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ON BALANCEDNESS AND D-COMPLETENESS OF THE SPACE OF SEMI-LIPSCHITZ FUNCTIONS

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Abstract. Let (X,d) be a quasi-metric space and (Y,q) be a quasi-normed linear space. We show that the normed cone of semi-Lipschitz functions from (X,d) to (Y,q) that vanish at a point $x_0 \in X$, is balanced. Moreover, it is complete in the sense of D. Doitchinov whenever (Y,q) is a biBanach space.

1. Introduction

In the last years the study of real-valued semi-Lipschitz functions defined on a T_0 quasi-pseudo-metric space has received a certain attention [11, 12, 16, 18]. In particular, it was shown in [16] that the set of real-valued semi-Lipschitz functions defined on a T_0 quasi-pseudo-metric space (X,d) that vanish at a point $x_0 \in X$ can be structured as a normed cone. Applications of semi-Lipschitz functions to questions on best approximation, global attractors on dynamical systems, and concentration of measure can be found in [13, 16], [17] and [22], respectively.

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In [21], semi-Lipschitz functions that are valued in a quasi-normed linear space have been discussed. This study was motivated, in great part, by the fact that quasi-normed linear spaces provide suitable mathematical models in the theory of computational complexity (see [4, 5, 20]).

Here we obtain some new properties of the space $SL_0(d,q)$ of semi-Lipschitz functions defined on the quasi-metric space (X,d) with values in the quasi-normed linear space (Y,q) and that vanish at a point $x_0 \in X$. We show the somewhat surprising fact that $SL_0(d,q)$ is balanced in the sense of Doitchinov [2]. We also prove that it is complete in the sense of Doitchinov whenever (Y,q) is a biBanach space. As an application of these results to asymmetric functional analysis, we deduce that the dual space of a T_1 quasi-normed linear space is balanced and Doitchinov complete. It is interesting to recall that the study of balanced quasi-metric spaces from a fuzzy point of view has been recently started in [8, 19], and that, on the other hand, some applications of balanced (extended) quasi-metrics to theoretical computer science have been given in [14, 15].

Throughout this paper the letters $\mathbb{R}^+$ and $\mathbb{N}$ will denote the set of non-negative real numbers and the set of positive integers numbers, respectively. Our basic reference for quasi-metric spaces is [3].

Next we recall some pertinent concepts.

As usual by a *monoid* we mean a semigroup $(X,+)$ with neutral element.

According to [9] a *cone* (*semilinear space* in [16]) is a triple $(X,+,\cdot)$ such that $(X,+)$ is an Abelian monoid, and $\cdot$ is a function from $\mathbb{R}^+ \times X$ to X such that for all $x,y \in X$ and $r,s \in \mathbb{R}^+$: (i) $r\cdot(s\cdot x) = (rs)\cdot x$; (ii) $r\cdot(x+y) = (r\cdot x)+(r\cdot y)$; (iii) $(r+s)\cdot x = (r\cdot x)+(s\cdot x)$; (iv) $1\cdot x = x$.

A *quasi-norm* on a cone $(X,+,\cdot)$ is [16, 18] a function $q : X \to \mathbb{R}^+$ such that for all $x,y \in X$ and $r \in \mathbb{R}^+$: (i) $x = \mathbf{0}$ if and only if there is $-x \in X$ and $q(x) = q(-x) = 0$; (ii) $q(r\cdot x) = rq(x)$; (iii) $q(x+y) \leqq q(x)+q(y)$.

If the quasi-norm q satisfies: (i$'$) $q(x) = 0$ if and only if $x = \mathbf{0}$, then q is called a *norm* on the cone $(X,+,\cdot)$.

A *(quasi-)normed cone* is a pair (X,q) such that X is a cone and q is a (quasi-)norm on X.

If $(X,+,\cdot)$ is a linear space and q is a quasi-norm on X, then the pair (X,q) is called a *quasi-normed linear space* (*asymmetric normed linear space* in [4]). Note that, in this case, the function $q^{-1} : X \to \mathbb{R}^+$ given by $q^{-1}(x) = q(-x)$ is also a quasi-norm on X and the function $q^s : X \to \mathbb{R}^+$ given by $q^s(x) = \max\{q(x), q(-x)\}$ is a norm on X. As in [6], we say that (X,q) is a *biBanach space* if (X,q^s) is a Banach space.

An easy but crucial example of a biBanach space is the pair $(\mathbb{R},u)$, where u is the quasi-norm on $\mathbb{R}$ given by $u(x) = \max\{x,0\}$ for all $x \in \mathbb{R}$. Note that $u^s(x) = |x|$ for all $x \in \mathbb{R}$, so $(\mathbb{R},u)$ is a biBanach space.

Let us recall that a quasi-pseudo-metric on a set X is a function $d: X \times X \to \mathbb{R}^+$ such that for all $x, y, z \in X$: (i) $d(x,x) = 0$; (ii) $d(x,z) \leqq d(x,y) + d(y,z)$. If d satisfies the additional condition: (iii) $d(x,y) = 0$ if and only if $x = y$, then we will say that d is a *quasi-metric* on X.

We will also consider *extended quasi-(pseudo-)metrics.* They satisfy the above three axioms, except that we allow $d(x,y) = +\infty$.

If d is a(n extended) quasi-(pseudo-)metric, then the function d^{-1} defined on $X \times X$ by $d^{-1}(x,y) = d(y,x)$ is also a(n extended) quasi-(pseudo-)metric called the *conjugate* of d and d^s defined on $X \times X$ by $d^s(x,y) = \max\{d(x,y), d(y,x)\}$, is a(n extended) (pseudo-)metric on X.

A(n *extended) quasi-(pseudo-)metric space* is a pair (X,d) such that X is a set and d is a(n *extended) quasi-(pseudo-)metric* on X.

Each (extended) quasi-pseudo-metric d on a set X generates a topology $\tau(d)$ on X which has as a base the family of open d-balls $\{B_d(x,r) : x \in X, r > 0\}$, where $B_d(x,r) = \{y \in X : d(x,y) < r\}$ for all $x \in X$ and $r > 0$. If the topology $\tau(d)$ is T_0 we say that (X,d) is a T_0 *(extended) quasi-pseudo-metric space.* Observe that if d is a(n extended) quasi-metric, then $\tau(d)$ is a T_1 topology on X.

It is well known that each quasi-norm q on a linear space X induces a T_0 quasi-pseudo-metric d_q on X given by $d_q(x,y) = q(x-y)$ for all $x, y \in X$.

2. The results

Let (X,d) be a quasi-metric space and let (Y,q) be a quasi-normed linear space. A function $f : X \to Y$ is called *semi-Lipschitz* if there is $k \geqq 0$ such that $q\big(f(x) - f(y)\big) \leqq k d(x,y)$ for all $x, y \in X$.

Given the quasi-metric space (X,d) and the quasi-normed linear space (Y,q), fix $x_0 \in X$ and put

$$SL_0(d,q) = \left\{ f : X \to Y : f(x_0) = 0 \quad \text{and} \quad \sup_{x \neq y} \frac{q\big(f(x) - f(y)\big)}{d(x,y)} < \infty \right\}.$$

Clearly $SL_0(d,q)$ is exactly the set of all semi-Lipschitz functions from (X,d) to (Y,q) that vanishes at x_0, and $(SL_0(d,q), +, \cdot)$ is a cone, where for each $f, g \in SL_0(d,q)$ and $r \in \mathbb{R}^+$ we define $f+g$ and $r \cdot f$ in the usual pointwise way [21].

Observe that the definition of $SL_0(d,q)$ given here is slightly different from the ones given in [18]. This is due to the fact that quasi-metric spaces of [18] correspond to our T_0 quasi-pseudo-metric spaces.

Now for each $f, g \in SL_0(d,q)$ define

$$\rho_{d,q}(f,g) = \sup_{x \neq y} \frac{q\big((f-g)(x) - (f-g)(y)\big)}{d(x,y)}.$$

Then $\rho_{d,q}$ is an extended quasi-metric on $SL_0(d,q)$ and the function $\|.\|_{d,q} : SL_0(d,q) \to \mathbb{R}^+$ given by $\|f\|_{d,q} = \rho_{d,q}(f, \mathbf{0})$, for all $f \in SL_0(d,q)$ is a norm on the cone $SL_0(d,q)$, (compare [16, 18, 21]).

In [2] Doitchinov introduced an important property of symmetry in quasi-metric spaces, namely *balancedness,* to develop a satisfactory theory of completion. He observed that paradigmatic examples of quasi-metric spaces, like the Sorgenfrey line, the Kofner plane and the Pixley-Roy spaces are balanced, and proved that every balanced quasi-metric generates a Hausdorff and completely regular topology.

Recall that an extended quasi-metric space (X,d) is *balanced* provided that for each pair of sequences $(y_n)_n$, $(x_n)_n$ in X such that $\lim_{n,m\to\infty} d(y_m,x_n) = 0$, and each $x, y \in X$ and $r_1, r_2 \in \mathbb{R}^+$ satisfying $d(x,x_n) \leqq r_1$ and $d(y_n,y) \leqq r_2$ for all $n \in \mathbb{N}$, it follows that $d(x,y) \leqq r_1 + r_2$. In this case, d is called a *balanced* extended quasi-metric.

We say that the normed cone $\big(SL_0(d,q), \|.\|_{d,q}\big)$ is *balanced* if the extended quasi-metric $\rho_{d,q}$ is balanced on $SL_0(d,q)$.

According to [2], by a *Cauchy* sequence in an extended quasi-metric space (X,d) we mean a sequence $(x_n)_n$ in X for which there is a sequence $(y_n)_n$ in X satisfying $\lim_{n,m\to\infty} d(y_m,x_n) = 0$. The extended quasi-metric space (X,d) is said to be *complete* if every Cauchy sequence is convergent with respect to $\tau(d)$.

Then, Doitchinov proved that each balanced quasi-metric space (X,d) is isometrically isomorphic to a $\tau(d)$ and $\tau(d^{-1})$-dense subspace of a balanced complete quasi-metric space.

Following the modern terminology [10], Cauchy sequences in the sense of Doitchinov will be called, in the sequel, *D-Cauchy* sequences and complete extended quasi-metric spaces will be called *D-(sequentially) complete* extended quasi-metric spaces. We say that the normed cone $\big(SL_0(d,q), \|.\|_{d,q}\big)$ is D-complete if the extended quasi-metric $\rho_{d,q}$ is D-complete.

THEOREM 1. *Let (X,d) be a quasi-metric space, (Y,q) a quasi-normed linear space and $x_0 \in X$. Then $\big(SL_0(d,q), \|.\|_{d,q}\big)$ is a balanced normed cone.*

PROOF. Let $(f_n)_n$, $(g_n)_n$ be sequences in $SL_0(d,q)$ with

$$\lim_{n,m\to\infty} \rho_{d,q}(g_m, f_n) = 0,$$

and let $f, g \in SL_0(d,q)$ and $r_1, r_2 \in \mathbb{R}^+$ such that $\rho_{d,q}(f, f_n) \leqq r_1$ and $\rho_{d,q}(g_n, g) \leqq r_2$ for all $n \in \mathbb{N}$. Choose $x, y \in X$ with $x \neq y$. Then

$$q\big((f-f_n)(x)-(f-f_n)(y)\big) \leqq r_1 d(x,y),$$

and

$$q\big((g_n-g)(x)-(g_n-g)(y)\big) \leqq r_2 d(x,y),$$

for all $n \in \mathbb{N}$. Moreover, for an arbitrary $\varepsilon > 0$ there is $n_0 \in \mathbb{N}$ such that

$$q\big((g_n-f_n)(y)-(g_n-f_n)(x)\big) < \varepsilon d(y,x),$$

for all $n \geqq n_0$. Consequently

$$q\big((f-g)(x)-(f-g)(y)\big) \leqq \Big\{q\Big(\big(f-f_{n_0}\big)(x)-\big(f-f_{n_0}\big)(y)\Big)$$

$$+\,q\Big(\big(f_{n_0}-g_{n_0}\big)(x)-\big(f_{n_0}-g_{n_0}\big)(y)\Big)+q\Big(\big(g_{n_0}-g\big)(x)-\big(g_{n_0}-g\big)(y)\Big)\Big\}$$

$$< r_1 d(x,y)+\varepsilon d(y,x)+r_2 d(x,y).$$

Since ε is arbitrary, it follows that

$$q\big((f-g)(x)-(f-g)(y)\big) \leqq r_1 d(x,y)+r_2 d(x,y).$$

Therefore $\rho_{d,q}(f,g) \leqq r_1 + r_2$. We conclude that $\big(SL_0(d,q), \|.\|_{d,q}\big)$ is balanced. □

COROLLARY. *Let (X,d) be a quasi-metric space, (Y,q) a quasi-normed linear space and $x_0 \in X$. Then $\big(SL_0(d,q), \tau(\rho_{d,q})\big)$ is a Hausdorff and completely regular topological space.*

THEOREM 2. *Let (X,d) be a quasi-metric space, (Y,q) a biBanach space and $x_0 \in X$. Then $\big(SL_0(d,q), \|.\|_{d,q}\big)$ is D-complete.*

PROOF. Let $(f_n)_n$ be a D-Cauchy sequence in $SL_0(d,q)$. Then, there is a sequence $(g_n)_n$ in $SL_0(d,q)$ such that $\lim_{n,m\to\infty} \rho_{d,q}(g_m, f_n) = 0$. Thus, given $\varepsilon > 0$ there is $n_0 \in \mathbb{N}$ such that $\rho_{d,q}(g_m, f_n) < \varepsilon$ for all $n, m \geqq n_0$.

Now fix $x \in X$. Then

$$q\big((g_m-f_n)(x)\big) < \varepsilon d(x,x_0) \quad \text{and} \quad q\big((f_n-g_m)(x)\big) < \varepsilon d(x_0,x),$$

so,

$$(*) \qquad q^s\big((g_m-f_n)(x)\big) < \varepsilon d^s(x,x_0) \quad \text{for all} \quad n,m \geqq n_0.$$

Therefore, for each $n, m \geqq n_0$,

$$q^s\big((f_n - f_m)(x)\big) \leqq q^s\big((f_n - g_{n_0})(x)\big) + q^s\big((g_{n_0} - f_m)(x)\big) < 2\varepsilon d^s(x, x_0),$$

and, since (Y, q) is a biBanach space, the sequence $\big(f_n(x)\big)_n$ is convergent in (Y, q^s). Then, we can construct a function $f : X \to Y$ such that $(f_n)_n$ is pointwise convergent to f with respect to the norm q^s. Observe that, by condition $(*)$, the sequence $(g_n)_n$ is also pointwise convergent to f with respect to q^s.

We shall prove that $f \in SL_0(d)$ and that $\lim_{n\to\infty} \rho_{d,q}(f, f_n) = 0$. Indeed, first note that $f(x_0) = \mathbf{0}$ because $f_n(x_0) = \mathbf{0}$ for all $n \in \mathbb{N}$. Now, for the given $\varepsilon > 0$, for $n \geqq n_0$ and for $x, y \in X$ with $x \neq y$, there exists $m \geqq n$ such that

$$q^s\big((f - g_m)(x)\big) < \varepsilon d(x, y) \quad \text{and} \quad q^s\big((f - g_m)(y)\big) < \varepsilon d(x, y).$$

Hence

$$\frac{q\big((f - f_n)(x) - (f - f_n)(y)\big)}{d(x, y)}$$

$$\leqq \frac{q\big((f - g_m)(x) - (f - g_m)(y)\big)}{d(x, y)} + \frac{q\big((g_m - f_n)(x) - (g_m - f_n)(y)\big)}{d(x, y)}$$

$$< \frac{q^s\big((f - g_m)(x)\big) + q^s\big((f - g_m)(y)\big)}{d(x, y)} + \varepsilon < 3\varepsilon.$$

It then follows that

$$\sup_{x\neq y} \frac{q\big(f(x) - f(y)\big)}{d(x, y)} \leqq 3\varepsilon + \sup_{x\neq y} \frac{q\big(f_{n_0}(x) - f_{n_0}(y)\big)}{d(x, y)}.$$

Thus, we have shown that $f \in SL_0(d, q)$ and $\rho_{d,q}(f, f_n) \leqq 3\varepsilon$ for all $n \geqq n_0$. Consequently $\big(SL_0(d, q), \|\cdot\|_{d,q}\big)$ is D-complete. $\square$

As an application of the above results we next show that if (X, p) is a T_1 quasi-normed linear space (i.e. the quasi-pseudo-metric d_p induced by the quasi-norm p is actually a quasi-metric), then the dual space (X^*, p^*) of (X, p) is balanced and D-complete in the natural sense that we explain in the following.

Let us recall [1, 6] that if (X, p) is a quasi-normed linear space then the so-called *dual algebraic* of (X, p) is the cone X^* consisting of all linear real-valued functions on X that are upper semicontinuous on $\big(X, \tau\big((d_p)^{-1}\big)\big)$. Equivalently, X^* consists of all linear real-valued functions on X that are lower semicontinuous on $\big(X,\tau(d_p)\big)$ [18, p. 58]. It immediately follows [13, 18] that $X^* = L(X) \cap SL_0(d_p,u)$, where $L(X)$ denotes the space of all linear real-valued functions on X and $SL_0(d_p,u)$ denotes the space of all semi-Lipschitz functions from (X,d_p) to the biBanach space $(\mathbb{R},u)$ (see Section 1) that vanish at $\mathbf{0}$. Note that in this case we have

$$\rho_{d_p,u}(f,g) = \sup_{x\neq y} \frac{\big((f-g)(x)-(f-g)(y)\big)\vee 0}{p(x-y)},$$

for all $f,g \in SL_0(d_p,u)$.

Let us also recall that p^* is the function from X^* to $\mathbb{R}^+$ defined by $p^*(f) = \sup\big\{\,f(x) : p(x) \leqq 1\big\}$ for all $f \in X^*$ [1, 6], and thus (X^*,p^*) is a normed cone which is said to be the *dual space* of (X,p). Furthermore $p*(f) = \|f\|_{d,u}$ for all $f \in X^*$ [18, p. 58], and clearly, $d_{p^*}(f,g) = \rho_{d_p,u}(f,g)$ for all $f,g \in X^*$, where, as in the case of quasi-normed linear spaces, we define $d_{p*}(f,g) = \sup\big\{\,(f-g)(x) : p(x) \leqq 1\big\}$.

THEOREM 3. *Let (X,p) be a T_1 quasi-normed linear space. Then (X^*,d_{p^*}) is a balanced D-complete extended quasi-metric space.*

PROOF. By Theorems 1 and 2, $\big(SL_0(d_p,u),\rho_{d_p,u}\big)$ is a balanced D-complete extended quasi-metric space. Since balancedness is a hereditary property, then $\big(X^*,d_{p^*}\big)$ is balanced. It remains to show that $\big(X^*,d_{p^*}\big)$ is D-complete. To this end, let $(f_n)_n$ be a D-Cauchy sequence in (X^*,d_{p*}). Then $(f_n)_n$ is a D-Cauchy sequence in $\big(SL_0(d_p,u),\rho_{d,u}\big)$, so there is $f \in SL_0(d_p,u)$ such that $\lim_{n\to\infty}\rho_{d,u}(f,f_n) = 0$. Moreover, and following the proof of Theorem 2, the sequence $\big(f_n(x)\big)_n$ is pointwise convergent to $f(x)$ with respect to the Euclidean norm u^s, for all $x \in X$. Taking into account this fact, it is routine to see that f is a linear function. We conclude that $f \in X^*$ and thus (X^*,d_{p^*}) is D-complete. □

In the light of Theorem 3 it seems interesting to recall that there exist T_1 (actually Hausdorff) quasi-normed nonnormable linear spaces in abundance (see, for instance, [7]).

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