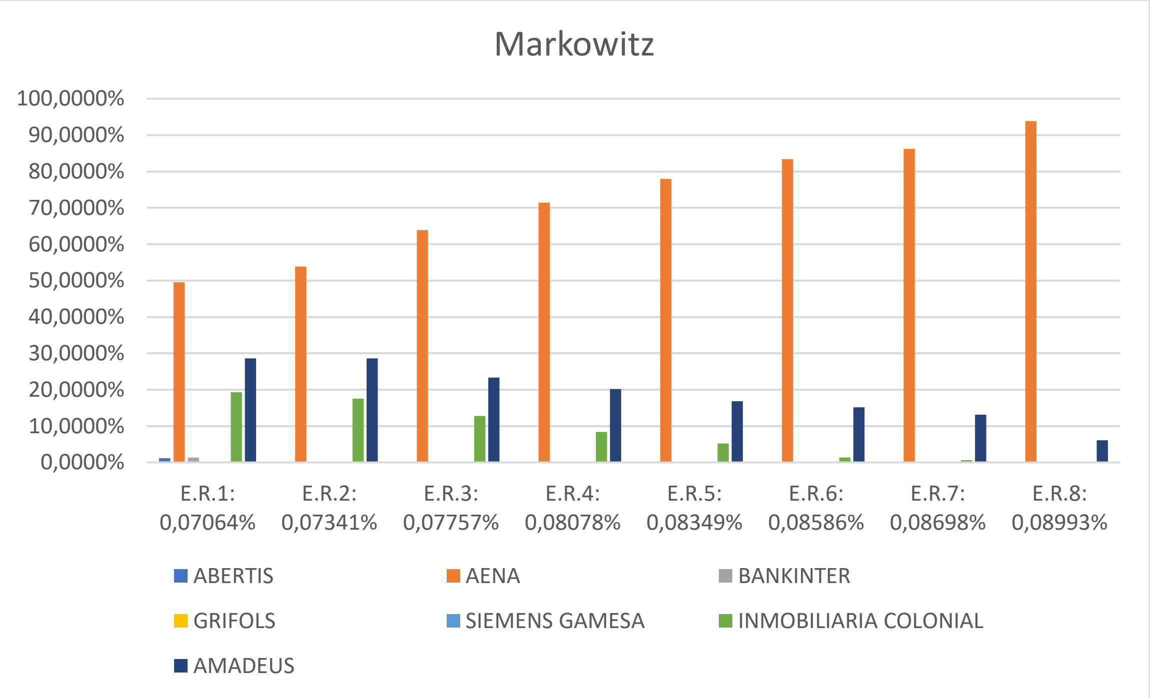


Source: Own development.

5.3. Markowitz – VaR 90% - VaR 95%

In order to understand better the difference between Markowitz's (1952) approach, Var approach at 90% of confidence level, and VaR approach at 95% of confidence level, these bar graphs were elaborated to observe those differences. In every portfolio there are includes all the assets with the weights founded by Solver. In the vertical axis is represented the proportion of weigh in the portfolio for every company, and in the horizontal axis the level of expected return, being the first the shortest and the last the largest.

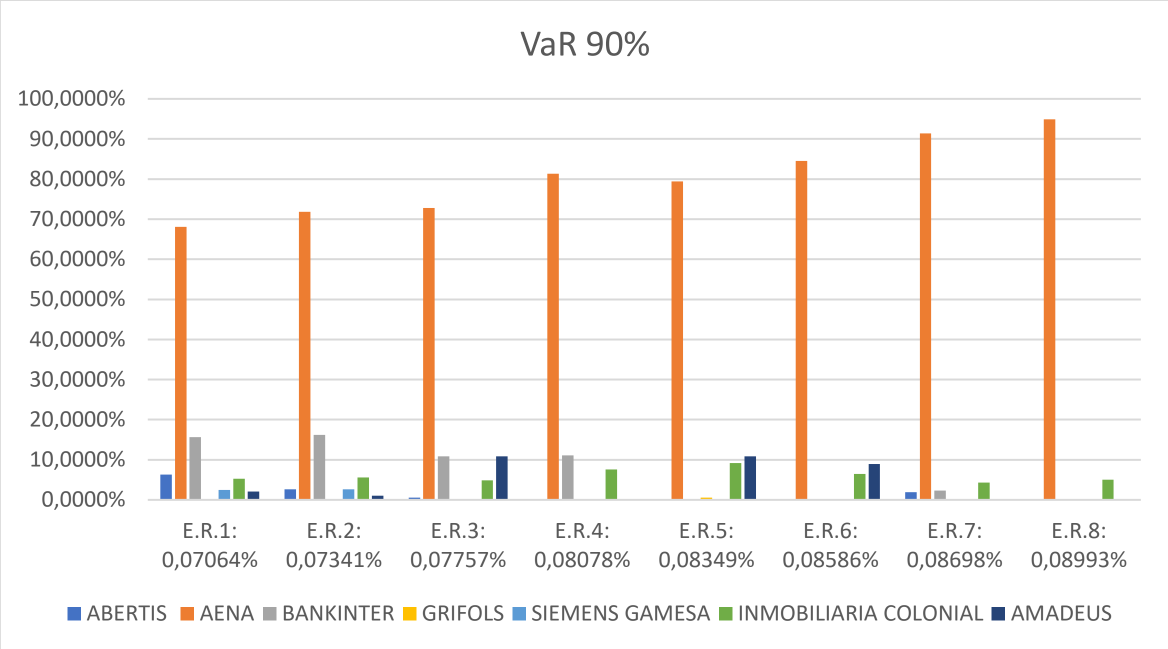
Figure 17: Graph of the eight portfolios using Markowitz



Source: Own development.

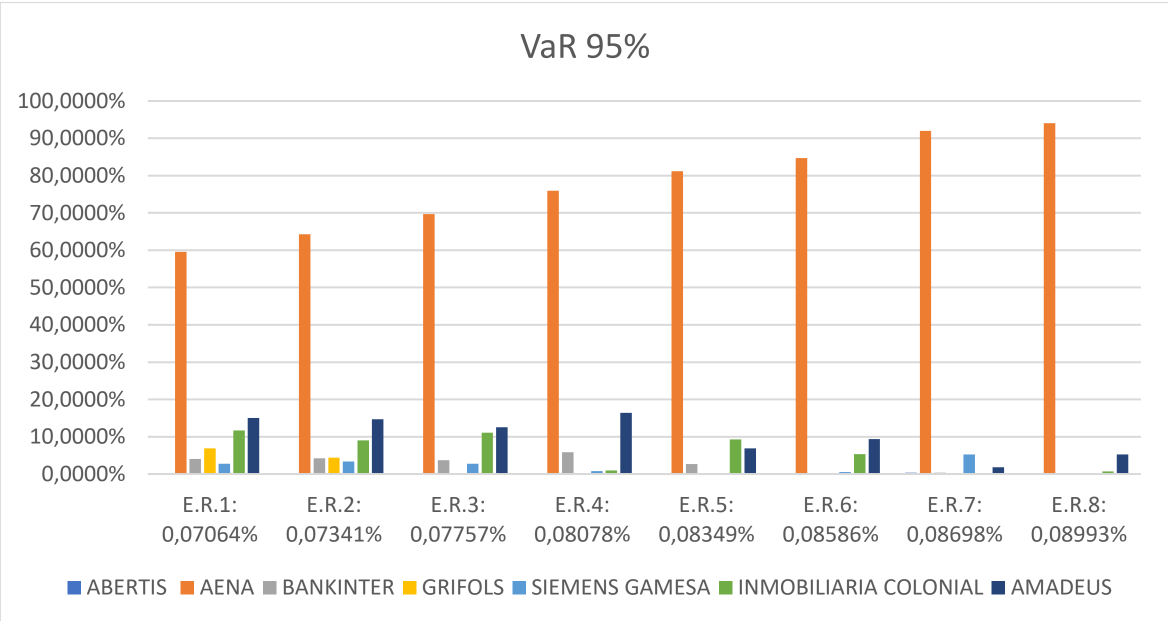
The graph begins with a short level of diversification, practically all the portfolio is concentrated in three companies: Aena, Inmobiliaria Colonial and Amadeus. This three companies are representatives until nearer the end. When the expected return is increasing and the risk assumed is major, Amadeus and Inmobiliaria Colonial are decreasing, in contrast to Aena which is becoming in one asset each time with more weight in the portfolios. It is remarkable than, apart of the lack of diversification, the movements in the graph, as increasing as decreasing have clean movements, thus the tendency is not broken in any way.

Figure 18: Graph of the eight portfolios using VaR 90%



Source: Own development.

Figure 19: Graph of the eight portfolios using VaR 95%



Source: Own development.

The results of the two approaches of VaR are similar but they present some differences. The VaR approach at 90% of confidence level presents a superior concentration at the beginning in Aena, as well give more importance to Bankinter and the importance of Amadeus and Inmobiliaria Colonial is lower than VaR approach at 95% of confidence level. In general, the results of VaR approach at 95% of confidence level are more similar to Markowitz's (1952) approach, (but in the case of two assets, Grifols and Siemens Gamesa), however with these approaches do not exist lineal tendencies. In general, their portfolios follow the tendencies but in some of them appear peaks in the weight of the assets (as set out in the previous points 5.1. and 5.2.). It is especially remarkable that the principal differences between Markowitz's (1952) approach and VaR seem to be that Markowitz's (1952) approach diversified in less assets but with more weight in their than VaR, and Markowitz's (1952) approach has a more clean and clear evolution when the expected return increases. In spite of these differences, especially with VaR approach at 95% of confidence level has obtained similar results using this alternative method.

6. DISCUSSION AND CONCLUSION: COMPARISON BETWEEN MARKOWITZ AND HISTORICAL APPROACH OF VALUE at RISK

Finally, in this section is described the main limitations of Markowitz's (1952) approach and VaR approach and the limitations of this final degree dissertation too, after that, I have written my own conclusions, to which I have get after the realization of all this final degree dissertation.

According to Michaud (1989) some of the main limitations that affect to Markowitz's (1952) approach are: the error maximization, the ignorance of important investment management considerations and unstable optimal solutions.

Error Maximization: When the method of Markowitz is used many "optimized portfolios" have an unintuitive character. The cause might be that "Mean-Variance" optimizers are "estimation-error maximizers". The "Expected Returns-Variance of returns (E-V)" or "Mean-Variance (M-V)" optimization overestimate the assets that have a major expected returns, lower or negative covariances, lower variances and underestimate the assets that do not have these characteristics, which leads to concentrate the portfolios in a few assets, decreasing the effect of diversification. For that reason, it is probably that in a significant form, the measure of diversifiable risk obtained by the optimizer, underestimate the real level of risk of the optimal portfolios.

Ignorance of investment management considerations: The restriction of liquidity in the portfolios changes significatively the composition of them, as in expected return as levels of risk. All the assets to invest in the financial markets do not have the same liquidity, depending on the preferences of the investor about the liquidity of his portfolio, the "efficient portfolio" according to Markowitz's (1952) approach might not be efficient to that investor. Moreover, the M-V do not analyse the percentage of the company that the portfolio is buying or selling. For example, if the portfolio belongs to a large company of investment and it buys shares of a small firm, the percentage of the company purchased might be able to change the value of the company, disturbing the results expected.

Unstable optimal solutions: sometimes, it was seen how little changes in the inputs of data could produce large changes in the solutions of the optimal portfolio, consequently the MV optimization are highly unstable. The reason behind of this phenomenon is an ill-conditioned of the covariance matrix, normally because the inputs assumptions do not show financially significative estimates for the period selected or it is induced by the use of insufficient historical data.

As Čorkalo (2011) observed, some of the limitations that affect to the VaR historical simulation are: the same ponderation to the recent data and older data, and in the most common situations of the markets, the simulations of Monte Carlo VaR are better.

Equivalent ponderation: the fact to the recent data and older have the same value in the calculation, could cause poor estimates due to some changes might had produced in recent trends as higher or lower volatility. The problem is aggravated by the reason that, with this method is interesting to incorporate more data with the objective to observe the strange events, but the current risk estimates do not be built with old market data.

Monte Carlo: Generally, the portfolios that are nonlinear over long time periods, which have a volatile historical data and non-stationary, and the normality assumption has not conformed (that it is the most common situation in financial markets), obtain a better results if the Monte Carlo approach is used instead of the historical approach to do it.

The limitations that have affected to this final degree dissertation are on the one hand, the lack of studies that compared Markowitz with the historical approach VaR. In other words, under my experience researching information of this theme, the majority of studies compare Markowitz with VaR using the approach of Monte Carlo. On the other hand, the problems resultants that has using Solver in the optimization of portfolios with VaR. Initially, near of thirty portfolios were calculated, but many of them were discarded because they do not have an adequate behaviour. Furthermore, when the final portfolios of VaR were selected with the 90% level of confidence, the VaR with 95% of confidence level was elaborated in order to compare these two VaR, and some previous portfolios selected with VaR at 90% of confidence level had to be also discarded for the same previous reason. Because of this, the VaR with 99% of confidence level was not elaborated, in order to keep the remaining portfolios to could compare their results and extract some conclusions.

To conclude this final degree dissertation, I expose my conclusions. One of them is that, with the collected information, it could not be said that Markowitz is better than Value at Risk and vice versa. Other conclusion is that, the results of Markowitz have more stable solutions in the behaviour of the portfolios solving the problem of optimization with Solver than the solutions founded for VaR by Solver. Finally, observing the results, it could be said that, in general (actually, in five of the seven assets considered to form the different portfolios), the VaR with the confidence level at 95% have a result more similar to Markowitz's (1952) approach than VaR with the 90% confidence level.

The general erratic behaviour of the weights of the different assets that minimize risk under VaR as risk measure could be related to the difficulties found when solving the optimization problem using Solver. These difficulties could be related to the series of stock prices employed to make the calculations together with the complexity of the optimization problem itself. This complexity comes from the fact that minimizing the risk measured by the historical approach to VaR implies, not only finding the position of the lowest value in the historical distribution of returns for a given level of confidence, but also interpolate to find the return in that position.

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