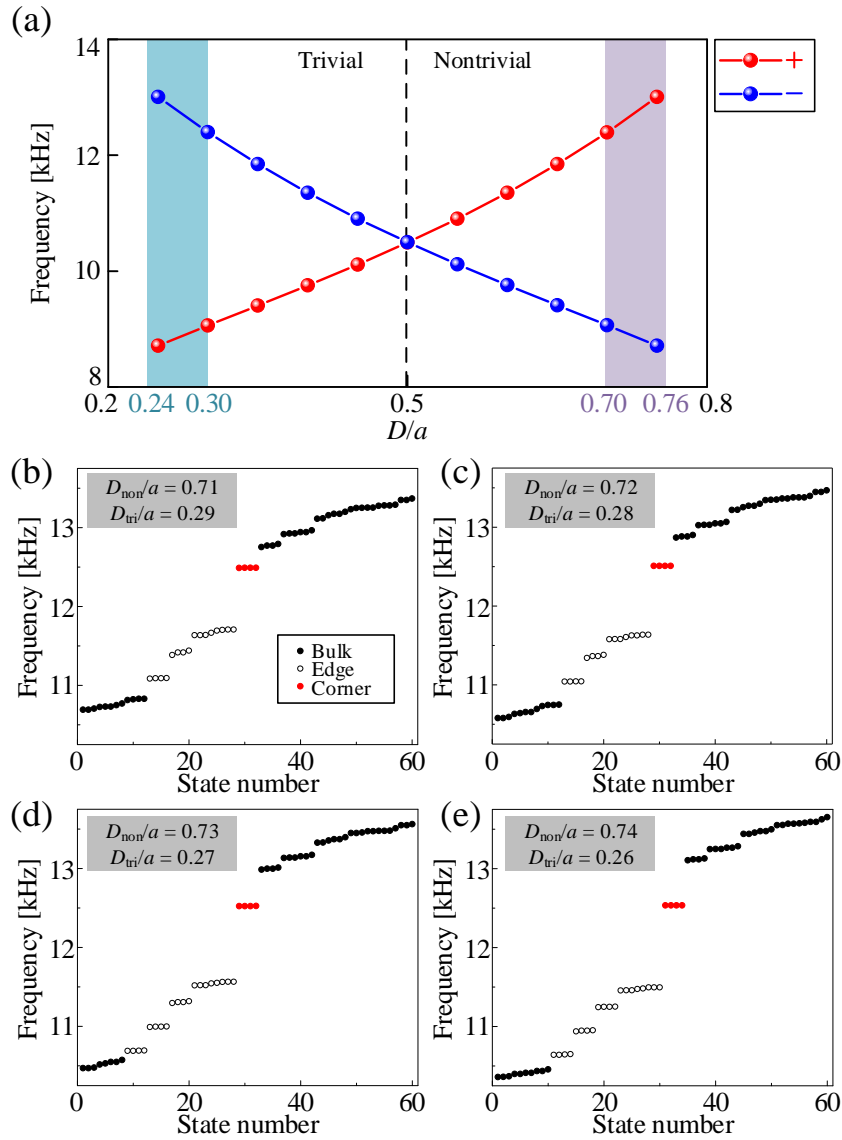


# I. SUPPLEMENTARY NOTE 1: GEOMETRICAL PARAMETERS



Supplementary Figure 1: (a) Eigenfrequencies of the acoustic states in dependence on the ratio  $D/a$  at the X point. Red/Blue dotted curves represent the modes with even/odd parity before and after the band inversion. Purple/Cyan regions represent the geometrical ranges of the nontrivial/trivial unit cells of the Concentric Square Crystal (CSC) supporting corner states within the complete band gap. (b)-(e) Eigenfrequencies of the finite CSC with randomly picked ratios  $D/a$  in the ranges indicated in (a).

Supplementary Figure 1(a) depicts the simulation of the topological phase transition against  $D/a$  where the regions of possible parameters are highlighted. In Supplementary Figure 1(b) to Supplementary Figure 1(e) we numerically construct four CSCs to verify the geometrical freedom in designing the topological corner states.