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DISCOUNT RATE DETERMINATION FOR INVESTMENT PROJECTS

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Abstract

The objective of this work is to present a review and collection of the main theories for the choice of the discount rate for investment projects in the private and public domain. This rate represents an abstract concept which becomes many times in one of the most important facts to be determinate when we are analyzing the viability for an investment project. Therefore, we will see in first place an explanation about the concept, next we will revise the different methods and variables that are used besides comments of the different advantages and disadvantages that may arise. Finally, the obtained conclusions will be presented.

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1. Introduction

From an economic-financial point of view Suárez, A. (2013) defines the company as a succession of investment and financing projects across time.

The expected incomes of the different periods can be considered constant or variable, in a finite or perpetual time horizon, constant or variable annually rate and, finally, the income can be interpreted as profits, dividends or cash flows.

The present work focuses on the different methodologies for the choice of the discount rate or cost of capital. There are some complications on the determination of this rate probably due to the subjectivity involved.

The amount of k rate will depend on the general situation of the country in which the firm is located and the future situation estimated for the company itself. When the environment expectations are unfavorable, the discount rate for update will be greater to collect the risk as will be detailed in later points of the work.

In the business sphere as well as for public projects, there is no standard methodology. Instead, the various theories have been refined over time, and multitude of authors have perform many guides or manuals, as in the case of public projects in Europe.

This work aims at the review and compilation of the different models and approaches most used to calculate the required rate of return or discount rate. For this purpose, the different models for calculate the discount rate are analyzed for a business environment according to the type of financing used and the capital structure used in order to apply a weighting between own and external resources for the calculation of the discount rate to be applied when carrying out the viability study of a business project. At the same it will be discuss the advantages or disadvantages that may arise according to several authors.

The first part of the work focuses on the business environment, while the second focuses on projects within the public sphere to end up making a contrast between them. For the methodology analysis for the used method to calculate the cost of capital, we proceed to the study of the different approaches on a guide elaborated by the Unit

Responsible for the Evaluation of the regional policy of the European Commission. Finally the conclusions obtained are exposed.

2. Theoretical concept of cost of capital

The discount rate represents the return that the investor demands on his investment. One possibility is to consider that this profitability is determined by the cost of capital. In this case, in order to obtain the objective unit value of the company, the interest rate to be used is the weighted average cost of capital (WACC) that the investor will have, which is an indicator of the minimum profitability that an entrepreneur will demand for his investments. The use of this cost implies ignoring the most profitable investment options that the entrepreneur could have.

Mascareñas, J. (2013) states: *"it should be the minimum rate of return for remunerate the various financial sources that make up their liabilities, in order to keep their investors satisfied while avoiding a stock exchanges decline"*. Therefore, this rate will be the minimum profitability limit that the company will require. Here the author tries to emphasize the importance of calculate an adequate discount rate, in fact, when a business project fails, the company goes into losses. This fact in turns into a lower valuation of the company shares in the market, as a result of the bad investment decision.

From an economic-financial point of view the company could be understood like a set of financing and investment projects so that an investment project will be carried out when it contributes to achieve the company objectives and if not, will be discarded. For this purpose, the company will need financial resources that will obtain through any of the means at its disposal: issuance of shares, request for credits, agreements with suppliers, etc. The means of these resources, depending on their origin, can be classified as internal financing sources or external financing sources.

Another classification regard to operative aspect of the cost of capital calculation, is based on the relation that these keep with the property of the company. From this point of view, we usually talk about own capitals or other capitals. Equity capital would be those contributions made by the shareholders either directly as equity capital or indirectly through self-financing, while the capital of others are composed of all funds contributed by outsiders not belonging to the company regardless of the time period of enforceability: loans, refundable grants, etc.

When we are talking about borrowed funds, it is easy to understand that we must bear expenses derived from their use in the form of interest, which constitute debts cost of capital. However, in the case of own resources we are not subject of interest payments, even not in some cases to make a payment of dividends to their contributors. It can not be said that this form of financing is free of cost, otherwise all companies would reject indebtedness and would only use their own capital. It is here where the calculation of the cost of capital becomes more difficult, when we speak about our own resources, since there don't have a explicit cost.

Now we assume another case, in which we no longer want to find out an objective value, now the project evaluator intends to obtain a subjective unit value. In this case, alternative investments that are presented to the specific subject of the valuation will have to be considered. Now in this scenario, the cost of capital will match with the profitability of investment opportunities. The models are the ones that the present work is going to expose next, for try to find this subjective value.

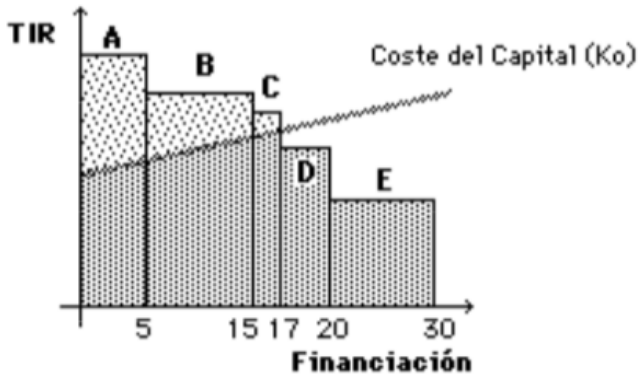
If we assume that the financial resources that are designated for an economic project are not used in this project, they can be dedicated for other investments that provide us with a certain return. To the profitability that we give up to carry out the original project, the opportunity cost that would suppose this, could be considered like a meaning of discount rate. Therefore, the cost of capital does not depend on the fact that there is an explicit cost, because exists from the moment that funds are designated to a particular investment, giving up other ones (López, F. 1993).

Sample of this connotation of discount rate referred to opportunity cost concept, its use in the internal rate of return (IRR) method. This process is one of the most used for the selection of investments under certainty conditions, and consists in update the future flows that the company expects to obtain or expected future cash flows. The discount rate used in this method (r) will be the one that cancels the current value of the collection and payment flows, comparing this one with the cost of capital (k). Under this method, investment projects whose IRR is higher than the cost of capital ($r > k$) will be accepted, and otherwise ($r < k$), the project in question would be rejected.

This definition focuses on the discount rate or cost of capital as the opportunity cost, if there is a project whose profitability is higher than the cost of capital of the company, the investment project would not be implemented existing alternative

investments with a higher return. For this cases, this new cost would be the cost of capital.

Mascareñas, J. (2013) shows in the following graphic illustration which is the minimum floor or cost of capital for a hypothetical company that arises the possibility of investment between five different projects (A, B, C, D and E), with equal degree of risk, ordered from highest to lowest IRR and with different volumes of funding:



Picture 1. Hypothetical case IRR & K_0

As picture 1 shows, only projects A, B and C have an expected return above the required minimum rate of return or cost of capital (represented by the letter K_0), while projects D and E do not exceed the minimum required return.

3. The importance of knowing the cost of capital and determining factors

Mascareñas, J. (2013) explains the main reasons as well as the factors that influence to know the cost of capital for a company:

1. The maximization of the value for a company that the senior managers must pursue implies the minimization of the cost for the production factors, including the financial capital. And to be able to minimize it, is necessary to know how to estimate it.

2. The analysis of the investment projects requires knowing the cost of the capital of the company with the objective of making adequate investments.

3. Other types of decisions, including those related to leasing, debt refinancing, and rotation fund management of the company, also are needed for estimate the cost of capital.

Next, continuing with the same author cited above, he summarizes the four main factors that affect the size of the cost of capital for a company:

Economic conditions: this factor determines the supply and demand of capital, as well as the expected level of inflation. The lower the capital offer is, the higher would be the cost of borrowing. When the demand varies, investors will vary their required rate of return in the case that the offer does not react proportionally. On the other side, if the supply of capital falls, or if prices are expected to fall, a lower level of rate of return will be required in each investment project. This variable is reflected by the risk-free interest rate since it is composed of the real interest rate paid by the state and the expected inflation rate.

Market risk: Investors will demand a higher rate of required return when the risk of a project increases, therefore, the issuance of securities will mean a higher cost for the company, and the opposite in the event that it descends.
The risk premium is called the difference between the required return on an investment project with risk, and another investment on the same but risk-free. The higher the risk, the higher the premium and vice versa.
Another very important factor for the companies is the liquidity premium, which is a premium that is incorporated into securities with lower liquidity to offset the less easy to sell it in the market.

Operational and financial conditions of the company: the decisions made by the company affect the risk or variability of the company's returns.
On the one hand there is the economic risk, which refers to the change in the performance of the company's assets and depends on the investment decisions. On the other hand, the financial risk, which refers to the return obtained by ordinary shareholders of the company. Whenever these risks increase, the higher the required return and therefore the higher the cost of capital be for the company.

The amount of funding: the larger the volume of the company's financing needs, the cost of capital will increase. If a company requests a volume of financing of a size much larger than this, it will increase the cost since it could be doubted the capacity of the same to absorb it or when a very large volume of shares is emitted, this causes a fall down in the price of the same causing a higher cost. As an exception would be the costs of emission, that how much larger is the volume of the emission, the lower would be this cost.

4. Classification of the actualization rate: the risk premium method

According to Cruz, S., and Valls, M.C. (2002), if we use as a discount rate the cost of capital of the company we will be using the minimum profitability that the investor will demand and we will, consequently, obtain the maximum value of the company.

These authors consider that the required return must be higher than the cost of capital, so that the rate to be used will be higher. There are two options for this:

1. The k estimation should be the return on alternative investments. Thus, if an investor can obtain a profitability by placing his money in another type of investment, it will be said about cost of opportunity the percentage that will demand of his investment in the company.

2. Get the discount rate as the sum of a base rate plus a risk premium.

As mentioned above, the discount rate k is composed about a base rate, which we denote by b, plus a risk premium r.

$$k = b(1 + r)$$

Where:

k = is the discount rate
b = represents the base rate
r = represents the risk premium

4.1 Base Type

It represents the risk-free interest rate and there are several alternatives to choose from:

a) Long-term interest rate without risk due to the solvency of the debtor: such as the legal rate of money and the type of rediscount of the Bank of Spain, Types of national markets or international markets.
b) The average returns rate of the sector to which the company belongs.
c) The average interest rate of the country and the sector.
d) Type of average return of shares

4.2 Risk premium

The risk should be included in the discount rate. It is completely logical and normal that, given the uncertainty and risk surrounding the future, the investor demands to its projects a superior return, being this the norm that operates in the market: profitability and risk, vary as a directly proportional form.

The risks that may be included in this additional premium are as follows:

Economic Risk: the uncertainty in obtaining estimated future income as well as the risk of loss of capital inherent to any investment in a company. This risk refers to the change in the performance of the company's assets and depends on the investment decisions of the company.

Financial Risk: derived from the financial structure of the company, the higher the level of indebtedness the lower the value of the company, since the greater the risk assumed.

Risk of illiquidity: investment in listed companies that are not publicly traded will be much less liquid than investment in companies that are listed on the stock exchange. Investment in companies, like all other investments, will require a return that is inversely proportional to the level of liquidity.

Note: The effect of inflation is not explained as it is not an indispensable object of the present work and to try not to extend the explanation too much. In the practice for this model, it is recommended to take care for such effect.

As an alternative to the classical formulation we have just analyzed, and in order to minimize some deficiencies and limitations that it presents, that there are some models that have emerged in the specialized literature, as we will see in the following points.

5. Financial models for determinate the discount rate

Once seen the classical formulation of discount rate and the incorporation of the risk, we will pass now to analyze the different financial models for the calculation of the discount rate for the corporations sphere as well as the variables that compose them, taking into account the capital structure for the last step, which is to weight them in the calculation for obtain the cost of capital for the company.

For calculating the discount rate we must include each financial source weighted by the weight that each one has over the total weight for the company. Therefore we might say that we are only interested in the necessary resources to carry out the investment, plus the necessary ones that can be derived from a possible increase of the rotation fund that the investment will need. The only exception would be if short-term resources are used for a company as usual to finance long-term investments. In such cases, the cost of such short-term debt should be included as a part of the cost of capital.

The objective is to determine the minimum rate of return that the company must obtain from its investments in order to satisfy the required return of its investors and at the same time not to drop down the market price of its securities, in other words, find out the cost of capital. Next, we will review the most used models for the calculation of the rate for both, the external financial resources and the own financial resources. Once determined individually, the percentage of each financial source will be determined over the total financing of the future investments. And finally with each one we can obtain the weighted average cost.

Assuming that one of the objectives of the company is to maximize the value for ordinary shareholders, profits will be retained only if the investment in the company is so attractive, at least as the best investment opportunity shareholders may have. Otherwise, in order to maximize shareholders values, dividend benefits should be distributed. To conclude we will say that said cost rate must be equal to the expected return on the best available investment. The cost of retained earnings is an implicit cost for which there are four main models to measure such return: the dividend growth model,

the financial asset valuation model (CAPM), the three-factor model of Fama and French, and the valuation model through arbitration (APM).

5.1 Dividend Growth Model

One of the models for calculating the required rate of return on investment in ordinary shares is the Gordon-Shapiro model. This model is a variation of the discounted cash flow analysis model and is based on the assumption that the theoretical price of a share is equal to the present value of future dividends that it is expected to be able to provide. These dividends will grow at a constant and cumulative rate (g) for an indefinite period.

If we call the market price of the share P_0, next year's dividend $D_1$and the rate of return required by shareholders K_e, we have the following expression:

$$P_0 = \frac{D_1}{(1+K_e)} + \frac{D_2}{(1+K_e)^2} + \cdots + \frac{D_n}{(1+K_e)^n} + \cdots = \sum_{t=1}^{t=\infty} \frac{D_t}{(1+K_e)^t}$$

Operating in summation, taking into account that it is the sum of a infinite series of numbers and whose ratio is (1+g)/(1+ K_e), the sum of series are equal to 1/(K_e- g). So the Gordon-Shapiro model would look like shows down in the box:

$$P_0 = \frac{D_1}{(K_e - g)} \quad \rightarrow \quad \boxed{K_e = \frac{D_1}{P_0} + g}$$

Where:

P_0: represents the market price of the share.
D_1: represents the expected dividend.
K_e: represents the rate of the required return by shareholders
g: cumulative rate

According this model, the rate of return for the ordinary shareholders clearing K_e in the Gordon-Shapiro model is equal to the result of adding the return for the dividends plus the cumulative rate of growth for this one.

The main drawback of this model lies in the calculation of the average annual growth rate (g), because the calculation of the dividend of the first year is equal to the last dividend spread increased (1 + g) times.

To perform the calculation of the g is usually performed by multiplying the coefficient of retention of profits (b) by the return on equity (ROE). The profit sharing rate, is the ratio of dividends to earnings per share, whereas ROE is obtained by dividing earnings per share after taxes by the book value of the share. According to the model we will assume that both variables will remain constant indefinitely.

5.2 Financial asset valuation model (CAPM)

The financial asset valuation model was initially developed by William Sharpe (1964) and John Lintner (1965), based on earlier works by Harry Markowitz based on modern portfolio theory and diversification (resulting in a Nobel Prize for Sharpe in 1990). This model is based on the fact that the required rate of return of an investor is equal to the rate of return without risk plus a risk premium, where systematic risk is the most important risk. This risk is measured by the volatility coefficient known as beta (ß), and measures the variation of the returns or volatility of a security respect to market variation. If ß takes a value of one, it means that the asset will oscillate in tune with the market, whereas the variation will be smaller as it is less than 1 and on the contrary if ß is greater than one.

The general expression for the model is as follows:

$$K_e = R_f + \left[E_M - R_f\right] ß$$

Where:

R_f: represents risk-free return
E_M: represents the market returns
ß: represents the volatility respect the market

R_f represents the risk-free asset for which it is advisable to use state obligations for ten years because three reasons as Mascareñas, J. (1997) explains: because the duration of the issue is similar with the duration for the cash flows of a company, the duration of the asset resembles the market index for calculate the returns

of the same and its coefficient ß, and because it is usually less volatile and also have a higher liquidity compared to similar longer-term issues. It can be said that by duration is meant the weighted average term of the life of the financial asset.

E_M indicates the expected returns for the market during a period in question and $\left[E_M - R_f\right]$ indicates the value of the market risk premium, which depends on the variance of the underlying economy, political risk and market structure. This means, with greater volatility, greater political instability, or the smaller and riskier the companies are listed on the stock market, the greater the risk premium.

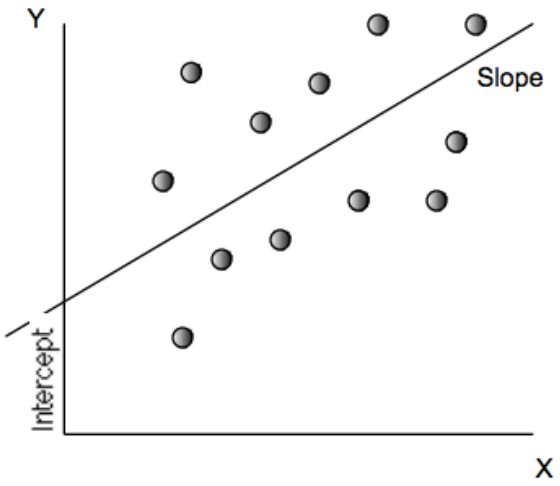
For the calculation of the beta (ß), it must be taken into account that it depends on three factors (see Mascareñas, J. 2007):

1. The type of business: the more sensitive the business could be, more greater could be the beta. In addition, if it is a business set, the beta will be the weighted average of each of them over the market value for all of them.

2. The operating leverage of the company, because a bigger variability of earnings before interest and taxes will make a bigger beta.

3. Financial leverage. A higher degree of indebtedness, a greater financial risk so a greater beta in consequence.

Damodaran, A. (2001) explains the calculation of the beta in practice. This one is done through a simple regression between the returns of the portfolio, and the returns of the stock index taken as reference. The tangent line of the angle for the regression line, represents the value of beta as shown in *picture 2* below.



Picture 2. Beta Graphical Representation

The returns of the asset are represented on the Y-axis and the returns belonging to the index on the X-axis.

Assuming that the beta for debt is null, the beta of a leveraged company ($ß_L$) in relation to the beta of an unleveraged company ($ß_U$) is defined as follows:

$$ß_L = ß_U \left[1 + (1 - t)(D/A) \right]$$

Where:

$ß_L$: represents the beta for a leveraged company.
$ß_U$: represents the beta for an unleveraged enterprise.
D: represents debt.
A: represents shares.
t: represents tax rate.

D/A is the debt/equity ratio at market value or, failing this, the book value and t are the tax rate. Now if the beta of the debt was not null, the expression would be:

$$ß_U = ß_L\ (A/V) + ß_D\ (D/V), \text{ donde } V = A + D\ (1 - t)$$

Where:

$ß_D$: represents the beta of the debt.
$ß_L$: represents the beta for a leveraged company.
$ß_U$: represents the beta for an unleveraged enterprise.
D: represents debt.
A: represents shares.
V: represents the value of the company
t: represents the tax rate.

The literature shows increasingly more detractors who disagree about the predictive capacity of Beta (see at: Fernández, P. 2015). According to the author, using calculated betas with historical data to calculate the required return on shares or to manage a securities portfolio is *"a huge mistake"* and explains the reasons which I describe in a summarized below:

1. There is a very strong daily volatility in the market and in the sector.
Authors like Aswath Damoraran (2001) recognize that the betas of the companies oscillate a lot but affirm that the sectorial betas composed by the beta of the companies of the same sector oscillate less.
2. It depends on which stock index is taken for reference.
3. It depends on the historical period used for its calculation.
In author's words, this fact demonstrates that *"there is no beta for a company's market"* because this one does not exist.
4. It depends on which returns are used to calculate them (daily, monthly, etc.).
5. The correlation and the regressions to calculate betas are very small.
6. They have a very little relation to the subsequent profitability of the shares.

There are several papers that try to evaluate if the predictions of the CAPM are actually fulfilled. Authors like Jagannathan and Wang (1999) argue that "*CAPM is alive and well"* while others, such as Fama and French (2003) say that *"empirical problems of CAPM probably invalidate their use in practical applications"* (see: Fernández, P. 2015). Now to continue we are going to see a brief explanation for the data used on the study for the model of Fama and French (1992), also known as the three-factor model (beta, price/book value and size) explained below.

5.3 The Fama and French Three-Factor Model

There are studies, the most important being conducted by Eugene Fama and Kenneth French of the University of Chicago in 1992, who have shown weaknesses in the beta for predicting the expected returns of financial securities or portfolios. The study shows that the returns of securities are inversely related to the size of the company measured through its market capitalization, in addition to an indirect relation with the book value / market ratio. And they manage with both relationships to explain financial returns more accurately than the beta does.

The study carried out by Eugene Fama and Kenneth French (1992) shows that in the period 1963-1990 the correlation between the ratio of the returns of the shares to their betas was very small, and in the other hand the ratio of returns to the size of companies with the ratio price / book value keep a stronger correlation. The authors divided the shares into 10 portfolios according to the size of the companies, the companies beta and the book price ratio. Pablo Fernández (2015) summarizes the results obtained in the tables and graphs shown below.

Company Size	Average Beta	Annual Average Profitability
1 (High)	0,93	10,7%
2	1,02	11,4%
3	1,08	13,2%
4	1,16	12,8%
5	1,22	14,0%
6	1,24	15,5%
7	1,33	15,0%
8	1,34	14,9%
9	1,39	15,5%
10 (Low)	1,44	18,2%

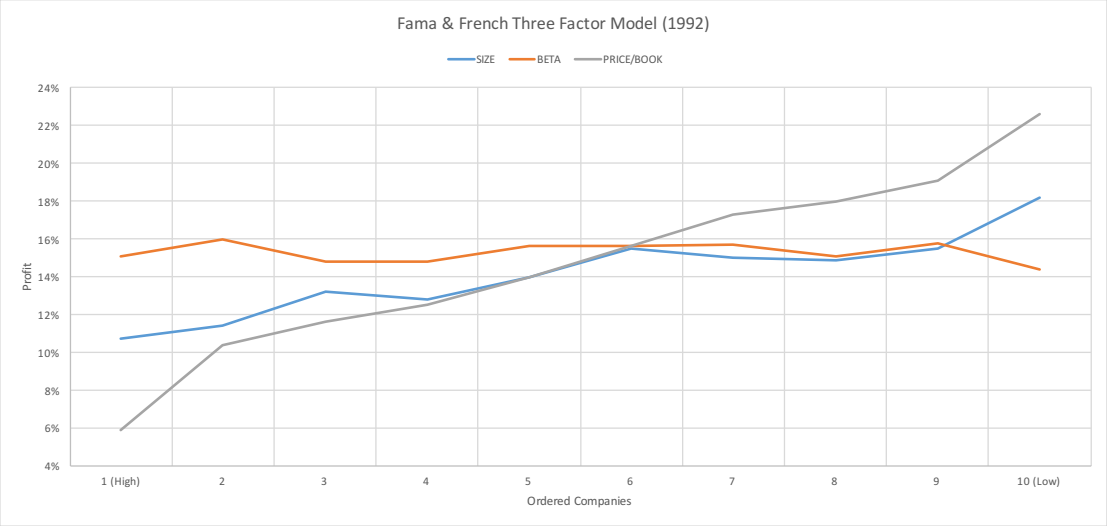
Table 1.1. Company Size Results

Companies Beta	Average Beta	Annual Average Profitability
1 (High)	1,68	15,1%
2	1,52	16,0%
3	1,41	14,8%
4	1,32	14,8%
5	1,26	15,6%
6	1,19	15,6%
7	1,13	15,7%
8	1,04	15,1%
9	0,92	15,8%
10 (Low)	0,8	14,4%

Table 1.2. Beta Results

Price/Book Value	Average Beta	Annual Average Profitability
1 (High)	1,35	5,9%
2	1,32	10,4%
3	1,3	11,6%
4	1,28	12,5%
5	1,27	14,0%
6	1,27	15,6%
7	1,27	17,3%
8	1,27	18,0%
9	1,29	19,1%
10 (Low)	1,34	22,6%

Table 1.3. Price/Book Results



Picture 3. Fama & French (1992) Results

Picture 3 shows the results obtained from the data sample extracted from the COMPUSTAT database belonging to the period 1963-1990 used by Fama and French (1992). In this figure I have captured the data of the three tables above with the purpose of explaining the previous data. As the chart shows, the variables selected in the study show a higher correlation than the beta on the profitability of the portfolios. The color blue represents the profitability respect to its size, in which it is observed that the smaller companies were more profitable. The gray line represents the comparison with the price/book value ratio in which it is observed that the companies with a lower ratio were more profitable. Both relationships show an indirect relationship with a positive trend. The study also reveals how the graph shows a virtually null correlation between the beta (represented in orange on the chart) and the different profits. The three-factor model of Fama and French (1992) is as follows in the next formula:

$$K_e = R_f + \left(E_M - R_f\right) ß + (E_P - E_G)\, s + (E_{ra} - E_{rb})\, h$$

Where:

R_f: represents risk-free return.

E_M: represents the market returns.

ß: represents the volatility with respect to the market.

E_P: represents the companies with the lowest market capitalization.

E_G: represents the companies with a greater market capitalization.
s: coefficient of regression pertaining to market capitalization.
E_{ra}: represents the ratio of companies with high book to market ratio.
E_{rb}: represents the ratio of companies with low book to market ratio.
h: regression coefficient corresponding to book to market.

It is observed that the first two summands are the same with those of the CAPM model analyzed in the previous point. The third adding indicates the risk premium expected among the companies with the lowest market capitalization, less those with a greater market capitalization. This is calculated by subtracting the average annual profits $(E_P - E_G)$, multiplied by the regression coefficient s. Finally, the fourth sum shows the expected risk premium among companies with higher and lower book price/book value ratio, through the difference of their historical profits $(E_{ra} - E_{rb})$. To do so, we analyze the differences between the book value of the net worth of the companies and the value given by the market (book to market ratio), all multiplied by the regression coefficient h.
The regression coefficients are determined by linear regressions. These factors are calculated using combinations of portfolios composed of series of classified stocks and available market data, available on the Kenneth French website. (See at: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).

5.4 The arbitration valuation model (APM)

This theory was developed by Stephen Ross, and is based on the idea that in a competitive financial market, arbitration will ensure the expected return on risk-free assets. In the arbitrage mechanism, bond prices adjust as investors build portfolios with the objective of making profits by buying assets in a market where they are cheaper to simultaneously, sell it in a more expensive one and get a risk-free profit. According to this theory, when such opportunities are exhausted, the equilibrium in the prices of financial assets will be reached.

The novelty introduced by this model is that in addition to the specific perturbations of each particular company, it adds another series of exogenous variables from a couple of macroeconomic factors, on which they also depends the profitability of each action.

The five factors most used by the model are:

1. The level of industrial activity or change in GDP.
2. The short-term real interest rate, measured by the difference between the yield of the Treasury Bills and the Consumer Price Index (CPI).
3. The short-term inflation rate, measured by changes in the CPI.
4. The long-term inflation rate, measured by the long-term return of the Public Debt minus the short-term one.
5. The insolvency risk, measured by the difference in long and short yield of corporate bonds rated AAA and BBB.

In the APM model the expected risk premium of a stock ($K_e - R_f$) depends on the risk premium associated with each of the macroeconomic factors in addition to the sensitivity of the profitability of each factor. The expression for the expected yield of any title (K_e) would be as follows:

$$K_e = R_f + ß_1 \lambda_1 + ß_2 \lambda_2 \ldots + B_n + \lambda_n$$

Where:

R_f: represents the return for the risk-free asset.
B_n: represents the macroeconomic factors.
λ_n: represents the different risk premiums or betas associated with the different factors.

R_f indicates the risk-free return of the asset and λ_i indicates the risk premium associated with each of the factors ($\lambda_i = E_i - R_f$). The APM betas depend on the same variables as in the CAPM: business type, financial and operating leverage. Once the factors were defined we would obtain the betas of each factor through the multivariate regression model. In this way, the value of the expected return of each share or opportunity cost of the capital is obtained, which, if its necessary, emission costs must be added.

5.5 Calculation of the weighted average cost of capital (WACC)

According to Beatriz Herrera (2008) in the practice, the determination of the discount rate or cost of capital of the financial asset or investment project is determined

using the CAPM for calculate the cost of equity and debt, and to combine both rates in a single cost are made by using the weighted average cost of capital (WACC).

Throughout the present work we have analyzed the different models of capital costs for calculate the cost of both financial components used, internal and external financial sources. The project analyst must be able to choose what model should implement in order to calculate the cost rate, and in case of not being able to apply any of the financial ones, a possible solution would be to use the risk premium method explained at point four. Finally, to calculate the discount rate that is required to apply in an investment project, we use the weighted average cost of the different financial components reflecting the weight of each type over the total financing used in the project.

After calculating the costs of the various financial sources and their weights we pass to calculate the average weighted cost of capital (WACC) using the following expression:

$$\text{WACC} = \text{Cost of Equity}\ (\text{Equity}\ /(\text{Debt}\ +\ \text{Equity})\ +\ \text{Cost of Debt}\ (\text{Debt}/(\text{Debt} +\ \text{Equity})$$

Where:

Equity: total equity
Debt: total debt
Cost of Equity: represents the percentage of equity.
Cost Debt: represents the percentage of external capital.

Finally, the WACC result can be applied as the discount rate that updates the flow of the expected cash flows in the investment projects. Suárez, A. (2013) states that in order for the company to survive and, if necessary, to grow, the rate of return or internal rate of return (IRR) of the investments made, must exceed the weighted average cost of the financial resources used for their funding.

In conclusion, the essence of the business function lies in discover investment opportunities whose expected average profitability exceeds from the cost of the capital used for finance them.

6. Selection of the discount rate in public investment projects

As discussed above, the objective of the present work is to analyze the different methodologies and application models as well as their variables with the final purpose towards the estimation of the cost rate in investment projects and which are currently carried out in the practice. To continue, once we have seen the procedures and dynamics followed by a company from any class belonging to the private sector, we go on to see the procedure used for the analysis of public projects domain to analyze how the cost rate is obtained so we can make a contrast between models and variables. For the analysis of the methodology I have reviewed the different approaches presented in the annexes of the guide prepared by the Responsible Unit for the Evaluation of the regional policy of the European Commission for the calculation of the cost of capital for investment projects on the public domain.

For public investment projects is used the cost-benefit analysis (CBA), which aims to determine the suitability of a project by enumerating and subsequently assessing in monetary terms all the costs and benefits directly and indirectly derived from such investment project.

The European Union (EU) who regulates the Structural Funds (FFEE), the Cohesion Fund (CF) and the Structural Instrument for Pre-Accession (ISPA), expressly require a cost-benefit analysis for large investment projects whose budget exceeds 5, 10 and 50 million euros. Depending on their financial dimension, different thresholds are set with the intention of avoiding excessive fragmentation of projects and ensuring an integrated and systematic use of funds. Those subject to FFEE are those whose cost may not be less than 50 million, 10 million euros in the case of the Cohesion Fund and 5 million in respect of the ISPA. To investigate the practical method of capital cost, we mainly analyze the procedure set out in a guide whose purpose serves as a memorandum for the evaluation of projects prepared for the Responsible Unit for the Evaluation of regional policy of the European Commission. This is the one in charge of evaluating the quality of the analysis carried out by the Public State before approving a co-financing or co-financing percentage on the same. The material of this guide is focused on European officials as well as external consultants or other interested parties for what constitutes a very practical material for the present work (see at: http://ec.europa.eu/regional_policy/sources/docgener/guides/cost/guide02_en.pdf).

Infrastructure investments and productive investments are those in which the guide is centered and can be financed through one or more community financial instruments: unsecured subsidies (FFEE, Cohesion Fund) and also reimbursable aid,

such as granted by ISPA (Instrument for Structural Policies for Pre-Accession). In addition, loans and other financial instruments could be purchased from the European Investment Bank (EIB), so the public administrations may choose financing sources such as public funds or community co-financing.

The structure for this type of project, which is determined in the guide, is listed by seven stages necessary for the co-financing developer requesting: definition of objectives, project identification, feasibility and analysis options, financial analysis, economic analysis, multicriteria analysis and sensitivity and risk analysis. For the study of the selection of the discount rate we will focus on the stage that requires it, this is the economic analysis. For economic analysis, a social discount rate is determined for which benefits are assessed from the social perspective. Before going on to see the models of practical application, it will be discussed below how a social project is valued and what possible variables will have to be considered when a social project is evaluated versus a private one.

6.1 The social evaluation of projects

First of all, it should be highlighted the difference between market prices and commercial valuation respect shadow prices, and social analysis or evaluation. Zabalza Martí, A. (1974) defines the concept of cost-benefit analysis (CBA) as a method of evaluating projects based on the use of social objectives. In case that the prices generated by the market are consistent with these objectives, in other words that these prices reflect social costs and benefits, there were no difference between a social or a commercial evaluation. However, so far as they do not meet those conditions, market prices can not be used to measure the social consequences of a particular project. Other types of prices, with properties that are generated by the market lack should be used, this ones are the *shadow prices*.

In the evaluation of a private project profitability is the most important objective for the investor, and the cost of capital is used to compare the cash flows in each period. This reflects the best alternative of use on some funds for an investor.

To adequate de project to the investor, the current value of the net benefit flow is calculated and, if this is one results positive, the investment in the project entails for the investor an increase in its wealth greater than the best alternative investment it could obtain from using those funds.

Ernesto R. Fontaine (2008) explains that for an absence loans project, the annual benefits will come from the sale of products (X_i , P_i) and costs will come from the purchase of inputs (Y_j , P_j); these flows will be received for *n* years, being the cost of capital r_t during the year t. Thus, the present value of the net private benefits of the project will be:

$$VABNP = \sum_{t=0}^{n} \frac{\sum_{i=1}^{n} X_{it} \cdot P_{it} - \sum_{j=1}^{m} Y_{jt} \cdot P_{jt}}{(1 + r_t)}$$

Where:

X_{it} = Number of products sold in year t
P_{it} = Sales price in year t
Y_{jt} = Quantity of products purchased in year t
P_{jt} = Price of raw materials in year t
r_t = Cost of capital during year t

This NPV of private benefits measures the increase in wealth of the project owner.

Once seen the valuation for private projects, we move to the social valuation of the same.The social evaluation of projects consists in compare the benefits with the costs that these projects imply for the country; that is to say, it consists in determining the effect that the execution of the project will have on the welfare of the society (social welfare).

According to the author Ernesto R. Fontaine (2008) the social welfare of a community will depend on the quantity of goods and services available (product or national income), the relative quantity of goods and services received by each member (personal distribution of that national income); political freedoms, respect for the property rights, institutions and the exercise of other human rights; of social mobility; of the military powers of the bordering countries; of alliances, compromises and disagreements with other countries; the composition and amount of foreign investment, and other factors that could be listed.

The evaluation of social projects is limited to consider only the effect that the project has on the amount and distribution of national income over time versus what would it happened with it, if the project had not been implemented.

In contrast to the evaluation of private projects, for the social scenario the variable for *social price* or *shadow price* is defined how: that price of goods and services of final consumption, produced by a project P_i^* and the social price or shadow of the inputs or raw materials used for the project P_j^*. With this variables we obtained that the Direct Net Social Benefit for any *" t "* year is:

$$BSN_t = \sum_{i=1}^{\tilde{n}} X_i \, P_i^* - \sum_{j=1}^{m} Y_j \, P_j^*$$

Where:

X_i = Quantity of products sold
P_i^* = Shadow sales price
Y_j = Quantity of products purchased
P_j^* = Shadow price of raw materials

In contrast to private projects, we have that the *current social value* of the direct net social benefit flow of the project, will be obtained using a *social rate* or *shadow discount rate* for each year (r_t^*). This is equal to the *social cost of capital* for a country country:

$$VABNSD = \sum_{t=0}^{n} \frac{[\sum_{i=1}^{\tilde{n}} X_{it} \cdot P_{it}^* - \sum_{j=1}^{m} Y_{jt} \cdot P_{jt}^*]}{(1 + r_t^*)}$$

Where:

X_{it} = Quantity of products sold for the year t
P_{it}^* = Shadow sales price for the year t
Y_j = Quantity of products purchased for the year t

P_j^* = Shadow price of raw materials for the year t

r_t^* = Social discount rate for the year t

Summarizing, the difference between the cost of capital for a private investor differs from the cost of public capital (social discount rate) due to market imperfections which reflect aspects that differ in the setting for social costs and benefits for a project. For example, in the case of public goods, where the norm is a private price equal to zero, projects that provide services such as national defense, roads, streets and other projects in which it is difficult to obtain a collection from people who use this goods or services generated. Imperfections in the market for goods and services from monopoly and monopsony situations are another reason why the social evaluation of a project can yield different or contradictory results to those obtained from its private or financial evaluation. Taxes and subsidies, quotas, prohibitions, customs tariffs or tax exemptions that come to favor the entrepreneur activities that may not be profitable from the point of view of the whole country.

According to the author Ernesto R. Fontaine (2008), the cases discussed above are known as *direct costs and benefits* (related to the first purchases and sales carried out by the project) *and indirect social benefits and costs* whose costs and benefits are not included in the formulas above. Continuing with the author, social evaluation should contain the *measurable and valuable externalities* that it generates, this is, costs and benefits that are borne by third parties, which are not compensated by the costs or do not pay for the receiving benefits. And finally, the existence of externalities that are *not measurable or not valuable*. The latter ones refer to intangible externalities.

According with the author, the Current Value of Total Net Social Benefits (VABNST) is:

$$VABNST = \sum \frac{[\,BNSD_t + BNSI_t + EMV_t + W_t]}{(1 + r_k^*)}$$

Where:

$BNSD_t$: direct social net benefit

$BNSI_t$: indirect social net benefit

EMV_t: measurable and valuable externalities

W_t: value assigned to intangible benefits

r_t^*: social discount rate for the year t

6.2 The social discount rate for large projects in the European Union

In the economic analysis of investment projects, the social discount rate is intended to reflect from the social perspective, how the future benefits and costs should be assessed against them at present. This rate may not match with the financial discount rate because the capital market is imperfect. The economic analysis assesses the contribution of the project to the economic wellness of the region or country considered.

There are several theoretical and political approaches to the interpretation and choice of the value of the social discount rate. The main ones are:

The first is based on a traditional concept, where marginal public investments must have the same returns as private investments in the case of substitute projects. This method yields higher rates than those obtained by the method explained below, because the capital markets submit the future to a more severe upgrade, towards short-term targeting.

The second approach is a formula based on the long-term growth rate of the economy, the approximate expression found would be:

$$r = ng + p$$

Where:

r: real social rate of public funds
n: represents the elasticity of social welfare.
g: represents the growth rate of public spending.
p: pure intertemporal preference rate.

The real social discount rate of public funds (r) expressed in a particular currency, would be equal to n, which constitutes the elasticity of social welfare versus public expenditure, multiplied by the rate of growth of public expenditure (g) and adding a pure intertemporal preference rate (p). The latter one, reflects the fact that an individual can care less for the future than for the present (selfish, if p > 0). Francisco Correa (2006) states that the purpose of using the temporary preference rate as a social

discount rate is to reflect the preferences of the government adequately on the efficiency prices, in relation to current and future consumption.

A practical example could be (see at: Guide of cost-benefit analysis of investment projects, p119, 2003) assuming that public cost for assistance to the most disadvantaged people grow at a real annual rate equal to the average per capita consumption, for example 2%, and that the elasticity of social welfare against this type of cost is between 1 and 2, if the pure intertemporal preference is around 1%, the real social discount rate will be between 3% and 5%.

7. Conclusion

The discount rate for private-sector projects is based on a weighted average between an opportunity cost and a required return. There are several models that start from hypotheses to simplify reality and study it better even though the risk premium of the market, together with Beta are probably the two financial parameters most investigated and controversial, and also the most confusing. In short, when we talk about private projects, the real complication is based on introducing methods and variables for risk aggregation so that the models get as closer to reality as possible, in order to establish the most exactly way a comparison between the cost of capital and the expected average profitability when we are evaluating investment projects as well as for making business decisions.

On the other hand, the social evaluation of projects and the social discount rate seek to measure the true contribution of the projects to the economic growth of the country. This information must be taken into account by the decision makers in order to be able to program the investments so that the investment has its greatest impact on the national product. However, because social assessment can not measure all the costs and benefits of projects, the final decision will also depend on other considerations such as economic, political and social ones. This implies, for example, that there could be projects with negative measures but high social intangible benefits that could not be carried out. Projects in which social evaluation is very useful, since it throws the information that is the most pertinent for the decision making and is not carried out by other considerations. The market is the one who guides private investment through profits and losses, but for public investment there is no efficient and clear mechanism of incentives and control for decision makers. A more equitable mechanism should be implanted in place so that the cost of poorly taken public decisions does not fall only on the citizenship.

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