Tuning and robustness analysis of event-based PID controllers under different event generation strategies

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ABSTRACT

In this article we propose practical rules for tuning event based PID controllers with two sampling strategies: symmetric send-on-delta (SSOD) and regular quantification (RQ). We present a detailed analysis about the effect of the derivative term of the controller when using SSOD or RQ and some guide lines are given to select the derivative filter coefficient. The two sampling strategies are compared, showing that, even when both of them lead to similar controlled output response, systems with RQ have better robustness properties than those with SSOD. The study is based on the describing function and the results are applicable to processes with dynamic responses of different types: with time delays, non-minimum phase, under-damped response, etc. The rules presented here are given in terms of phase and gain margins that are measures of robustness used in the design of continuous PID controllers. This allows the application of conventional PID tuning methods to the case of event-based PID. The tuning rules are very simple and can be used for tuning PID, PI, PD and other controller structures.

KEYWORDS
PID, event-based control, quantification, symmetric send-on-delta

1. Introduction

In recent years several studies have been published in which event-based PID control algorithms are proposed. Those works were carried out within the context of networked control systems and they attempt to make use of the well-known advantages of PID control, while at the same time reducing the amount of traffic through the communications network that interconnects the different hardware units that make up the control system, i.e. sensors, controllers and actuators.

One of the most widely used event-generation strategies is based on transmitting the value of the signal only when it crosses levels or thresholds δ. This strategy is known as *send on delta* (SOD) and its effectiveness in terms of controlling and reducing communications has been widely tested and proven, Dormido, Sanchez, and Kofman (2008); Ploennigs, Vasyutynskyy, and Kabitzsch (2010).

A variation on the SOD strategy applied to PI control was put forward in Beschi,

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Dormido, Sanchez, and Visioli (2012). In that proposal the sampled signal is quantified in amounts that are multiples of a threshold δ, so that the relation between the input and output of the event generator is symmetric with respect to the origin. This strategy is known as symmetric send-on-delta (SSOD). Some important results in the study of SSOD-based PI controllers have recently been published in Beschi et al. (2012),Chacón, Sánchez, Visioli, Yebra, and Dormido (2013). In Beschi and Visioli (2013) the tuning of PI controllers was addressed using an SSOD sampling strategy for the control of first-order systems with delay , the results obtained thus being limited to this kind of model. More general results about this kind of systems were presented in Beschi, Dormido, Sanchez, and Visioli (2013), where it was established the conditions for the DC gain of the open loop transfer function which ensured the absence of oscillations without assuming any specific model structure.

In Romero, Sanchis, and Penarrocha (2014) the authors presented a simple rule for tuning PID controllers when SSOD is used in the control loop. The proposal is based on the describing function (DF) technique. The rule presented in that paper entails with the concept of phase margin, one of the most traditional measures of relative stability in closed-loop control systems, and its application is easy and intuitive. On the other hand, in Romero, Sanchis, and Arrebola (2015) the authors presented a new sampling strategy different from SSOD. The proposal is inspired by the Regular Quantization (RQ) of a signal when an Analog to Digital converter is used, that is why we call it RQ sampling strategy. The reliability of RQ as an alternative to SSOD was demonstrated through laboratory experiments in that article.

Most of the studies about symmetric send-on-delta (SSOD) based control systems are limited to PI controllers and, conversely, there is a lack of results concerning the event based PID algorithm. In this article we present an exhaustive study of the behavior of event-based control systems when using SSOD or RQ in combination with a PID controller. We propose tuning rules with the aim of avoiding permanent oscillations due to limit cycles. It is shown that in the case of RQ the rule for controller tuning only depends on the gain margin of the system, while in the SSOD sampling the tuning rule is given in terms of the phase margin. We also present a detailed analysis about the effect of the derivative term of the controller when using SSOD or RQ and some guide lines are given to select the derivative filter coefficient. The two sampling strategies are compared and we prove that, even when both of them lead to similar controlled output response, systems with RQ are more robust to the appearance of oscillations than those with SSOD. The study is based on the describing function and the results are applicable to processes with different types of dynamic responses, as time delays, non-minimum phase, under-damped response, etc. The rules presented here are given in terms of phase and gain margins, that are measures of robustness used in the design of continuous PID controllers. This allows applying conventional PID controller tuning methods to the case of event-based PID. The rules are very simple to apply and can be used for tuning PID, PD and PI controllers.

2. Statement of the problem

Let us consider the networked control systems shown in Figure 1, where $C(s)$ and $G(s)$ are the transfer functions of the controller and the process respectively, the block EG represents the event generator and the block ZOH is a zero order holder. Additionally, y_r is the reference, y is the controlled output and p is a disturbance input.

These two control schemes were proposed in Beschi et al. (2012), where it was

(a) EG-$C(s)$ architecture

(b) $C(s)$-EG architecture
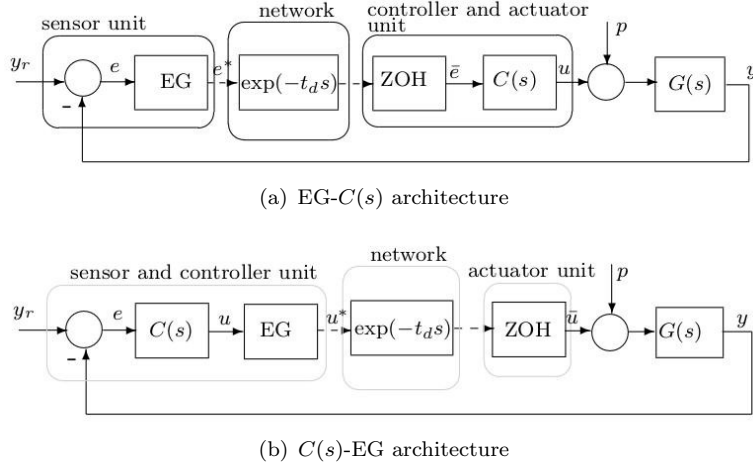
Figure 1. Networked control system with event generator sampling strategies.

considered that $C(s)$ is a PI controller and that the event generator is based on SSOD. This is why the authors called them SSOD-PI and PI-SSOD architectures. The present work is not limited to the study of PI controllers with SSOD mechanism, and we have therefore renamed these architectures as EG-$C(s)$ and $C(s)$-EG, respectively.

In the scheme presented in Figure 1(a) it is assumed that the controller is located close to the actuator and that the sensor sends measurements e^* of the error $e = y_r - y$ to the controller via a communications network. The data are sent by means of the event generator (EG) block. The ZOH block holds the latest value of $\bar{e}$ in e^* until a new value is sent. On the other hand, the scheme shown in Figure 1(b) assumes that the sensor and the controller are in the same unit and that communication between the controller and the actuator takes place via a communications network. In this case, new control actions u^* are sent to the actuator by means of the EG block. The ZOH in the actuator holds the value of the latest control action sent in $\bar{u}$ until a new value is sent.

Regarding the behavior of the communications network, it is assumed that the delay in the transmission is negligible or known and constant with a value t_d. This delay is represented by the term $e^{-t_d s}$ on the left of the ZOH blocks in Figure 1. This consideration is based on the fact that today there are protocols that significantly reduce the number of collisions among packages, which are the fundamental causes of the variable delays in communication networks Leva and Terraneo (2013).

The drawback of the $C(s)$-EG architecture depicted in Figure 1(b), as demonstrated by Beschi et al. (2012), is that the simple existence of the integrator leads to an oscillatory behavior unless one of the delta thresholds coincide with the exact input needed in steady state to maintain the output in the required setpoint. As this is not a realistic assumption in real applications, the $C(s)$-EG scheme always produce persistent oscillations, no matter how the controller is tuned. The only way to avoid oscillations in that case (see Beschi et al. (2012)) is to include a dead band in the error signal. That dead band would make the direct application of the proposed describing function analysis not valid. For those reasons the study of the $C(s)$-EG scheme will be left for future works, and will not be considered in this paper.

Under the assumption that the delays in the network are negligible or known and constant, the control scheme in Figure 1(a) can be depicted as shown in Figure 2. Here the EG-ZOH block represents the combination of the EG and ZOH blocks, while

$G_{ol}(s)$ is the open loop transfer function that corresponds to the linear part, that is to say:

$$G_{ol}(s) = C(s)G(s)e^{-t_d s} \tag{1}$$
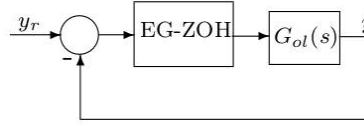
Figure 2. Equivalent system of that in Figure 1(a).

The behaviour of the control system represented in Figure 2 for a known $G(s)$ depends on both the event generator and the tuning of the $C(s)$ controller. In the sections that follow we will see the extent to which each of these elements affects the overall performance of the system.

The PID algorithm considered in this study is given by the equation (2), where K_p is the proportional gain, T_d and T_i are the derivative and integral times respectively, and N is the derivative filter coefficient.

$$C(s) = K_p \left(1 + \frac{1}{T_i s} + \frac{N T_d s}{N + T_d s} \right) \tag{2}$$

It is worth mentioning that if $N = 0$ the derivative term is canceled and the equation (2) is reduced to a PI controller. On the other hand, a PID with pure derivative action $T_d s$ is obtained as $N \to \infty$. Therefore, intermediate values of N let us regulate the contribution of the derivative term to the overall control action. Despite some authors fix the value of N at a given value (for example $N = 10$), recent works have shown how using N as a free tuning parameter allows to balance the noise amplification and the closed loop performance (see Graebe and Isaksson (2002); Kristiansson and Lennartson (2006)). In the event based system presented in Figure 1(a), the PID input is a noise free stairs like signal with steps of magnitude δ. Those steps are amplified by the controller in the same amount as a high frequency noise, resulting in control action jumps that depend on N. Hence, the selection of N is a compromise between control action jumps and the amount of derivative action effectively used. One of the objectives of this paper is to analyze the performance of the two event based sampling strategies with different values of filter parameter N.

3. Analysis of the event generators

In this section we characterize the sampling strategies under study. First we present the input/output equations of SSOD and RQ and then the describing functions of both samplers are shown and compared.

3.1. SSOD-based event generator

The input-output characteristic of the SSOD-based event generator can be seen in Figure 3. The small circles represent the x^* values sent to the ZOH. A new value $x^* = i\delta$, $i \in \mathbb{Z}$ is sent when x crosses the $i\delta$ levels. The thick horizontal lines on the circles highlight the fact that $\bar{x}$ maintains its value for $\pm\delta$ variations around the $i\delta$ levels. Equation (3) describes this behavior.
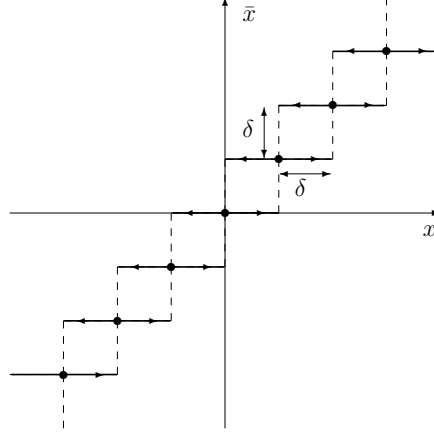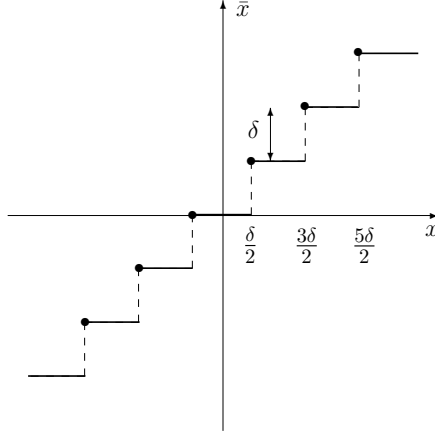

Figure 3. Input-Output characteristics of the SSOD based EG-ZOH block.

$$\bar{x}(t) = \begin{cases} (i+1)\delta & \text{if } (\bar{x}(t^-) = i\delta) \,\&\, (x(t) \geq (i+1)\delta) \\ (i-1)\delta & \text{if } (\bar{x}(t^-) = i\delta) \,\&\, (x(t) \leq (i-1)\delta) \\ i\delta & \text{if } x(t) \in [(i-1)\delta, (i+1)\delta] \end{cases} \quad (3)$$

3.2. RQ-based event generator

The input-output characteristic of the RQ-based event generator can be seen in Figure 4. The small circles represent the x^* values sent to the ZOH. A new value $x^* = i\delta$, $i \in \mathbb{Z}$ is sent when x crosses the levels $\frac{(2i-1)}{2}\delta$. The thick horizontal lines on the circles highlight the fact that $\bar{x}$ maintains its $i\delta$ value if $x(t) \in [\frac{2i-1}{2}\delta, \frac{2i+1}{2}\delta]$. Equation (4) describes this behavior.

$$\bar{x}(t) = \begin{cases} (i+1)\delta & \text{if } (\bar{x}(t^-) = i\delta) \,\&\, (x(t) \geq \frac{2i+1}{2}\delta) \\ (i-1)\delta & \text{if } (\bar{x}(t^-) = i\delta) \,\&\, (x(t) \leq \frac{2i-1}{2}\delta) \\ i\delta & \text{if } x(t) \in [\frac{2i-1}{2}\delta, \frac{2i+1}{2}\delta] \end{cases} \quad (4)$$

3.3. Describing functions

In Gelb and VanderVelde (1968) the describing function of the RQ was obtained, which is given by equation (5) for a sinusoidal input with an amplitude A such that $\frac{2m-1}{2}\delta \leq A \leq \frac{2m+1}{2}\delta$.

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**Figure 4.** Input-Output characteristics of the RQ based EG-ZOH block.

$$N_q = \frac{4\delta}{\pi A} \sum_{k=1}^{m} \sqrt{1 - \left(\frac{(2k-1)}{2} \frac{\delta}{A} \right)^2} \tag{5}$$

Following Gelb and VanderVelde (1968) it can be shown that the describing function of the SSOD is given by equation (6) for a sinusoidal input whose amplitude A is such that $m = \lfloor \frac{A}{\delta} \rfloor$. In that equation $j = \sqrt{-1}$.

$$
\begin{aligned}
N_s = \frac{2\delta}{\pi A} &\left[1 + \sqrt{1 - \left(m \frac{\delta}{A} \right)^2} \right. \\
&\left. + 2 \sum_{k=1}^{m-1} \sqrt{1 - \left(k \frac{\delta}{A} \right)^2} \right] - jm \frac{2\delta^2}{\pi A^2}
\end{aligned} \tag{6}
$$

As can be seen, the two describing functions are dependent on the quotient $\frac{\delta}{A}$. Figure 3 shows how, in the case of SSOD, the output is zero for $A < \delta$. Therefore, it only makes sense to evaluate the describing function (6) for $\frac{\delta}{A} \in [0,1]$. By performing the same analysis for RQ, Figure 4, it can be seen that for $A < \delta/2$ the output is null, and therefore it only makes sense to evaluate the describing function given by equation (5) for $\frac{\delta}{A} \in [0,2]$.

Figures 5(a) and 5(b) show the graphs of module and argument of the describing functions N_q and N_s as a function of δ/A respectively. There are some major differences between them. The module of N_q can take values greater, equal or less than 1. On the other hand, the module of N_s is always less than 1. That means that, depending on δ/A, the amplitude of the RQ sampler output could be either greater, equal or less than its input. Contrarily, the output amplitude of the SSOD sampler will be always less than its input. Regarding the argument of describing functions, it is always zero for N_q and negative for N_s. Consequently, the RQ sampler does not introduce any phase lag to its input, while the SSOD strategy delays its input between 0° and 45°, depending on the relation δ/A. One common characteristic of N_q and N_s is that whether $\frac{\delta}{A} \to 0$,

(a) N_q

(b) N_s
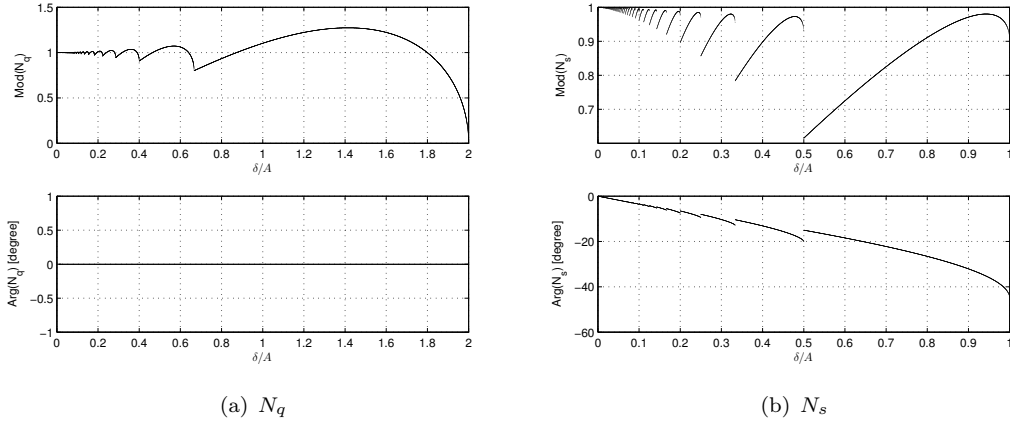
Figure 5. Module and argument of the RQ and SSOD samplers describing functions.

(a) RQ

(b) SSOD
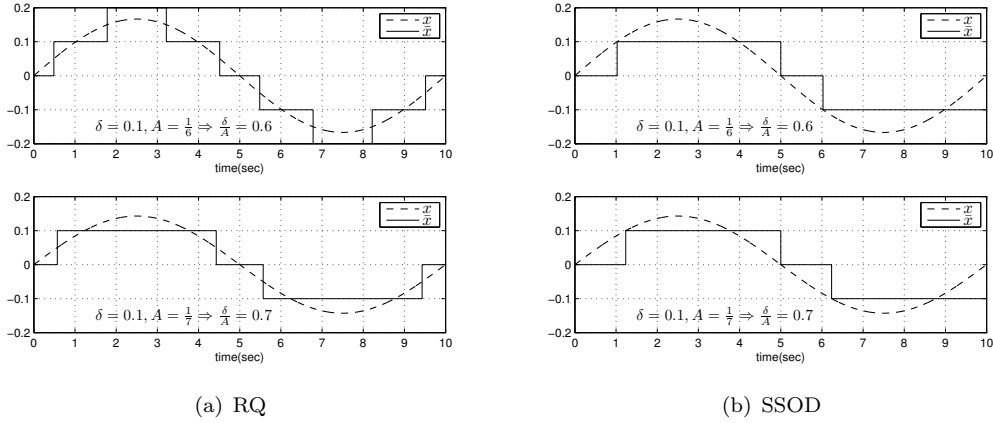
Figure 6. Time response of RQ and SSOD samplers to sinusoidal inputs with amplitude $\frac{1}{6}$, $\frac{1}{7}$ and $\delta = 0.1$

then $\mathrm{Mod}(N) \to 1$ and $\mathrm{Arg}(N) \to 0$. That is to say, for infinitesimal values of δ both event generators behave as unitary gains.

In order to better illustrate the behaviour of both samplers, Figure 6 shows the time responses of the RQ and SSOD strategies to sinusoidal inputs with amplitudes of 1/6 and 1/7 when $\delta = 0.1$, that is, δ/A equal to 0.6 and 0.7. As can be noted, for $\delta/A = 0.6$ the amplitude of the RQ output is greater than its input. This is not possible for the SSOD strategy since the output always changes to values less than the input (see equation (3)), conversely, the output of the RQ sampler after the sampling instant is always greater than the input (see equation (4)). It is also important to note the difference in the phase lag between the outputs of RQ and SSOD samplers. As predicted by the describing functions, unlike the RQ sampler, the SSOD introduce a phase lag to its input. It is more clearly appreciated in the case of $\delta/A = 0.7$ (graphics on the bottom of Figure 6).

4. Design of the controller

In this section we analyze the design of the $C(s)$ controller under the two event-generation strategies described in the previous section. We will consider $C(s)$ to be

the widelly used PID controller defined in equation (2), although the analysis can be applied to other controller structures.

In order to analyze the robustness properties of the event sampling strategies, any PID tuning procedure could be adequate. The PID design strategy chosen in this work is the one described in Sanchis, Romero, and Balaguer (2010), where the PID parameters are those that maximize the integral gain while fulfilling an equality constraint on the phase margin and an inequality constraint on the gain margin. However, the results would be similar with other tuning methods, as will be shown in example 3. The selected PID design procedure can be expressed as

$$\max_{a} \quad K_i \tag{7}$$
$$s.t. \quad \Phi_m = \Phi_d$$
$$\gamma_m \geq \gamma_d$$

where $a = T_i \omega_g$ is an adimensional tuning parameter, $K_i = K_p/T_i$ is the integral gain, Φ_d is the desired phase margin, Φ_m is the achieved phase margin, γ_d is the minimum required gain margin, and γ_m is the achieved gain margin. For a given value of a, there is a unique PI controller that fulfils the phase margin, and also a unique PID controller if parameters N and T_i/T_d are fixed. The use of that adimensional parameter simplifies the optimization problem, as the optimum value of a lies between 0 and 10 independently of the time constants of the process (see Sanchis et al. (2010) for more details).

The maximization of integral gain is equivalent to the minimization of the disturbance IE, as $IE = \frac{1}{K_i}$, and is approximately equivalent to the minimization of the disturbance IAE, if the response is not very oscillatory (guaranteed if adequate phase and gain margins are imposed).

In the case of the PID controller, the value of the filter parameter, N, and the relation between integral and derivative times, T_i/T_d are asumed to be selected to the desired values before performing the previous optimization. A value of $T_i/T_d = 4$ is set by default. The optimization of this relation could improve the performance, hence this is a suboptimal tuning, but the robustness analysis would be similar. On the other hand, the value of N is selected as a compromise between noise amplification and performance. In the case of the event based sampling (either RQ or SSOD), the value of N has a direct effect on the control action discontinuities due to the jumps on the sampled output. The larger the value of N, the better performance (the lower IAE), but the higher jumps in the control action due to the event based sampling.

If the difference between the last and the new value of e is δ, it can be easily proved (e.g. by applying the initial value theorem) that the control action is suddenly increased by $\Delta_u(t_e)$ given by the equation (8).

$$\Delta u(t_e) = K_p(1 + N)\delta \tag{8}$$

In any case, before the selection of N, the desired robustness must be decided, in the form of a required phase margin and a minimum required gain margin. The presence of the event based sampling affects the selection of robustness margins due to the possible appearance of limit cycles. The analysis of this problem is based on the use of the describing functions N_q and N_s for event generators based on RQ and on SSOD.

These describing functions can be used to analyse the existence of limit cycles in the system shown in Figure 2. It is well known that this approach is an approximation, since it assumes that high order harmonics in the output are negligible, i.e. that the dynamics of the process filter out the high order harmonics of the oscillatory input when the limit cycle occurs. This approximation relies on the low pass frequency characteristic of most processes.

The condition of the existence of limit cycles is given by equation (9), which corresponds to the intersection of $G_{ol}(j\omega)$ and $-1/N_i$, where N_i refers to both N_q and N_s. The term on the right depends on the quotient $\frac{\delta}{A}$, and therefore the form of $-\frac{1}{N_q}$ and $-\frac{1}{N_s}$ does not depend on the value of δ used in the event generators. This means that the inexistence of intersections must be guaranteed through the term on the left, more specifically through the tuning of the $C(j\omega)$ controller.

$$G_{ol}(j\omega) = -\frac{1}{N_i} \tag{9}$$

Figure 7 shows the graph of $-\frac{1}{N_q}$, which is located upon the real axis with a maximum value of -0.78, represented on the graph by point C. On the left, the curve tends towards $-\infty$, an extreme that is reached when $\frac{\delta}{A} \to 2$.
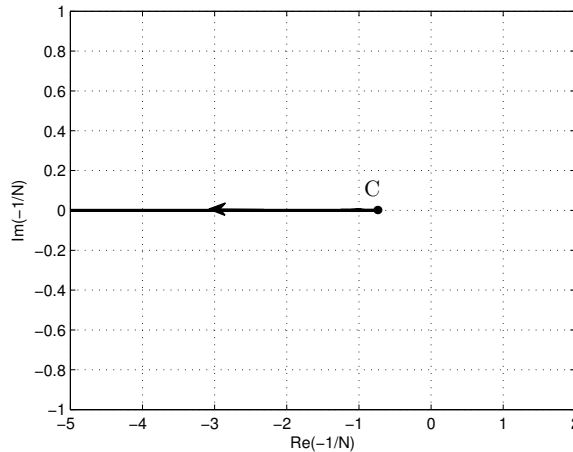
Figure 7. Graph of $-\frac{1}{N_q}$ (for RQ sampling)

Figure 8 shows the graph of $-\frac{1}{N_s}$. The curve is significantly different from that of the RQ case. It is made up of several branches, each of which is obtained for a value of $m = \lfloor \frac{A}{\delta} \rfloor$. It can be observed that the maximum value of the curve on the real axis is the point (-1,0), which is obtained when $\frac{\delta}{A} \to 0$. As in the case of RQ, the maximum real value of the curve is -0.78, which, for the SSOD case, corresponds also to an imaginary value of -0.78, represented on the graph by point C.

For both event generation strategies robustness criteria will be established that will allow the $C(s)$ controller to be tuned not only to guarantee the stability of the controlled system, but also to prevent limit cycles.
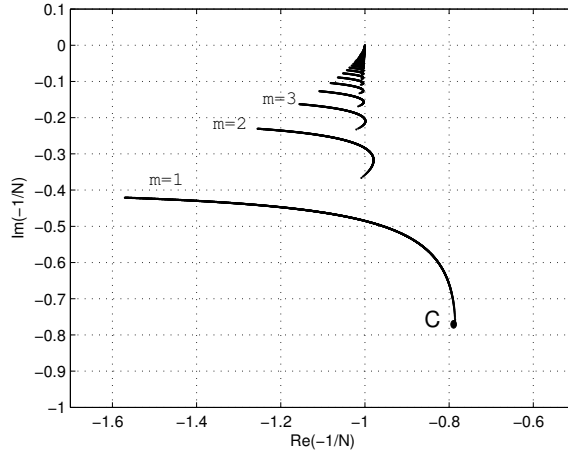
Figure 8. Graph of $-\frac{1}{N_s}$ (for SSOD sampling)

4.1. *Design of the controller for RQ-based EG*

In accordance with the shape of the graph of $-\frac{1}{N_q}$ shown in Figure 7, to prevent the presence of limit cycles when event generators based on RQ are used, all that needs to be done is to design the $C(s)$ controller in such a way that the Nyquist plot lies on the right of the point (0;-0.78). This can be seen in Figure 9, where there is no intersection between $G_{ol}(j\omega)$ and $-\frac{1}{N_q}$.

Taking into account that the point (0;-0.78) define the stability bound when the RQ sampler is used, new robustness margins can be defined to measure the distance from $G_{ol}(\omega)$ to this point. The first new robustness measure, named as RQ-gain-margin, is denoted by γ_{rq} and is given by the following equation:

$$\gamma_{rq} = \frac{0.78}{|G_{ol}(\omega_f)|} \tag{10}$$

where ω_f is the lowest value of frequency in which $\arg(G_{ol}(\omega)) = -180^o$. Using the equation (10) and the definition of gain margin ($\gamma_m = \frac{1}{|G_{ol}(\omega_f)|}$), the relationship between γ_m and γ_{rq} can be easily obtained: $\gamma_{rq} = 0.78\gamma_m$, that expressed in decibels is:

$$\gamma_{rq_{db}} = \gamma_{m_{db}} - 2.15db \tag{11}$$

This equation means that the introduction of the RQ sampler reduces the traditional gain margin $\gamma_{m_{db}}$ in 2.15db, no matter which are the controller and process models.

The second new robustness measure, named as RQ-phase-margin, is denoted by ϕ_{rq} and is given by the following expression:

$$\phi_{rq} = 180 + \arg(G_{ol}(\omega'_g)) \tag{12}$$

where ω'_g is the frequency where $|G_{ol}(\omega'_g)| = 0.78$. The relationship of ϕ_{rq} with the

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original phase margin, ϕ_d, is not simple, since it depends on the process model and controller, but for most processes $\phi_{rq} < \phi_d$.

Taking into account the previous expressions, the proposed PID design criterion to avoid the limit cycle oscillations, is simply:

$$\gamma_{d_{db}} > 2.15db \tag{13}$$

No further condition must be imposed on ϕ_d, because the fullfilment of (13) guarantees that $\phi_{rq} > 0$. Furthermore, in practice, condition (13) is not restrictive at all, since any reasonable design always impose a gain margin larger than 2.15 db.
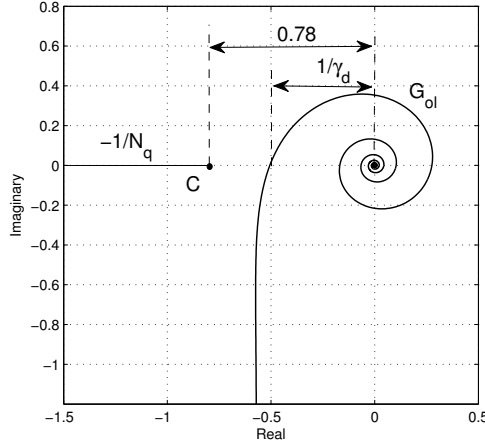
Figure 9. Constraint of the gain margin γ_d for the case of RQ-based event generators.

4.2. *Design of the controller for SSOD-based EG*

According to the shape of the Nyquist diagrams of usual processes, the most critical point in the plot in order to avoid the intersection is the point C, as can be seen in Figure 10. As commented earlier, point C represents the maximum real value and the minimum imaginary value of the curve $-\frac{1}{N_s}$, that lies on the left of point C and outside the circle of unit radius. Point C has coordinates (-0.78, -0.78), which correspond to a module of 1.1. New robustness margins can be defined to measure the distance from $G_{ol}(\omega)$ to C. The first new robustness measure, named as SSOD-phase-margin, is denoted by ϕ_{ssod} and is given by the following equation:

$$\phi_{ssod} = \arg(G_{ol}(\omega_c')) - (-135^o) \tag{14}$$

where ω_c' is the frequency in which $|G_{ol}(j\omega)|$ is equal to 1.1, see Figure 10.

The relationship between ϕ_{ssod} and ϕ_d is as follows:

$$\phi_{ssod} = \phi_d + \Delta\phi_d - 45^o \tag{15}$$

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where

$$\Delta\phi_d = \arg(G_{ol}(\omega'_c)) - \arg(G_{ol}(\omega_g)) \tag{16}$$

The second new robustness measure, named as SSOD-gain-margin, is denoted by γ_{ssod} and is given by the following equation:

$$\gamma_{ssod} = \frac{1.1}{|G_{ol}(\omega'_f)|} \tag{17}$$

where ω'_f is the frequency in which $arg(G_{ol}(j\omega)) = -135^o$, see Figure 10.

Using the previous definitions, the condition to avoid limit cycle oscillations is,

$$\phi_{ssod} > 0^o \tag{18}$$

Using (15), the design criterion (18) to avoid limit cycles can be rewritten in terms of ϕ_d, leading to:

$$\phi_d > 45^o - \Delta\phi_d \tag{19}$$

The value of $\Delta\phi_d$ is unknown prior to the controller design. Consequently, condition (19) can not be used in the design of the controller to guarantee a given robustness performance when SSOD sampling is implemented. A reasonable assumption, as was shown by the authors in Romero et al. (2014), is to consider $|G_{ol}(j\omega'_c)| \approx |G_{ol}(j\omega_c)| \approx 1$, being ω_c the value of ω for which $|G_{ol}(j\omega)| = 1$. Under this premise $\Delta\phi_d \approx 0$ and the condition (19) is reduced to:

$$\phi_d > 45^o \tag{20}$$

It is important to note that no further design constraint must be imposed to the conventional gain margin, since the fulfillment of condition (20) guarantees that $\gamma_{ssod} > 0$. SSOD condition (20) is more restrictive than RQ condition (13) since phase margins below 45° can be needed in applications where a very fast response is required.

4.3. Comparison of robustness measures for SSOD and RQ

In the previous sections the conditions for tuning PID when using SSOD or RQ sampling schemes have been presented, equations (13) and (20). These conditions are given in terms of the traditional robustness measures in the design of continuous PID controller: phase margin for SSOD and gain margin for RQ respectively. However, the actual margins to avoid the limit cycle oscillations are considerably reduced with respect to the values of ϕ_d and γ_d used in the design of the PID. In particular, when the RQ sampler is used, the actual gain margin expressed in decibels is given by $\gamma_{rq_{db}}$ (equation (11)), and the actual phase margin is given by ϕ_{rq} (equation (12)), whereas
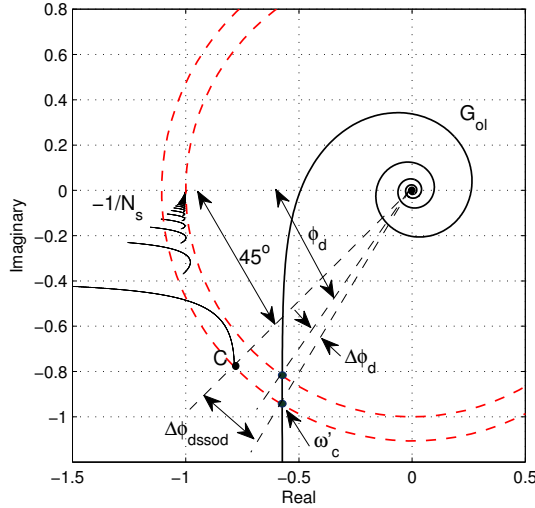
Figure 10. Constraint of the phase margin ϕ_d to avoid oscillations for the case of SSOD-based event generators

for the SSOD strategy the actual phase margin is given by ϕ_{ssod} (equation (15)), and the actual gain margin by γ_{ssod} (equation (17)).

The aim of this section is to compare the effect of the RQ and SSOD in the robustness to oscillations of the system with respect to the margins used in the PID design. To do this, the relative values of $\gamma_{rq_{db}}$, ϕ_{rq}, $\gamma_{ssod_{db}}$ and ϕ_{ssod} with respect to $\gamma_{d_{db}}$ and ϕ_d are calculated by the equations (21), (22), (23) and (24). These values give the percentage represented by the actual margins ($\gamma_{rq_{db}}$, ϕ_{rq}, $\gamma_{ssod_{db}}$ and ϕ_{ssod}) with respect to the design margins ($\gamma_{d_{db}}$ and ϕ_d), after introducing the RQ or SSOD respectively.

$$\gamma_{rq_r} = \frac{\gamma_{rq_{db}}}{\gamma_{d_{db}}} 100\% \tag{21}$$

$$\phi_{rq_r} = \frac{\phi_{rq}}{\phi_d} 100\% \tag{22}$$

$$\gamma_{ssod_r} = \frac{\gamma_{ssod_{db}}}{\gamma_{d_{db}}} 100\% \tag{23}$$

$$\phi_{ssod_r} = \frac{\phi_{ssod}}{\phi_d} 100\% \tag{24}$$

All four relative margins depend on the shape of $G_{ol}(j\omega)$, and therefore, on the process and the controller. In order to gain insight about the values of those relative margins, and to compare the resulting robustness of the RQ versus the SSOD sampling strategy, we have studied the behaviour of these parameters in an extensive batch of 134 models described in Åström and Hägglund (2006), which represent the most

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common dynamics in real applications. For all those models, PID controllers were designed using the tuning method presented in Sanchis et al. (2010) and summarized by equation (8), with $[\phi_d = 60^o,\ \gamma_{d_{db}} \geq 8]$, and filter coefficients $N = 0$ (corresponding to the PI controller), $N = 3$ and $N = 10$. The figures 11, 12 and 13 show the values of the relative margins for RQ and SSOD for the four cases. Furthermore, in order to make the analysis easier, graphics with quotients $\gamma_{rq\_ssod} = \gamma_{rq_r}/\gamma_{ssod_r}$ and $\phi_{rq\_ssod} = \phi_{rq_r}/\phi_{ssod_r}$ have been also included. After analyzing those figures one can conclude:

- The relative phase margin to oscillations is much higher in the RQ than in the SSOD ($\phi_{rq\_ssod} > 1$) for all the models and all the designed PID.
- The relative gain margin to oscillations is also higher in the RQ than in the SSOD ($\gamma_{rq\_ssod} > 1$) for all the models in the PID designed with $N = 0$ (PI), and is also higher for most of the models in the PID designed with $N = 3$ and $N = 10$.
- There are some exceptions where the relative gain margin in the SSOD in slightly higher than in the RQ ($\gamma_{rq\_ssod} < 1$). Those cases can be observed in figures 12 and 13, and correspond to a PID designed with $N = 3$ and $N = 10$ (high derivative effect) for models that are very near to pure first order systems. For example, a first order plus delay system where the time constant is more than 100 times the delay. In fact those systems can be made almost arbitrarily fast with a PI controller and therefore, a PID with N=3 or 10 is not a reasonable choice for those systems in practice.

As a summary, the use of RQ event sampling results in a much smaller alteration of the robustness achieved by the conventional controller if compared to the use of SSOD.

5. Effect of δ parameter

The describing functions N_q and N_s, equations (5) and (6), are dependent upon the quotient $\frac{\delta}{A}$ and none of them contain a term that depends only on δ. This means that both N_q and N_s have normalised expressions independent of the value of δ and that, consequently, the shape of the graphs of $-\frac{1}{N_q}$ and $-\frac{1}{N_s}$ in Figures 7 and 8 are not affected by the change in the value of δ. This means that once the $C(s)$ controller has been designed to prevent the intersections between $G_{ol}(j\omega)$ and $-\frac{1}{N_q}$ or $-\frac{1}{N_s}$, then that condition will hold regardless of the value of the δ parameter that is set in the event generator. Hence, neither the stability nor the robustness to oscillations of the control system in Figure 1 will be affected by δ.

The value of δ does have an influence, however, on the steady state error, due to the fact that the event generators based on both RQ and SSOD introduce a dead zone effect. It is clear that the higher the δ, the higher the error. On the other hand, the value of δ is directly related to the number of events generated in each of the strategies: the higher the δ, the lower the number of events. Taking this into account, the selection of δ implies a trade-off between the desired precision in the control and the number of events.

Another important issue about δ when a PID controller is used in the system depicted in Figure 1(a) concerns the control action jumps each time (t_e) a new value of e is sent by the EG block. If the difference between the last and the new value of e is δ, as shown in section 4, the control action is suddenly increased by $\Delta_u(t_e) = K_p\delta(1 + N)$.

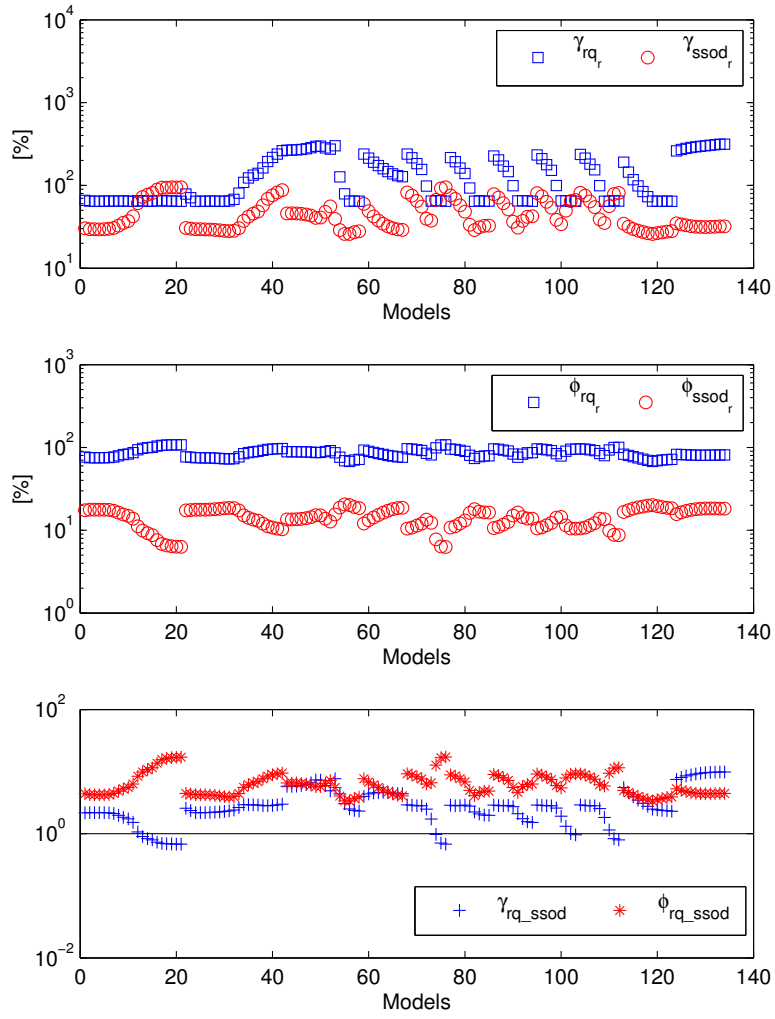If $N = 0$, that is for the PI controller case, $\Delta_u(t_e)$ depends only on the proportional
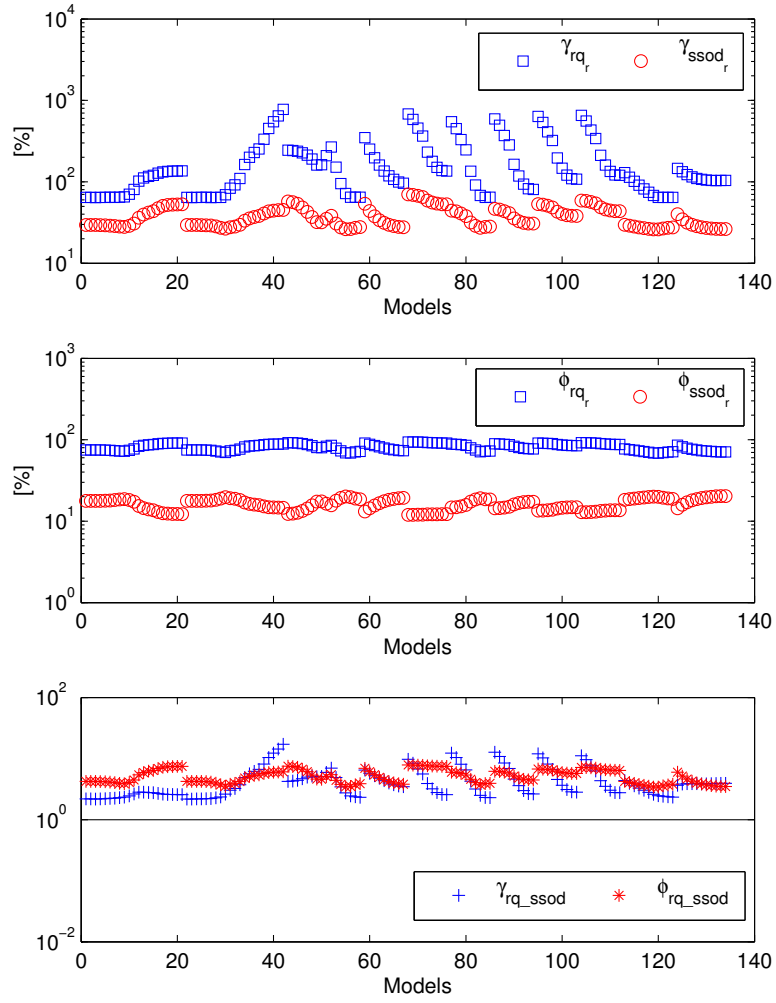
Figure 11. Relative gain and phase margins for RQ and SSOD for the batch of models. PID designed for $\phi_d = 60^o$, $\gamma_{d_{db}} > 6db$ and $N = 0$.
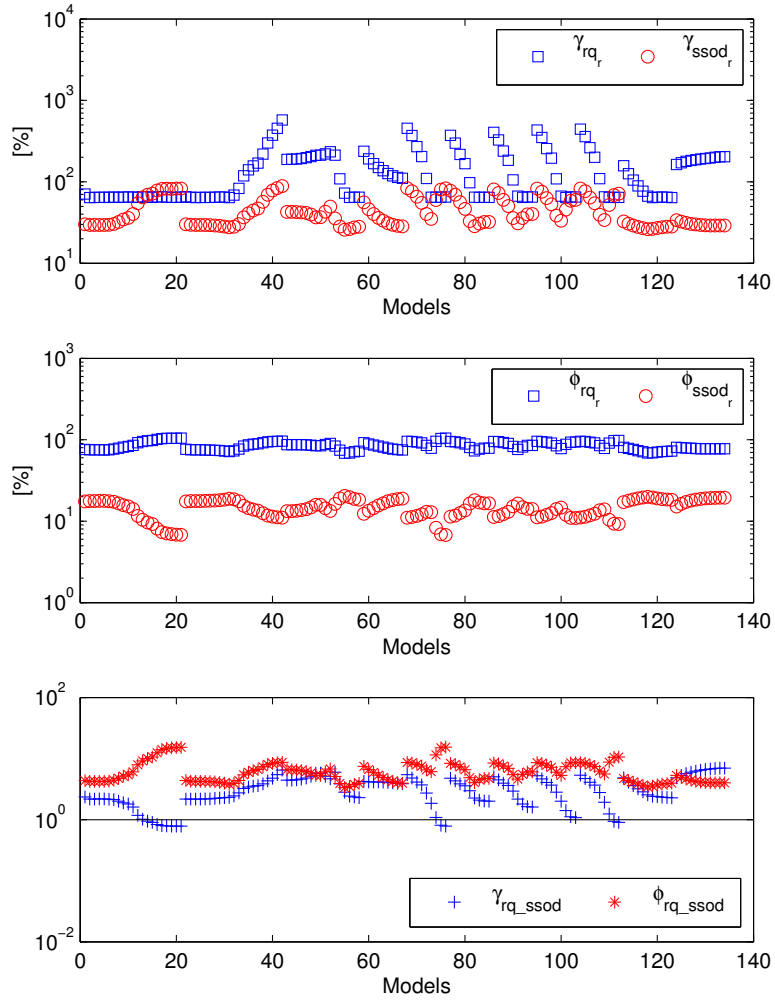
Figure 12. Relative gain and phase margins for RQ and SSOD for the batch of models. PID designed for $\phi_d = 60^o$, $\gamma_{d_{db}} > 6db$ and $N = 3$.

Figure 13. Relative gain and phase margins for RQ and SSOD for the batch of models. PID designed for $\phi_d = 60^o$, $\gamma_{d_{db}} > 6db$ and $N = 10$.

gain K_p. On the other hand, when the derivative term is considered in the controller, the magnitude of control action jumps is proportional to both K_p and N. Therefore, the parameter N could be used to limit the jumps in the control action for a given value of δ. However, a lower N results in a worse performance (in terms of IAE). Therefore, the selection of N is a compromise between control action jumps and performance. In order to do so, one should use a PID from a vendor that allows to freely choose the value of N. Of course, one can also select the value of N if the PID algorithm is implemented in a programmable platform. Unfortunately, not all PID vendors allow to choose the value of N. In those cases, a series low pass filter could be used to smooth the control action, but this is out of the scope of this paper.

It is also important to highlight that the value of δ has to be properly selected to avoid unnecessary events due to the measurement noise. Owing to the hysteresis, the SSOD sampler keeps in general a good performance even if the error signal is noisy, that is, no events are generated due to the noise. For that, it is just necessary to set δ higher than the noise amplitude. In the case of the RQ generator, however, the noise could produce bursts of events when the error signal is near the thresholds. This drawback can be partially overcome by filtering the sensor output with a suitable filter in order to reduce the noise amplitude as much as possible. Obviously, for the controller tuning the filter has to be taken into account as part of G_{ol}.

It is worth pointing out that meanwhile some values of the error signal are outside of the band where the error is zero ($[-\delta/2, \delta/2]$ for RQ and $[-\delta, \delta]$ for SSOD) the integral action of the controller is recalculated to compensate the error. Therefore, in steady state the error will be necessarily inside the forementioned band, provided this band is wider than the noise amplitude.

6.   Simulation examples

In this section some simulation examples are developed to show the efectiveness of the proposed PID tuning rules for RQ and SSOD, to compare the resulting robustness to oscillations of both approaches, and to illustrate how the values of δ and N are related to the number of events, the final error, the performance (in terms of IAE) and the control action jumps.

To tune the controllers we will use the method presented in Sanchis et al. (2010), summarized in procedure (8), which is equivalent to minimizing the IE, while guaranteeing the exact desired phase margin and a gain margin higher than the minimum required.

6.1.   *Example 1*

Consider the second order plus time delay system with transfer function given by equation (25). For this system five PID controllers were designed for values of N equal to 0, 3, 10, 15 and 20. The case with $N = 0$ is a PI controller. In all cases phase margin ϕ_d of 50° and gain margin γ equal or greater than $2(6dB)$ were considered. Therefore, the designs fulfill the robustness restrictions introduced by both SSOD and RQ sampling strategies. Figure 14 shows the graphs of $-\frac{1}{N_s}$ and $-\frac{1}{N_q}$ together with the G_{ol} which is achieved by tuning the PID for each design. As can be seen, the open loop transfer function does not intersect neither $-\frac{1}{N_s}$ nor $-\frac{1}{N_q}$. In this example $\delta = 0.1$ was considered for SSOD and RQ. The PID parameters and the Δ_u values predicted

by equation (8) are shown in Table 1.

$$G_1(s) = \frac{e^{-0.5s}}{(s+1)^2} \tag{25}$$
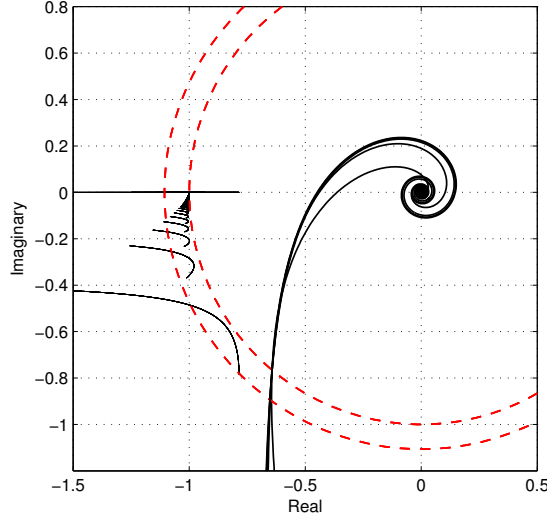
Figure 14. Graph of $\frac{1}{N_d}$, $\frac{1}{N_s}$ and G_{ol} for the PID designed for $G_1(s)$ with $\phi_d = 50^o$, $\gamma > 2$ and filter coefficients $N = 0$, $N = 3$, $N = 10$, $N = 15$ and $N = 20$.

In order to evaluate the robustness of the designs with each event sampling strategy, the values of the relative margins ϕ_{ssod_r}, γ_{ssod_r}, ϕ_{rq_r} and γ_{rq_r} were calculated. The results are presented in Table 1. In all cases the relative margins resulting from RQ are much larger than those of SSOD. Consequently, the closed loop system with RQ sampler might admit a wider variation in the model parameters before the limit cycle appears. To further emphasize this fact, the time delay of model G_1 was varied to study the effect in the shape of G_{ol} and in the appearing of limit cycles. Table 1 shows the percentage of change in the time delay of $G1$ that produces a limit cycle in the closed loop with each nominal designed controller and each event sampling approach. In all the cases the higher robustness of the RQ event sampling strategy is clear. This idea is further emphasized in Figure 15, where the Nyquist plots for $N = 15$ are shown for the nominal model case and for the two limit cases that lead to a limit cycle.

Simulations for three of the controllers ($N = 0$, $N = 3$ and $N = 15$) with the nominal model were carried out using SSOD and RQ sampling. Figures 16, 18 and 20 show the time responses of the output of the controlled system in the presence of step changes in the reference and disturbance when the event generator is based on RQ (upper graph) and on SSOD (lower graph). The control action of all these cases is shown in Figures 17, 19 and 21. Moreover, the events generated in each case are also shown by vertical blue lines. $\delta = 0.1$ was used in all the examples. In order to compare the system responses the integral of absolute error (IAE) and the number of events were calculated. The IAE is computed in the simulation only till the instant when the output enters the δ band, since the event based sampling leads to a steady state error. The results are summarized in Table 1.

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Table 1. PID design and simulation results obtained for model G_1 with $\delta = 0.1$ and different values of N

N		0	3	10	15	20
K_p		1.18	2.15	2.50	2.59	2.65
T_i		1.76	1.92	1.95	1.97	1.99
T_d		0.00	0.41	0.47	0.48	0.49
Δ_u		0.12	0.86	2.75	4.16	5.57
$\phi_{ssod_r}[\%]$		18.19	17.77	17.28	17.35	17.41
$\gamma_{ssod_r}[\%]$		29.67	29.74	30.73	30.58	30.43
$\phi_{rq_r}[\%]$		76.91	76.06	77.44	77.21	76.98
$\gamma_{rq_r}[\%]$		110.48	80.45	81.82	80.68	79.78
Admissible delay change [%]	SSOD	50	30	28	27	27
	RQ	150	90	84	81	79
IAE	SSOD	1.54	0.89	0.78	0.76	0.75
	RQ	1.44	0.91	0.75	0.74	0.73
Events	SSOD	27	21	21	21	21
	RQ	29	27	23	23	23
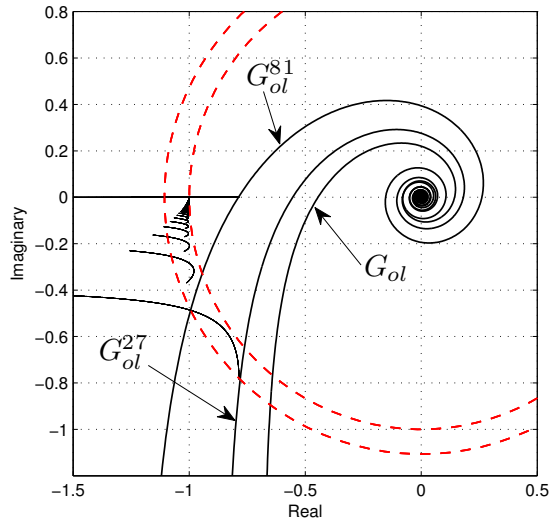
Figure 15. Effect over G_{ol} of variations in the time delay of model G_1 for PID designed with $N = 15$. G_{ol}^x represents the open loop transfer function for variation of x per cent of the time delay.

From the figures and values in Table 1, it can be noted that with SSOD or RQ quite similar system responses are obtained for each design. The increment in the value of N leads to a reduction in the IAE at the expense of greater jumps in the control action (Δ_u) whenever a new event is generated. However, for values of N larger than 3 (for example 10, 15 or 20), the reduction of IAE is small (almost negligible) while the magnitude of the control action jumps grows rapidly with N. It is interesting to note that for the values of N that are more common in commercial PID (as $N = 10$ or $N = 20$), the control action jumps due to the event sampling are really high. One can conclude that for event based PID it should be better to use a value of N lower than those usually used in commercial PID. Some PID vendors allow to freely chose the value of parameter N and, of course, one can set it also freely if the PID algorithm is implemented in a programmable platform. Because the value of δ is the same in all cases, significant variations are not observed in the number of events.
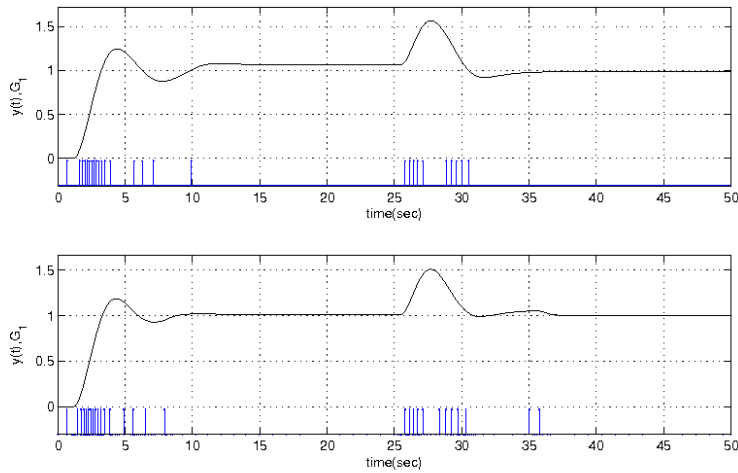
Figure 16. Behaviour of the controlled output y in the presence of a change in the reference y_r and disturbance p for the PID designed for $G_1(s)$ with $\phi_d = 50^o$, $\gamma > 2$ and filter coefficients $N = 0$, with an event generator based on SSOD (top) and RQ (bottom).

6.2. *Example 2*

In this case a different situation from that presented in the example 1 is considered: now a constraint is imposed to the control action jumps: $\Delta_u \leq \Delta_{umax}$ and the value of δ will be selected to guarantee this condition. The system to be controlled has an under-damped response with transfer function given by equation (26) which has a single real pole and a pair of complex conjugate poles.

Five PID were designed for this system with the same tuning parameters considered in the example 1: $\phi_d = 50^o$, $\gamma \geq 2$ and N equal to 0, 3, 10, 15 and 20. Figure 22 shows the graphs of $-\frac{1}{N_s}$ and $-\frac{1}{N_q}$ together with the G_{ol} which is achieved by tuning the PID for each design. As can be seen, the open loop transfer function does not intersect neither $-\frac{1}{N_s}$ nor $-\frac{1}{N_q}$, therefore the designs fulfill the robustness restrictions introduced by both SSOD and RQ. The PID parameters are shown in Table 2.
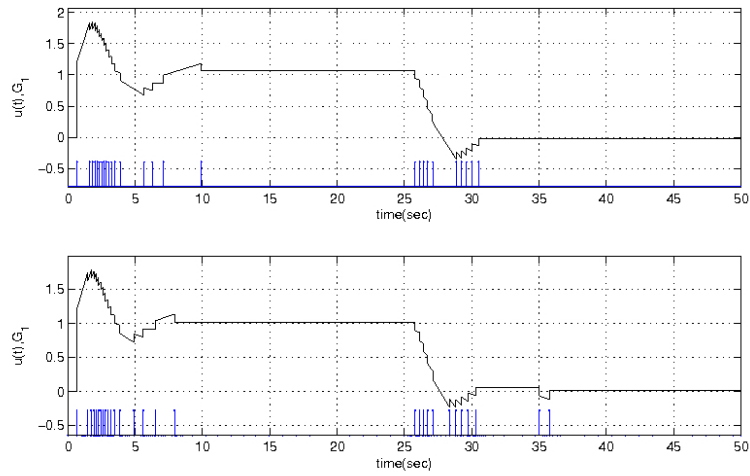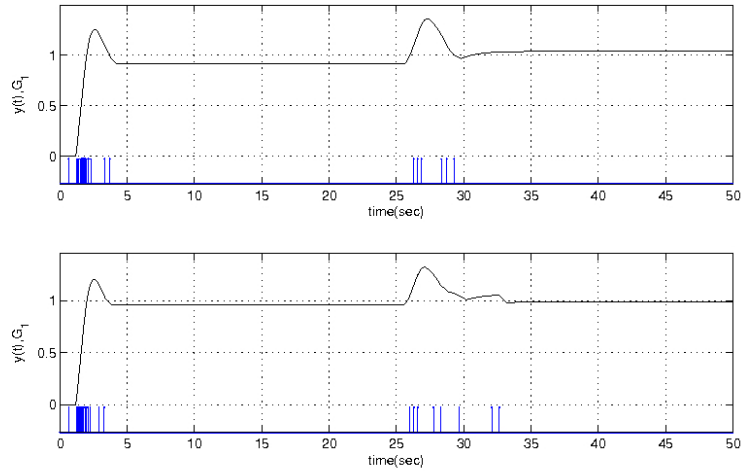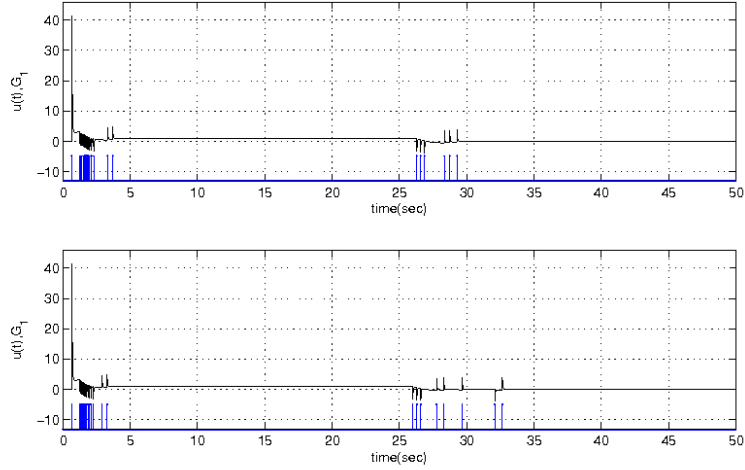
Figure 17. Behaviour of the control action u in the presence of a change in the reference y_r and disturbance p for the PID designed for $G_1(s)$ with $\phi_d = 50^o$, $\gamma > 2$ and filter coefficients $N = 0$, with an event generator based on SSOD (top) and RQ (bottom).
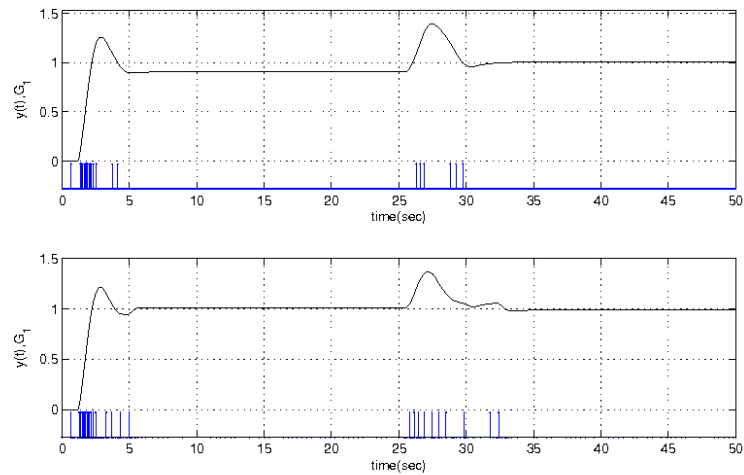

Figure 18. Behaviour of the controlled output y in the presence of a change in the reference y_r and disturbance p for the PID designed for $G_1(s)$ with $\phi_d = 50^o$, $\gamma > 2$ and filter coefficients $N = 3$, with an event generator based on SSOD (top) and RQ (bottom).

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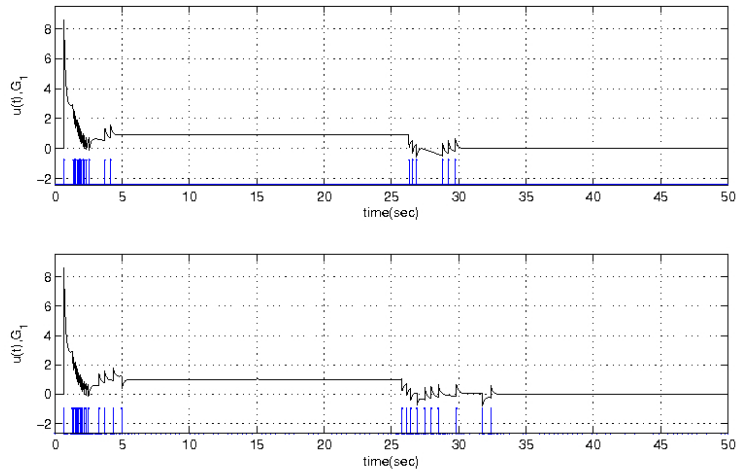
Figure 19. Behaviour of the control action u in the presence of a change in the reference y_r and disturbance p for the PID designed for $G_1(s)$ with $\phi_d = 50^o$, $\gamma > 2$ and filter coefficients $N = 3$, with an event generator based on SSOD (top) and RQ (bottom).

Figure 20. Behaviour of the controlled output y in the presence of a change in the reference y_r and disturbance p for the PID designed for $G_1(s)$ with $\phi_d = 50^o$, $\gamma > 2$ and filter coefficients $N = 15$, with an event generator based on SSOD (top) and RQ (bottom).

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Figure 21. Behaviour of the control action u in the presence of a change in the reference y_r and disturbance p for the PID designed for $G_1(s)$ with $\phi_d = 50^o$, $\gamma > 2$ and filter coefficients $N = 15$, with an event generator based on SSOD (top) and RQ (bottom).

$$G_2(s) = \frac{5}{(s+5)(s^2+s+1)} \tag{26}$$
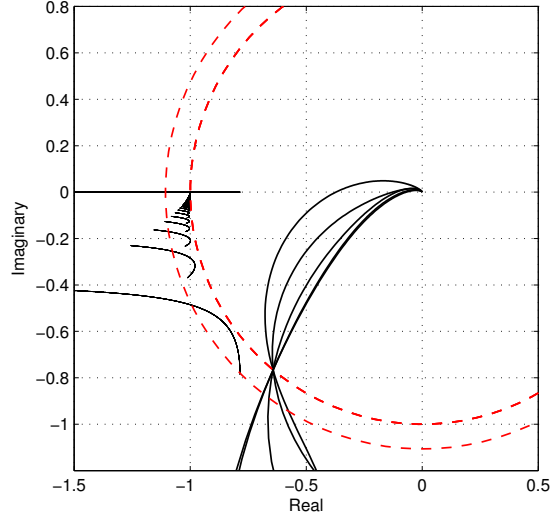
Figure 22. Graph of $\frac{1}{N_d}$, $\frac{1}{N_s}$ and G_{ol} for the PID designed for $G_2(s)$ with $\phi_d = 50^o$, $\gamma > 2$ and filter coefficients $N = 0$, $N = 3$, $N = 10$, $N = 15$ and $N = 20$.

As in example 1, in order to evaluate the robustness, the values of the relative margins ϕ_{ssod_r}, γ_{ssod_r}, ϕ_{rq_r} and γ_{rq_r} were calculated. The results are presented in Table 2. In all cases the relative margins resulting from RQ are also much larger than those of SSOD. To illustrate this fact, the gain of model G_2 was varied to study the effect in the shape of G_{ol} and in the appearing of limit cycles. Table 2 shows the

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Table 2. PID design and simulation results obtained for model G_2 with $\Delta_u = 0.1$ and different values of N

| N		0	3	10	15	20
K_p		0.49	1.61	2.81	3.26	3.25
T_i		0.83	2.00	2.36	2.45	2.47
T_d		0.00	0.43	0.57	0.59	0.60
δ		0.20	0.01	0.0032	0.0019	0.0013
$\phi_{ssod_r}[\%]$		26.28	16.31	13.61	13.29	13.12
$\gamma_{ssod_r}[\%]$		22.42	39.27	53.52	55.91	57.49
$\phi_{rq_r}[\%]$		62.76	86.42	90.98	91.52	91.84
$\gamma_{rq_r}[\%]$		106.95	282.80	304.89	327.80	350.50
Admissible gain change [%]	SSOD	16	30	45	47	49
	RQ	105	600	720	900	1020
IAE	SSOD	2.80	1.24	0.85	0.75	0.70
	RQ	2.67	1.23	0.85	0.75	0.70
Events	SSOD	21	149	637	1029	1447
	RQ	26	162	652	1049	1471

percentage of change in the gain of $G2$ that produces a limit cycle in the closed loop with each nominal designed controller and each event sampling approach. Again, the higher robustness of the RQ event sampling strategy is evident. Figure 23 shows the nyquist plots for $N = 15$ for the nominal model case and for the two limit cases that lead to a limit cycle.

As commented before, in this case we impose the restriction $\Delta_u \leq \Delta_{umax}$. In order to satisfy it, equation (8) is applied to calculate the values of δ to be used with each controller. The calculation is based on the PID parameter N and K_p according to the equation (27). The PID parameters and the values of δ for each controller are shown in Table 2.

$$\delta = \frac{\Delta_{umax}}{K_p(N + 1)} \tag{27}$$

Simulations for three of the PID ($N = 0$, $N = 3$ and $N = 15$) with their respective values of δ were carried out using SSOD and RQ sampling. Figures 24, 26 and 28 show the time responses of the output of the controlled system in the presence of step changes in the reference and disturbance when the event generator is based on RQ (upper graph) and on SSOD (lower graph). The control action of all these cases are shown in Figures 25, 27 and 29. Moreover, the events generated in each case are also shown by vertical blue lines. In order to compare the system responses the integral of absolute error (IAE) and the number of events were calculated. The results are summarized in Table 2.

Similar to example 1, for each value of N the system has quite similar responses despite the sampling strategy used: SSOD or RQ. In all the cases the restriction ($\Delta_u \leq \Delta_{umax}$) is fulfilled at the expense of reductions in δ as the N increases. As a result, the number of events rises significantly. Hence, in this approach the selection of N for the controller design is a trade-off between the closed loop speed (the IAE decreases as N increases) and the use of the communication network. As in the previous example, for the most common values of N used by PID vendors (10 to 20) the IAE does not improve significantly while the number of events rises rapidly with N.
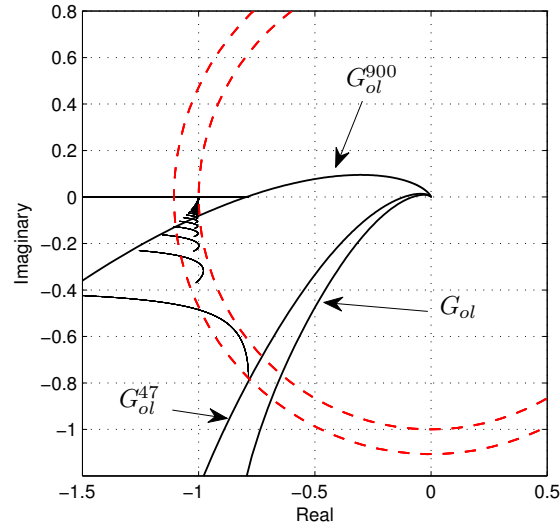
Figure 23. Effect over G_{ol} of variations in the time delay of model G_2 for PID designed with $N = 15$. G_{ol}^x represents the open loop transfer function for variation of x per cent of the system gain.

Therefore, the same conclusion can be obtained, for event based PID, low values of N should be used.
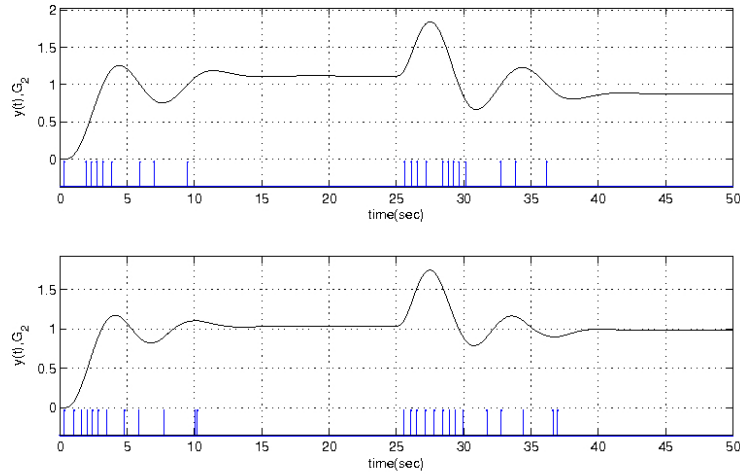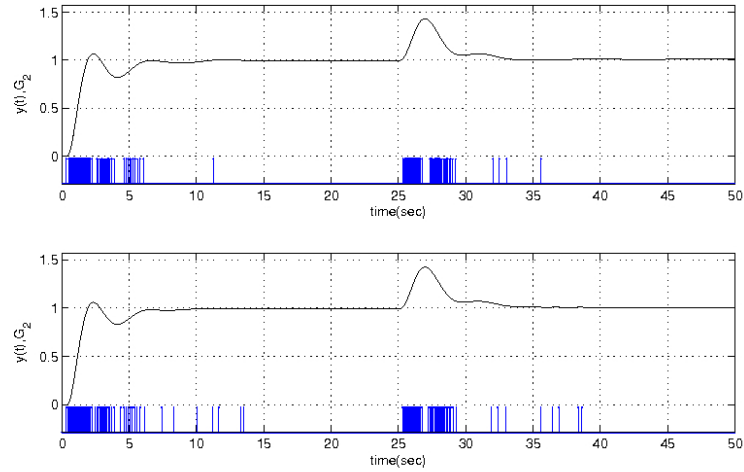

Figure 24. Behaviour of the controlled output y in the presence of a change in the reference y_r and disturbance p for the PID designed for $G_2(s)$ with $\phi_d = 50^o$, $\gamma > 2$ and filter coefficient $N = 0$, with an event generator based on SSOD (top) and RQ (bottom).

6.3.   *Example 3: Robustness analysis with other tuning methods*

As has been already commented, the approach presented in this paper allows to analyse the robustness of the event-based control system in Figure 1(a) no matter the method used for tuning the controller. In order to illustrate this fact, this section presents a robustness study of controllers designed with three well known PID tuning meth-
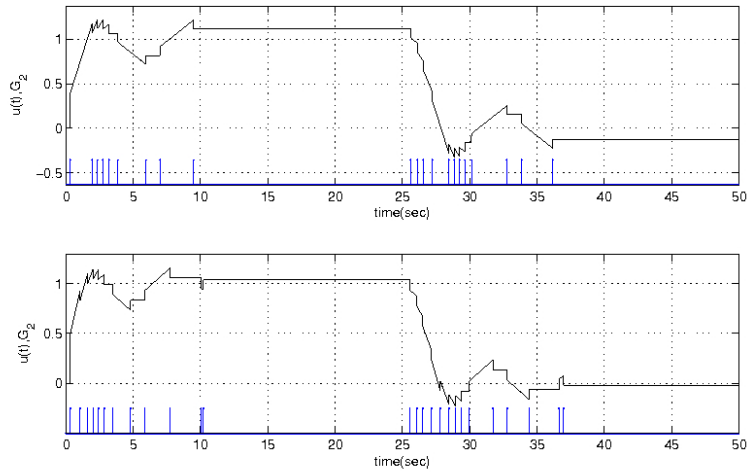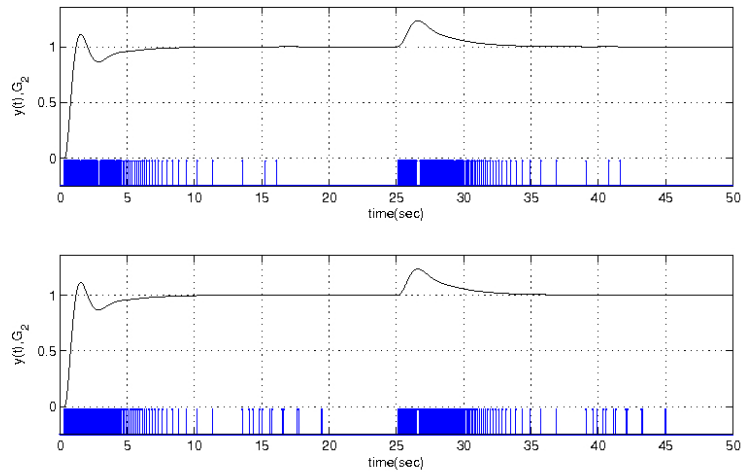
Figure 25. Behaviour of the control action u in the presence of a change in the reference y_r and disturbance p for the PID designed for $G_2(s)$ with $\phi_d = 50^o$, $\gamma > 2$ and filter coefficient $N = 0$, with an event generator based on SSOD (top) and RQ (bottom).

Figure 26. Behaviour of the controlled output y in the presence of a change in the reference y_r and disturbance p for the PID designed for $G_2(s)$ with $\phi_d = 50^o$, $\gamma > 2$ and filter coefficient $N = 3$, with an event generator based on SSOD (top) and RQ (bottom).
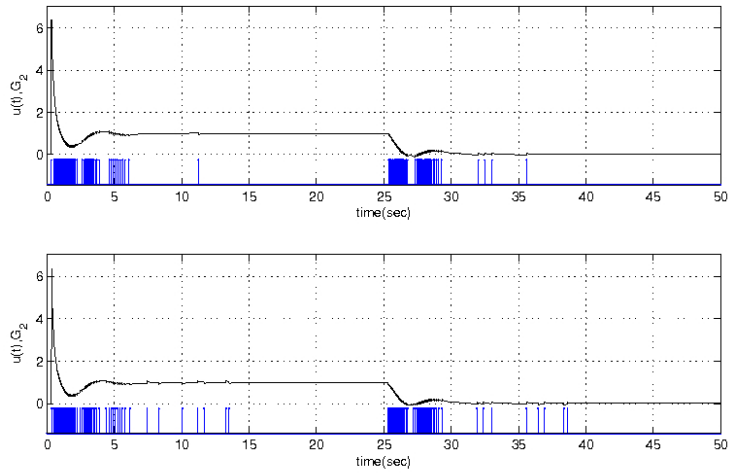
Figure 27. Behaviour of the control action u in the presence of a change in the reference y_r and disturbance p for the PID designed for $G_2(s)$ with $\phi_d = 50^o$, $\gamma > 2$ and filter coefficient $N = 3$, with an event generator based on SSOD (top) and RQ (bottom).
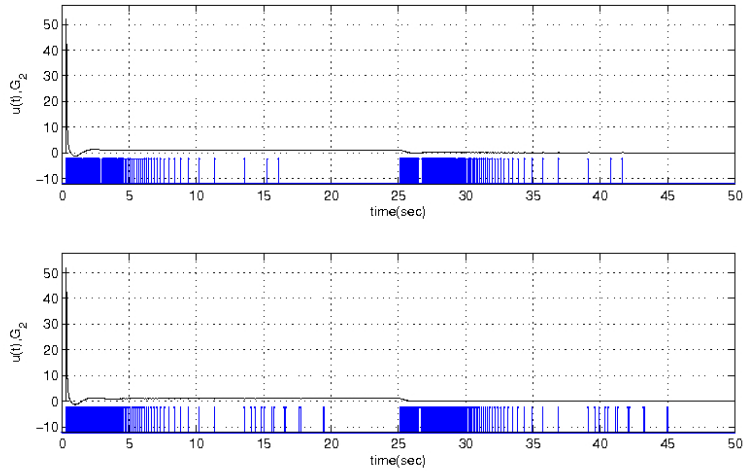

Figure 28. Behaviour of the controlled output y in the presence of a change in the reference y_r and disturbance p for the PID designed for $G_2(s)$ with $\phi_d = 50^o$, $\gamma > 2$ and filter coefficient $N = 15$, with an event generator based on SSOD (top) and RQ (bottom).

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Figure 29. Behaviour of the control action u in the presence of a change in the reference y_r and disturbance p for the PID designed for $G_2(s)$ with $\phi_d = 50^o$, $\gamma > 2$ and filter coefficient $N = 15$, with an event generator based on SSOD (top) and RQ (bottom).

ods. Specifically we have considered the Ziegler-Nichols method (Ziegler and Nichols (1942)), the method proposed in Åström and Hägglund (1988) (page 60), which guarantees gain and phase margins greater than $6db$ and 45° respectively, and the AMIGO method (Åström and Hägglund (2004)) where the robustness is specified by means of the maximum value of the sensitivity function (M_{s_d}) within a range between 1.4 and 2.

Three PID controllers for the process with transfer function $G_1(s)$, equation (25), were designed using these methods (with $M_{s_d} = 1.7$ in the case of the AMIGO). In this case a fixed value of $N = 10$ was used in the controllers. The results are summarized in Table 3 and Figure 30 which shows the polar plot of G_{ol} for the three controllers. Since the designs are not based on pre-fixed phase and gain margins (γ_d and ϕ_d), the values γ_{rq}, ϕ_{rq}, γ_{ssod} and ϕ_{ssod} are used to measure the robustness instead of the relative margins given by equations (21)-(24).

The values presented in Table 3 show that in all cases $\gamma_{rq} > \gamma_{ssod}$ and $\phi_{rq} > \phi_{ssod}$. This confirms the better robustness properties of the RQ sampler in comparison to the SSOD one, despite the tuning method. According to the values of the RQ margins, all the controllers avoid limit cycles oscillations under RQ sampling strategy. On the other hand, the design obtained with Z-N presents negative margins for the SSOD sampler which predicts the existence of a limit cycle. This behaviour is verified by the simulations presented in Figure 31 where the responses of the system outputs to step change in the reference are shown for all the controllers and $\delta = 0.1$. It can be noted the oscillatory response of the PID obtained by the Ziegler-Nichol method when used with SSOD sampling. The rest of cases have non-oscillatory responses, as predicted by the values of the robustness margins in Table 3.

7. Conclusions

In this article we have presented a set of rules for designing controllers for networked control systems in which event generators are used to reduce the amount of data
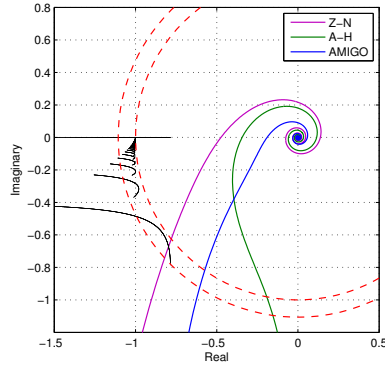
Figure 30. Graph of $\frac{1}{N_d}$, $\frac{1}{N_s}$ and G_{ol} for the PID designed for $G_1(s)$ with different well-known tuning methods and filter coefficient $N = 10$. Z-N: Ziegler and Nichols (1942) , A-H: Åström and Hägglund (1988), AMIGO: Åström and Hägglund (2004).
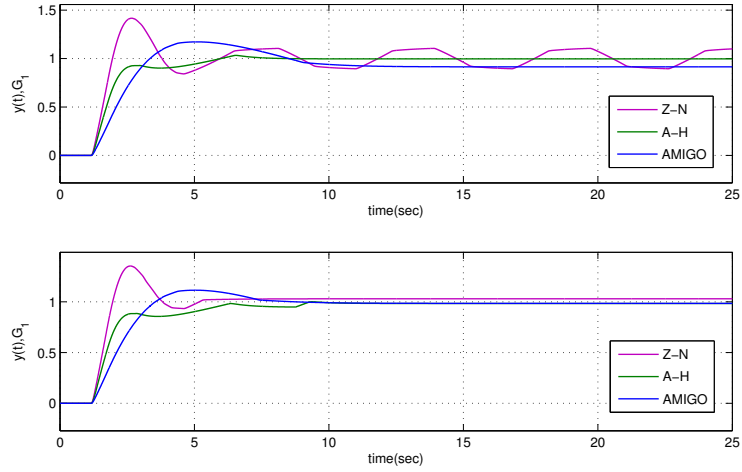
Figure 31. Behaviour of the controlled output y in the presence of a change in the reference y_r for PIDs designed for $G_1(s)$ using well known tuning methods, with an event generator based on SSOD (top) and RQ (bottom). Z-N: Ziegler and Nichols (1942) , A-H: Åström and Hägglund (1988), AMIGO: Åström and Hägglund (2004).

Table 3. PID design results obtained for model G_1 with different well-known tuning methods.

Tuning Method	Z-N	Å-H	AMIGO
K_p	2.81	1.64	1.11
T_i	1.63	2.52	1.36
T_d	0.40	0.62	0.44
Δ_u	3.09	1.80	1.22
$\phi_{ssod}[^o]$	-3.08	36.82	11.68
$\gamma_{ssod}[db]$	-0.87	5.94	5.58
$\phi_{rq}[^o]$	29.00	67.85	50.49
$\gamma_{rq}[db]$	4.32	7.11	12.26

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sent via the communications network. Two event generation mechanisms have been studied: one based on regular quantification (RQ), and another based on symmetric send-on-data (SSOD).

The rules are given in terms of constraints on the classical robustness margins of the continuous control systems, such as the gain margin for the case of RQ and the phase margin for the case of SSOD. The rules allow us to use the available methods for tuning continuous PID controllers, based on the phase and gain margins, for the case of event-based control systems.

Furthermore, new robustness margins to oscillations have been introduced for RQ and SSOD event sampling, that allow to evaluate the resulting robustness of the designed controller when the event based sampling is used. Based on those new robustness margins, an analysis carried out over an extensive batch of models allows to conclude that, for similar performance, the use of RQ event sampling results in a much more robust controlled system if compared to the use of SSOD. However, the measurement noise affects more the RQ due to the lack of hysteresis, tending to produce bursts of events in thresholds crossings.

Moreover, it is shown that selection of the δ parameter of the event generator, for both the RQ and the SSOD, do not affect the stability and robustness of the control system, that are ensured by the appropriate controller tuning. The selection of δ is related to the steady state error and the number of events. The value of δ also determines the amplitude of the control action jumps. For the PID, those jumps also depend on the derivative filter parameter, N, i.e., the higher the N the higher the jumps. However, the higher the N, the higher the performance (in terms of IAE for example). Therefore, the selection of δ and N also represents a compromise between control action jumps and performance. The values of N used for most vendors ($N = 10$ to 20) tend to produce very abrupt changes in the control action. Therefore, for PID with event based sampling, it is better to use smaller values of N (as $N = 3$ for example), since the deterioration in terms of IAE is not too important, while the control action jumps are reduced drastically.

The results have been illustrated by simulations. Processes with different types of dynamic responses have been considered, including complex poles and time delays. Furthermore, it has been demonstrated how the new margins to oscillation introduced in the article can be applied to measure the robustness to limit cycles of event based control systems despite the tuning method used for the controller design. In this sense, three well known tuning methods have been compared when using in this kind of event-based-PID control system.

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