Appendices

Appendix 1. DVM as a time series

If we represent each respective time value from Table 1 and from Equation 1 by means of the convention:

$$Y_1, Y_2, Y_3, \ldots, Y_{t-2}, Y_{t-1}, Y_t, Y_{t+1}, Y_{t+2}, \ldots, Y_{n-2}, Y_{n-1}, Y_n. \qquad \text{(A1.1)}$$

where each value of the number of DVM is represented, in general terms, by Y_t, the series is represented by the first frequency of DVM (Y_1) until the last value (Y_n). Each DVM value for any month, Y_t, will thus correspond to the value of the previous month, Y_{t-1}, and the following month, Y_{t+1}.

For example, in Table 1, there are 226 monthly items of data ($n = 226$) between January 2000 and October 2018. For the first month ($t = 1$), $Y_1 = 6$ (that is, six DVM committed in January 2000); the last value ($t = 226$), $Y_{226} = 4$ (four DVM in June 2018); the item of data in 42nd place ($t = 42$) has a DVM value of 8, $Y_{42} = 8$ (eight DVM in June 2003), and a previous value ($t = 41$) of 6 ($Y_{41} = 6$), and so forth.

Previous values of *DVM* can be expressed as an autoregressive equation, and thus in Equation 1, according to our hypothesis, any value of Y_t, is a function of the number of *DVM* in the previous month, Y_{t-1}, and of the same month in the previous year or, in other words, 12 months before, Y_{t-12}; functionally, this would be:

$$Y_t = f(Y_{t-1}, Y_{t-12}) \qquad \text{(A1.2)}$$

Appendix 2. The hypothesis

Functional form of the hypothesis

The functional form of the hypothesis would be:

$$Y_t = f\,(\textit{Previous values of DVM, Law Effect, Trend, [Law Effect·Trend]}) \qquad \text{(A2.1)}$$

In Equation 1 we have written the interaction between *Law effect* and *Trend* in brackets to highlight the fact that it is a special (product) variable. In the following we will explain how the components of functional Equation 1 can be represented statistically.

Statistical formulation of the hypothesis

Thus, on replacing in Equation A1.1 *Previous values of DVM* (Y_{t-1}, Y_{t-12}), *Trend* (*1/t*), *Law Effect* (as a dummy variable: *DLE*) and *Law Effect·Trend* (in the dummy variable for Law Effect: *DLE·(1/t)*):

$$Y_t = \beta_0 + \beta_1 \cdot Y_{t-1} + \beta_2 \cdot Y_{t-12} + \beta_3 \cdot DLE + \beta_4 \cdot (1/t) + \beta_5 \cdot [DLE \cdot (1/t)] + e_t \qquad \text{(A2.2)}$$

where β_0 is the value of the y-intercept; β_1 and β_2 are the autoregressive coefficients of Y_{t-1} and Y_{t-12}; β_3, is the coefficient of the dummy variable *Law Effect* (*DLE*); β_4 is the coefficient of the variable *Trend* (*1/t*); β_5 is the coefficient of the interaction between the variables *Law Effect·Trend* (*DLE·(1/t)*), which will reflect the change in trend of DVM between the periods before and after the Law Effect; and e_t is the forecasting error of the equation (or residuals of the model).

Equation A2.2 can be disaggregated into two equations: (a) corresponding to before the implementation of the Law, substituting DLE = 0 in Equation A3.1; and (b) the equation referring to the interval after the implementation of the Law is obtained by making DLE = 1. We will see this in the two final simplified equations, for before the Law (DLE = 0):

$$\begin{aligned} Y_t &= \beta_0 + \beta_1 \cdot Y_{t-1} + \beta_2 \cdot Y_{t-12} + \beta_3 \cdot 0 + \beta_4 \cdot (1/t) + \beta_5 \cdot [0 \cdot (1/t)] + e_t \\ &= \beta_0 + \beta_1 \cdot Y_{t-1} + \beta_2 \cdot Y_{t-12} + \beta_4 \cdot (1/t) + e_t \qquad \text{(A2.3)} \end{aligned}$$

and for after the Law (DLE = 1):

$$Y_t = \beta_0 + \beta_1 \cdot Y_{t-1} + \beta_2 \cdot Y_{t-12} + \beta_3 \cdot 1 + \beta_4 \cdot (1/t) + \beta_5 \cdot [1 \cdot (1/t)] + e_t$$

$$= (\beta_0 + \beta_3) + \beta_1 \cdot Y_{t-1} + \beta_2 \cdot Y_{t-12} + (\beta_4 + \beta_5) \cdot (1/t) + e_t \qquad \text{(A2.4)}$$

General results

The hypothesis in Equation A2.2 and its statistical representation with estimated values is therefore confirmed (except for the β_1 coefficient, not significant; but the overall fit probability of the total equation is .001), the equation being (rounding to two decimal places):

$$Y_t = 6.96 - .01 \cdot Y_{t-1} - .15 \cdot Y_{t-12} - 4.08 \cdot DLE - 35.05 \cdot (1/t) + 443.17 \cdot [DLE \cdot (1/t)] + e_t \qquad \text{(A2.5)}$$

A caveat on the degrees of freedom in Table 2 should be noted. In Table 2 we can see that the Total df is 213 because the number of months used in the analysis is 226 – 12 = 214, since, when we use lag variables, we lose the maximum number of lags of the IVs introduced into the equation. Therefore, if we use Y_{t-12}, we have lost 12 values, and so the Total df is $n - 1$ (214 – 1 = 213).

Simplified Equations

Hence, Equation A2.5 can be simplified into two sub-equations: for before and for after the Law Effect.

Bearing in mind that *before the Law* the dummy variable *DLE* has a value equal to zero ($DLE = 0$), Equation A2.5 is reduced to:

$$Y_t = 6.96 - .01 \cdot Y_{t-1} - .15 \cdot Y_{t-12} - 4.08 \cdot 0 - 35.05 \cdot (1/t) + 443.17 \cdot [0 \cdot (1/t)] + e_t$$

$$= 6.96 - .01 \cdot Y_{t-1} - .15 \cdot Y_{t-12} - 35.05 \cdot (1/t) \qquad \text{(A2.6)}$$

In order to calculate the simplified equation *after implementation of the Law*, using Equation 6, and bearing in mind that the dummy variable Law Effect has a value of one ($DLE = 1$), it is transformed into:

$$Y_t = 6.96 - .01 \cdot Y_{t-1} - .15 \cdot Y_{t-12} - 4.08 \cdot 1 - 35.05 \cdot (1/t) + 443.17 \cdot [1 \cdot (1/t)] + e_t$$

$$= (6.96 - 4.08) - .01 \cdot Y_{t-1} - .15 \cdot Y_{t-12} + (-35.05 + 443.17) \cdot (1/t) + e_t$$

$$= 2.88 - .01 \cdot Y_{t-1} - .15 \cdot Y_{t-12} + 408.12 \cdot (1/t) + e_t, \quad \text{(A2.7)}$$

Note that Equations A2.6 and A2.7 are the respective developments of Equations A2.3 and A2.4, which in turn stem from Equation A2.2.

Appendix 3. Long-run trend

Formulation

The LRT in time series (Gujarati, Porter, and Gunasekar, 2013; Huckfeldt, Kohfeld, and Likens, 1982) is achieved when

(a) $t \to \infty$,

(b) $Y_t = Y_{t-1} = Y_{t-2} = \ldots = Y_{t-12} = Y^*$,

(c) $e_t = 0$, (A3.1)

LRT Results

We are interested in determining the LRT of the series in each of its components, before and after implementation of the Law. Thus, *before the Law*, using Equation (A2.6) with the properties of Equations (A3.1), becomes:

$$Y^* = 6.96 - .01 \cdot Y^* - .15 \cdot Y^* - 35.05 \cdot (1/\infty)$$

$$= 6.96 - .16 \cdot Y^* - 35.05 \cdot (0) = 6.96 - .16 \cdot Y^* \quad \text{(A3.2)}$$

or, if $Y^* = 6.96 - .16 \cdot Y^*$, leaving the Y^* terms on one side of the equation gives $1.16 \cdot Y^* = 6.96$, that is, by solving for Y^* it can be seen that: $Y^* = 6.96/1.16 = 6.00$, which indicates that the trend in DVM before the implementation of the Law would be around *6 murders per month*.

The LRT *after implementation of the Law*, using the values from Equation A2.7 and applying properties of Equations A3.1, will be:

$$Y^* = 2.88 - .01 \cdot Y^* - .15 \cdot Y^* + 408.12 \cdot (1/\infty)$$

$$= 2.88 - .16 \cdot Y^* + 408.12 \cdot (0) = 2.88 - .16 \cdot Y^* \quad \text{(A3.3)}$$

leaving: $Y^* = 2.88 - .16 \cdot Y^*$, or $1.16 \cdot Y^* = 2.88$, and solving for Y^* results in:

$Y^* = 2.88/1.16 = 2.48$, thus indicating that the trend after implementation of the Law is approximately *2.48 DVM per month*.