Business Cycle Fluctuations, Economic Growth & Granular Behavior: An Agent-Based Approach [‡]

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Abstract. This work proposes a generalization of Delli Gatti et al.'s (2005) model in order to study the granular behavior observed empirically (Gabaix, 2011), namely a few very large firms are able to account for a very high fraction of macroeconomic fluctuations. A sample made of 40000 Spanish firms during the period ranging from 1995 to 2015 is employed in order to carry out the empirical analysis. It is found that the original model with constant-return-to-scale technology is able to generate a degree of heterogeneity closer to that observed empirically than that produced by the model with decreasing-return-to-scale technology. Nevertheless, the lower degree of heterogeneity produced by the latter makes it possible to reduce the market share of the largest company, which, in the original model, is far from what is observed.

Keywords: Business cycle; Economic growth; Agent-based model; Power-law distribution; Granular behavior.

JEL Codes: E32; C63; C82.

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1. Introduction

THE PRESENT WORK is aimed to study the granular hypothesis developed by Gabaix (2011), namely a few very large firms are able to account for a very large fraction of macroeconomic fluctuations, in the Spanish economy. This author finds that the idiosyncratic movements of the largest 100 firms in the United States explain about one-third of variations in output growth.

The granular hypothesis has been employed in several empirical works in order to study the volatility in the economic activity. Blank et al. (2009), using an early approach of the granular residual (Gabaix, 2009), show that fat-tailed distribution in the German banking sector affects its own stability. Wagner (2012) test the hypothesis in a sample of German manufacturing industry. The author finds that very few firms are able to account for a significant amount of the industry's sales growth, the estimated coefficient of determination is approximately 45%. Other works, however, have focused on studying variations in exports through the granular approach. This is the case of Di Giovanni and Levchenko (2012) and del Rosal (2013). The first authors show that trade openness can increase the explanatory power of the granular measure on the aggregate volatility, whilst the second author reaches the conclusion that the granular behavior can be present in the exports of all UE countries, causing significant impact on the aggregate output.

In addition, it is also our purpose to try to explain this behavior theoretically. The model developed by Delli Gatti et al. (2005) has been chosen as a base for the theoretical analysis. It employs the agent-based (AB) approach which allows heterogeneous agents (multiple firms and a bank) to interact with each other. This interaction is able to create scaling laws, which are widely observed in nature, along with self-sustained growth and business cycle fluctuations. Concretely, a generalization of the model is proposed, in which the production function is not linear but exponential, with an exponent smaller than one. By doing this, we can analyze the behavior of the model when decreasing-returns-to-scale technology is introduced.

The remainder of the work is organized into five sections. Section 2 presents the related literature. It is divided into two subsections. The first one briefly reviews some of the works that constitute the literature known as "financial accelerator" hypothesis as well as some works using the AB approach to study this hypothesis. The second one is precisely dedicated to comment on the paradigm shift that is taking place in macroeconomics thanks to the AB approach. Section 3 presents the data set employed and the empirical analysis carried out with them. Section 4 introduces the generalization of Delli Gatti et al.'s (2005) model proposed and develops the deterministic analysis of it. Section 5 shows the simulation results, comparing the ones obtained with the original model to the ones obtained with the generalization. Finally, Section 6 concludes.

2. Related literature

2.1. Macroeconomic models and the "financial accelerator" hypothesis

Until the 1990s, mainstream macroeconomics had adopted the assumptions underlaying the Modigliani-Miller theorem (see Modigliani and Miller (1958)), namely: frictionless markets, competitive markets, individuals and firms can undertake financial transactions at the same prices, no asymmetries of information, no taxes and no bankruptcy cost. These assumptions imply that financial structure is both indeterminate and irrelevant to real economic outcomes (Bernanke et al., 1999). However, a large body of literature, referred as "financial accelerator" hypothesis (FAH), has shown that financial factors have a significant impact on business cycle fluctuations and on the transmission of monetary shocks. Concretely, when credit markets are characterized by asymmetric information and agency problems, they are able to propagate and amplify shocks to the macroeconomy (Bernanke et al., 1996, 1999), hence the Modigliani-Miller irrelevance theorem no longer applies.

It shall be presented below some works finding evidence that the credit market is relevant to explain aggregate fluctuations. One of the first works using the "financial accelerator" hypothesis is the one carried out by Bernanke and Gertler (1989). These authors, by introducing asymmetry of information between the entrepreneurs (borrowers) and the savers (lenders) in a RBC model, show that borrowers' net worth and the agency costs of investment are inversely correlated. According to them, the implication of this finding is two-fold. First, the procyclicality of borrowers' net worth causes a decline in agency costs in booms and a rise in recessions, which is able to generate investment fluctuations and cyclical persistence, hence a kind of accelerator effect emerges. Second, shocks to borrowers' net worth will be an initiating source of real fluctuations. This makes that those individuals with the most direct access to investment projects become un-creditworthy. The resulting fall in investment has negative effects on both aggregate demand and aggregate supply.

Following the approach adopted by Bernanke and Gertler (1989), Bernanke et al. (1999) show that asymmetries of information play a key role the relationship between lenders and borrowers. They show that the existence of credit-market imperfections creates an inverse relationship between the external finance premium, namely the difference between the cost of funds raised externally and the opportunity cost of funds internal to the firm, and borrowers' net worth. This fact occurs because, when borrowers cannot contribute to the financing of projects due to the lack of wealth, the divergence of interests between the borrower and the suppliers of external funds increases, leading to an increase in agency costs; in equilibrium, lenders must be compensated for higher agency costs by a larger premium. Since borrowers' net worth is found to be procyclical, the external finance premium is countercyclical, which enhances the swings in borrowing and thus in investment, spending, and production. Therefore, this leads the authors to conclude that the addition of credit-market effects raises the possibility that relatively small changes in entrepreneurial wealth could be an important source of cyclical fluctuations.

Focusing also in the asymmetries of information as an amplifying source of economic cycles, Bernanke and Gertler (1990) study an economy in which entrepreneurs evaluate potential investment projects and select those that seem most worthwhile. Entrepreneurs, who are also borrowers, are assumed to know more about the project's probability of success than the potential lender. According to the authors, This informational asymmetry introduces an Akerlof "lemon" problem (see Akerlof (1970)) in the issuance of securities. The consequence of this problem is an increase in the prospective financing costs and thus the entrepreneurs' willingness to evaluate projects in the first place. The results obtained suggest that, in general equilibrium, both the quantity of investment spending and its expected return will be sensitive to the net worth positions of potential borrowers, i.e., the entrepreneurs. Furthermore, it is found that a decline in borrowers' net worth below an endogenously determined limit will precipitate a complete collapse of credit markets and investment.

Greenwald and Stiglitz (1993) use the existence of informational imperfections to interfere in the appropriate distribution of risks among economic agents. In other words, the role played by information imperfections is to restrict a firm's ability to raise equity funds in external capital markets. In the framework employed by the authors, firms make their economic decisions taking into account the risk consequences and their willingness to undertake risks is affected both by their total net worth and their stock of liquid assets. The results point out that the level and distribution of net worth among firms has real macroeconomic implications, namely changes in firms' perceptions of the risks which they face and in their net worth position have potentially large effects on their willingness to produce.

Despite the fact that these kind of models were able to shed light on how financial factors can have a real impact on the economy, Gallegati et al. (2003) suggest two drawbacks that characterize the "financial accelerator" macroeconomic models: (1) the dynamics of the variance of their financial position is not analyzed and (2) the lack of interaction direct interaction among agents. Based on these facts, they decide to resort to the AB approach. The authors create an AB economy in which heterogeneous agents (multiple firms and one bank) interact in the financial markets. Allowing the interaction of the agents causes the model to be able to generate empirical regularities such as: every recession is forestalled by a sensible rise of the ratio between debt commitments to profits and the ratio between debt and capital, firm sizes are left-skewed distributed, growth rates are Laplace distributed and small idiosyncratic shocks to firms generate large aggregate fluctuations. In addition, the model may also develop bankruptcy cascades. When a firm goes bankrupt, it leaves the market and cannot pay back its debt as well as the debt commitments to the bank. Therefore, the bank's balance sheet deteriorates because of the capital loss. As a consequence of the existing link between bank capital and the credit supply, this last shrinks, triggering an interest rate rise. Since debt commitments rise, firms insolvency increases more, thus self-reinforcing this dynamic.

Battiston et al. (2007), focusing on credit inter-linkages, also study the link between the financial acceleration and financial contagion. They show that financial acceleration offsets the stabilizing role of risk sharing and amplifies the effects of a shock to a single agent of the network, leading to a full fledged systemic crisis. The authors suggest that above an intermediate level of connectivity, a further increase may have the perverse effect of amplifying financial distress through the financial acceleration and to increase systemic risk.

More recent work also points in this direction. For instance, Grilli et al. (2014), using the AB approach, find that when an interbank connectivity exceeds a certain threshold, it increases agents' financial fragility and generates larger bankruptcy cascades because it triggers a larger systemic risk which in turn prevails over sharing risk. The authors define this threshold as pseudo-optimal and claim that it depends critically on the random network topology modeled.

2.2. Agent-based models: the alternative paradigm

The Lucas' critique (Lucas, 1976) of econometric models argues that they are not suitable to evaluate economic policies because they lack a microeconomic foundations, namely tastes and technology, which may cause the estimated coefficients to vary when there are changes in policy regimes.

This idea was incorporated into the *general equilibrium* (GE) framework, in which the equilibrium between supply and demand is achieved by maximizing the behavior of agents. Concretely, price-taking firms who produce goods and services of known type and quality maximize their profit, price-taking consumers with exogenously determined preferences maximize their utility and the Walrasian Auctioneer that determines prices to ensure each market clears (Tesfatsion and Judd, 2006). These optimizing agents are assumed to have rational expectations (Hoover, 1990), which, according to Kryvtsov and Petersen (2015), implies that: they consider all the information available, understand how the economy works and the future consequences of their actions and make optimal decisions that are time consistent. Thus future policy changes can be discounted and have an effect on the choice of today. Therefore, changes in economic policies are analyzed dynamically.

Nevertheless, this approach poses problems in terms of aggregation. Models are not tractable due to strong non-linearities. The representative-agent (RA) was used to overcome this problem. The existence of this representative maximizing agent whose choices coincide with the aggregate choices of the heterogeneous individuals allows to compute the *optimal* aggregate solution by means of a summation of the choices made by each agent (Delli Gatti et al., 2007). Moreover, its dynamics is identical to that of each single unit. Therefore, the RA is able to provide microfoundations for aggregate behavior and a framework in which equilibria are unique and stable (Kirman, 1992).

The RA assumption has aroused much criticism. Kirman (1992), for instance, argues that reducing the behavior of a heterogeneous group of agents is both unjustified and leads to misleading conclusions. The author sets out four reasons why the use of the RA in macroeconomic models is not sustainable.

1. Individual maximization does not imply collective rationality, nor a certain degree of rationality observed in the collective implies that individuals act rationally.
2. Even if the first point is overlooked and it is accepted that the choices of the aggregate are those of a maximizing individual, the reaction of the RA to parameter changes may not coincide with the aggregate reactions of the agents represented.
3. Assuming that the two previous points do not apply, there may still be cases where, given two situations, the RA prefers the first to the second, whilst every individual prefers the second to the first.
4. At the empirical level, the RA assumption causes this individual to have unnatural characteristics, because his behavior has the complicated dynamics of a group of heterogenous individuals.

In addition, according to Fagiolo and Roventini (2012), the presence of a Walrasian auctioneer, setting prices before exchanges take place, along with the RA assumption rule out almost by definition the possibility of interactions carried out by heterogeneous individuals.

Geweke (1985), using the neoclassical production and several representative agents (one for production, one for factor demand and one for supply), shows that the perils (incorrect evaluations of the effect of a policy change) of ignoring aggregation are of the same order of magnitude as those of ignoring expectations, and proposes to treat aggregation more explicitly than is usually done. Kirman (1992) claims that:

> [...] to develop appropriate microfoundations for macroeconomics is not to be found by starting from the study of individuals in isolation, but rests in an essential way on studying the aggregate activity resulting from the direct interaction between different individuals.

Trying to break away from the *reductionist* paradigm, which is based on the RA assumption, some economists have adopted the *holistic* approach. According to this approach, the aggregate is different from the sum of its components because of the interaction between them. Interactions create emergent properties at aggregate level, hence the properties of the sub-units can be grasped only analyzing the behavior of the aggregate as a whole

(Delli Gatti et al., 2005). In this approach the concept of equilibrium differs dramatically from the mainstream equilibrium concept. The equilibrium of a system does not require that every element is in equilibrium, but the aggregate is quasi-stable, namely all influences acting on the system offset each other so that the system is in an unchanging condition (Tesfatsion and Judd, 2006). Thus the aggregate equilibrium is compatible with the individual disequilibrium (Feller, 1957).

The holistic approach seems more suitable to study a complex system, which is the sign of human societies, institutions and organizations (Gilbert, 2004), since they are characterized by the presence of a wide number of mutually interacting elements whose interactions produce non-linear dynamics (Grilli et al., 2014) that generate a feedback process where causes and effects are no longer proportional to each other (Helbing, 2012).[1] In a complex system the dynamics at the micro-level are chaotic [2]. However, from this chaotic interaction of heterogeneous individuals *natural laws* emerge at the aggregate level as the outcome of a self-organizing process (Grilli et al., 2014).

A modeling strategy based on the RA is not able to reproduce either the persistent heterogeneity of the agents or their interactions, hence the search for natural laws in economics does not require the adoption of the reductionist paradigm. Some authors (see, for instance, Amaral et al. (1997), Marsili and Zhang (1998) and Stanley et al. (1995)) have shown that scaling laws are generated by a system with strong heterogeneous interacting agents (HIA), which is incompatible with the reductionist approach employed by mainstream economics (Delli Gatti et al., 2005).

Delli Gatti et al. (2005) suggest that the agent-based modeling (ABM) strategy should be adopted in order to study HIA, because ABMs have been developed to study the interaction of many heterogeneous agents. They are based on new microfoundations, whose relevance and reliability are grounded in the empirical evidence they can account for. Therefore, microfoundations are solid if they produce an economic behavior coherent with the empirical evidence, not with some optimizing principle.

ABMs applied to economics are called *Agent-based Computational Economics* (ACE). ACE is defined by Tesfatsion and Judd (2006) as the computational study of economic processes modeled as dynamic systems of interacting agents, where the term “agent” refers to bundled data and behavioral methods representing an entity constituting part of a computationally constructed world.

[1] Despite the fact that there is consensus on the characteristics of a complex system, namely interacting units and emergent properties (Flake, 1998), no consensus has been reach on its definition. Tesfatsion and Judd (2006) gather up to three different definitions used in the complex systems literature.

[2] According to Ott (2002), a system is chaotic if it shows an exponential sensitivity to initial conditions.

Fagiolo and Roventini (2012) enumerate ten characteristics defining ACE models. First, *bottom-up perspective*, which means that aggregate properties must be obtained as the macro-outcome of a possibly unconstrained micro-dynamics at the level of basic agents. Second, *heterogeneity* of agents. Third, the *evolving complex system approach*, namely aggregate properties emerge out of repeated interactions among agents. Fourth, *nonlinearity* of interactions, and nonlinear feedback between micro- and macro-levels. Fifth, *direct endogenous interactions*, namely the decisions undertaken today by an agent directly depend on the past choices made by other agents in the population. Sixth, *bounded rationality*, which implies that agents have some local and partial principles of rationality both in time and space. Seventh, *the nature of learning* consequence of dynamically changing environments. Eighth, *"true" dynamics*, i.e., true, non-reversible dynamics generated by adaptive expectations Ninth, *endogenous and persistent novelty* that forces agents to learn and adapt. Tenth, *selection-based market mechanisms*, which are complex and span a number of dimensions.

Fagiolo and Roventini (2012) also explain the methodology followed by ACE models. According to these authors the evolution of the system is observed in discrete time steps ($t = 1, 2, \ldots$), and over time the size of agents, which populate the economy, may change. At each time t every agent i is characterized by a finite number of microeconomic variables, which may change across time, and by a vector of microeconomic parameters, that are fixed in time. In addition, the economy may be characterized by some macroeconomic (fixed) parameters.

Given the microeconomic variables and parameters specified at $t = 0$, at $t > 0$ some agents are randomly chosen to update their variables. The agents selected collect their available information about the current and past state of a subset of other agents, typically those they directly interact with, and plug this information into heuristics, routines and other algorithmic behavioral rules, which, in turn, are designed to mimic empirical and experimental knowledge found in the literature. Once the update is completed, a new set of microeconomic variables is fed into the economy. Aggregate variables are computed by summing up or averaging individual characteristics.

Finally, because of stochastic components in decision rules, expectations or interactions, the dynamics of the variables can be explained by a stochastic process parameterized by micro- and macro-parameters. Nevertheless, the existence of nonlinearities makes the researcher to have to resort to computer simulations in order to analyze the behavior of the model.

3. Data set

The data employed is taken from the database *Sistema de Análisis de Balances Ibéricos* (SABI), elaborated by INFORMA D&B in collaboration with Bureau Van Dijk, which contains more than two million of Spanish firms and five hundred thousand Portuguese firms. Yet we are only interested in the Spanish firms. As can be seen in Table 1, the sample obtained is made of forty thousand firms from all sectors, except Public Administration. However, only seven thousand have observations for each year, which represent 17.5% of the total sample. The most populated sectors in our sample are: Wholesale Trade - Durable Goods, Wholesale Trade - Nondurable Goods, General Building Contractors, Food & Kindred Products and Business Services. In order to assess whether the sample is representative of the Spanish economy, the out-degree and in-degree distribution of the input-output (IO) network of the whole economy have been calculated. It is found that the largest sectors are Wholesale, Construction and Food Products, which coincides with our sample. Other important sectors are Real State and Accommodation and Food Services.[3] Moreover, when plotting the decumulative distribution function of number of firms by sector, this resembles to the in-degree and out-degree distributions (see Fig. 1).

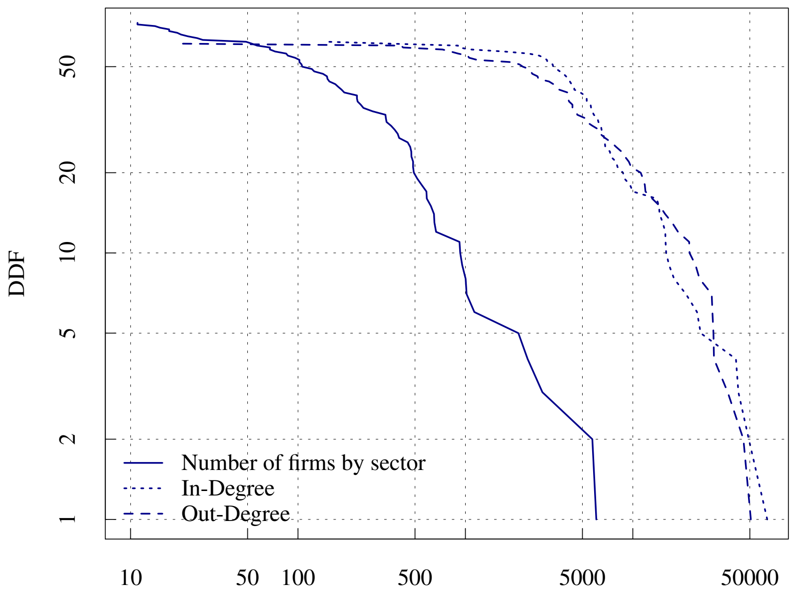


Fig. 1. Decumulative distribution function (DDF) of number of firms by sector, in-degree and out-degree of IO network of year 2010.

[3]The IO tables can be found at `http://www.ine.es/dyngs/INEbase/es/operacion.htm?c=Estadistica_C&cid=1254736165950&menu=enlaces&idp=1254735576581`.

Table 1. Sector definitions and number of firms in each sector. Firms operating in more than one sector are classified according to the first SIC code they provided.[4]The fourth column refers to the firms for which there are observations each year.

Division	SIC/Sector	Sample	Complete cases
Agriculture, Forestry, & Fishing	01 Agricultural Production - Crops	182	13
	02 Agricultural Production - Livestock	360	57
	07 Agricultural Services	73	6
	08 Forestry	22	3
	09 Fishing, Hunting, & Trapping	49	16
Mining	10 Metal, Mining	14	3
	13 Oil & Gas Extraction	17	1
	12 Coal Mining	19	5
	14 Nonmetallic Minerals, Except Fuels	141	33
Construction	15 General Building Contractors	2884	172
	16 Heavy Construction, Except Building	333	101
	17 Special Trade Contractors	922	147
Manufacturing	20 Food & Kindred Products	2353	554
	21 Tobacco Products	15	4
	22 Textile Mill Products	225	59
	23 Apparel & Other Textile Products	167	47
	24 Lumber & Wood Products	225	42
	25 Furniture & Fixtures	149	35
	26 Paper & Allied Products	487	161
	28 Chemical & Allied Products	452	126
	29 Petroleum & Coal Products	931	305
	27 Printing & Publishing	11	6
	30 Rubber & Miscellaneous Plastics Products	548	165
	31 Leather & Leather Products	57	14
	32 Stone, Clay, & Glass Products	622	160
	33 Primary Metal Industries	487	112
	34 Fabricated Metal Products	957	232
	35 Industrial Machinery & Equipment	653	166
	36 Electronic & Other Electric Equipment	475	140
	37 Transportation Equipment	585	141
	38 Instruments & Related Products	102	18
	39 Miscellaneous Manufacturing Industries	102	33
Transportation & Public Utilities	40 Railroad Transportation	25	4
	41 Local & Interurban Passenger Transit	394	101
	42 Trucking & Warehousing	1003	153
	43 U.S. Postal Service	27	3
	44 Water Transportation	335	63
	45 Transportation by Air	125	7
	46 Pipelines, Except Natural Gas	17	2
	47 Transportation Services	649	93
	48 Communications	338	25
	49 Electric, Gas & Sanitary Services	1017	105
Wholesale Trade	50 Wholesale Trade - Durable Goods	6052	1242
	51 Wholesale Trade - Nondurable Goods	5730	994
Retail Trade	52 Building Materials & Gardening Supplies	53	10
	53 General Merchandise Stores	87	11
	54 Food Stores	476	86
	55 Automative Dealers & Service Stations	516	59
	56 Apparel & Accessory Stores	227	28
	57 Furniture & Homefurnishings Stores	279	19
	58 Eating & Drinking Places	237	33
	59 Miscellaneous Retail	468	74
Finance, Insurance & Real Estate	60 Depository Institutions	85	9
	61 Nondepository Institutions	174	17
	62 Security & Commodity Brokers	68	
	63 Insurance Carriers	96	1
	64 Insurance Agents, Brokers & Service	150	17
	65 Real State	588	61
	67 Holding & Other Investment Offices	492	49
Services	70 Hotels & Other Lodging Places	668	90
	72 Personal Services	105	18
	73 Business Services	2072	187
	75 Auto Repair, Services, & Parking	246	37
	76 Miscellaneous Repair Services	106	19
	78 Motion Pictures	154	8
	79 Amusement & Recreation Services	378	59
	80 Health Services	402	65
	81 Legal Services	68	2
	82 Educational Services	189	23
	83 Social Services	121	6
	84 Museums, Botanical, Zoological Gardens	20	1
	86 Membership Organizations	11	
	87 Engineering & Management Services	1131	137
Total No. Firms		**40000**	**6995**

The period under study ranges from 1995 to 2015, namely a lapse of time of 21 years. As it is well-known, Spain went through the worst crisis of its recent history during this period. From year 1995, when Spain joins the Monetary Union, to 2008 took place the expansion phase (see Fig. 2 upper panel). A direct consequence of belonging to the Monetary Union was the reduction of interest rates and the absence of exchange rate risk, triggering a sharp increase in credit supply (Fernandez-Villaverde et al., 2013), which subsequently led to an increase in consumption and investment. The expansion phase was followed by a deep recession (Fig. 2), which began by the influence of the financial crisis of 2008 and continued because of the bubble in the construction sector, which had a major role in the Spanish economy and the accumulated household debt during the boom.

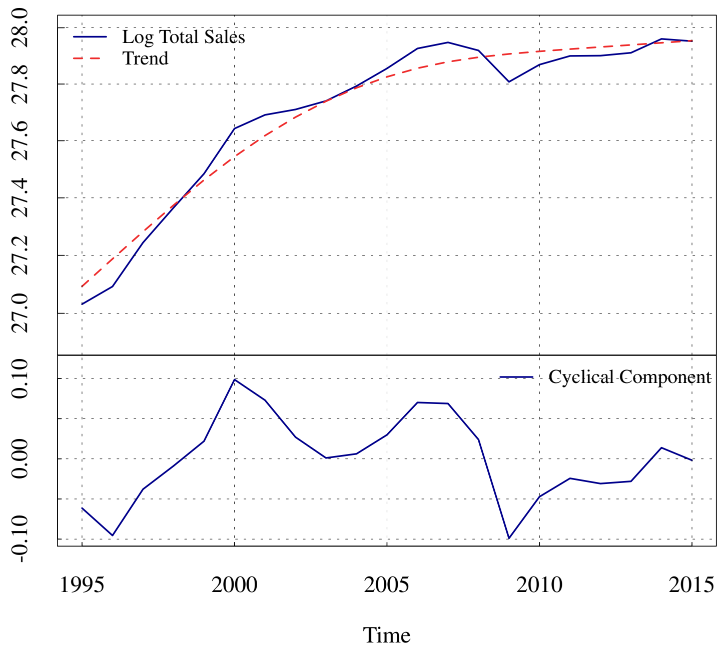


Fig. 2. Hodrick-Prescott smoothed series and cyclical component of logarithm of total sales.[5]

The only variable employed in the analysis is the amount sales. In order to make the values of sales in different years comparable, all values have been adjusted to 2010 euros by the GDP deflator, which can be found at Eurostat's database.

[4]The Standard Industrial Classification (SIC) is a system for classifying industries through a four-digit code used by government agencies of different countries, including United States and United Kingdom. In this work, it is used the version with two digits.

[5]Hodrick and Prescott (1997) proposed the following filter:

$$\min_{\{g_t\}_{t=-1}^{T}} \left\{ \sum_{t=1}^{T} (y_t - g_t)^2 + \lambda \sum_{t=1}^{T} [(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]^2 \right\},$$

in order to remove the cyclical component of a time series from raw data. The multiplier λ allows to adjust the sensitivity of the trend to short-term fluctuations. Since the frequency of the time series employed here is annual, λ has been set equal to 100.

Following Gabaix (2011), the sum of the sales of the top 50 and 100 firms in the sample as a fraction of total sales has been calculated. It is found that, on average, the 100 largest firms account for 24.8 % of the total sales and the 50 largest firms account for 18.2% (see Fig. 3), whilst they only represent, on average, 0.39% and 0.19% of the total volume of sales, respectively. This is a first piece of evidence suggesting that there exist a large heterogeneity in the sample. Aiming at characterizing the firms' size distribution, in the following section it is used statistical test and measures applied to the volume of sales.

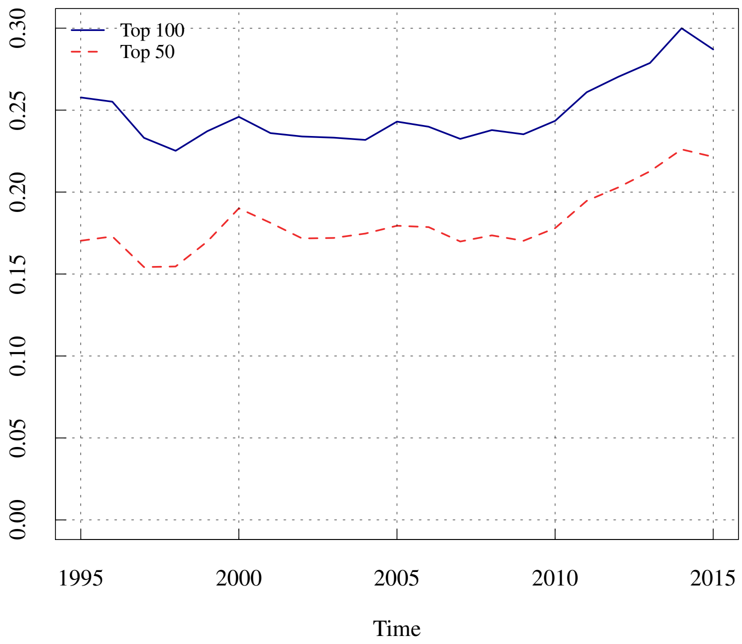


Fig. 3. Sum of the sales of the top 50 and 100 firms in the sample, as a fraction of total sales.

3.1. Empirical distributions

The graphical representation of the firms' size distribution, measured by the volume of sales, has been made using the kernel density estimation, which is a nonparametric method that estimates probability density function (PDF). The kernel density estimator is:

$$\widehat{f}_h(x) = (nh)^{-1} \sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right),$$

where the kernel K used is the gaussian function and h is the *bandwidth*, which is set equal to 0.5 (as Segarra and Teruel (2012)). The left panel of Fig. 4 shows that the tails of the firms' size distribution are fatter than the normal distribution. In addition, the Q-Q plot (left panel of Fig. 4) points out that the tails of the distribution are too fatter to be identified as gaussian.

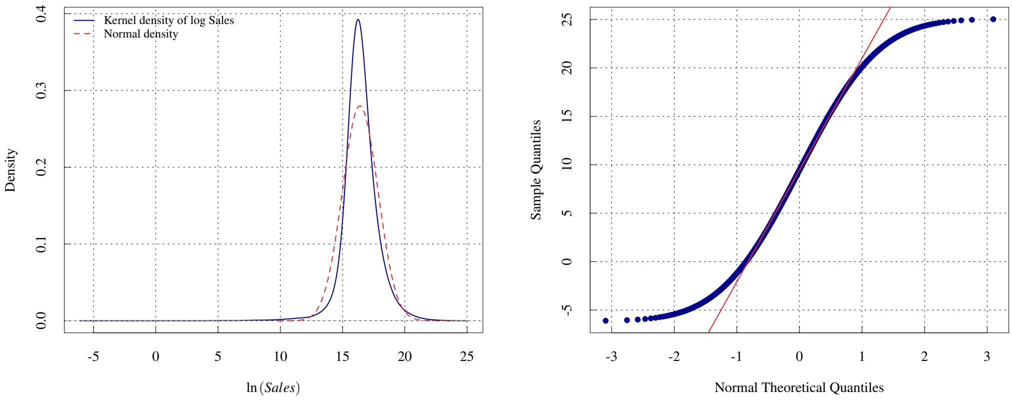


Fig. 4. Firms size distribution (left panel) and Q-Q Plot (right panel), year 2010.

In order to assess also whether the rest of the years have a firms' size distribution characterize by fatter tails, it has been resorted to several of the most employed normality tests. All of them have in common the null hypothesis (H_0), namely data is normally distributed, whereas the alternative hypothesis (H_1) states that data is not normally distributed.

The first test is the Shapiro-Wilk test. The test statistic is

$$W = \frac{\left(\sum_{i=1}^{n} a_i x_{(i)}\right)^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2},$$

where $x_{(i)}$ is the ith order statistic, i.e., the ith-smallest number in the sample, a_i are constants, and $\bar{x}$ is the sample mean. According to Razali et al. (2011), the Shapiro-Wilk test is the most powerful normality test, followed by the Anderson-Darling test and Lilliefors test. Regarding the Anderson-Darling test, the test statistic is

$$A^2 = -n - \sum_{i=1}^{n} \frac{2i-1}{n} \left[\ln(p_{(i)}) + \ln(1 - p_{(n-i+1)})\right],$$

where $p_{(i)} = \Phi\left(\left[x_{(i)} - \bar{x}\right]/s\right)$. Φ is the cumulative distribution function of the standard normal distribution and s is standard deviation of the data values. Finally, the test statistic used in the Lilliefors test is

$$D = \max\{D^+, D^-\}$$

where $D^+ = \max_{i=1,...,n}\left\{i/n - p_{(i)}\right\}$ and $D^- = \max_{i=1,...,n}\left\{p_{(i)} - (i-1)/n\right\}$.

Table 2 presents the test statistics along with the skewness and kurtosis for each year. As can be seen, the skewness has negative sign, except for years 2014 and 2015, indicating that the tail on the left side is fatter than the right side, namely the mass of the distribution

Table 2. Skewness and kurtosis, along with the normality tests: Shapiro-Wilk (SW), Anderson-Darling (AD) and Lilliefors. N is the number of complete cases. * Significant at 0%.

Year	N	Skewness	Kurtosis	SW Test[6]	AD Test	Lilliefors Test
1995	18112	-0.86	8.81	0.826*	184.062*	0.069*
1996	20064	-0.79	8.29	0.826*	179.248*	0.065*
1997	21434	-1.01	10.15	0.824*	233.032*	0.074*
1998	22975	-0.92	7.96	0.821*	265.032*	0.077*
1999	24403	-1.08	10.00	0.820*	309.455*	0.080*
2000	25809	-1.24	11.53	0.820*	375.400*	0.085*
2001	26844	-1.14	10.41	0.818*	382.598*	0.086*
2002	27664	-1.24	11.37	0.820*	435.826*	0.094*
2003	28189	-1.29	12.14	0.818*	464.674*	0.095*
2004	28503	-1.11	11.5	0.820*	428.090*	0.090*
2005	28927	-1.47	14.57	0.818*	556.807*	0.104*
2006	29420	-1.25	12.18	0.822*	560.416*	0.101*
2007	28147	-1.38	14.72	0.824*	555.931*	0.101*
2008	27496	-1.67	17.66	0.830*	639.296*	0.109*
2009	27936	-1.08	13.72	0.833*	555.512*	0.099*
2010	27997	-0.89	14.31	0.833*	581.467*	0.099*
2011	27726	-1.46	24.22	0.833*	675.035*	0.109*
2012	27451	-0.72	17.57	0.834*	646.682*	0.105*
2013	27427	-0.7	20.26	0.832*	758.450*	0.114*
2014	27221	0.39	12.69	0.827*	854.325*	0.118*
2015	25721	1.74	8.14	0.827*	1116.651*	0.149*

is concentrated on the right hand side. The kurtosis is higher than three for all years, which is the value observed in the univariate normal distribution, hence the distribution of firms' size is *leptokurtic*. This implies that the distributions produce more outliers than the normal distribution. In regard to the normality tests, all of them have a p-value equal to zero, so that the null hypothesis of normality can be rejected in favor of the alternative hypothesis. This finding is in line with Ganugi et al. (2005) and Reichstein and Jensen (2005), who rejected that the firm's size distribution, measured by the volume of sales, is characterized by a lognormal distribution.

Following Newman (2005), it has been decided to use a rank/frequency plot of the data in order to illustrate the degree of heterogeneity in the firms' size distribution. To make the plot of the DDF $\mathrm{P}(x)$,[7] it is needed to sort the volume of sales in decreasing order and rank them, then by definition there are n volume of sales with frequency greater than or equal to the nth most common volume of sales. Thus the DDF $\mathrm{P}(x)$ is proportional to the rank n of a volume of sales. The resulting graph of the process explained above can be seen in the Fig. 5. It is shown how a very small number of firms had a very large volume of sales, whilst a very large number of firms had a very small volume of sales. Thus the distribution

[6]Due to a maximun imposed by the R package *Stats* in the number of observations that can be used to compute the SW Test, it has been used the first 5000 firms each year.

[7]Plot of the probability $\mathrm{P}(x)$ that x has a value greater or equal to x: $\mathrm{P}(x) = \int_x^\infty \mathrm{p}(x')\,\mathrm{d}x'$.

is characterized by an upper fatter tail.

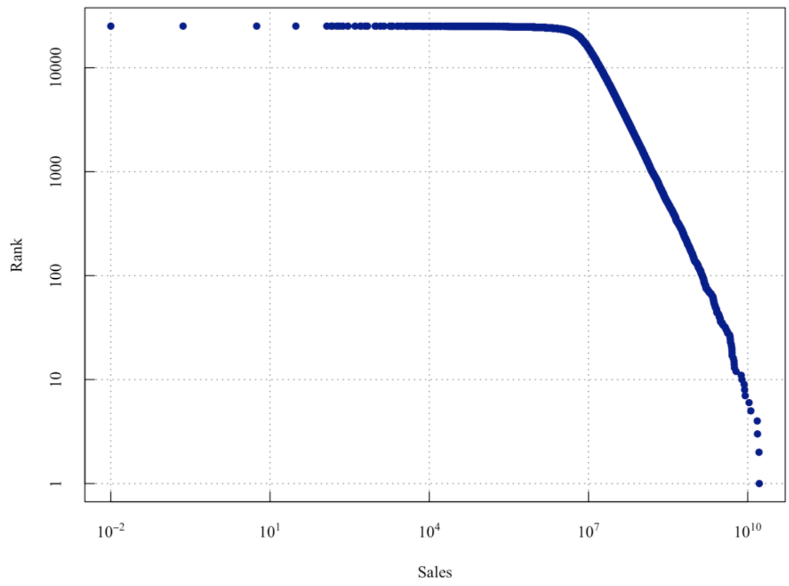


Fig. 5. Rank/frequency plot with logarithmic scales, year 2010.

Moreover, the plot in Fig. 5 follows closely a straight line for a volume of sales exceeding 10^7 (Zipf, 1949). This implies that the fraction of firms with a volume of sales between x and $x + \mathrm{d}x$, $\mathrm{p}(x)$, can be adjusted by $\mathrm{p}(x) = \mathrm{c} - \zeta \ln x$. Taking exponential of both sides,

$$\mathrm{p}(x) = \mathrm{e}^{c} x^{-\zeta},$$

which is a kind of distributions considered to follow a *power-law*.

Based on this evidence, next section is devoted to assess whether the firms' size distribution follows a power-law.

3.2. Power-law

Clauset et al. (2009) states that the density presented above cannot hold for all $x \geq 0$, since it diverges as $x \to 0$. Therefore, it is essential to have a lower bound to the power-law behavior, which is denoted by them as x_{min}. Provided $\zeta > 1$, they find that the continuous power-law distribution is

$$p(x) = \frac{\alpha - 1}{x_{min}} \left(\frac{x}{x_{min}} \right)^{-\zeta}.$$

Aiming at estimating the scaling parameter ζ, the authors use the *method of maximum likelihood* (MLE). Assuming that the data employed is drawn from a power-law distribution for $x \geq x_{min}$, they found that the MLE for the continuous case is

$$\widehat{\zeta} = 1 + n \left[\sum_{i=1}^{n} \ln \left(\frac{x_i}{x_{min}} \right) \right]^{-1},$$

where x_i are the observed values of x such that $x_i \geq x_{min}$. This MLE for the scaling parameter is equivalent to the Hill estimator (Hill, 1975). The standard error associated is $(\widehat{\zeta} - 1)/\sqrt{n} + \mathrm{O}(1/n)$.

Regarding the lower bound, x_{min}, it is estimated by choosing the estimated lower bound ($\hat{x}_{min}$) that makes the probability distributions of the data and the best-fit power-law model similar as possible above $\hat{x}_{min}$ (Clauset et al., 2007). To quantify the distance between two probability distributions, Clauset et al. (2009) decided to use the Kolmogorov-Smirnov (KS) statistic, which is the maximum distance between the cumulative distribution functions (CDF) of the data and the fitted model:

$$D = \max_{x \geq x_{min}} |\mathrm{S}(x) - \mathrm{P}(x)|,$$

where $\mathrm{S}(x)$ is the CDF of the data for those observations with value at least x_{min}, and $\mathrm{P}(x)$ is the CDF for the power-law model that best fits the data in the region $x \geq x_{min}$. Therefore, $\hat{x}_{min}$ is the value of the lower bound that minimizes the distance.

Finally, Clauset et al. (2009) propose a *goodness-of-fit test* based on measurement of the distance between the distribution of the empirical data and the hypothesized model. First, a large number of power-law distributed synthetic data sets with scaling parameter $\hat{\zeta}$ and lower bound $\hat{x}_{min}$ are generate. Then, each synthetic data set is fitted to its own power-law model, and the KS statistic for each one is calculated. The fraction of the synthetic distance that are larger than the empirical distance is the p-value. The null hypothesis (H_0) states that data is generated from a power law distribution, whereas the alternative hypothesis (H_1) states that data is not generated from a power law distribution.

The results are presented in Table 3. As can be seen, the estimated scaling parameter is close to two, which is the number that reflects the maximum heterogeneity. It is also worth noting that this scaling parameter has been calculated using, on average, 28% of the observation, namely the number of firms with a volume of sales above the lower bound. The p-value obtained from the goodness-of-fit indicates that the power-law model is a plausible fit to the data, except for years: 2011, 2012 and 2013. Nonetheless, these results do not mean that the firms' size distribution can only be fitted by a power-law. Due to the fact that it is always hard to differentiate between log-normal and power-law behavior in small

Table 3. Estimated scaling parameter ($\hat{\zeta}$) and its standard error ($Se(\hat{\zeta})$), maximum volume of sales (max), total number of observations (N), number of observations in the region $x \geq x_{min}$ (n), and the p-value generated by the goodness-of-test. Statistically significant values are denoted in **bold**.[9]

Year	N	n	max	$\hat{x}_{min}$	$\hat{\zeta}$	$Se(\hat{\zeta})$	p-value
1995	18112	3180	$8.86 \cdot 10^9$	$2.00 \cdot 10^7$	2.13	0.020	**0.98**
1996	20064	3446	$9.50 \cdot 10^9$	$1.90 \cdot 10^7$	2.12	0.019	**0.84**
1997	21434	4000	$1.03 \cdot 10^{10}$	$1.89 \cdot 10^7$	2.12	0.018	**0.52**
1998	22975	4835	$1.06 \cdot 10^{10}$	$1.45 \cdot 10^7$	2.12	0.016	**0.10**
1999	24403	5596	$1.01 \cdot 10^{10}$	$1.83 \cdot 10^7$	2.12	0.015	**0.93**
2000	25809	6694	$1.29 \cdot 10^{10}$	$1.79 \cdot 10^7$	2.11	0.014	**0.86**
2001	26844	7094	$1.16 \cdot 10^{10}$	$2.09 \cdot 10^7$	2.11	0.013	**0.40**
2002	27664	7325	$1.05 \cdot 10^{10}$	$2.40 \cdot 10^7$	2.12	0.013	**0.71**
2003	28189	7722	$1.09 \cdot 10^{10}$	$2.32 \cdot 10^7$	2.12	0.013	**0.38**
2004	28503	8151	$1.25 \cdot 10^{10}$	$2.73 \cdot 10^7$	2.11	0.012	**0.46**
2005	28927	8650	$1.64 \cdot 10^{10}$	$2.88 \cdot 10^7$	2.11	0.012	**0.39**
2006	29420	9315	$1.83 \cdot 10^{10}$	$3.34 \cdot 10^7$	2.11	0.011	**0.64**
2007	28147	9607	$1.85 \cdot 10^{10}$	$2.81 \cdot 10^7$	2.10	0.011	**0.45**
2008	27496	9105	$2.03 \cdot 10^{10}$	$2.82 \cdot 10^7$	2.06	0.011	**0.69**
2009	27936	8441	$1.44 \cdot 10^{10}$	$1.30 \cdot 10^7$	2.03	0.011	**0.11**
2010	27997	8592	$1.65 \cdot 10^{10}$	$2.69 \cdot 10^7$	2.05	0.011	**0.90**
2011	27726	9125	$2.18 \cdot 10^{10}$	$1.11 \cdot 10^7$	2.02	0.011	0.06
2012	27451	9178	$2.60 \cdot 10^{10}$	$1.03 \cdot 10^7$	2.01	0.011	0.02
2013	27427	9460	$2.43 \cdot 10^{10}$	$1.39 \cdot 10^7$	2.01	0.010	0.05
2014	27221	9621	$2.25 \cdot 10^{10}$	$4.16 \cdot 10^7$	2.06	0.011	**0.91**
2015	25721	9736	$1.91 \cdot 10^{10}$	$4.53 \cdot 10^7$	2.07	0.011	**0.98**

samples such as the one employed here, it is decided to apply the direct comparison of models proposed by Clauset et al. (2009). These authors propose a *likelihood ratio test* intended to directly compare two distribution against each other. The procedure consist in compute the ratio of the likelihoods, one for each data. The logarithm of this ratio, $\mathfrak{R}$, can be positive, negative or zero. Nevertheless, the sign of the log-likelihood ratio is not enough to indicate which model is the better fit because it depends on statistical fluctuations. In order to identify whether the sign of $\mathfrak{R}$ is sufficiently positive or negative that could not be the result of a chance fluctuation, the authors employ the method proposed by Vuong (1989), which gives a p-value indicating whether the observed sign of $\mathfrak{R}$ is statistically significant. Only if $p < 0.1$, the sign is a reliable indicator of which model is the better to fit the data.

The alternative distributions considered in the analysis are the log-normal and the exponential. The results presented in Table 4 suggest that the exponential distribution can be ruled out for each year. However, for most of the years the test cannot differentiate between log-normal and power-law.[8] The exception are the years 2011, 2012 and 2013, for which the log-normal distributions is a better fit to the data.

[8] In a large enough sample, 5.5 million of firms, Axtell (2001) does find that the firms' size distribution follows a power-law.

[9] The results presented in Table 3 and Table 4 were calculated using the `R` package *poweRlaw*, developed by Gillespie (2014).

Table 4. Likelihood ratio test $\mathfrak{R}$ and p-values. The p-values of the power-law distribution are the same as Table 3.

	Power-law	Log-normal		Exponential	
Year	p	$\mathfrak{R}$	p	$\mathfrak{R}$	p
1995	**0.98**	0.22	0.83	9.62	0.00
1996	**0.84**	-0.31	0.75	9.55	0.00
1997	**0.52**	-0.27	0.79	10.10	0.00
1998	**0.10**	-0.31	0.76	11.71	0.00
1999	**0.93**	-0.05	0.96	12.10	0.00
2000	**0.86**	-0.14	0.89	12.54	0.00
2001	**0.40**	-0.06	0.95	13.22	0.00
2002	**0.71**	0.00	1.00	13.44	0.00
2003	**0.38**	0.05	0.96	13.88	0.00
2004	**0.46**	-0.21	0.84	13.33	0.00
2005	**0.39**	0.05	0.96	13.18	0.00
2006	**0.64**	0.02	0.99	12.85	0.00
2007	**0.45**	-0.09	0.93	14.56	0.00
2008	**0.69**	-0.73	0.46	13.95	0.00
2009	**0.11**	-1.32	0.19	17.55	0.00
2010	**0.90**	-0.85	0.40	14.43	0.00
2011	0.06	-1.61	0.11	16.91	0.00
2012	0.02	-1.23	0.22	16.38	0.00
2013	0.05	-1.51	0.13	15.45	0.00
2014	**0.91**	-0.54	0.59	12.42	0.00
2015	**0.98**	-0.50	0.62	12.99	0.00

To conclude with the power-law analysis, let us present Fig. 6, which can summarize the whole process outlined above. Both the power-law and the log-normal fits are almost identical in the region $x \geq x_{min}$, differing slightly in the upper tail. Moreover, it is emphasized that the exponential distribution is not able to describe properly the firms' size distribution.

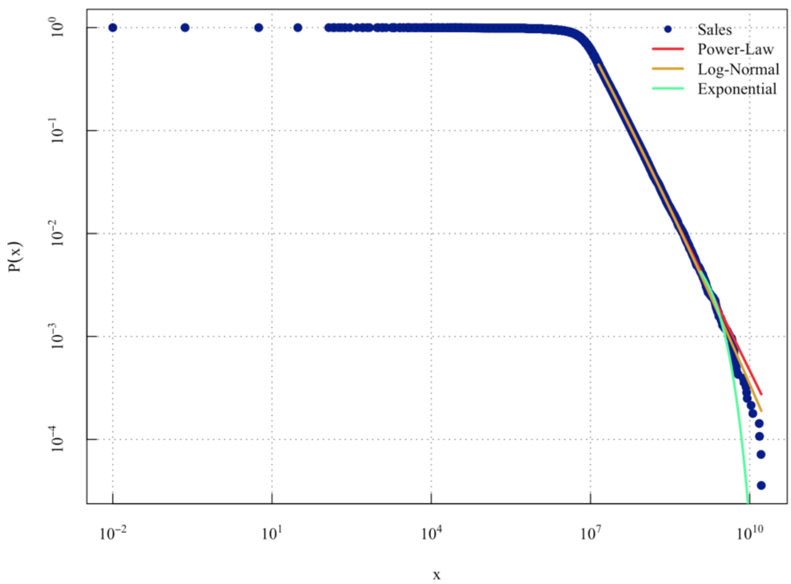


Fig. 6. The CDF $P(x)$ and its maximum likelihood power-law fit, sales in year 2010.

3.3. "Islands" Economy model

Having showed that the firms' size distribution is characterized by a great heterogeneity, it is now our aim to assess how this fact can impact at macroeconomic level. To achieve this goal we follow Gabaix (2011). According to this author, when the firms' size distribution is not uniform, the idiosyncratic shocks of the largest 100 firms are able to explain one-third of the fluctuations of GDP.

The "granular hypothesis" is illustrated by Gabaix (2011) by using the "Islands Economy" model. The model presented below has been adapted to our case of study.

It is considered an economy populated by N firms which do not have linkages between them. Firm i produces and sells in year t a quantity S_{it} of the consumption good. Firm i's growth rate can be expressed as

$$g_{it} = \frac{\Delta S_{it+1}}{S_{it}} = \frac{S_{it+1} - S_{it}}{S_{it}} = \sigma_i \varepsilon_{it+1}, \tag{3.1}$$

where σ_i is firm i's volatility and ε_{it+1} are uncorrelated random shocks with mean 0 and variance 1. Total sales is $S_t = \sum_{i=1}^{N} S_{i,t}$, and its growth is

$$\frac{\Delta S_{t+1}}{S_t} = \frac{1}{S_t} \sum_{i=1}^{N} \Delta S_{i,t+1} = \sum_{i=1}^{N} \frac{S_{it}}{S_t} \sigma_i \varepsilon_{it+1}, \tag{3.2}$$

Due to the fact that ε_{it+1} are uncorrelated, if we assume that firms all have the same volatility $\sigma_i = \sigma$, and define $\sigma_S = \sqrt{var \frac{\Delta S_{t+1}}{S_t}}$, then the standard deviation of sales can be expressed as

$$\sigma_S = \sigma \sqrt{\sum_{i=1}^{N} \left(\frac{S_{it}}{S_t} \right)^2} = \sigma h, \tag{3.3}$$

where h is the square root of the Herfindhal-Hirschman Index (HHI). Therefore, the impact of sales volatility on the aggregate depends on the firms' size distribution. This is finding is at odds with the "diversification argument" (Acemoglu et al., 2012), which states that the large number of firms populating the economy ensures that idiosyncratic shocks average out in the aggregate (Lucas, 1977). If firms all have the same weight in the economy, namely $S_{it}/S_t = 1/N$, eq. 3.3 becomes $\sigma/\sqrt{N}$, hence individual volatility is negligible. Nonetheless, it has been showed that there exist a great deal of heterogeneity. Particularly, the estimated scaling parameter α is found to be close to 2, namely the maximum heterogeneity (Zipf's law). In this case, Gabaix (2011) shows that the aggregate volatility decays much more slowly, $1/\ln N$ instead of $1/\sqrt{N}$. This implies that individual volatility has a non-negligible impact on the aggregate.

The average square root of the HHI and sales volatility have been calculated in order to illustrate the argument exposed above. Moreover, according to *Directorio Central de Empresas*, the number of firms in year 2015 is approximately 3 million.[10] Following the diversification argument the estimated volatility of sales is 0.0002 ($\sigma = 35.1\%$), whereas taking into account the existing heterogeneity the estimated volatility is 1.57 ($h = 4.5\%$), which is much more close to the actual one, 4.9.[11]

As regards to the empirical implementation, we aim to quantify not the impact of idiosyncratic shocks of the largest firms on the aggregate, but the explanatory power of the growth rate, so that unlike Gabaix (2011), we do not subtract the average growth rate from the individual growth rate. The idea it is to keep the analysis as simple as possible. The resulting measure is

$$\Gamma_t = \sum_{i=1}^{\kappa} \frac{S_{it}}{S_t} g_{it}, \tag{3.4}$$

where κ are the largest firms ($\kappa \leq N$).

The sales growth of firms has been winsorized in order to handle the outliers. Concretely, values above 100% and below -100% have been replaced with this value.[12]

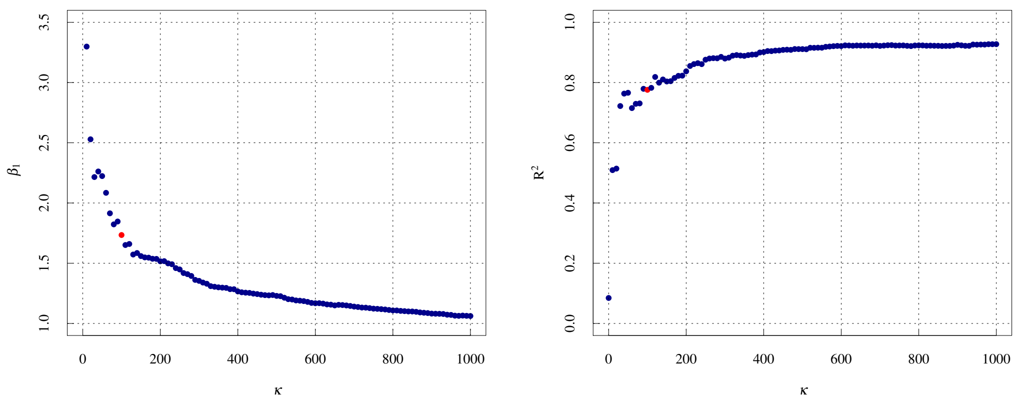


Fig. 7. β_1 and R^2 as function of the κ largest firms.

We run the growth rate of aggregate sales on eq. 3.4 in order to assess whether there exists granular behavior in our sample made of Spanish firms.[13] It can be expresses as

[10]More information can be found at `http://www.ine.es/jaxiT3/Datos.htm?t=299`.

[11]The effect of linkages between firms is estimated to have a multiplicative effect close to 2.6, in such case the estimated volatility would be 4.1%.

[12]g_{it} has been winsorized at $M = 100\%$ by replacing it by $T(g_{it})$, where $T(x) = x$ if $|x| \leq M$, and $T(x) = sign(x)M$ if $|x| \geq M$. Results do not change significantly when the threshold M changes.

[13]It has been calculated using the 1000 largest firms in the sample in order to make it comparable with the results obtained from the model.

$$g = \beta_0 + \beta_1 \Gamma + \mu, \tag{3.5}$$

where $g = \Delta S_{t+1}/S_t$. Fig. 7 shows the estimated β_1 and the determination coefficient (R^2) for different values of κ. This analysis considers the largest one thousand firms, in steps of ten firms, namely $\kappa \in \{1, 10, 20, \ldots, 1000\}$. As can be seen, the estimated β_1 tends to 1 as the number of firms used in computing eq. 3.4 increases. In addition, the evolution of R^2 indicates that there is a very strong granular behaviour in our sample. For instance, the growth rate of sales of the largest 100 firms is able to explain approximately 80% of the aggregate growth rate of one thousand firms (red dot in Fig. 7).

To sum up, in this section we have shown that our sample, made of Spanish firms, presents a large heterogeneity. The firms' size distribution cannot be described by a gaussian distribution because the probability of having extreme values is much higher. Then, we have tried to contrast whether the distribution follows a power-law distribution by using the approach suggested by Clauset et al. (2009). It is found that both power-law and log-normal can fit the distribution. Finally, following Gabaix (2011), we found that there is granular behavior in our sample, namely a few very large firms are able to account for an important part of aggregate behavior.

4. Structure of the Model

The model presented in this section aims to be a generalization of Delli Gatti et al.'s (2005) financial accelerator model. The authors consider a sequential economy populated by many firms and the banking sector ("the bank"), which undertake decisions at each time period t, where $t = 1, 2, \ldots, T$. Two markets are opened: the goods market for an homogeneous good and the credit market. In the goods market, output is supply-driven, following the *leveraged aggregate supply* class of models developed by Greenwald and Stiglitz (1990, 1993), which implies that firms sell all the output they optimally decide to produce. Due to the fact that the only input used to produce is capital, output follows the evolution over time of the capital stock, which in turn is determined by investment. Investment depends on the interest rate and the degree of financial fragility. The higher the net worth, the lower the probability of bankruptcy and the higher the level of supply and investment.

Imperfect information in the equity market is assumed to cause firms to only raise funds in the credit market. Credit demand depends on investment expenditures, which is therefore dependent on the bank's interest rate. Credit supply is a multiple of the bank's net worth, which is negatively affected as borrowing firms go bankrupt.

When firms go bankrupt, aggregate output decreases and the net worth is eroded by "bad debt". Consequently, credit supply diminishes, raising the interest rate to each firm. Fragile firms will default and leave the market, whilst the surviving ones will reduce investment and production. Bankruptcies will spread and a domino effect will follow.

As noted by Delli Gatti et al. (2007): "the source of the domino effect is the positive feedback of bankruptcy on aggregate financial fragility, which in turn is a consequence of the direct interaction of firms through the banking sector".

4.1. Firms

In each time period t, the economy is populated by large finite number of competitive firms indexed by $i = 1, \ldots, N$. It is assumed that firms have no relation between them, namely the authors assume that each firm is located in an island. At each time t firms produce an homogeneous output (Y) using as input capital (K). The firm's production functions is:

$$Y_{it} = \phi K_{it}^{\beta}, \tag{4.1}$$

where capital productivity (ϕ) is constant and uniform across firms and β is less than or equal to one. Delli Gatti et al. (2005) consider the case in which β is equal to one, hence they assume constant-returns-to-scale technology. We, however, decide to analyze the model assuming the existence of decreasing-returns-to-scale technology, namely output increases by less than that proportional change in capital.

At each time period t, firms set their individual selling price through a random process around the average market price of output P_t, according to the law

$$P_{it} = u_{it} P_t, \tag{4.2}$$

with expected value $\mathbb{E}(u_{it}) = 1$ and finite variance σ^2. Moreover, $u_{it} = P_{it}/P_t$ is a random variable uniformly distributed in the range $[0, 2]$.

The authors assume that firms are fully rationed on the equity market, so that the only external source of finance they have at their disposal is credit. Firms can finance their capital stock through either net worth (A) or bank loans (L), according to the balance sheet identity $K_{it} = A_{it} + L_{it}$. Assuming that firms and bank hold a long-term contractual relationship, the debt commitment in real term is $r_{it} L_{it}$, where r_{it} is the real interest rate and the return on net worth. Thus, financial cost faced by firms are equal to $r_{it}(A_{it} + L_{it}) = r_{it}K_{it}$. Total variable costs are equal to $gr_{it}K_{it}$, with $g > 1$. g represents other financial costs related to the capital. Therefore, profits in real terms (π) are

$$\pi_{it} = \frac{P_{it}Y_{it} - P_t gr_{it}(L_{it} + A_{it})}{P_t} = u_{it}\phi K_{it}^{\beta} - gr_{it}K_{it}, \tag{4.3}$$

and the expected profit is $\mathbb{E}(\pi_{it}) = \phi K_{it}^{\beta} - gr_{it}K_{it}$.

The law of motion of net worth at each time period t is

$$A_{it} = A_{it-1} + \pi_{it}, \tag{4.4}$$

and firms go bankrupt when their net worth becomes negative, $A_{it} < 0$. Inserting eq. 4.3 into eq. 4.4 and rearranging, it is obtained that bankruptcy occurs when

$$u_{it} < \frac{1}{\phi}\left(gr_{it}K_{it}^{1-\beta} - \frac{A_{it-1}}{K_{it}^{\beta}}\right) \equiv \bar{u}_{it}. \tag{4.5}$$

The probability of bankruptcy is therefore

$$Pr(u_{it} < \bar{u}_{it}) = \frac{1}{2}\bar{u}_{it} = \frac{1}{2\phi}\left(gr_{it}K_{it}^{1-\beta} - \frac{A_{it-1}}{K_{it}^{\beta}}\right). \tag{4.6}$$

Following Greenwald and Stiglitz (1990, 1993), Delli Gatti et al. (2005) incorporate the probability of bankruptcy into the firm's profit function, since going bankrupt costs, and this cost is increasing in the firm's output. In addition, it is assumed that bankruptcy costs are quadratic, cY_{it}^2, with $c > 0$. The objective function is as follows

$$\Gamma_{it} = \phi K_{it}^{\beta} - gr_{it}K_{it} - \frac{c\phi}{2}K_{it}^{\beta}(gr_{it}K_{it} - A_{it-1}). \tag{4.7}$$

The first order condition is

$$\frac{\partial \Gamma_{it}}{\partial K_{it}} = \phi\beta K_{it}^{\beta-1} - gr_{it} - \frac{c\phi\beta}{2}K_{it}^{\beta-1}(gr_{it}K_{it} - A_{it-1}) - \frac{c\phi gr_{it}}{2}K_{it}^{\beta} = 0.$$

Rearranging it, we obtain the following equation

$$2\beta\phi + c\beta\phi A_{it-1} = 2grK_{it}^{1-\beta} + (\beta+1)cgr_{it}\phi K_{it} \tag{4.8}$$

In order to find the desired capital, namely the optimal capital stock, it is assumed that $K^{\beta-1}$ in eq. 4.8 is equal to one. The resulting expression is

$$K_{it}^{d} = \frac{2(\phi\beta - gr_{it})}{(\beta+1)c\phi gr_{it}} + \frac{\beta A_{it-1}}{(\beta+1)gr_{it}} \tag{4.9}$$

Desired investment in time t is the difference between the desired capital stock and the capital stock inherited from the previous period, $I_{it} = K_{it}^{d} - K_{it-1}$. To finance this investment, firms recur to retained profits and to new mortgaged debt, $I_{it} = \pi_{it-1} + \Delta L_{it}$, where $\Delta L_{it} = L_{it} - L_{it-1}$. Therefore, the demand for credit is: $L_{it}^{d} = K_{it}^{d} - \pi_{it-1} - A_{it-1}$. Introducing eq. 4.9 into the previous expression and rearranging, it is obtained that the demand for credit is given by:

$$L_{it}^{d} = \frac{2(\phi\beta - gr_{it})}{(\beta+1)c\phi gr_{it}} - \pi_{it-1} + \frac{(\beta - 2gr_{it})A_{it-1}}{(\beta+1)gr_{it}} \tag{4.10}$$

4.2. Banks

In Delli Gatti et al.'s (2005) model, it is assumed that banks are lumped together in a vertically integrated banking sector. Credit supply (L^{s}) at each time period t is determined by the sum of bank's equity (E) and deposits (D), which are determined as a residual. This is $L_{t}^{s} = E_{t} + D_{t}$. To determine the aggregate level of credit supply, the authors assume that the bank is subject to a prudential rule such that

$$L_{t}^{s} = \frac{E_{t-1}}{\nu}, \tag{4.11}$$

where the risk coefficient ν is constant. Therefore, the more financial robustness the bank has, the higher the credit supply.

Credit is allotted to each individual firm i on the basis of the mortgage it offers, which is proportional to its size, and to the amount of cash available to serve debt according to the following rule:

$$L_{it}^{s} = \lambda L_{t}^{s}\Phi_{it-1} + (1-\lambda)L_{t}^{s}\Lambda_{it-1}, \tag{4.12}$$

where $\Phi_{it-1} = K_{it-1}/K_{t-1}$ and $\Lambda_{it-1} = A_{it-1}/A_{t-1}$, with $K_{t} = \sum_{i=1}^{N_{t-1}} K_{it-1}$, $A_{t} = \sum_{i=1}^{N_{t-1}} A_{it-1}$ and $0 < \lambda < 1$. The equilibrium interest rate is determined as credit demand (eq. 4.10) equals credit supply (eq. 4.12), that is:

$$r_{it} = \frac{\beta\,(2 + cA_{it-1})}{gc\,((2/c\phi) + (1+\beta)A_{it-1} + (1+\beta)\pi_{it-1} + (1+\beta)L^s_{it})} \tag{4.13}$$

Assuming that the return on bank's equity is given by the average of lending interest rate $\bar{r}_t$ and deposits are remunerated with the borrowing rate r^A_{it}, the bank's profit (π^B) at time t is given by:

$$\pi^B_t = \sum_{i \in N} r_{it} L^s_{it} - \bar{r}_t\,[(1-\omega)\,D_{t-1} + E_{t-1}]\,, \tag{4.14}$$

where ω captures the degree of competition in the banking sector and $1/(1-\omega)$ is the spread between lending and borrowing interest rates.

The law of motion of E_t at each time period t is

$$E_t = \pi^B_t + E_{t-1} - \sum_{i \in \Omega_{t-1}} B_{t-1}, \tag{4.15}$$

where Ω_{t-1} is a set containing the bankrupt firms and B_{t-1} represents the *bad debt* of these firms. According to eq. 4.15, idiosyncratic real shocks lead to systemic consequences. An increase in bad debts makes equity decrease, which in turn leads to a lower credit supply. Consequently, financial costs rise due to a higher interest rate. Firms' net worth distribution affects the average lending interest rate, which in turn influences the bank's profits and, hence, credit supply. Thus, the firms affect each other trough indirect interactions.

4.3. Firms' demography

As pointed above, firms go bankrupt when their net worth becomes negative, so that they leave the market. Delli Gatti et al. (2005) specify that new entrants, replacing bankrupted firms, are determined by the following endogenous mechanism:

$$N^{entry}_t = \bar{N}\mathbb{P}\,(entry) = \frac{\bar{N}}{1 + \exp\left[d\,(\bar{r}_{t-1} - e)\right]}, \tag{4.16}$$

where $\bar{N} > 1$, d and e are constants. The higher is the interest rate, the higher are firms debt commitments, and the lower is the number of entries. In addition, it is assumed that entrants are on average smaller than incumbents (Bartelsman et al., 2005; Caves, 1998). New firms' capital stock is a fraction of the average of incumbents' stocks. Therefore, entrants' size in terms of their capital stock is drawn from a uniform distribution centered around the mode of the size distribution of incumbent firms.

4.4. Deterministic analysis and dynamics

The deterministic analysis takes only into account one bank and one firm ($i = 1$), which sets a price of the homogeneous good equal to the expected one. Thus, the representative firm is unaffected by shocks on price, $u_t = 1$. This fact implies that the firm's profit is the same as the expected profit in the stochastic version. The idea is to consider the deterministic version as a benchmark to later explain the stochastic dynamic.

Following Pulcini (2017), it is first compute the return on equity (ROE), namely the measure of a firm's profitability indicating how much profit it generates without recourse to external sources of financing. It is defined as $ROE^{firm} = \pi/A_{t-1}$. Rearranging eq. 4.9, it is found that the net worth is expressed as

$$A_{t-1} = \frac{K_t(\beta+1)gr_t}{\beta} - \frac{2(\phi\beta - gr_t)}{\beta c\phi}. \tag{4.17}$$

Considering that the first component of eq. 4.17 grows exponentially because of K, whereas the second component changes much more slowly, the equation can be approximated as follows

$$A_{t-1} \approx \frac{K_t(\beta+1)gr_t}{\beta}. \tag{4.18}$$

Therefore, the representative firm's ROE is

$$ROE^{firm} = \frac{\beta\left(\phi K_t^{\beta} - gr_t K_t\right)}{K_t(\beta+1)gr_t}. \tag{4.19}$$

Due to the fact that the bank side of the model has not been modified, we employ the bank's ROE, defined as $ROE^{bank} = \pi^b/A_{t-1}^b$, computed by Pulcini (2017). It is as follows

$$ROE^{bank} = r_t\omega\gamma, \tag{4.20}$$

where $\gamma = 1/\nu - 1$. As can be see, while ROE^{bank} depends positively and linearly on the interest rate, ROE^{firm} does it negatively and non-linearly.

The equilibrium interest rate $r^\star$ presented in eq. 4.21 is obtained by Equalizing eq. 4.19 and 4.20 and solving for r.

$$r^\star = \frac{\beta}{1+\beta}\frac{1}{2\gamma\omega}\left(\sqrt{1+4K_t^{\beta-1}\frac{\gamma\omega\phi}{g}\left(\frac{1+\beta}{\beta}\right)}-1\right). \tag{4.21}$$

Making use of the following Taylor expansion: $\sqrt{1+x} \approx 1 + \frac{1}{2}x - \frac{1}{8}x^2 + o(x)$, the equilibrium interest rate can be expressed as

$$r^{\star} = \frac{\phi}{g} K_t^{\beta-1} - \frac{\phi^2}{g^2} \gamma\omega \left(\frac{1+\beta}{\beta} \right) K_t^{2\beta-2}. \tag{4.22}$$

Inserting eq. 4.22 into the deterministic version of firm's profits, $\pi = \phi K^{\beta} - g r^{\star} K$, we find that the second term of the expansion is essential for the economy to grow (see eq. 4.23).

$$\pi = \frac{\phi^2}{g} \gamma\omega \left(\frac{1+\beta}{\beta} \right) K_t^{2\beta-1}. \tag{4.23}$$

Now, eq. 4.22 is inserted into eq. 4.18,[14] so that we obtain

$$A_{t-1} = \frac{\beta+1}{\beta} \phi K_t^{\beta} \tag{4.24}$$

According to 4.24, representative firm's equity is its revenue times the mark-up $(\beta+1)/\beta$. Using eq. 4.23 and eq. 4.24, we can compute the analytic ROE^{firm}, which is

$$ROE_t^{firm} = \frac{\phi}{g} \gamma\omega K_t^{\beta-1}. \tag{4.25}$$

Following the same procedure for calculating the bank's ROE, we find that this is

$$ROE_t^{bank} = \frac{\phi}{g} \gamma\omega K_t^{\beta-1}. \tag{4.26}$$

Then, in equilibrium, $ROE^{firm} = ROE^{bank}$. In additon, recalling that law of motion of firm's equity is the equity of the previous period plus profits (see eq. 4.4), the growth rate of the bank's and the firm's equity are the ROE^{bank} and ROE^{firm}, respectively. If we calculate the growth rate of eq. 4.24 and divide it by the equity in $t-1$ (eq. 4.24), it is obtained that the growth rate of equity is equal to the growth rate of capital times β (eq. 4.27).

$$\frac{\Delta A}{A} = \frac{\frac{\beta+1}{\beta} \beta \phi K^{\beta-1} \Delta K}{\frac{\beta+1}{\beta} \phi K^{\beta}} = \beta \frac{\Delta K}{K} \tag{4.27}$$

Equalizing ROE^{firm} and the growth rate of capital times β, we find that growth rate of

[14] For simplicity, we only have taken into account the first term of the eq. 4.22.

capital is explained as

$$\frac{\Delta K}{K} = \chi K^{\beta-1}, \tag{4.28}$$

where $\chi = \frac{\phi\gamma\omega}{\beta g}$. Thus, the growth rate of the economy depends on the financial parameters γ and ω, the real parameters ϕ and g, and β.

Rearranging and solving eq. 4.28, we obtain the theoretical capital:

$$K_t = K_0 \left(1 + \chi \frac{(1-\beta)\,t}{K_0^{1-\beta}}\right)^{\frac{1}{1-\beta}}. \tag{4.29}$$

For $\beta \to 1$, eq. 4.29 can be expressed as: $K_t = K_0 \exp\left(\frac{\phi\gamma\omega}{g}t\right)$.

We now insert eq. 4.29 into eq. 4.22, taking into account the first term of the expansion, in order to obtain the theoretical interest rate:

$$r_t = \frac{\phi}{g} K_0^{\beta-1} \left(1 + \chi \frac{(1-\beta)\,t}{K_0^{1-\beta}}\right)^{-1} \tag{4.30}$$

It is worth noting that when β is set equal to one, $r_t = \phi/g$, which is the interest rate based on perfect competitive equilibrium found in the original model (see Pulcini (2017)).

5. Simulations

This section is devoted to present the simulation results generated by the generalization of Delli Gatti et al.'s (2005) model we propose in this work. It is considered both the endogenous entry mechanism and the constant entry mechanism. In addition, we will discuss which one is able to get closer to the empirical evidence presented in section 3.

In the simulations reported, the initial number of firms (N_0) is set to 1000 and the number of iterations (T) to 1000. When the endogenous entry mechanism is employed, $\bar{N}$ is set to 18, $d = 100$ and $e = 0.1$. Concerning the parameters, we follow Delli Gatti et al. (2005). For the firm they are set as follows: $\phi = 0.1$, $c = 1$, $\nu = 0.08$ and $g = 1.1$. For the bank, they are: $\omega = 0.002$ and $\lambda = 0.3$.

Simulations start at time $t = 1$. To perform calculations in period one for each firm we must set the following initial conditions: $K_{i0} = 100$, $A_{i0} = 20$, $L_{i0} = 80$, $\pi_{i0} = 0$ and $B_{i0} = 0$. These initial conditions are uniform across firms. Therefore in period zero firms are identical.

5.1. Comparison between stochastic and deterministic case

The left panel of Figure 8 shows the evolution of aggregate capital in the stochastic case with constant number of firms and the theoretical capital when $\beta = 1$. It can be seen that the aggregate capital grows as fast as the theoretical version. This fact implies that the latter is able to replicate the growth rate of the former.

In regard to the interest rate (see Figure 8 right panel), it fluctuates and evolves over time around the perfect competitive equilibrium interest rate, $r_t = \phi/g$, hence it can explain the behavior of the stochastic interest rate in the long-run.

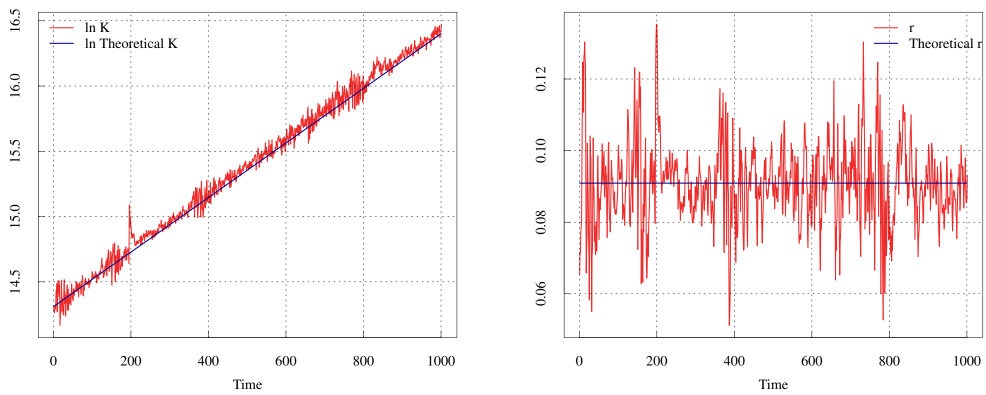


Fig. 8. Long-run theoretical K (left) and r (right) and the stochastic version when $\beta = 1$.

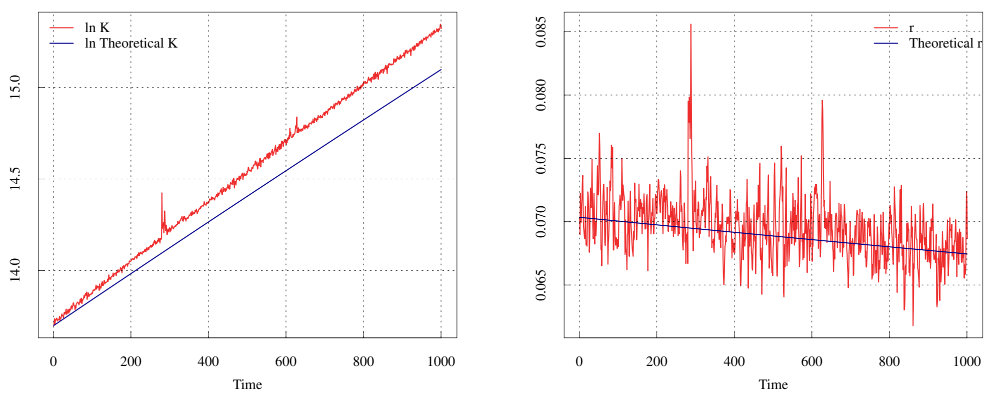


Fig. 9. Long-run theoretical K (left) and r (right) and the stochastic version when $\beta = 0.97$.

When considering $\beta < 1$, namely decreasing-return-to-scale technology, the theoretical capital does not seem to explain the dynamics of the stochastic version as well as the previous case (Figure 9 left panel). Particularly, when β is set to 0.97, it can be seen that the theoretical capital diverges from the stochastic version as time goes by. It has also been found that the difference between the two capitals increases substantially as β decreases. The reason for this behavior will be studied in subsequent works.

It is also presented the interest when $\beta = 0.97$ in the right panel of Figure 9. As can be seen, it presents a fairly reasonable adjustment of the interest rate obtained in the stochastic model.

Finally, it is interesting to focus the attention on the differences between the long-run stochastic K and r when $\beta = 1$ and when $\beta = 0.97$. Fist, the long-run K with $\beta = 1$ generates much more fluctuations than the long-run K with with $\beta = 0.97$. Second, K with $\beta = 1$ grows slightly less than K with $\beta = 0.97$, which is expected. It was not expected, however, the self-sustained growth in the very long-run generated by the model with decreasing-return-to-scale. We believe that this behavior is consequence of the third difference observed, namely the decreasing trend in r when $\beta = 0.97$. A lower interest rate makes external financing for firms more affordable, so that they can invest and produce more over time.

5.2. Simulation results with dynamic reintroduction

5.2.1. Constant-returns-to-scale technology

It is first presented the results generated by the model when β is set equal to one, namely there exists a constant-returns-to-scale technology and the entry process is determined by the endogenous mechanism exposed above.

In the left panel of Figure 10 is shown the aggregate output which is characterize by sizable fluctuations. For instance, from the simulation period 800 to 900 the growth rate of aggregate output goes from an increase of 10% to a decrease of 30%. It also reflects the existence of volatility clustering, namely large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes (Mandelbrot, 1963).

Regarding the interest rate, it fluctuates around the perfect competitive equilibrium interest rate (middle panel of Figure 10). Moreover, in periods of instability it increases substantially its volatility.

Right panel of Figure 10 reports the evolution of the number of firms as time goes on and the ratio of failed firms to new entrants. Considering that the initial number of firms was set to 1000, almost half of the population has disappear by the period 150. Then the number grows, but most of the time it is below the initial number of firms. The ratio of failed firms to new entrants fluctuates around one. It shows clearly the periods in which a large number of firms fail and the number of entrants is not enough to replace them, these are the two picks around period 800 and 1000.

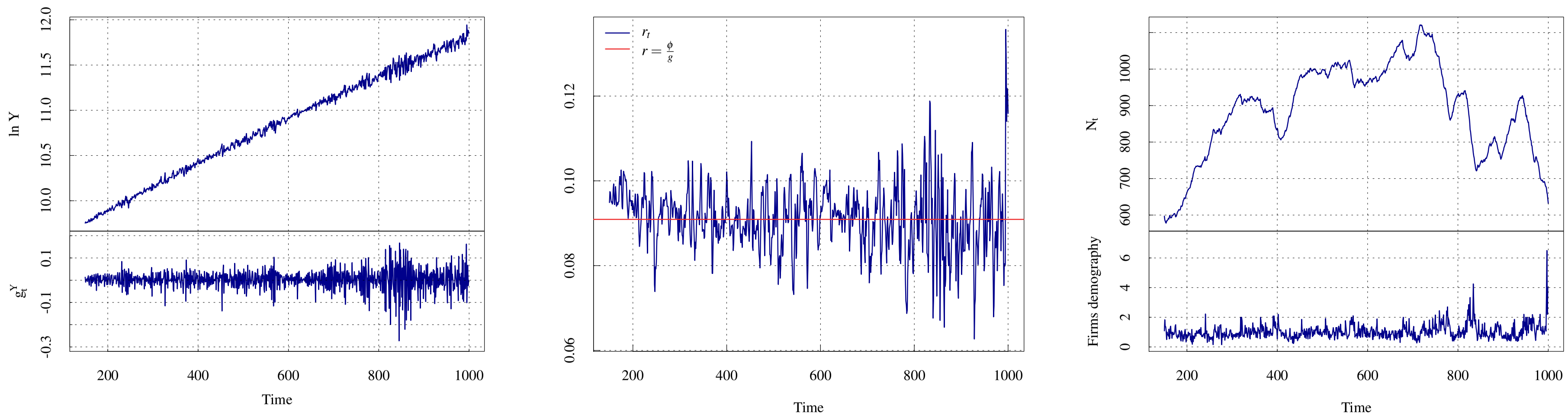


Fig. 10. Left: logarithm of the aggregate output (top) and growth rates of aggregate output (bottom). Middle: interest rate and interest rate with perfect competition in the goods market (red line), defined as $r = \phi/g$. Right: number of firms (top) and the ratio of failed firms to new firms in the bottom. Constant-returns-to-scale technology, $\beta = 1$.

We now turn our attention to the analysis of heterogeneity in firms' size distribution. Following the same procedure explained in Section 3, we aim at measuring how heterogeneous is the firms' size distribution each simulated period. The right panel of Figure 11 presents the estimated scaling parameter of the distribution. It fluctuates around the value two and has an average of 2.1, which is consistent with the values observed empirically. In addition, it is well-fitted by a power-law distribution (right panel of Figure 11). It is worth recalling that all the firms starts with the same values of capital, equity and loan, and it is the model which generates this heterogeneity. As mentioned in the literature, form the interaction of heterogeneous agents emerge naturals laws such as the power-law.

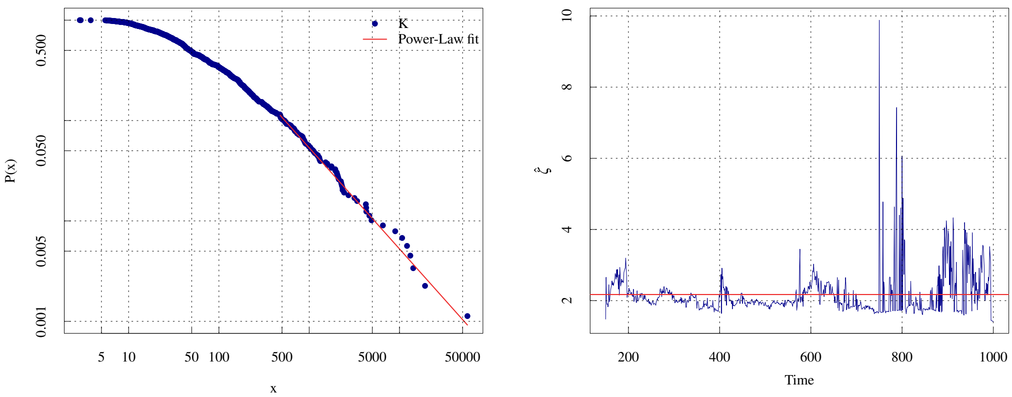


Fig. 11. The CDF $P(x)$ with its power-law fit (left) and the estimated scaling parameter ζ with its average value (right) when $\beta = 1$.

Other measures of heterogeneity are the square root of the HHI and the granular residual (Gabaix, 2011) – both have been introduced in Section 3. The first measure has been calculated as

$$h_t^K = \sqrt{\sum_{i=1}^{N} \left(\frac{K_{it}}{K_t} \right)^2}.$$

The second measure, taking into account that the output is supply-driven and hence all the output produced is sold, has been calculated using the production of each firm.

$$\Gamma_t^Y = \sum_{i=1}^{K} \frac{Y_{it}}{Y_t} g_{it}^Y,$$

where $g_{it}^Y = (Y_{it} - Y_{it-1}) / Y_{it}$.

The results are presented in Figure 12. The left panel displays the time series of the square root of the HHI. As can be seen, the values produced are much more large than the ones observed empirically ($h = 10\%$ when considering 1000 firms). It is also worth noting the existing gap between the this measure with the largest firm and without it during the period of instability. Based on this, it can be said that the fail of the largest firm triggers the recession observed in the aggregate output and the pronounced drop in the growth rate.

The explanatory power of granularity measurement is presented in the right panel of Figure 12. It has been calculated using a lapse of time 21 simulation periods, so that we can compare this result with the one observed empirically. Concretely, we have used periods ranging from 350 to 371. The results suggest that the model with constant-return-to-scale is able to generate a granular behavior that is close to the one observed. Both measures, empirical and theoretical, point out that when the number of firms used in the calculation is increased above 200, the additional explanatory power is very low.

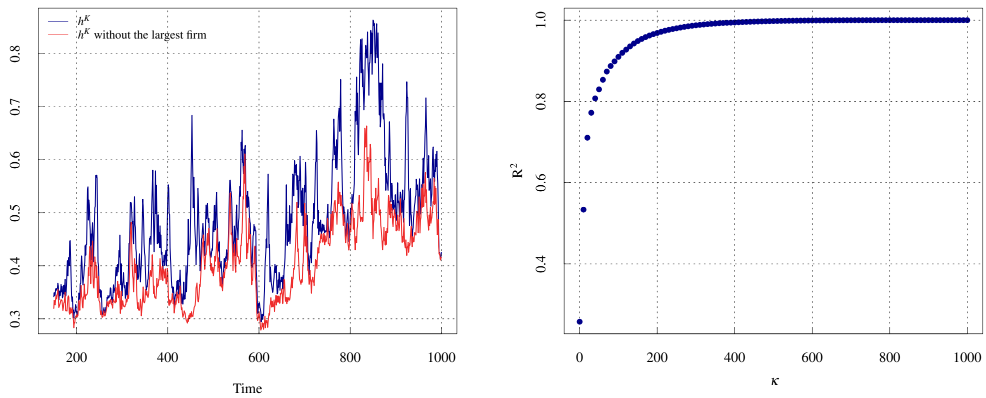


Fig. 12. h^K over time (left) and R^2 as a function of the κ largest firms (right) when $\beta = 1$.

5.2.2. Decreasing-returns-to-scale technology

The simulation results of the model with decreasing-returns-to-scale technology, β is set equal to 0.97, and dynamic reintroduction of firms are presented below.

The left panel of Figure 13 shows the aggregate output and its growth rate. It can be seen that aggregate fluctuations are much softer, in the range $\pm 4\%$, and the occurrence of expansions and recessions have disappeared. Furthermore, the economy grows at a slightly slower rate.

The interest rate (middle panel of Figure 13) seems to decrease over time and fluctuates in a narrower range than the model with decreasing-returns-to-scale technology. This has an direct impact on the population of firms (right panel of Figure 13). The number of firms grows in time, doubling the population. The economy suffers episodes where few companies disappear, but are followed by massive entries due to the fact that the interest rate is lower as time goes on. The ratio fail firms to new entries indicates that new entries are more in number that exists.

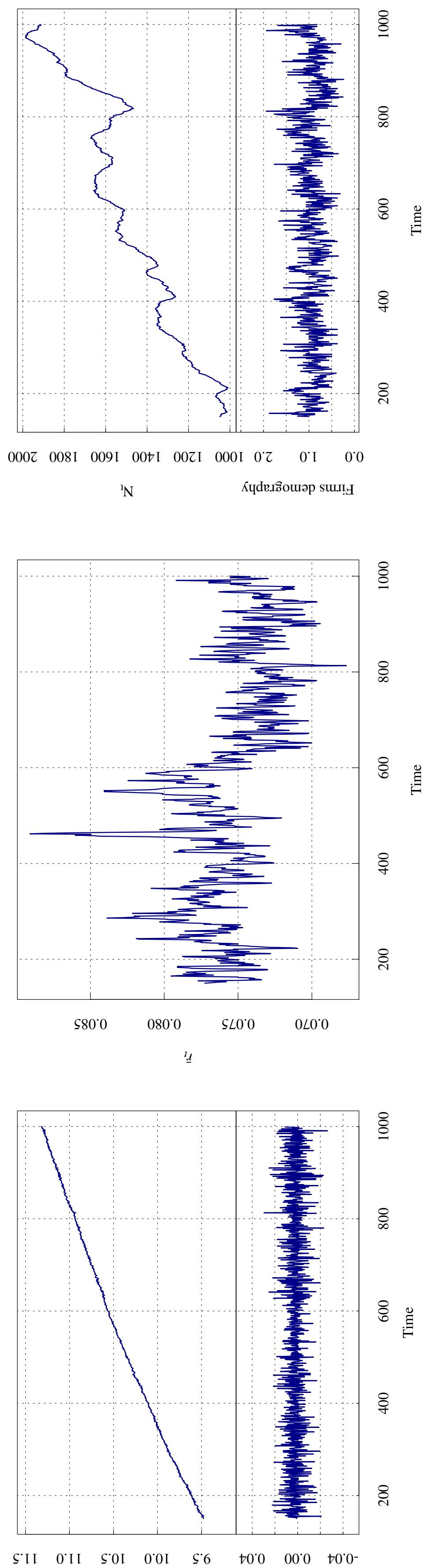
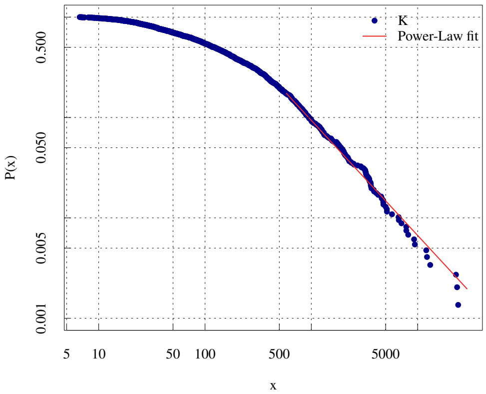


Fig. 13. Left: logarithm of the aggregate output (top) and growth rates of aggregate output (bottom). Middle: interest rate. Right: number of firms (top) and the ratio of failed firms to new firms in the bottom. Decreasing-returns-to-scale technology, $\beta = 0.97$.

Despite the fact that the model with $\beta = 0.97$ is able to generate power-law firms' size distributions, as shown in the left panel of Figure 14, these are not common, because the heterogeneity is reduced considerably. This can be seen in average estimated scaling parameter displayed in the right panel of Figure 14, which increases to 2.6.



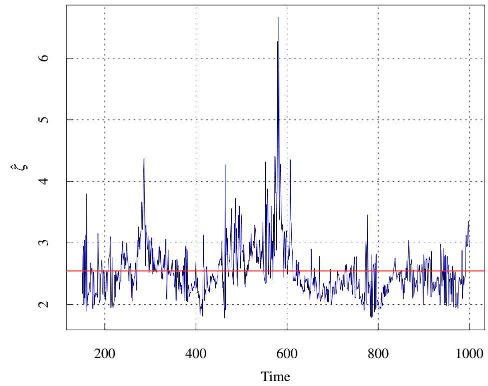
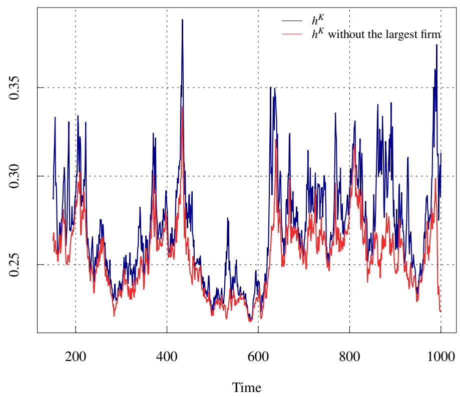


Fig. 14. The CDF $\mathrm{P}(x)$ with its power-law fit (left) and the estimated scaling parameter ζ with its average value (right) when $\beta = 0.97$.

The decrease in the heterogeneity is also reflected by the time series of the square root of the HHI (left panel of Figure 15). Now, it is closer to the empirical than it was when β was set to one. Moreover, the gap between the index with and without the largest firm has been substantially reduced.

Nevertheless, the measure employed to quantify the existence of granular behavior presents much less heterogeneity than observed (right panel of Figure 15). As can be seen, the explanatory power increases substantially until around 600 firms are considered in the calculation. Yet empirically and the case with constant performance, there was an increase in the explanatory power practically negligible from 200 companies onwards.



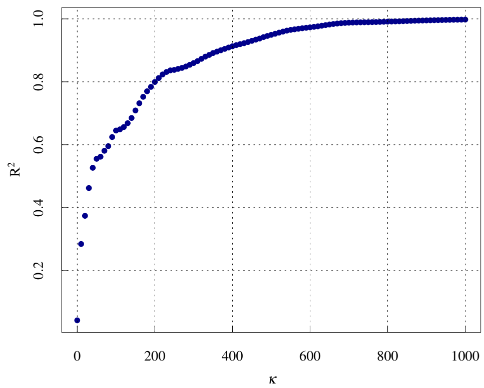


Fig. 15. h^K over time (left) and R^2 as a function of the κ largest firms (right) when $\beta = 0.97$.

6. Conclusion

This work has assessed the existence of granular behavior in the Spanish economy, namely a few very large firms account for a very large fraction of the macroeconomic fluctuations (Gabaix, 2011). Using the approach proposed by Clauset et al. (2009), it is found that the firms' size distribution can be fitted by a power-law distribution, which implies the existence of a high degree of heterogeneity. This, in turn, causes that the sales growth of the largest firms in the sample are able to account for most of the growth in aggregate sales.

Aiming at replicating the granular behavior theoretically, we resort to Delli Gatti et al.'s (2005) model, of which a generalization is proposed. Concretely, we employ an exponential production function instead of a linear one. This change allows us to study how the output produced by the model varies when decreasing-return-to-scale technology is taken into account.

We find that the original model is able to generate, thanks to the interaction among heterogeneous agents, a degree of heterogeneity close to the one observed empirically. For instance, the average estimated scaling parameter is very similar to one observed in the sample. Moreover, the explanatory power of the measure proposed to quantify the granular behavior evolves similarly as the number of firms employed to calculate it increases. Nevertheless, it is also observed that the model with constant-return-to-scale technology creates a gigantic firm able to capture a disproportionate market share.

When there is decreasing-return-to-scale technology, the model produces almost no heterogeneity. This is quantify by using the estimated scaling parameter of the firms' size distribution, which increases considerably and moves away from value 2, and the evolution of the explanatory power of the granular measure, which needs more firms to be taken into account to explain the same amount that is explained by many fewer companies in the original model. The model also reduces the market share of the largest firm.

It is also worth noting that it is observed that the model with decreasing-return-to-scale technology has self-sustained growth in the very long-run. We believe that this behavior is consequence of an interest rate that decreases over time, which makes external financing for firms more affordable, so that they can invest and invest more over time.

Finally, I would like propose a future line of research. This work could be enriched by using a model with multiple banks which interact with each other, competing for granting credit to firms.

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A. Simulation results with constant reintroduction

A.1. Constant-returns-to-scale technology

With constant reintroduction, namely the number of failing firms is replaced by the same number of entries, the model with constant-returns-to-scale technology generates aggregate fluctuations more frequently (see left panel of Figure 16). Unlike its dynamic reintroduction version, this version presents four clear periods of instability, in which the interest rate oscillates much more (right panel of Figure 16).

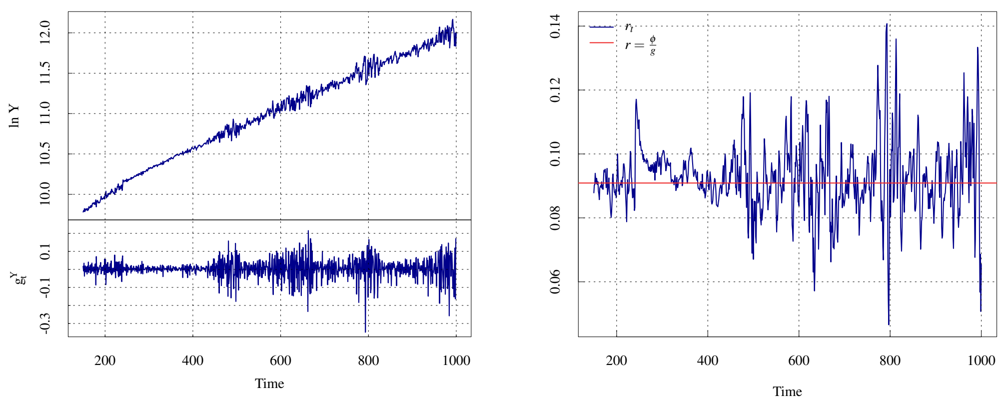

Fig. 16. Left: logarithm of the aggregate output (top) and growth rates of aggregate output (bottom). Right: interest rate. Constant reintroduction of firms. Constant-returns-to-scale technology, $\beta = 1$.

In regard to the power-law fit of firms' distribution, this version of the model is able to generate power-law distribution (left panel of 17), but, however, due to the fact that there are more periods of instability, they are less frequent. This can be see in the evolution of the estimated scaling parameter over time (right panel of 17). There are few values around the value 2.

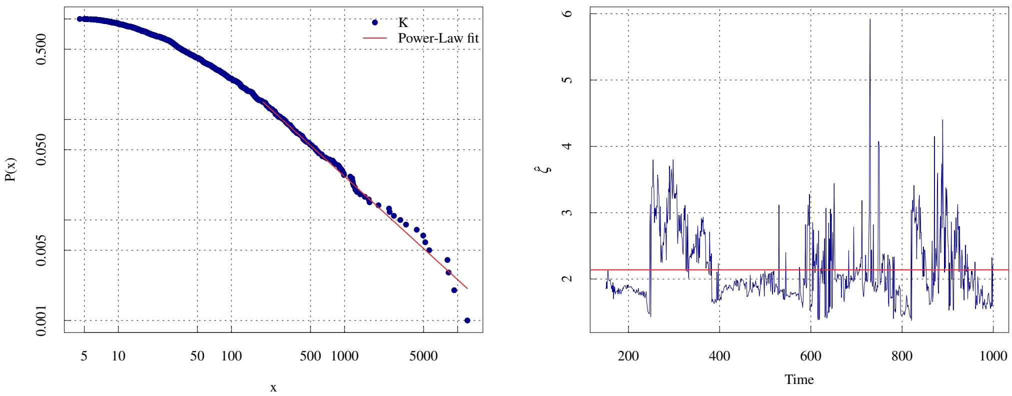

Fig. 17. The CDF $\mathrm{P}(x)$ with its power-law fit (left) and the estimated scaling parameter ζ with its average value (right) when $\beta = 1$ with constant reintroduction.

The square root of the HHI is characterize by the same behavior as the dynamic reintroduction version (see left panel of Figure 18). It produces values that are much more large than the ones observed empirically and it creates an enormous gap between the index with and without the largest firm. Concretely, it can be seen that in three of the four periods of instability, the largest firm accumulates a disproportionate market share, and its failure triggers the recession.

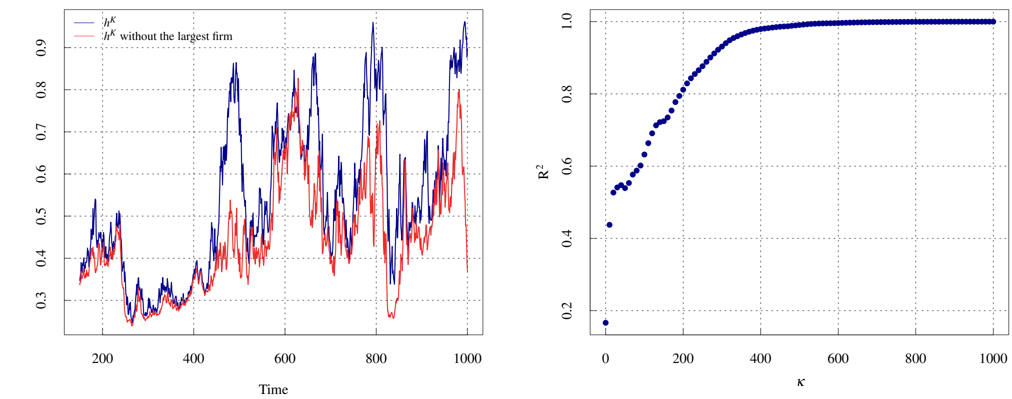

Fig. 18. h^K over time (left) and R^2 as a function of the κ largest firms (right) when $\beta = 1$ with constant reintroduction.

The granular measure indicates that when considering constant reintroduction the degree of heterogeneity is reduced substantially (see right panel of Figure 18). the number of firms from which the explanatory power increases in an almost negligible way is much higher than in the dynamic case.

A.2. Decreasing-returns-to-scale technology

The simulation results generated by the model with decreasing-returns-to-scale technology and constant reintroduction is presented in this subsection.

The model is very similar to the version with dynamic reintroduction in terms of aggregate production and interest rate (see Figure 19). It does not present pronounced aggregate fluctuations and the interest decreases over time.

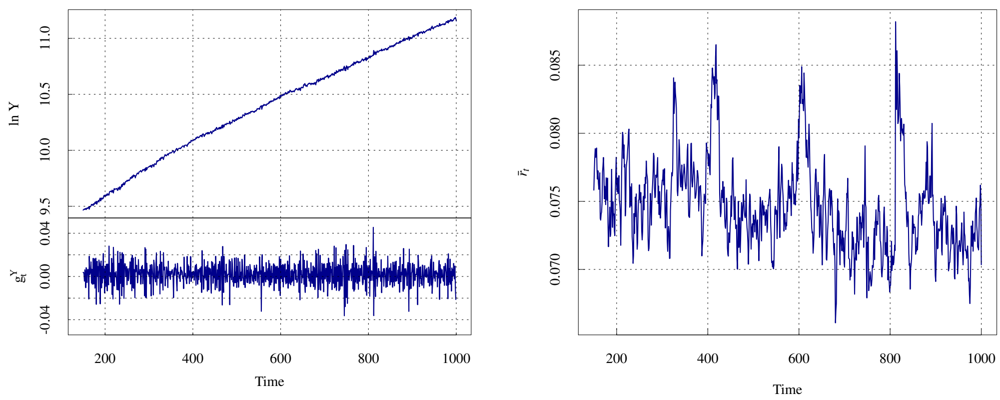

Fig. 19. Left: logarithm of the aggregate output (top) and growth rates of aggregate output (bottom). Right: interest rate. Constant reintroduction of firms. Decreasing-returns-to-scale technology, $\beta = 0.97$.

The model is able to generate firms' size distributions that can be fitted by a power-law (left panel of Figure 19), but they are rare cases, since in most of the periods the scaling parameter is very far from the value 2 (right panel of Figure 19). Concretely, the average value is 2.5.

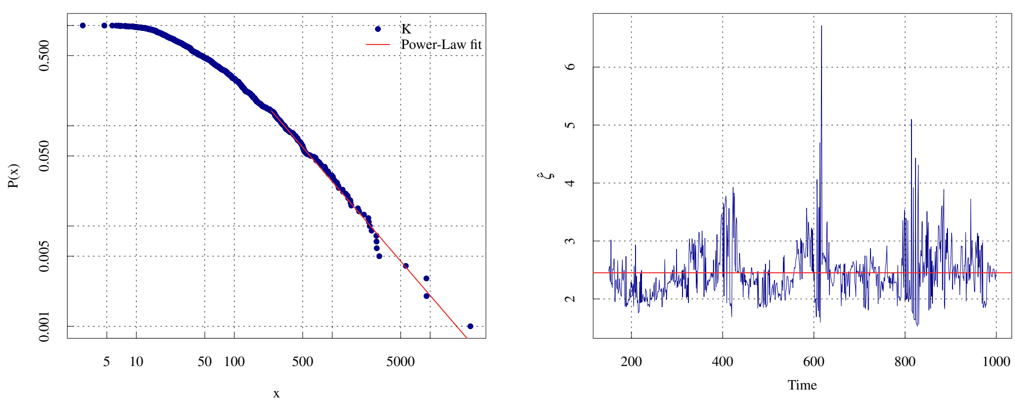

Fig. 20. The CDF $\mathrm{P}(x)$ with its power-law fit (left) and the estimated scaling parameter ζ with its average value (right) when $\beta = 0.97$ with constant reintroduction.

Finally, we show the evolution of the square root of the HHI over time in the left panel of Figure 21 and the explanatory power of the granular measure in the right panel. As can be seen, the heterogeneity is slightly higher than that generated by the model with dynamic reintroduction. Concretely, the average value of the h index is higher, and the largest firm seem to be able to accumulate a disproportionate market share. On its part, the measure of the granularity, presented in the right panel of Figure 21, indicates that heterogeneity decreases great with respect to the dynamic case. In view of this discrepancy between the two measures, the change in the degree of granularity seems more significant.

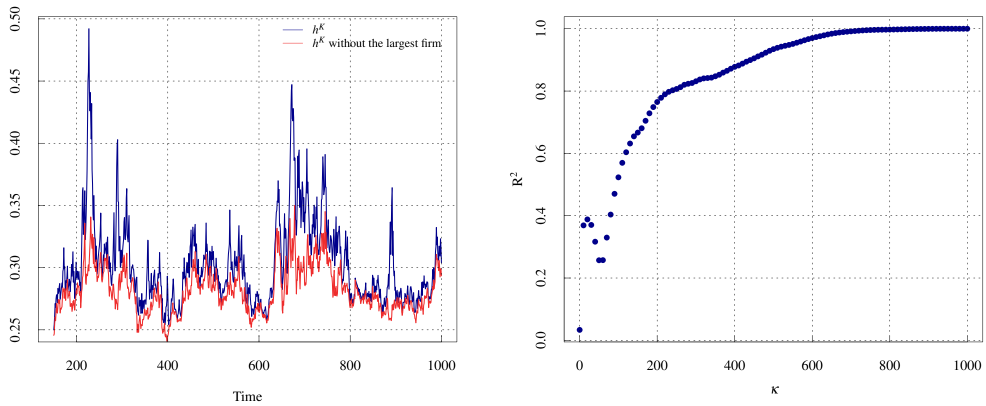

Fig. 21. h^K over time (left) and R^2 as a function of the κ largest firms (right) when $\beta = 1$ with constant reintroduction.