Available online at www.sciencedirect.com

ScienceDirect

Procedia Manufacturing 41 (2019) 547–554

Procedia MANUFACTURING

www.elsevier.com/locate/procedia

8th Manufacturing Engineering Society International Conference

# Incorporation of form deviations into the matrix transformation method for tolerance analysis in assemblies

Serrano-Mira, J.[a]*; Rosado-Castellano, P.[a]; Romero-Subirón, F.[a]; Bruscas-Bellido, G.M.[a]; Abellán-Nebot, J.V.[a]

[a]*Universitat Jaume I, Dpt of Industrial Systems and Engineering Design, Av. Vicent Sos Baynat, Castellón E-12071, Spain*

**Abstract**

Mathematical models for tolerance representation are used to assess how the geometrical variation of a specific component feature propagates along the assembly, so that tolerance analysis in assemblies can be carried out using a specific tolerance propagation method. Several methods for tolerance analysis have been proposed in the literature, being some of them implemented in CAD systems. All these methods require modelling the geometrical variations of the component surfaces: parametric models, variational models, DoF models, etc. One of the most commonly used models is the DoF model, which is employed in a number of tolerance analysis methods: Small Displacement Torsor (SDT), Technologically and Topologically Related Surfaces (TTRS), Matrix Transformation, Unified Jacobian–Torsor model. However, none of the DoF-based tolerance analysis methods incorporates the effect of form deviations. Among the non DoF-based methods, there are two that include form tolerances: the Vector Loop or Kinematic method and the Tolerance Map (T-Map) model, although the latter is still under development. In this work, a proposal to incorporate form deviations into the matrix transformation method for tolerance analysis in assemblies is developed using a geometrical variation model based on the DoF model. The proposal is evaluated applying it to a 2D case study with components that only have flat surfaces, but the proposal can be extrapolated to 3D cases.



*Keywords:* Tolerance analysis; form tolerances; matrix transformation method; DoF-based tolerance model

* Corresponding author. Tel.: +37 964 72 8110; fax: +37 964 72 8170.
*E-mail address:* jserrano@uji.es

© 2019 The Authors. Published by Elsevier B.V.
This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/)
Peer-review under responsibility of the scientific committee of the 8th Manufacturing Engineering Society International Conference.

## 1. Introduction

In 1D tolerance analysis, procedures used for the specification of the variability of a characteristic can be performed by dimension (size) or by tolerance zone. Usually the first one is used, while the specification by tolerance zone is used when specifications other than dimensional are also included in the analysis. In this case, to incorporate the dimensional, orientation and form variabilities, the tolerance zone is established as a linear interval in the analysis direction in which the element (surface, line, axis, etc.) is supposed to be projected. The addition of the variabilities in the same chain of tolerances (tolerance chain) is done linearly using some of the tolerance stack-up models (worst case, statistical model, method of moments, simulation, convolution, etc.).

However, 2D and 3D tolerance analysis are much more complex since, in addition to the dimensional variations, the geometric variations cause small kinematic adjustments of the components (or parts) in the assembly. In these cases, it is necessary to use a tolerance representation model in order to model the geometrical variations of each element with respect to its nominal geometry. Additionally, a tolerance propagation model (or method) to evaluate the way in which the variability of each component is propagated to the rest of the components in the assembly is required. Each tolerance propagation method is based on a single tolerance representation model.

There are many proposals of tolerance representation models. All of them are able to deal with the dimensional and geometrical orientation tolerances, but not all of them can handle form tolerances. The Degree of Freedom (DoF) tolerance model is a tolerance representation model used by different tolerance propagation methods. This tolerance model does not allow modelling form tolerances, which are considered negligible by most authors. However, when mating components in an assembly, form deviations can cause small kinematic adjustments that are added up to those caused by size and orientation tolerances. These additional kinematic displacements increase the final variability and their contribution may be relevant in some particular cases (e.g. small components in contact with much larger surfaces).

There is a plethora of proposals of tolerance representation model and tolerance propagation methods for 3D tolerance analysis. However, misunderstanding of the two concepts is present in literature, and often they mixed up or they are distinguished, being both considered under terms such as “mathematical models for tolerance representation” or similar terms. Khan [1] carries out an interesting review work in this field, although the difference between models and methods is not made.

From the works of Shah [2], Case [3], Khan [1], and other author’s proposals, the *tolerance representations models* can be established: Offset models, Parametric models, Variational Surface model (VSM), DoF models, Multi-variate region models, etc. According to the way in which each model represents the tolerances, not all of them are capable of treating the form tolerances and not all of them are able to differentiate among different types of tolerances (size, orientation, form).

Regarding the methods for tolerance propagation, there are also multiple proposals. Each propagation method uses a specific representation model, but the same representation model can be used by different propagation methods. The main methods for tolerance propagation are: Vector Loop or Direct Linearization Method (DLM) [4, 5]; Small Displacement Torsor (SDT) [6]; Technologically and Topologically Related Surfaces (TTRS) [7]; Homogeneous Transformation Matrices method (HTM); Tolerance Map (T-Map) [8, 9], Unified Jacobian-torsor method [10], and others. SDT, TTRS, HTM, and Jacobian Torsor are methods based on the DoF model. On the other hand, only the DLM and the T-Map incorporate the treatment of form tolerance, and the Vector Loop and TTRS methods are implemented in commercial CAT (Computer Aided Tolerancing) systems.

In this work, a proposal to incorporate form tolerances in the matrix transformation method for tolerance propagation is presented. The tolerance propagation method is based on the DoF tolerance representation model. The work begins by reviewing the main existing proposals of tolerance representation models and tolerance propagation methods. Next, section 3 describes the DoF tolerance model and the matrix transformation method. In section 4, the proposal to incorporate form tolerances in the matrix transformation method is described. Then, in section 5, the proposal is evaluated applying it to a case study. For simplicity, the case study used in this work is a 2D one with components that only have flat surfaces, but the proposal can be extrapolated to 3D cases.

## 2. The matrix transformation method for tolerance analysis

The matrix transformation method for tolerance analysis is based on the work of Whitney [11] for the modeling of the resulting tolerances in an assembly due to its own action. The method was proposed by Gerbino and Serrano [12] and developed by Serrano [13]. It is based on the DoF tolerance model, and makes use of homogeneous transformation matrices, with some particularisations, for tolerance propagation.

In the DoF tolerance model, the geometric element (feature) affected by the tolerance keeps its ideal form, but changes its position and orientation. The feature has a local reference system to which the position and orientation variations are applied (three translations and three rotations in the most general case). The space covered by the feature when these variations are applied determines the tolerance zone. This substitution principle of the real feature is shown in Fig. 1. The real element (with complex errors in form, position and orientation) is approximated by a substitute feature. The geometry of the substitute feature has the same nature as the nominal (a plane remains a plane, a cylinder remains a cylinder, etc.), but its position and orientation are fitted to that of the real feature, that is, the form is kept invariant, but not the position or orientation.

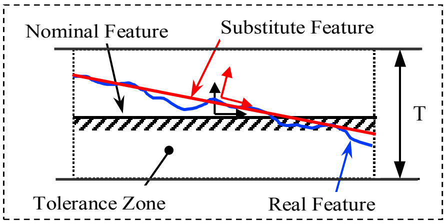


Fig. 1. DoF tolerance model: approximation of a real geometrical feature to the nominal substitute feature with modification of position and orientation, and the resulting tolerance zone (the geometry variations are shown in aggrandized way) [12].

The position and orientation of the substitute feature is obtained by multiplying the homogeneous transformation matrix that defines the reference system of the original ideal feature by a homogeneous transformation matrix M referred to the local reference system. In the matrix M, the parameters (u, v, w) and (α, β, γ) represent the three translations and the three rotations/angles with respect to the local reference system (x, y, z). In the case of the variations induced by the tolerances, it can be assumed that (α, β, γ) << 1, so that in the matrix M the small deviations between the nominal feature and the substitute feature can be approximated to the equivalent infinitesimals. By introducing them into the matrix M, the matrix of variations (or deviations) MV is obtained as shown in Eq. 1, where s = sin() and c = cos(). The components of this MV matrix indicate the DoFs that do not keep the geometry of the feature invariant. This type of tolerance modelling enables the representation of position, size and orientation tolerances, but not of form tolerances.

$$M = \left[\begin{array}{ccc|c} c\gamma c\beta & -s\gamma c\alpha + c\gamma s\beta s\alpha & s\gamma s\alpha + c\gamma s\beta c\alpha & u \\ s\gamma c\beta & c\gamma c\alpha + s\gamma s\beta s\alpha & -c\gamma s\alpha + s\gamma s\beta c\alpha & v \\ -s\beta & c\beta s\alpha & c\beta c\alpha & w \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \Rightarrow MV = \left[\begin{array}{ccc|c} 1 & -\gamma & \beta & u \\ \gamma & 1 & -\alpha & v \\ -\beta & \alpha & 1 & w \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \quad (1)$$

The matrix transformation method for tolerance analysis uses 4x4 homogenous transformation matrices to locate in the space each individual component in relation to a coordinate system. The matrix locating each component in terms of position (p) and orientation (R) is referred as matrix of position (MP). Each MP uses six variables, three translations (according to the X, Y, Z axes) and three rotations around the same axes, and its expression is identical to the matrix M in equation (1). In this way, each component is assigned a reference system that locates it in a space through an MP. Additionally, each component feature is assigned a reference system that is located with respect to the component reference system through its corresponding MP. Each MP is referred as $MP_{a,b}$, indicating that it is the matrix of position of the reference system of element b with respect to the reference system of element a.

The matrices of position define and locate each component in an assembly, but they are not sufficient to describe completely the assembly, since the contact relations between components have not been defined. To do this, matrices linking two parts through a contact relationship between their surfaces is defined. Each of these matrices, which have the same structure as the matrices of position, is referred as Matrix Link (ML). Each ML locates a feature of a component with respect to a feature in contact of the other component. In this way, the location of one part with respect to another part can be known according to the contact restrictions between them. For example, in Fig. 1 the location of part 2 (its reference system) with respect to part 1 through feature Ai and Bk of part 1 and 2 respectively is as follows: $MP_{P1,P2} = MP_{A,Ai} \cdot ML_{Ai,Bk} \cdot MP_{B,Bk}^{-1}$.

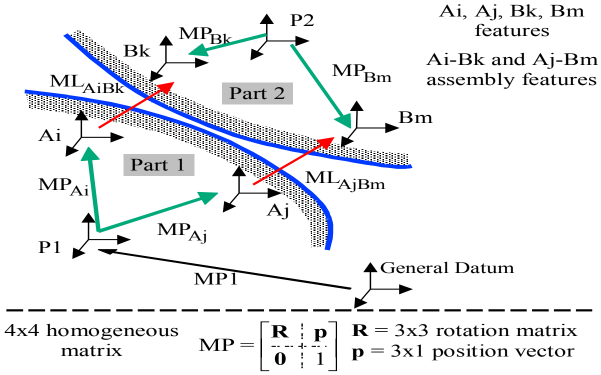


Fig. 2. Two parts with their reference systems and link matrices ML that allow moving from one to another through two different paths. The structure of the MP (matrices of position) is also shown.

These ML matrices are directional, so that ML1→2 ≠ ML2→1. Therefore, it is necessary to specify the direction of the link (e.g. in previous example $ML_{Ai,Bk} \neq ML_{Bk,Ai}$). For this reason, a link must specify which is the support part and which is the supporting part (master and slave in the relationship).

On the other hand, when a variation occurs in any of the surfaces that make up the assembly, a redistribution of the components takes place. This means that the ML relating each pair of surfaces will be modified according to the free DoF in the relationship, that is, a kinematic variation will occur between each pair of surfaces. This variation is introduced in the link through the Kinematic Matrix (MK) that establishes the fixed and free DoF between each pair of surfaces in contact. In this way, the new link matrix will be ML '= ML • MK. For example, in the case of a plane-plane contact relation, the Z axis must be kept invariant, so the kinematic matrix is the one indicated in Eq. 2.

$$MK = \left[\begin{array}{ccc|c} 1 & -f\gamma & C\beta & fx \\ f\gamma & 1 & -C\alpha & fy \\ -C\beta & C\alpha & 1 & Cz \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \tag{2}$$

The position with respect to the Z axis (Cz) and the angles with respect to the X and Y axes (Cβ y Cα) are fixed (constants), while the small displacements with respect to the X and Y axes (fx and fy) and the angle with respect to the Z axis (fγ) are free. Therefore, the free parameters will be unknown and must be resolved according to the rest of the kinematic restrictions of the assembly.

## 3. Proposal of incorporation of form tolerances in the matrix transformation method for tolerance analysis

The DoF tolerance model is based on the fact that the involved real element affected is replaced by a substitute feature. This substitute feature keeps its ideal form (the same of the nominal feature), but changes its orientation and position, varying both within a tolerance zone, as shown in Fig. 1. This allows to model and control the size, orientation and position variations of the real feature, but not its form. Several authors explain that controlling the form is not necessary, since its effect on a tolerance chain is much less than those due to size, orientation and position variations. Form tolerances are assumed to be of less magnitude and are included in the former.

However, on certain occasions additional variations due to form tolerances may be relatively important, particularly when contact surfaces have significant size differences. Typically, variation allowances in a feature are consistent with its size for the same quality requirement. For that reason, the orientation and position of a small element in size in contact with a much larger element in size can be altered due to the form variations of the latter, as shown in Fig. 3. In this figure, the variation of the dimensional characteristic C owned to the form variations of the contact surface with part P1 can be seen.

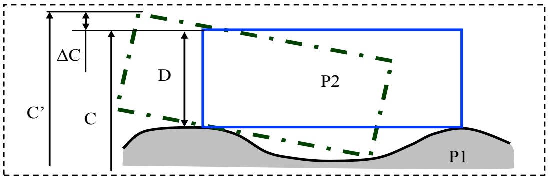


Fig. 3. Influence of form tolerances in the final value of a dimension in the case of contact between two surfaces of the same quality and very different size (form errors in part P2 of smaller size have not been included).

Some authors propose the incorporation of additional DoF to the kinematic ones in the joint, such as a size DoF and a shape DoF with the aim of setting the tolerances completely: form, size and location. However, this incorporation is proposed to be considered in a support system to tolerance specification and consistency analysis (tolerance form must be smaller than dimensional and orientation tolerances, etc.), but not to be applied to assess the form tolerance influence on tolerance propagation. An important issue to be considered is how the extraction of the substitute feature from the real feature will be done. This process is very much related to the inspection system to be used. Since the substitute feature must be the best approximation to the real feature, the criterion used is similar to the one employed in CMM machines, that is, minimisation of the fitting error, giving the position of the centre (or reference) and its orientation (Fig.1).

In addition, the way of including the influence of form tolerances depends on the type of contact between surfaces (type of surfaces in contact and DoF limited by the contact). For that reason, reference contacts will be distinguished from support contacts. The former are contacts limiting 1 DoF, particularly 1 translation. Support contacts are contacts limiting 2 DoF in 2D (1 translation and 1 rotation), and 2 or 3 in 3D (1 translation and 1 rotation, or 1 translation and 2 rotations).

In the case of the reference contacts, the form error will result in the contact not being made on the substitute features, but slightly displaced. Therefore, form errors influence must be expressed as an additional translation. In the case of the support contacts, in addition to the translation, an extra rotation in relation to the rotations locked in the contact. In both cases, an additional translation or rotation must be considered. Hence, the form errors contribution must be realised in the MK matrix, since in fact they mean a change in the DoF or in the value of their contribution in relation to what it was initially set in that matrix.

Once the contribution of the form errors and how this contribution is incorporated in the tolerance propagation method have been established, the next step is to quantify the value of the contribution. In the case of the reference contact, the additional translation will depend on the points of the surfaces where the real contact may be produced in relation to the theoretical contact of the substitute features and on the value of the form tolerances. In the borderline case, the contact could be produced between two waviness peaks, one peak and one valley, or even two valleys. Thus, the translation would be equal to the sum of the form variations of each surface. However, it is highly unlikely that this borderline case occurs, so the squared contribution pointed out in Eq. 3 is proposed. This contribution is specified by the reference contact between surfaces 2A and 3B (the surface 2 of a part A and the surface 3 of mating part B), which is less penalising.

The estimation of the contribution of the form tolerances in the case of support surfaces is much more complex, since the contact is influenced by the relative surface size and by the order of the value of the form variation relative to the surface size. When sizes are similar, the contact will not appear between the valleys of both surfaces and it is very unlikely that the contact is produced between a valley and a peak. In these cases, the contact will be produced mostly between peaks. However, when difference in size is significant, the probability of contact between valleys and peaks increases. For that reason, to consider the additional translation and rotation due to the form deviations $\Delta f$

(flatness, straightness, roundness, …), the values Δpf and Δrf specified in Eq. 4 are proposed. These values have been specified for the support contact in 2D between surfaces 2A and 3B, being $L_{2A}$ and $L_{3B}$ the characteristic size of each surface respectively, $L_{2A}$ the smallest surface and having surface 3B the rotation.

$$\Delta pf_{2A,3B} = \sqrt{\Delta f_{2A}^2 + \Delta f_{3B}^2} \tag{3}$$

$$\Delta pf_{2A,3B} = \Delta pf_{2A,3B}^{0} \pm \Delta pf_{2A,3B}^{v} = \left[\frac{(\Delta f_{2A} + \Delta f_{3B})}{2}\right] \pm \left[\left(\frac{L_{2A}}{L_{2A} + L_{3B}} - 0{,}5\right)(\Delta f_{2A} + \Delta f_{3B})\right]$$
$$\Delta rf_{3B} = \left(2 - \frac{L_{2A} + L_{3B}}{L_{2A}}\right) \cdot \frac{(\Delta f_{2A} + \Delta f_{3B})}{L_{3B}/2} \tag{4}$$

The above described proposal of incorporation of form tolerances is based on the one developed by Serrano [13] to incorporate form tolerances in an Integrated Model for Tolerance Management in Design and Manufacturing in 3D Systems. In this work, the proposal is described in more detail.

## 4. Case Study

In this section the previously proposal is applied to the assembly composed of parts A and B, shown in Fig. 4 which includes the functional representation using “assembly graph (A-graph)” [14]. The function requirement FR1 implies a distance relationship between a point on the surface 4 of the piece A ($P_{4A}$) and another point belonging to surface 4 of part B ($P_{4B}$). This distance will be measured in the direction established by the FR1.

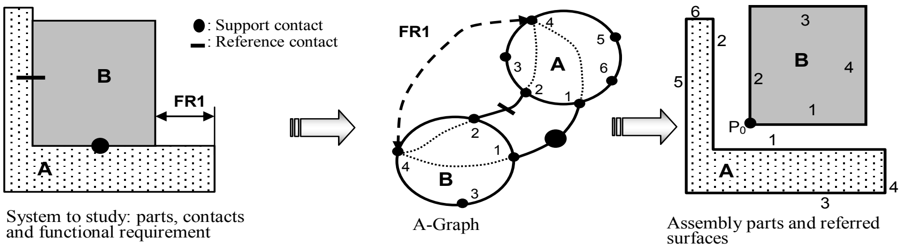


Fig. 4. Assembly with part relations and functional requirement (FR1), A-graph and part surfaces identification.

Therefore, if the FR is defined in the reference system of part A according to Eq. 5, where $P'_{A,4A}$ and $P'_{A,4B}$, written out in the part A reference system, are obtained from points $P_{4A}$ and $P_{4B}$, according to the Eq. 6 that include: 1) the variation matrices $MV_{4A}$ and $MV_{4B}$, quantifying the deviations of features 4A y 4B induced by the tolerances; 2) the position matrices $MP_{A,4A}$ and $MP_{B,4B}$, positioning features 4A y 4B with the part A and B references systems; and 3) the position matrix $MP'_{A,B}$, positioning the part B reference system with respect to part A reference system. In the Eq. 6, the matrices MV present the form shown in Eq. 7, where $ty_{\#,X}$ and $rz_{\#,X}$ are the displacement and angle deviations of surface # for the part X.

In order to compute the position matrix $MP'_{A,B}$, the two possible paths throw the part features in contact are considered. These paths result in the Eq. 8, where the matrices MK are expressed as a function of four variables, corresponding to the four free DoF in the contacts between the parts A and B. Specifically, a free DoF (displacement in X) in the contact between surfaces 1B and 1A and three DoF (displacement in X and Y and rotation in Z) in the contact between surfaces 2A and 2B. For this case, the matrices MK have the expressions (9).

$$|FR1| = \overline{\left(P'_{A,4A} P'_{A,4B}\right)} \cdot \overrightarrow{n_{A,FR1}} \tag{5}$$

$$
\begin{aligned}
P'_{A,4A} &= MP_{A,4A} \cdot MV_{4A} \cdot P_{4A} \\
P'_{A,4B} &= MP'_{A,B} \cdot MP_{B,4B} \cdot MV_{4B} \cdot P_{4B}
\end{aligned}
\tag{6}
$$

$$
MV_{\#,X} = \begin{pmatrix} 1 & -rz_{\#,X} & 0 & 0 \\ rz_{\#,X} & 1 & 0 & ty_{\#,X} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\tag{7}
$$

$$
MP'_{A,B} \Rightarrow \begin{Bmatrix} MP'_{A,B} = MP_{2A} \cdot MV_{2A} \cdot ML_{A2B2} \cdot MK_{A2B2} \cdot MV_{2B}^{-1} \cdot MP_{2B}^{-1} \\ MP'_{A,B} = MP_{1A} \cdot MV_{1A} \cdot ML_{A1B1} \cdot MK_{A1B1} \cdot MV_{1B}^{-1} \cdot MP_{1B}^{-1} \end{Bmatrix}
\tag{8}
$$

$$
MK_{1A,1B} = \begin{pmatrix} 1 & 0 & 0 & fx_{1A.1B} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad MK_{2A,2B} = \begin{pmatrix} 1 & -frz_{2A,2B} & 0 & fx_{2A.1B} \\ frz_{2A,2B} & 1 & 0 & fy_{2A.1B} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\tag{9}
$$

In order to obtain these four variables, four equations are needed. Two equations result from equalling the positions in X and Y obtained by the two paths shown in A-Graph (Fig. 1) for the $MP'_{A,B}$, which must be unique. The other two equations are obtained from proposing the uniqueness of the contact point of surfaces 2A and 2B (point $P_0$ shown in Fig. 1 is considered, since it is the one that penalizes the most). From the four equations, the variables corresponding to the four free DoF are obtained as a function of the feature variations for the part A and B.

Once the variables for the free DoF are known, the FR1 can be expressed as in the Eq. 10 where $|FR1|_0$ is the nominal value, when all feature deviations are zero, and dFR1 is the functional requirement variation corresponding to a certain combination of deviations in the part features. The term dFR1 is linearized, expressing it as a linear combination of feature deviations weighted with sensitive coefficients. These sensitive coefficients quantify the isolated effect of each feature deviation and they are calculated, using the partial derivatives. Its expression is shown in Eq. 11, where $MV_{[i,j]}$ are the components of the variation matrices. In the case at hand, the expression of dFR1 is shown in Eq. 12.

$$
|FR1|' = \overrightarrow{(P'_{A4A} P'_{A4B})} \cdot \vec{n}_{A,FR1} = |FR1|_0 + dFR1
\tag{10}
$$

$$
dFR_1 = \sum_{\forall i,j} \frac{\partial dFR_1}{\partial MV_{[i,j]}} \cdot MV_{[i,j]}
\tag{11}
$$

$$
\begin{aligned}
dFR_1 = &\left[ -ty_{2A} - ty_{2B} + ty_{4A} - ty_{4B} - rz_{2A}\frac{L_{2A}}{2} + rz_{2B}\frac{L_{2B}}{2} + rz_{1A}\frac{L_{2B}}{2} - rz_{1B}\frac{L_{2B}}{2} \right] \\
&+ \left[ xp_{4A}.rz_{4A} - xp_{4B}.rz_{4B} + xp_{4B}.rz_{1B} - xp_{4B}.rz_{1A} \right]
\end{aligned}
\tag{12}
$$

In order to include the form tolerances, the kinematic matrices for the support contact (1A-1B) and for the reference contact (2A–2B) must include the contributions previously explained (Eq. 4). For the support contact between 1A-1B the kinematic matrix will be (Eq. 13).

For the reference contact, the term $\Delta pf_{2A,2B}$ (explained Eq. 4) should be included in the kinematic matrix, but this is not possible because its position corresponds to a free DoF. In these cases, the kinematic matrix does not change, and this condition is imposed in the equation relating the contact point between surfaces 2A and 2B. In a similar way, the variables corresponding to the four free DoF are obtained as a function of the feature variations for part A and B, and the functional requirement in expressed as in Eq. 10. The dFR1 is linearized likewise, but including the terms added for the form tolerances. The final expression for dFR1 including form tolerances effect is in Eq. 14.

$$MK_{1A,1B} = \begin{pmatrix} 1 & -\Delta rf_{1B} & 0 & fx_{1A,1B} \\ \Delta rf_{1B} & 1 & 0 & \Delta pf^{0}_{1A,1B} + \Delta pf^{v}_{1A,1B} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{13}$$

$$\begin{aligned} dFR_1 = &\left[ -ty_{2A} - ty_{2B} + ty_{4A} - ty_{4B} - rz_{2A}\frac{L_{2A}}{2} + rz_{2B}\frac{L_{2B}}{2} + rz_{1A}\frac{L_{2B}}{2} - rz_{1B}\frac{L_{2B}}{2} + \Delta rf_{1B}\frac{L_{2B}}{2} \right] \\ &+ \left[ -\Delta pf_{2A,2B} + xp_{4A}.rz_{4A} - xp_{4B}.rz_{4B} + xp_{4B}.rz_{1B} - xp_{4B}.rz_{1A} - xp_{4B}.\Delta rf_{1B} \right] \end{aligned} \tag{14}$$

## 5. Conclusions and future works

The DoF tolerance model does not incorporate form tolerances and tolerance propagation methods based on DoF suppose form tolerances negligible. However, as has been justified, in certain cases its influence may be relevant. In this work a proposal to consider form variations in the HTM method, based on DoF model, has been presented. To this end, additional translations and rotations are included, which has been quantified for the worst case. The proposal has been applied and validated on a 2D case study, that allows to show the additional variation provided by the form deviations.

In future work, the application of the proposal will be generalized to the 3D problems and it will be generalized to include other geometries (cylinders, spheres, any surfaces, etc.). At the same, the inclusion of form tolerances in other methods (SDT, TTRS, Jacobian Torsor) will be investigated.

## References

[1] N.S. Khan, Generalized Statistical Tolerance Analysis and Three Dimensional Model for Manufacturing Tolerance Transfer in Manufacturing Process Planning, PhD Thesis, Arizona State University, (2011)

[2] J.J. Shah, M. Mäntylä, Parametric and Feature-Based CAD/CAM: Concepts, Techniques, and Applications, Jhon Wiley & Sons, Inc.,(1995)

[3] K. W. Chase, J. Gao, S.P. Magleby, C.D. Sorenson, Including geometric feature variations in tolerance analysis of mechanical assemblies; IIE transactions, 28 (1996) 795-808

[4] K. W. Chase, S.P. Magleby, J. Gao, Tolerance analysis of two-and three-dimensional mechanical assemblies with small kinematic adjustments, Advanced tolerancing techniques 5 (1997)

[5] K. W. Chase, S.P. Magleby,, C. Glancy, A comprehensive system for computer-aided tolerance analysis of 2-D and 3-D mechanical assemblies, Geometric Design Tolerancing: Theories, Standards and Applications, Chapman & Hall, (1998) 294-307

[6] A. Clément, A. Desrochers, A. Rivière, Theory and practice of 3D tolerancing for assembly, 2nd CIRP Int. Seminar Computer-Aided Tolerancing, 1 (1991) 25–55

[7] A. Desrochers, A. Clément, A dimensioning and tolerancing assistance model for CAD systems, Int J Adv Manuf Technol, 93 (1994) 52-61.

[8] J.K. Davidson, A. Mujezinović, J.J. Shah, A new mathematical model for geometric tolerances as applied to round faces, ASME J Mech Des, 124 (2002), 609–22.

[9] A. Mujezinović, J.K. Davidson, J.J. Shah, A new mathematical model for geometric tolerances as applied to polygonal faces, ASME J Mech Des, 126 (2004) 504–18.

[10] W. Ghie, L. Laperriere, A. Desrochers, A unified Jacobian-torsor model for analysis in computer aided tolerancing, Recent Advances in Integrated Design and Manufacturing in Mechanical Engineering, (2003) 63-72.

[11] D. E. Whitney, O. L. Gilbert, M. Jastrzebski, Representation of Geometric Variations Using Matrix Transforms for Statistical Tolerance Analysis in Assemblies, Research in Engineering Design, 6, 4 (1994) 191-210.

[12] G. Gerbino, J. Serrano-Mira, A feature-based tolerancing model for functional analysis in assemblies of rigid parts, Proceedings of 2nd CIRP Int. Sem. on Intelligent Computation in Manufacturing Engineering (ICME 2000), Naples (Italy), (2000) 239-244.

[13] J. Serrano-Mira, De la Función a la Fabricación: Metodología para el Tratamiento Integrado de Tolerancias en Conjuntos Mecánicos, PhD Thesis, Jaume I University, Castellón (Spain) (2016)

[14] J. Serrano-Mira, J. V. Abellan-Nebot, F. Romero-Subiron, From Function to Manufacturing: a Framework for Optimum Tolerancing in Multi-Stage Machining Processes, Proc. of IDMME – Virtual Concept 2010, Bordeaux, France, (2010)