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**Título artículo / Títol article:**

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**Autores / Autors**

**María Amparo Camarero Olivas, Javier Ordóñez Monfort**

**Revista:**

**Economic Modelling**

**Versión / Versió:**

**Preprint**

**Cita bibliográfica / Cita bibliogràfica (ISO 690):**

**CAMARERO OLIVAS, María Amparo; ORDÓÑEZ MONFORT, Javier. Nonlinear adjustment in the real dollar-euro exchange rate: The role of the productivity differential as a fundamental. *Economic Modelling*, 2012 , vol. 29, no 2.**

**url Repositori UJI:**

**<http://hdl.handle.net/10234/63715>**

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# Nonlinear adjustment in the real dollar-euro exchange rate: the role of the productivity differential as a fundamental

Mariam Camarero and Javier Ordóñez\*

Department of Economics

Jaume I University

October 2011

## Abstract

In this paper we analyze the influence of productivity differentials in the dynamics of the real dollar-euro exchange rate. Using nonlinear procedures for the estimation and testing of ESTAR models during the period 1970-2009 we find that the dollar-euro real exchange rate shows nonlinear mean reversion towards the fundamentals represented by the productivity differential. In addition, we provide evidence about the ability of this variable to capture the overvaluation and undervaluation of the dollar against the euro.

**JEL classification:** C22, F31.

**Keywords:** nonlinearities, real exchange rate, productivity differential fundamental.

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\*Address for comments: Javier Ordóñez. Department of Economics. Jaume I University. Campus del Riu Sec. E-12080 Castellón (Spain). e-mail: jmonfort@eco.uji.es. Ph.: 34-964728605. Fax.: 34-964728590. The authors gratefully acknowledge the financial support from the CICYT project ECO2011-30260-C03-01 and the Generalitat Valenciana project PROMETEO/2009/098. They also thank the comments and suggestions from Cecilio Tamarit and an anonymous referee.

# 1 Introduction

The purchasing power parity (PPP) theory postulates that national price levels should be equal when expressed in a common currency. Since the real exchange rate is the nominal exchange rate adjusted for relative national price levels, variations in the real exchange rate represent deviations from PPP. It has become something of a stylized fact that the PPP does not hold continuously. Thus deviations of spot exchange rates away from PPP are persistent and this is consistent with a unit root or near-unit root behavior of the real exchange rates. The persistent divergence from equilibrium causes that linear PPP-based fundamentals exchange rates models do not perform well in predicting or explaining future or past exchange rate movements (Frankel and Rose, 1995, Taylor, 1995, Sarno and Taylor, 2002). In addition, Haidar (2011b) showed that certain measurements of currency valuation are misleading for economies whose markets are structurally different from the benchmark currency countries. Other authors still believe that some form of PPP does in fact hold at least as a long run relationship (MacDonald, 1999, 2004). The issue of whether the real exchange rate tends to revert towards a long-run equilibrium has been a topic of considerable debate in the literature (e.g. Lothian and Taylor, 1996, Lothian and Taylor, 1997, and Taylor and Taylor 2004, Lothian and Taylor, 2008, among others). Panel unit root and long-run studies have reported evidence favourable to parity reversion (see Taylor, 1995, for a survey), however, as pointed out by Rogoff (1996) and Obstfeld and Rogoff (2001), it is impossible to reconcile the high short-term volatility of real exchange rates with the slow rate at which shocks in the real exchange rate appear to die out in those studies. This conclusion, known as the PPP-Puzzle, constitutes one of the most controversial issues related to real exchange rates.

The relatively recent literature on nonlinearities and exchange rates can be considered a possible solution to such puzzles. Taylor, Peel and Sarno (2001) and Kilian and Taylor (2003) argued that allowing for nonlinearities in real exchange rate adjustment are key both to detect mean reversion in the real exchange rate and to solve the PPP-puzzle. Moreover, Imbs et al. (2003) found that mean reversion speed increases using TAR models and sectoral disaggregated price data. Following their argument, the further away the real exchange rate is from its long-run equilibrium, the stronger will be the forces driving it back towards equilibrium. Another way to reconsider the linear PPP-based models is to integrate in this basic model the impact of shocks coming from real variables<sup>1</sup>. Thus, persistent shocks might be supply-

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<sup>1</sup>Deviations from the PPP base real exchange definition can be also justified en terms

related and incorporate, for example, the Harrod-Balassa-Samuelson (HBS) effect which postulates that productivity shocks affect the equilibrium real exchange rate. From the empirical evidence, it seems that the productivity differential plays a very important role in explaining some real exchange rate movements. Alquist and Chinn (2002) found supporting evidence for the productivity differential as the most important fundamental that explains the behavior of the real dollar-euro exchange rate since the mid 1980s. Furthermore, they argued that the magnitude of the correlation between the two variables is much larger than what would predict the HBS effect. In a previous paper, Camarero, Ordóñez and Tamarit (2005) estimated a long-run model for a synthetic pre-euro-dollar exchange rate, finding that the main factor explaining the dynamic adjustment in the error correction model was the productivity differential. In contrast, Schnatz et al. (2004) found that although the productivity differential was an important determinant of the real dollar-euro exchange rate, its ability to explain the real depreciation of the euro in the late nineties could be considered very limited. However, their sample ends in 2002, so that it only includes three years of euro data. Lothian and Taylor (2008) investigated the influence of productivity differentials on the equilibrium level of the pound-dollar and pound-franc real exchange rates. Although these authors found statistically significant evidence of the HBS effect for the pound-dollar real exchange rate, they failed to find any significant evidence of the HBS effect for the pound-franc real exchange rate.

In this paper we focus on testing for and estimating some form of nonlinear adjustment in the real dollar-euro exchange rate towards the productivity differential. This paper aims at bringing together the literature based on linear fundamentals-based models and the assessment of non-linear mean-reversion to purchasing power parity. For this purpose, the analysis looks at the ability of an economy-wide productivity measure as a useful fundamental to capture the dollar-euro exchange rate behavior. It can be considered an extension of Camarero, Ordóñez and Tamarit (2005), as it includes data from the euro-years and concentrates on the fundamental with the strongest dynamic effect.

The remainder of this paper is organized as follows. Section 2 briefly describes the methodology used in the empirical analysis. In Section 3 we present the data, as well as the estimated nonlinear model. We also analyze the adjustment of the real exchange rate towards the productivity differential. The last section concludes.

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of changes in the terms of trade. See Haidar (2011a) for a discussion of how a currency crisis transmits through international trade.

## 2 Methodology

A number of authors has reported evidence of nonlinear adjustment in the real exchange rate<sup>2</sup>. Such nonlinearities can be modelled using a smooth transition autoregressive (STAR) process, proposed by Granger and Teräsvirta (1993). In this model, the adjustment takes place in every period at a speed that varies with the extent of deviation from equilibrium. A STAR model can be formulated as follows:

$$y_t = \left(\alpha + \sum_{i=1}^p \phi_i y_{t-i}\right) [1 - G(\gamma, y_{t-d} - c)] + \left(\tilde{\alpha} + \sum_{i=1}^p \tilde{\phi}_i y_{t-i}\right) G(\gamma, y_{t-d} - c) + \varepsilon_t \quad (1)$$

where  $\alpha$ ,  $\tilde{\alpha}$ ,  $\gamma$  and  $c$  are constant terms;  $\varepsilon_t$  is an i.i.d. error term with zero mean and constant variance  $\sigma^2$ . The transition function  $G(y_{t-d}; \gamma, c)$  is continuous and bounded between 0 and 1.

The STAR models can be considered a variety of regime-switching as they allow for two regimes associated with the extreme values of the transition function  $G(y_{t-d}; \gamma, c) = 1$  and  $G(y_{t-d}; \gamma, c) = 0$ , where the transition between these two regimes is smooth.

Two popular choices of transition functions are the first-order logistic function,

$$\text{LSTAR: } G(\gamma, y_{t-d} - c) = (1 + \exp\{-\gamma(y_{t-d} - c)\})^{-1}, \quad \gamma > 0, \quad (2)$$

and the exponential function,

$$\text{ESTAR: } G(\gamma, y_{t-d} - c) = 1 - \exp\{-\gamma(y_{t-d} - c)^2\}, \quad \gamma > 0. \quad (3)$$

where  $c$  is the equilibrium level of  $y_t$  and  $\gamma$  the transition parameter, which determines the speed of transition between the two extreme regimes, with higher values of  $\gamma$  implying faster transition.

The first one delivers the logistic STAR (LSTAR) model. When  $\gamma \rightarrow \infty$ , the logistic function approaches 1 and the LSTAR model becomes a two-regime threshold autoregressive (TAR) model, whereas when  $\gamma = 0$ , the LSTAR model reduces to a linear AR model. The second one delivers the exponential STAR or ESTAR model. The exponential function is symmetric and U-shaped around zero. The ESTAR model collapses to a linear AR(p) model for either  $\gamma \rightarrow 0$  or  $\gamma \rightarrow \infty$ , and it is therefore useful to capture

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<sup>2</sup>See for example Taylor (2006) for a recent overview of the real exchange rate and purchasing power parity debate.

symmetric adjustment of the endogenous variable above and below the equilibrium level. According to the empirical literature, the ESTAR model is one particular statistical characterization of nonlinear adjustment, which appears to work well for exchange rates.

In our research, we will use the procedure developed by Granger and Teräsvirta (1993) and Teräsvirta (1994) for the specification and estimation of parametric STAR models. Their technique consists of the “specific-to-general” strategy for building nonlinear time series models suggested by Granger (1993) and, as indicated by van Dijk et al. (2002), it comprises the following steps: (a) specify a linear AR model of order  $p$  for the time series under investigation; (b) test for the null hypothesis of linearity against the alternative of STAR nonlinearity; (c) if linearity is rejected, select the appropriate transition variable and the form of the transition function; (d) estimate and evaluate the model; (e) use the model for descriptive or forecasting purposes.

Testing for linearity against a STAR is a complex matter because, under the null of linearity, the parameters in the STAR model are not identified. Luukkonen et al. (1988) and Teräsvirta (1994) proposed a sequence of tests to evaluate the null of an AR model against the alternative of a STAR model. These tests are conducted by estimating the following auxiliary regression for a chosen set of values of the delay parameter  $d$ , with  $1 < d < p$ :<sup>3</sup>

$$y_t = \beta_0 + \sum_{i=1}^p \beta_{1i} y_{t-i} + \sum_{i=1}^p \beta_{2i} y_{t-i} y_{t-d} + \sum_{i=1}^p \beta_{3i} y_{t-i} y_{t-d}^2 + \sum_{i=1}^p \beta_{4i} y_{t-i} y_{t-d}^3 + \epsilon_t. \quad (4)$$

The null of linearity against a STAR model corresponds to:  $H_0 : \beta_{2i} = \beta_{3i} = \beta_{4i} = 0$  for  $i = 1, 2, \dots, p$ . The corresponding LM test has an asymptotic  $\chi^2$  distribution with  $3(p+1)$  degrees of freedom under the null of linearity. If linearity is rejected for more than one value of  $d$ , the value of  $d$  corresponding to the lowest  $p$ -value of the joint test is chosen. In small samples, it is advisable to use  $F$ -versions of the LM test statistics because these have better size properties than the  $\chi^2$  variants (the latter may be heavily oversized in small samples). Under the null hypothesis, the  $F$  version of the test is approximately  $F$  distributed with  $3(p+1)$  and  $T-4(p+1)$  degrees of freedom.

If linearity is rejected, we need to test for LSTAR against ESTAR nonlinearity. For this purpose, Granger and Teräsvirta (1993) and Teräsvirta (1994) proposed the following sequence of tests within the auxiliary regression (4):

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<sup>3</sup>Equation (4) is obtained by replacing the transition function in the STAR model (1) by a suitable Taylor series approximation (see Granger and Teräsvirta, 1993).

$$\begin{aligned}
H_{03} &: \beta_{4i} = 0 \quad i = 1, 2, \dots, p \\
H_{02} &: \beta_{3i} = 0 | \beta_{4i} = 0 \quad i = 1, 2, \dots, p \\
H_{01} &: \beta_{2i} = 0 | \beta_{3i} = \beta_{4i} = 0 \quad i = 1, 2, \dots, p.
\end{aligned}$$

An ESTAR model is selected if  $H_{02}$  has the smallest p-value, otherwise the selected model is the LSTAR.

Since this type of linearity tests assume stationarity we first need to check whether  $y_t$  is a stationary variable. Kapetanios, Shin and Snell (2003) proposed a framework to test for nonstationarity against nonlinear but globally stationary exponential smooth transition autoregressive processes. Consider a univariate smooth transition autoregressive of order 1, STAR(1) model:

$$y_t = \phi y_{t-1} + \tilde{\phi} y_{t-1} (1 - \exp\{-\gamma y_{t-d}^2\}) + \varepsilon_t \quad (5)$$

As suggest by Kapetanios, et al. (2003), KSS hereafter, equation (5) can be conveniently reparameterised as:

$$\Delta y_t = \beta y_{t-1} + \tilde{\phi} y_{t-1} (1 - \exp\{-\gamma y_{t-d}^2\}) + \varepsilon_t \quad (6)$$

where  $\beta = \phi - 1$ . Imposing  $\beta = 0$  (that is, the variable is a nonstationary process in the central regime) and  $d = 1$ , our specific ESTAR model is:

$$\Delta y_t = \tilde{\phi} y_{t-1} (1 - \exp\{-\gamma y_{t-1}^2\}) + \varepsilon_t \quad (7)$$

where  $\varepsilon_t \sim iid(0, \sigma^2)$ . In order to test for the null hypothesis of a unit root  $H_0 : \gamma = 0$  against  $H_1 : \gamma > 0$  outside of the threshold<sup>4</sup>, Kapetanios et al. (2003) proposed a Taylor approximation of the ESTAR model since, in practice, the coefficient  $\gamma$  cannot be identified under  $H_0$ . Thus, under the null hypothesis, the model becomes

$$\Delta y_t = \delta y_{t-1}^3 + \eta_t \quad (8)$$

where  $\eta_t$  is an error term. Now, it is possible to apply a  $t$ -test to analyze whether  $y_t$  is a nonstationary process,  $H_0 : \delta = 0$ , or whether it is a nonlinear stationary process, such that  $H_1 : \delta < 0$ .

Equation (7) can be extended to include a constant and a trend as well as the more general case where the errors are serially correlated so that equation (8) becomes:

$$\Delta y_t = \sum_{i=1}^p \rho_i \Delta y_{t-i} + \delta y_{t-1}^3 + \eta_t \quad (9)$$

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<sup>4</sup>The process is globally stationary provided that  $-2 < \tilde{\phi} < 0$ .

Once nonlinearities are proved to be significant, the adequacy of the estimated STAR model can be evaluated using the tests suggested by Eitrheim and Teräsvirta (1996). They proposed three LM tests for the hypotheses of no error autocorrelation, no remaining nonlinearity and parameter constancy.

## 3 Empirical results

### 3.1 Data

The data is quarterly and covers the period 1970:Q1 to 2009:Q2. We use the series from Camarero, Ordóñez and Tamarit (2005) for the nominal (synthetic) dollar-euro exchange rate data for the period 1970:Q1 to 1997:Q4 and from the European Central Bank Monthly Bulletin for the rest of the sample. It deserves further attention the description of the real exchange rate we are using. Given its launch in 1999, the short history of the euro exchange rate does not allow to analyze its long-term evolution. To overcome this problem it has become common practice in the literature to use a proxy measure for the euro, either the Deutsche mark or the so called synthetic euro exchange rate.<sup>5</sup> Thus, we make use of the synthetic euro exchange rate and, after 1999, the euro itself. Then, to compute the real variable we use consumer price indices obtained from the OECD Main Economic Indicators database for the US and from the European Central Bank Monthly Bulletin for the EMU. The productivity differential is proxied by labor productivity differential, computed as GDP per employed person. The data of employment and GDP are taken from the OECD Main Economic Indicators with the exception of the German labor data for the period 2001:Q1 to 2006:Q4, which have been obtained from the German Statistisches Bundesamt. European productivity is a weighted average based on the relative GDP of the four largest euro-area economies, with fixed weights<sup>6</sup> and base year 2005.<sup>7</sup> The productivity differential is plotted in Figure 1. All the variables are in natural logarithms.

In this paper we assess to what extent the productivity differential governs the real dollar-euro exchange rate behavior and whether deviations of the exchange rate from its productivity differential may follow a nonlinear

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<sup>5</sup>Nautz and Offermans (2006) provided empirical evidence that the synthetic euro exchange rate constitutes a better proxy for the euro prior to 1999 as compared with the Deutsche Mark.

<sup>6</sup>The weights are 0.37 for Germany, 0.26 for France, 0.25 for Italy and 0.12 for Spain.

<sup>7</sup>The choice of the countries used for aggregation is mainly due to problems of data availability. However, even if we consider only four countries, Germany, France, Italy and Spain account for over 80% of the euro-area GDP.



process. Thus, we focus on  $y_t = rer_t - difpro_t$  where  $rer_t$  and  $difpro_t$  denote respectively the real dollar-euro exchange rate and the productivity differential between the Euro Area and the US. Furthermore, this choice of the variable of interest will allow us to gauge the degree of overvaluation of the US dollar relative to the Euro through the time path of the transition function as demonstrated in the following section.

Previous to the STAR modelling we test whether  $rer_t$ ,  $difpro_t$  and  $y_t$  are stationary processes. For this purpose we use the Kapetanios, et al (2003) test for a unit root in the nonlinear STAR framework. Table (1) presents the results for the KSS test applied to  $rer_t$ ,  $difpro_t$  and  $y_t$  allowing for a constant and a constant plus a trend. The lag length for  $i$  in equation (9) can be chosen using an information criteria (AIC, BIC, HQ and MAIC in Table 1). Critical values have been obtained by Monte Carlo simulation for a sample of 150 observations and 50,000 replications and are shown at the bottom of Table 1. We conclude from the results that  $difpro_t$  is nonstationary when allowing for a trend, and  $rer_t$  is stationary only at 10% significance level. The variable  $y_t$  is, however, clearly stationary in levels. In addition to the unit root test, we have formally tested for cointegration between the real exchange rate and the productivity differential using the Johansen procedure. According to the results, the cointegration vector can be identified by imposing a restriction of the productivity parameter to one with a Barlett corrected test of 1.038 (p-value: 0.308). A dummy variable intended to capture the currency union was included in the cointegration analysis. However the dummy was not significant. In addition, we have also checked for the stability of the cointegration relation and no significant break was found. Figure 2 shows the results for the test for beta constancy developed by Hansen and Johansen (1999).

### 3.2 Nonlinear estimation results

Once we have concluded that the variable of interest,  $y_t$ , is stationary, we can test for linearity, since the linearity tests we apply are only valid under this assumption. Table 2 reports values of the test statistics  $H_0$ ,  $H_{01}$ ,  $H_{02}$  and  $H_{03}$ . Given the quarterly frequency of the data employed, we consider  $d=1, \dots, 8$  as plausible values for the delay parameter<sup>89</sup>. From Table 2 we conclude

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<sup>8</sup>Table 2 reports the linearity test only for  $p = 2$ , since for this lag length we have obtained the lowest p-values. Linearity test for different values of  $p = 2$  are available upon request.

<sup>9</sup>Regarding the choice of  $d$ , economic intuition suggests that it should not take very long for the real exchange rate to adjust in response to a shock (Lothian and Taylor, 2008). Thus, we test for low values of  $d$ .

that the hypothesis of linearity is rejected at 5% level of significance when  $d=5$  and  $6^{10}$ . Furthermore, according to the sequence of test statistics  $H_{01}$ ,  $H_{02}$  and  $H_{03}$  the ESTR representation of the data is preferred to the LSTAR, i.e.  $H_{02}$  presents the smallest p-value. Our results suggest that there is a significant evidence of nonlinearity in the exchange rate adjustment to its productivity fundamental which appears to be reasonably approximated by an ESTAR model with a delay of five or six.

Table 4 presents the estimated ESTAR model for  $d=6$ <sup>11</sup> as well as a series of misspecification tests suggested by Eitrheim and Teräsvirta (1996). Following Teräsvirta (1994),  $\gamma$  has been standardized to make easier to compare the speeds of adjustment when  $\gamma$  is divided by the standard deviation of  $y_t$ . Concerning the identification of this model we could not reject the four restrictions  $c = 0$ ,  $\alpha = \tilde{\alpha} = 0$ ,  $\phi_1 = -\tilde{\phi}_1$  and  $\phi_2 = -\tilde{\phi}_2$  with a likelihood ratio test p-value of 0.37. These restrictions imply an equilibrium level for  $y_t$  in the neighborhood of which  $y_t$  is close to a second-order unit root process, becoming increasingly mean reverting with the absolute size of the deviation from equilibrium. Thus, our model reconciles two apparently contradictory results commonly found in the literature: the real exchange rates behaves as a random walk in the neighborhood of the equilibrium fundamental in a model which is globally stationary implying that real exchange rates are mean-reverting. As pointed out by Taylor and Peel (2000), the “t-ratios” for the transition parameter  $\gamma$  should be interpreted with caution, since under the null hypothesis  $H_0 : \gamma = 0$ ,  $y_t$  follows a unit root process. We have therefore computed the empirical marginal significance level of  $\gamma$  using Monte Carlo methods. The empirical marginal significance level appears in square brackets under the estimated transition parameter in Table 4.

The adequacy of the model is proved through the evaluation tests proposed by Eithreim and Teräsvirta (1996). The results of the misspecification tests suggest that the model is well specified since there is no evidence of autocorrelation, all possible nonconstancies in the parameters have been properly captured by our model and there are no STAR-type nonlinearities in the data that have not been captured by the model. Thus, the nonlinear model is stable. A related topic is the effect of spanning different nominal exchange rate regimes and the need for allowing for shifts in volatility in the error term of the empirical model. In our paper, we allow for shifts in volatility in a general way by using heteroscedastic-robust estimation methods. The

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<sup>10</sup>Similar results are found using the  $\chi^2$  version of the LM statistic. See Table 3

<sup>11</sup>The model with  $d=5$  has been also estimated however it delivers slightly poorer forecast when compared with  $d=6$ , so that the last is preferred. In our case  $d=6$  implies a delay parameter of one year and a half. Similar values for  $d$  are found in previous empirical literature.

misspecification test reported in Table 4 also suggests that there is no ARCH effect not accounted in our nonlinear model.

To gain insight into the mean-reverting properties of the estimated nonlinear model, we have carried out a dynamic stochastic simulation. Figure 3 plots the impulse response function obtained by Monte Carlo simulation of shocks to  $y_t$  of sizes 25, 15, 10, 5 and 1%. The nonlinear nature of the process is clearly shown since the speed of mean reversion dramatically increases with the size of the shock.

Finally, we test whether the estimated ESTAR model can beat the AR(2) linear model in terms of out-of-sample forecasting. The relative forecast performance can also be used as a model selection criterion and thus, as a way to evaluate the estimated models. We use the data from 2006:Q3 up to 2009:Q2 to evaluate the forecasting performance of the estimated AR and ESTAR models. Thus, we carry out a fully recursive forecasting exercise, which takes only into account the information that was available at the time of the forecast. Once forecasts are obtained we test whether the linear or the nonlinear model performs better in terms of forecasting. For this purpose we use the Diebold-Mariano statistic to compare predictive accuracy. According to our results, we reject the null of equal predictive accuracy against the alternative that the nonlinear model provides better forecast with a test statistic of -4.91 (p-value: 0.000). Therefore, our estimated ESTAR model is preferred to the linear one for the estimation of the real dollar-euro exchange rate.

Finally, using the estimated transition function it is possible to obtain the degree of over- or undervaluation of the dollar relative to the euro according to the productivity fundamental. Taylor and Peel (2000) propose a series of transformations to the transition function, which allow to assess whether the dollar is overvalued or undervalued.<sup>12</sup> Panel (a) in figure 4 displays the time series plot of the transformed transition function. Values above the horizontal axis indicate dollar overvaluation and those below it show dollar undervaluation. From our results, it appears that the dollar has been most of the time undervalued against the euro. However, following the path described by the logarithm of the real dollar-euro exchange rate in panel (c) there are two periods of dollar appreciation, where our estimated model also indicates dollar overvaluation. The first one starts in 1981 and reverts in 1986 with the Plaza Agreement (1985) and the Louvre Accord (1987). The other one runs from 1999 to 2002, coinciding with the first three years after the introduction

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<sup>12</sup>They argue that the transition function itself cannot be used as an indicator of either overvaluation or undervaluation, as it is only a measure of the importance of the deviation from equilibrium regardless of the sign.

of the euro and giving rise to an interesting debate in the empirical literature of exchange rates (De Grauwe (2000), Meredith (2001), Alquist and Chinn (2002), Schnatz et al. (2004)). The fact that our model captures well these two important episodes, which characterized the real dollar-euro exchange rate during our sample period, highlights the importance of the nonlinear models against the linear ones, as well as the robustness of our estimated model. Panel (b) in figure 4 plots the the sum of the coefficients in the instantaneous AR(2) process  $y_t$ . As suggested by Taylor and Peel (2000) this sum can be viewed as a measure of the degree of mean reversion of the real exchange rate at a particular point in time. Mean reversion is strong, in our case, during the mid-seventies, the mid-eighties and from 2007 onwards.

## 4 Conclusions

In this paper we estimate a nonlinear model for the determination of the real dollar-euro exchange rate based on the productivity differential. As shown by the empirical literature on exchange rates, nonlinear models offer more satisfactory results in dealing with some of the real exchange rate puzzles, so that they are a better alternative to explain the persistent behavior of the real exchange rate.

Using quarterly data on the dollar-euro exchange rate and the associated productivity differential for the period 1970:Q1-2009:Q3, we find evidence of nonlinearities in the dynamics of the exchange rate. These nonlinearities, which are of the form of an exponential smooth transition model, allow real dollar-euro exchange rate deviations from equilibrium to be consistent with a long-run adjustment toward the productivity fundamental, despite the apparent persistent behavior of the series. Our results also indicate that the nonlinear model offers better forecasting performance than the linear alternatives.

In addition, the transformed transition function is able to capture the well-established dollar overvaluation in the mid 1980s and the weakness of the euro after its introduction in 1999. This fact reinforces the idea that the productivity differential is an adequate explanation of the behavior of the real dollar-euro exchange rate and that volatility in exchange rates should not be directly associated with disconnection from fundamentals.

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Figure 1: Euro Area-US productivity differential

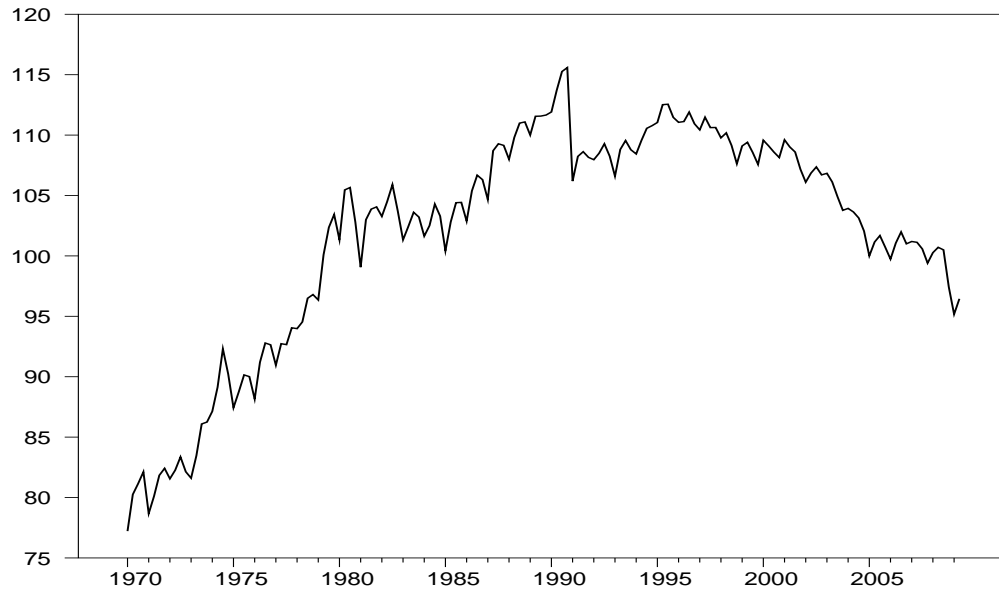




Figure 2: Tests for the stability of the cointegration space  
**Test of Beta Constancy**

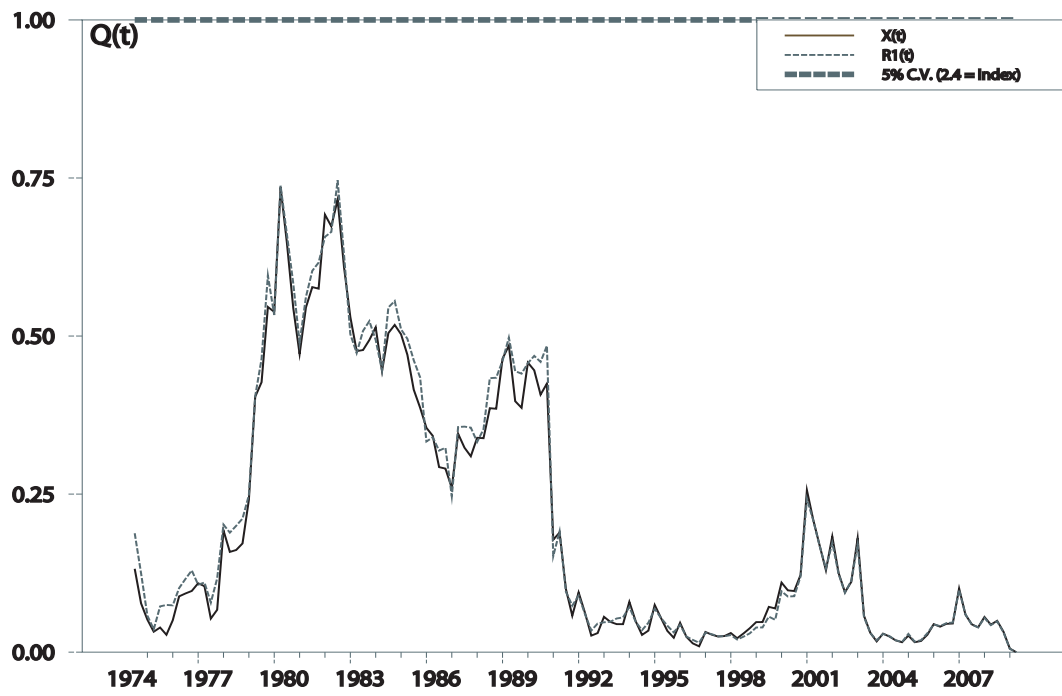


Figure 3: Impulse response functions

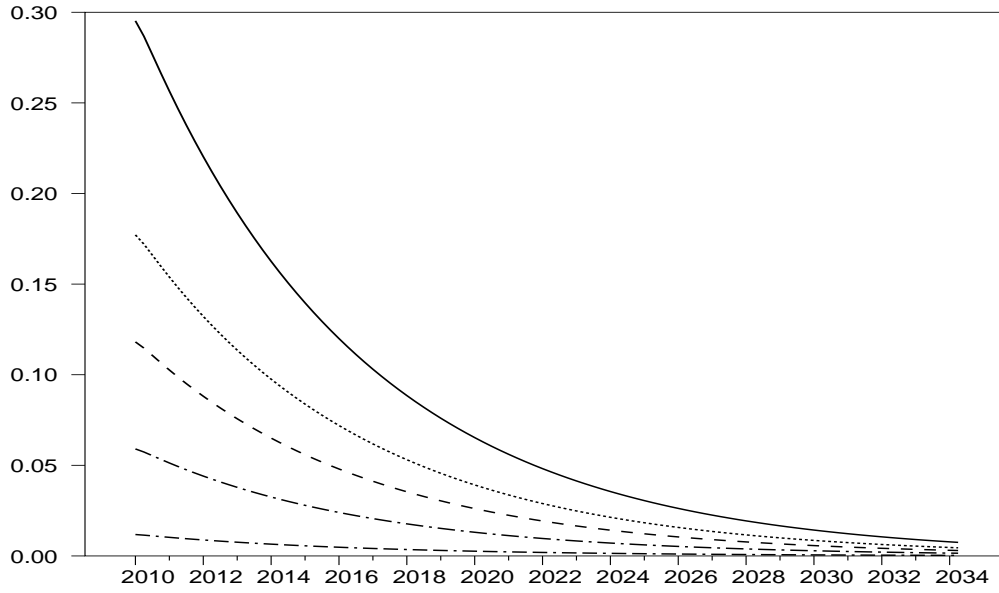
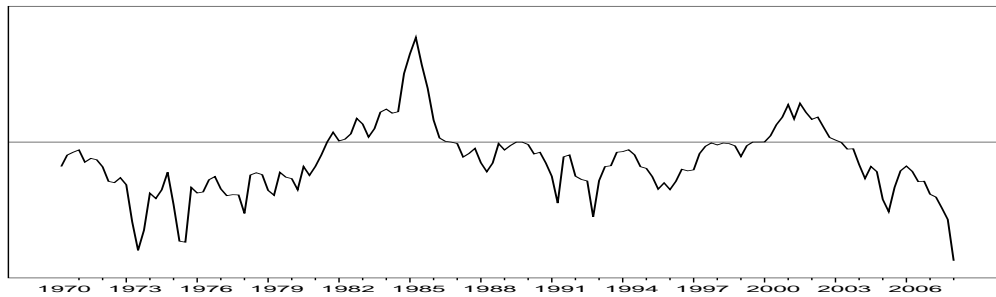
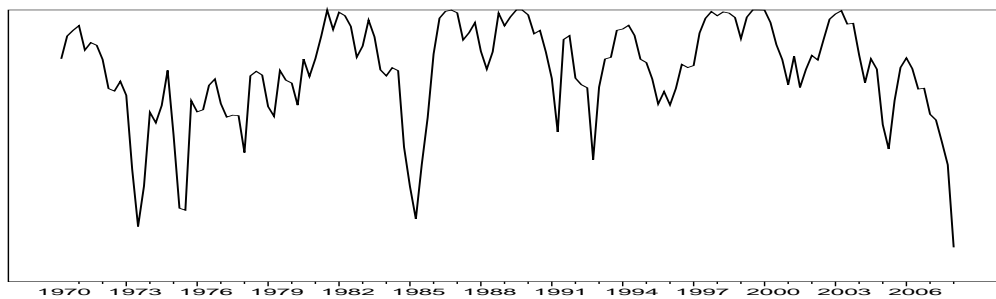


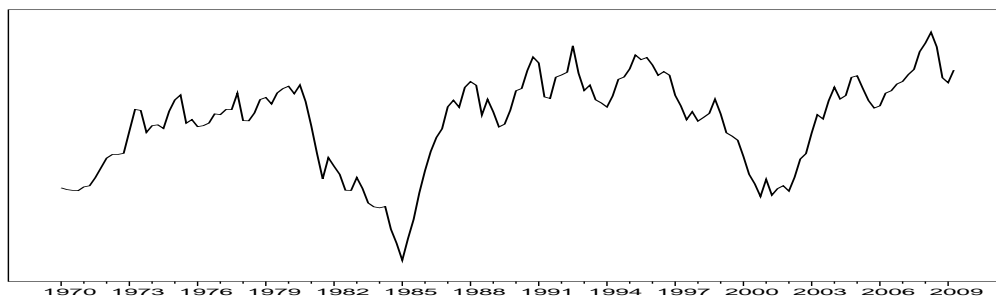
Figure 4: Estimated dollar overvaluation against euro



(a) dollar overvaluation relative to productivity fundamental



(b) mean reversion



(c) logarithm of the dollar-euro exchange rate

Table 1: KSS nonlinear unit root test

	Test with trend			Test with constant		
	$Rer_t$	$Difprod_t$	$y_t$	$Rer_t$	$Difprod_t$	$y_t$
AIC	-2.631*	-0.783	-3.203***	-2.497*	-2.769**	-3.190***
BIC	-2.631*	-0.962	-2.809**	-2.497*	-2.769**	-2.765**
HQ	-2.631*	-0.783	-3.203***	-2.497*	-2.769**	-3.190***
MAIC	-2.058	-0.300	-2.263	-1.973	-2.582**	-2.431*

Table shows the t-statistics of the null of unit root against nonlinear stationarity for different information criteria. \*, \*\* and \*\*\* denote rejection of the null at 10%, 5% and 1% respectively. Critical values have been tabulated by stochastic simulation with  $T = 150$  and 50,000 replications allowing for a constant (Case 1) and for a trend (Case 2) under the alternative.

Asymptotic critical values		
Fractile (%)	Case 1	Case 2
1	-3.16	-3.18
5	-2.50	-2.74
10	-2.11	-2.31

Table 2: P-values for the linearity test (F-variant)

---

Transition variable	$H_0$	$H_{01}$	$H_{02}$	$H_{03}$
$y_{t-1}$	0.50	0.24	0.77	0.34
$y_{t-2}$	0.07	0.02	0.35	0.33
$y_{t-3}$	0.06	0.08	0.04	0.77
$y_{t-4}$	0.07	0.21	0.02	0.92
$y_{t-5}$	0.01	0.14	0.00	0.66
$y_{t-6}$	0.01	0.42	0.00	0.82
$y_{t-7}$	0.11	0.42	0.01	0.70
$y_{t-8}$	0.92	0.62	0.62	0.94

---

Note: p-values of F variants of the LM-type tests for STAR nonlinearity of the quarterly deviation or the real dollar-euro exchange rate and the productivity differential between the Euro Area and the US euro-zone for the period 1970:Q1 to 2009:Q2. For a brief description of the test statistics see Section 2.

Table 3: P-values for the linearity test (Chi-variant)

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Transition variable	$H_0$	$H_{01}$	$H_{02}$	$H_{03}$
$y_{t-1}$	0.48	0.23	0.76	0.34
$y_{t-2}$	0.06	0.02	0.34	0.33
$y_{t-3}$	0.05	0.08	0.03	0.77
$y_{t-4}$	0.06	0.21	0.01	0.91
$y_{t-5}$	0.00	0.13	0.00	0.66
$y_{t-6}$	0.01	0.41	0.00	0.81
$y_{t-7}$	0.10	0.47	0.01	0.69
$y_{t-8}$	0.91	0.61	0.62	0.94

---

Note: p-values of Chi squared variants of the LM-type tests for STAR nonlinearity of the quarterly deviation or the real dollar-euro exchange rate and the productivity differential between the Euro Area and the US euro-zone for the period 1970:Q1 to 2009:Q2. For a brief description of the test statistics see Section 2.

Table 4: Estimated ESTAR model

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**Estimated model:**

$$y_t = \left( \underset{(0.000)}{1.215} y_{t-1} - \underset{(0.014)}{0.241} y_{t-2} \right) \left[ \exp \left( \underset{[0.002]}{-0.450} y_{t-6}^2 \right) \right]$$

Sample: 1970:Q1–2009:Q2

**Diagnostic tests:**

Autocorrelation 1-4:	1.896 [0.085]
ARCH 1-4:	6.726 [0.151]
Test for constancy of parameters:	0.601 [0.838]
Test for non remaining nonlinearity:	1.661 [0.134]

---

Note: Marginal significance levels for the “t-ratio” of the estimated transition parameter was calculated by Monte Carlo methods and are given in square brackets. Figures in parentheses below coefficient estimates denote the ratio of the estimated coefficient to the estimated standard error of the coefficient estimate. Autocorrelation 1-4 stands for the autocorrelation tests for the residuals up to 4 lags; ARCH 1-4 stand for autoregressive conditional heteroskedasticity tests (ARCH) up to order 4. Misspecification tests are constructed as Eitrheim and Teräsvirta (1996).