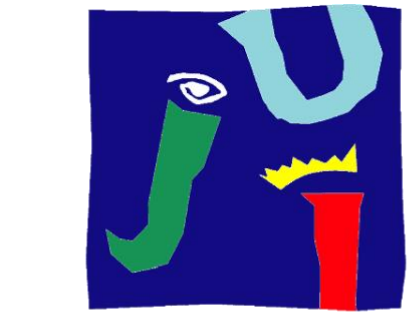

UNIVERSITAT JAUME·I

PORTFOLIO OPTIMIZATION: HISTORICAL VALUE AT RISK vs EXPECTED SHORTFALL

BACHELOR'S THESIS

FINANCIAL AND ACCOUNTING DEGREE

AUTHOR: SERGIO CASAÑA NOMDEDEU

ACADEMIC COURSE: 2020/2021

E-MAIL: al375177@uji.es

TUTOR: ALEJANDRO JOSÉ BARRACHINA MONFORT

ABSTRACT.

The main objective of this thesis is to compare the weightings of different efficient portfolios by using two different approaches to measure the risk: Historical Value at Risk (HVaR) and Expected Shortfall (ES) both for confidence levels of 95 % and 90%. In addition, the results of this study will be compared with the results obtained in previous works.

Based on the results obtained in the present study, we can surely confirm that: the weights of the efficient portfolios evaluated by HVaR have a more erratic behavior than those evaluated by ES; assets which expected return is higher have an upward trend in the evolution of weightings; Es is more similar to Markowitz in trend weights than HVaR; the similarity in the evolution of the weights according to Markowitz does not imply that they are also similar according to HVaR.

JEL CODE: G11

INDEX:

LIST OF TABLES

LIST OF FIGURES

1. INTRODUCTION.

The risk of a portfolio made up of different securities such as, for example, funds, assets, bonds, etc. These can be measured in different ways. The Modern Portfolio Theory published in 1952 by the renowned economist Harry Max Markowitz (August 24, 1927, Chicago, USA) states that the risk of an asset or a portfolio composed of these is measured by its variance (standard deviation). However, we can pose a problem to this risk measurement: the variance can only be considered sufficiently accurate and valid if the future value of the portfolio is approximately normally distributed, and this is a very strict condition. Otherwise, it is also sometimes more interesting for the investor to measure risk in monetary units. It is for these two reasons that this study will focus mainly on the risk analysis of different efficient portfolios using two alternatives to variance as risk measure: the Historical Value at Risk (HVaR) and the Expected Shortfall (ES), making a brief note about Markowitz.

The reason that led me to carry out this comparative analysis is, basically, due to my great interest in financial markets and to learn to use different methods that allow me to understand and analyze these markets in greater depth and, of course, to expand my knowledge on the valuation and construction of efficient portfolios of the assets found in these markets. For this, I consider that it is necessary, first of all, to know how to use risk assessment techniques for one or more investments that go beyond variance or standard deviation, a tool that has been studied as well as used for this purpose throughout of my stage in this degree in subjects such as markets and financial institutions or portfolio management.

Previous studies have already proposed portfolio optimization under the VaR approach as a risk measure, such as Campbell, Huisman and Koedijk (2001), Rizzi and Benati (2007) and Yoshida (2009). However, only the first one used the HVaR treatment, an approach that has been used in the present study. The main difference is that these authors seek to obtain an efficient portfolio frontier and this thesis is based mainly on the comparative analysis of the weightings of the selected assets. More recently, Camáñez (2019), carried out in his final degree thesis, the comparison in the weightings of efficient portfolios using the HVaR and the return volatility approach as a risk measure.

However, only two works were found employing ES as a risk measure, Isaksson (2016) and San Félix (2019). The first one, compares the results obtained with those that would result from applying the approach according to Markowitz, for a single expected return and for a single confidence level. Otherwise, the main objective of the work carried out by San Félix (2019) is to compare the results obtained in the weightings of different efficient portfolios using the ES and the approach according to Markowitz as a risk measure.

Among this current Project, we try to compare the weightings of different efficient portfolios by using two different risk measurement methods, Historical Value at Risk (HVaR) and Expected Shortfall (ES). Thus, so that the results can be compared with those obtained by Camáñez (2019) and San Félix (2019), we have used the same interval of expected return. From these results, it follows that ES follows a more defined pattern in the evolution of the weights with respect to HVaR for both confidence levels and, in addition, it will be observed that, although the evolutions in different studies according to Markowitz are very similar, it does not mean that it is also according to HVaR.

In the same way, some anomalies are found in this evolution that were also found by Camáñez (2019) for HVaR 95% but not for HVaR 90%. It is also concluded that the assets which expected return is higher in relation to the study period, their weights will have an upward trend and, for assets which expected return is lower, their weightings will have a downward trend for both risk measurement methods and for both confidence levels. Furthermore, it can be seen that the weights according to ES follow a more similar trend than HVaR with respect to the approach according to Markowitz.

Finally, the structure that this study follows is presented: in section 2 of the work, the theoretical foundations of both approaches will be discussed. In sections 3 and 4, the data used and the methodology of the present study will be presented, respectively. In sections 5 and 6, we will present the results obtained, which will later be analyzed and compared for confidence levels of 95% and 90%. In section 7 the behavior of the weights will be analyzed and a brief note will be made on the anomalies observed in the performance of this study. In section 8, conclusion of this analysis and the study in general.

2. THEORETICAL FOUNDATION.

2.1 RISK CONCEPT.

The risk associated with one or more financial operations refers to the uncertainty that occurs in the return of an investment. This uncertainty is due to the changes that may occur in the field of the sector in which it operates, to the non-return of the capital of one of the two parties that make up the financial operation carried out, as well as the existing instability of the financial markets (BBVA, 2020).

Hence, when evaluating the level of risk to which an individual is exposed when he wishes to carry out a financial transaction, it will be important to first assess his level of risk aversion. Precisely, being risk-averse means avoiding having a certain level of rejection of the uncertainty we talked about earlier. As a general rule, (although not always) investors will have a tendency to be risk averse. For instance: there are two types of assets, A and B. Both assets provide an expected return of 9%, however, asset A has a risk measured as the volatility of returns of 14% while asset B has a risk of 18%. Therefore, the risk-averse investor will choose for asset A which, for the same return, provides less risk. On the other hand, an investor considered “lover” to risk, might prefer asset B because, although its risk is higher, one of its possible returns may also be greater than all the returns on asset B. Otherwise, risk-neutral investor would be indifferent to choosing between the two assets because they have the same expected return and this type of investor is indifferent to the level of risk.

On the other hand, for a risk measure to be considered reasonable and therefore appropriate, it must necessarily meet the following four characteristics or properties (Cáceres, 2017):

1. Monotonicity: if, for instance, a portfolio named “X” always achieves better results than another portfolio named “Y”, then we can say that portfolio X will have less risk.

$$X, Y \ \varepsilon \ V, \quad X \geq Y \longrightarrow \sigma(X) \leq \sigma(Y)$$

2. Transitional invariance: if we add an amount of cash in a certain portfolio, the risk of this will be reduced by the same amount.

$$X \varepsilon V,\ c > R \longrightarrow \sigma(X + c) \leq \sigma(X) - c$$

3. Positive homogeneity: if we increase the size of a certain portfolio by a factor b, its risk will be multiplied by the same factor b.

$$X \varepsilon V,\ b > 0 \longrightarrow \sigma(bX) \leq \sigma(X)$$

4. Subadditivity: the risk of a portfolio must necessarily be equal to or less than the sum of the risks of the assets that comprise it.

$$X, Y, X + Y \ \varepsilon V \longrightarrow \sigma(X + Y) \leq \sigma(X) + \sigma(Y)$$

To conclude with, the contribution of each of the assets to the risk of a portfolio is provided by the degree of relationship that exists between the mentioned asset and the remaining members of the portfolio.

2.2 VALUE AT RISK.

Financial institutions have a wide variety of techniques to measure and manage the risk they bear. One of the most widely used and well-known tools is the Value at Risk (VaR), which is defined as the maximum expected loss that the position of a portfolio may have under normal market conditions and as long as there is no negotiation in the portfolio, under a confidence level and a defined time horizon (Nieto, 2016).

The concept of VaR emerged in the United States in the 1980s, being used by major financial institutions in the management of derivatives. However, it was not until the 90s when this tool was developed by statisticians and mathematicians from the financial services firm JP Morgan and it was quickly adapted by the rest of the Wall Street financial firms thanks to the great success that had in its initial phases already its simplicity of understanding. However, the concept of VaR takes on special importance as a risk measurement tool with the outbreak of the crisis in 2008, especially in cash management in financial institutions. Following the bank regulation reform initiatives published on December 10, 2010, with the aim of strengthening the financial system (Basel III), it makes the departments of financial institutions in charge of managing risks assign a daily, weekly and monthly VaR to instruments traded on financial markets.

According to Nieto (2016), it is important to know that, in order to calculate the VaR, it will be necessary to define the following parameters:

- Time horizon: it is about defining the period of time for which you want to carry out the analysis process of the maximum probable loss. The time horizon will be modified depending on the analysis portfolio itself. Normally, for positions with high liquidity a time horizon of one day is used, while on the other hand, for positions with low liquidity it is usual to use one month. However, the VaR calculation can also be carried out with a higher time horizon (by quarters, by years, etc.) Of course, assuming the risk that some modification is made to the portfolio that could alter the result obtained.

In addition, it is extremely important to take into account that for longer time horizons it is necessary to have access to a large amount of information that not all companies have.

- Confidence level: this is the probability that one or more figures are within an interval. To assign a confidence level, it is important to know what the purpose of the VaR is going to be. Thus, when the objective of estimating VaR is risk management of, for example, an investment, the commonly assigned confidence level will be 95%.

- Maximum loss: represents, in currency or percentage, the maximum loss that is expected from the investment itself.

Source: https://blog.mirai-advisory.com

Figure 1: Parametric model representation Value at Risk

Once the concept of VaR has been defined and the different parameters that must be taken into account when calculating it have been defined, we will proceed to give a brief overview of the possible existing methods for calculating it.

It is necessary to argue that there are two calculation models for this element of risk measurement: the parametric and the non-parametric model. Although in this study we are going to focus on the calculation of VaR for different efficient portfolios according to the non-parametric model, we consider it necessary to give a brief overview of all the possible calculation methods.

Parametric models consider that the future performance of a portfolio will follow a known probability distribution, that is, as a normal distribution. Within this section, we can differentiate two different calculation approaches:

- MonteCarlo model: this approach consists of predicting the future performance of the portfolio through the simulation of various scenarios of occurrence of risk factors, originated randomly and that have an impact on the value of the portfolio itself.

Through the difference between the current value of the portfolio and the appropriate value for each scenario, the total evaluation of the portfolio is carried out. It is important to know that historical data is not taken into account in this approach.

- Parametric model: it consists of a process that uses historical information to calculate volatilities and correlations of the market variables that are assigned to the current value of the portfolio with a statistical computation model, accepting that the returns of the assets are distributed in a normal way. This method is the easiest one to apply.

$$VaR = /R - z \cdot \delta / \cdot V$$

In this equation, R is the expected return, z the level of significance, δ the standard deviation and V the value of the investment.

On the other hand, non-parametric models are more general approaches, since it is not necessary to make assumptions about the distribution function and they can be used for any type:

- Historical simulation model: characterized in that the future performance of a portfolio can be predicted based on an analysis of its historical behavior. This approach reproduces the future performance of a portfolio based on the historically analyzed changes in a succession of variables applied to the portfolio.

The calculation of the VaR by historical simulation will always be slightly more laborious with respect to the parametric VaR and, however, less precise than the VaR by the MonteCarlo approach. As we have stated previously, in this study we will focus on calculating VaR using the historical approach for eighteen efficient portfolios made up of seven Ibex 35 companies.

2.3 EXPECTED SHORTFALL.

The Expected Shortfall, also known as conditional value at risk (CVaR), average value at risk (AVaR), tail VaR or expected tail loss (ETL), represents the expected loss for a portfolio in a given time horizon once the VaR has been exceeded evaluated by a previously established level of confidence, that is, it is a measure of risk of a decrease in the value of a portfolio that symbolizes the maximum loss expected of said portfolio (Peiro Ucha, 2015).

As we explained previously in section 2, for a risk measure to be considered reasonable and, therefore, appropriate, it must necessarily meet the properties of monotonicity, transitional invariance, positive homogeneity and subadditivity. VaR complies with all these measures except the last one, that is, this risk measure will be sub-additive only when the distribution of the return of a portfolio is normal. Therefore, we can affirm that the ES will be more appropriate in the risk measurement of a portfolio than the VaR. It is for this specific reason that one of the agreed international measures on market risk consisted in changing the VaR to the ES (Cáceres, 2017).

In addition, compared to VaR, the ES provides information on the magnitude that a portfolio's losses can reach, so we can state that the ES does describe the left tail of the distribution. It is for this and for the limitation stated in the previous paragraph that it is recommended to use the ES as a risk measure.

Source: https://blog.mirai-advisory.com

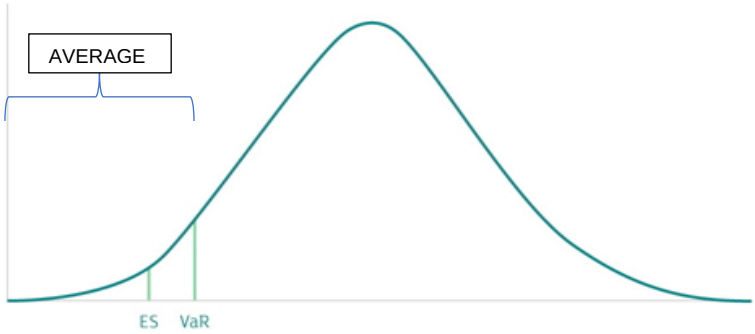

Figure 2: Expected Shortfall representation

Below, for a better understanding, a comparison of both risk measurement methods can be observed.

Source: ***www.researchgate.net***

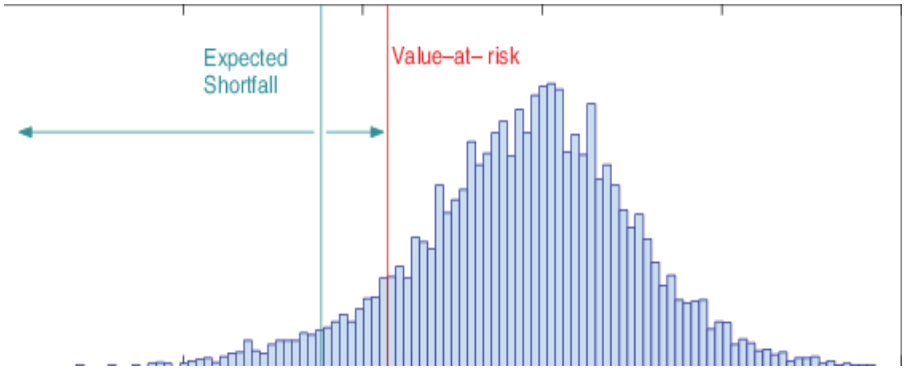

Figure 3: VaR - ES comparision

In closing, we can affirm that ES is an alternative risk assessment method to VaR and that it has greater sensitivity to the shape of the loss distribution in the left tail of the distribution. Furthermore, the ES, as can be seen in figure 1 and figure 2, will always be higher in absolute terms than the VaR.

3. EXTRACTION OF REAL DATA.

Before building efficient portfolios, under the two risk assessment methods set out above, we have chosen seven companies that are listed on the Spanish stock market and that belong to the “Ibex 35”. As a requirement for this election, it is important to bear in mind that the prices of these seven entities must provide a positive expected return average for the chosen period. This period will run from January 2, 2018, to February 28, 2020. In this case, the companies that we have selected and that provide a positive expected return for this period are the following:

- Acciona: it is a Spanish entity that is dedicated to the promotion and management of infrastructures (water, concessions, constructions and services) and renewable energies. It focuses mainly on renewable energy through wind energy projects, photovoltaic solar energy, thermal solar energy, hydraulic energy and biomass energy. In addition, it is also engaged in stock brokerage through financial entities such as "Bestinver".

- Inmobiliaria Colonial: it is a multinational company dedicated to the real estate sector. The shareholders of this company include entities such as: Royal Bank of Scotland (United Kingdom), Commerzbank (Germany), Credit Agricole (France), Coral Partners (Netherlands), La Caixa (Spain), Goldman Sachs (United States) and Banco Popular (Spain).

- Endesa: the acronym “ENDESA” stands for “National Electricity Company Sociedad Anonima” in Spanish, is an entity that carries out its operations in the electricity and gas sector.

- Ferrovial: it is a multinational company that carries out its productive activities in the field of infrastructures. That is why it operates, transport and services to cities, through four subdivisions: airports, highways, buildings and services.

- Iberdrola: Spanish company dedicated to the production, distribution and commercialization of energy. It is one of the most important companies at an international level, second in electricity production in our country and among the world leaders in this sector. It is based in Bilbao.

- Merlin Properties SOCIMI: It is a real estate investment company whose objective is focused on the acquisition, management and rental of properties for commercial purposes located mainly in Spain and with certain activity in Portugal.

- Cellnex Telecom: it is a Spanish company belonging to the telecommunications industry. Mainly, it carries out its activities in four areas: services for telecommunications infrastructures, audiovisual broadcasting networks, security and emergency network services, and intelligent management of urban infrastructures and services. The specialty of this company is to provide technology networks adapted to 5G, with a presence in the following countries: Spain, Italy, Holland, France, United Kingdom, Switzerland and Ireland.

Once the companies that will be part of this study have been determined, we proceed to extract their respective daily quotes within the time interval that we have proposed, in this case, from January 2, 2018, to February 28, 2020, at through the website "Yahoo Finance".

To calculate the daily returns included in this period, we could do it through two different approaches: the simple return and the compound return (using natural logarithms). In our study, we will use the yield by using natural logarithms, since we will not only take into account the difference between prices, but also the volatility between them.

$$Ri = ln(P_t / P_{t-1})$$

Following this approach, we obtain the respective daily return of each of the 7 companies using the Microsoft Excel spreadsheet (table 1) and we verify, using the “PROMEDIO” Excel function, that the expected daily returns for this period of each company is positive (table 2).

Table 1: Using the natural logarithm

	DAILY RETURNS 3/1/2018 - 28/2/2020						
DATE	ACCIONA	INM. COL	ENDESA	FERROVIAL	IBERDROLA	MERLÍN	CELLNEX
28/2/20	0,047436	-0,023167	-0,032609	-0,0375400	-0,0426807	-0,053480	-0,039554
27/2/20	-0,026234	-0,034184	-0,027939	-0,0390489	-0,0395956	-0,029661	-0,019528
26/2/20	0,010772	-0,001638	0,003653	-0,0042856	0,0498617	0,000790	-0,020628
25/2/20	-0,014337	-0,021854	-0,019731	-0,0222036	-0,0250129	-0,021893	0,009517
24/2/20	-0,041818	-0,028415	-0,029853	-0,0376241	-0,0226152	-0,025202	-0,032200
21/2/20	0,004275	0,004680	0,000774	0,0010076	0,0130544	0,000000	0,004950
20/2/20	-0,010230	-0,016286	-0,011552	-0,0229208	-0,0166259	-0,003012	-0,010900
19/2/20	0,030136	-0,000769	0,015821	0,0085753	0,0094024	0,012864	0,020666

Table 2: Companies expected returns

	EXPECTED RETURN
ACCIONA	0,093%
INM. COLONIAL	0,061%
ENDESA	0,049%
FERROVIAL	0,055%
IBERDROLA	0,086%
MERLÍN	0,005%
CELLNEX	0,151%

As we can appreciate, each of these companies meet the only and main requirement that we exposed previously, and that is that all of these provide a positive expected return for the chosen period.

4. METHODOLOGY OF THE STUDY.

Once the asset selection process has been exposed and their expected daily returns have been calculated for the chosen period, we will proceed to explain how we have carried out the process of creating optimal portfolios under the VaR and ES approaches.

It is important to take in mind again, that the approach we are going to use to calculate VaR is the historical simulation approach (explained above in point 2.2 and denoted HVaR from now on). Having remembered this question, the next thing to know is that the calculation of the expected daily return of a portfolio is calculated as the sum weighted by the relative weight of each share that makes up the portfolio of the expected daily returns of these stocks,

$$E[R_p] = X_1E[R_1] + X_2E[R_2] +\ldots+ X_nE[R_n]$$

In which X_n is the weighting of the asset n, where n ϶ {1,..,N} and $E[R_n]$ is the expected daily return of the asset n.

However, it is also important to say, with respect to the relative weights of those assets in the portfolio, that they must necessarily meet two restrictions: budgetary restriction, that is, all the money is invested and, therefore, the sum of all the relative weights will be equal to 1, and the condition of non-negativity, that is, no weight can be negative.

Once we have exposed these basic assumptions for calculating the expected daily return of a portfolio, we will proceed to explain the process followed to obtain optimal or efficient portfolios using HVaR and ES as risk measures based on the data described in the previous section. As we will see below, obtaining an efficient portfolio comes from solving a complex optimization problem. For the approach and resolution of this optimization problem, the Microsoft Excel spreadsheet and its complementary optimization tool Solver have been used.

As a starting point, the balanced portfolio has been taken, that is, the one in which each of the seven stocks considered (see previous section) has the same relative weight. That is, 100%/7, so the relative weight of these assets in the initial portfolio will be 14’29%.

We specify the relative weights of the shares in a row of the Excel sheet in which we have calculated the daily returns of the seven stocks chosen in accordance with what was stated in the previous section.

Table 3: Balanced porfolio weights

	ACCIONA	INM. COL	ENDESA	FERROVIAL	IBERDROLA	MERLÍN	CELLNEX	
WEIGHTS	14,29%	14,29%	14,29%	14,29%	14,29%	14,29%	14,29%	100%

As we can see, in this way the two restrictions that we exposed previously are fulfilled, that is, all the weights are positive and they all add up to 100%, so all the money is invested.

Following the indications given in the previous paragraphs when calculating the expected daily return of a portfolio, the "SUMAPRODUCTO" function provided by the Excel program can be applied to each row of the spreadsheet that shows the daily return of each share and the row in which the relative weights of the same in the balanced portfolio are specified to obtain in a column of the spreadsheet the daily returns of our balanced portfolio.

Table 4: Daily returns of the balanced portfolio

SUMA ✕ ✓ *fx* =SUMAPRODUCTO(B3:H3;$N$2:$T$2)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
1		DAILY RETURNS 3/1/2018 - 28/2/2020												ACCIONA	INM. COL	ENDESA	FERROVIAL	IBERDROLA	MERLÍN	CELLNEX
2	DATE	ACCIONA	INM. COL	ENDESA	FERROVIAL	IBERDROLA	MERLÍN	CELLNEX			PORTFOLIO		WEIGHTS	14,29%	14,29%	14,29%	14,29%	14,29%	14,29%	14,29%
3	28/2/20	0,047436	-0,023167	-0,032609	-0,0375400	-0,0426807	-0,053480	-0,039554			$T$2)									
4	27/2/20	-0,026234	-0,034184	-0,027939	-0,0390489	-0,0395956	-0,029661	-0,019528			-0,030884									
5	26/2/20	0,010772	-0,001638	0,003653	-0,0042856	0,0498617	0,000790	-0,020628			0,005504									
6	25/2/20	-0,014337	-0,021854	-0,019731	-0,0222036	-0,0250129	-0,021893	0,009517			-0,016502									
7	24/2/20	-0,041818	-0,028415	-0,029853	-0,0376241	-0,0226152	-0,025202	-0,032200			-0,031104									
8	21/2/20	0,004275	0,004680	0,000774	0,0010076	0,0130544	0,000000	0,004950			0,004106									
9	20/2/20	-0,010230	-0,016286	-0,011552	-0,0229208	-0,0166259	-0,003012	-0,010900			-0,013075									
10	19/2/20	0,030136	-0,000769	0,015821	0,0085753	0,0094024	0,012864	0,020666			0,013814									
11	18/2/20	0,027472	0,000769	0,009771	-0,0019854	0,0049606	-0,012864	0,010074			0,005457									
12	17/2/20	0,009025	-0,012992	0,014241	0,0126415	0,0187092	-0,002253	0,000000			0,005625									
13	14/2/20	0,011856	0,006857	0,004792	0,0128034	0,0247156	0,013595	0,027370			0,014570									
14	13/2/20	-0,012762	-0,001528	-0,001600	-0,0020325	0,0152239	-0,011342	-0,001083			-0,002161									
15	12/2/20	0,011845	0,005358	-0,003591	-0,0043897	-0,0071651	0,010582	0,004558			0,002457									
16	11/2/20	0,010134	0,017809	0,004390	0,0180185	0,0119705	0,013772	0,008739			0,012119									
17	10/2/20	0,001854	0,005484	0,000400	0,0065394	0,0101671	0,005408	0,004399			0,004893									
18	7/2/20	0,022515	0,026265	-0,003196	0,0013821	-0,0101671	0,000775	0,002869			0,005778									
19	6/2/20	0,007619	0,012171	0,009217	-0,0065484	0,0174932	-0,000775	-0,007926			0,004464									
20	5/2/20	0,027133	0,000817	0,008490	0,0096653	0,0252173	0,000775	0,002195			0,010613									
21	4/2/20	0,005911	0,013158	-0,005264	0,0076602	0,0044337	0,001552	-0,006572			0,002983									
22	3/2/20	-0,011788	-0,000827	-0,000404	-0,0017461	0,0032369	0,006231	0,019401			0,002015									
23	31/1/20	0,001955	0,001656	-0,001614	-0,0083392	-0,0030349	-0,003899	0,025021			0,001678									
24	30/1/20	0,007859	-0,005783	0,001210	-0,0110118	0,0030349	-0,012374	0,000228			-0,002405									

Using the “PROMEDIO” function that we have already seen, we dedicate a cell of the spreadsheet to obtain the expected daily return of the starting balanced portfolio, which in our case will be 0'0715%.

We dedicate another cell of the spreadsheet to obtain the HVaR for our, for the moment, balanced portfolio. To obtain this HVaR, the first thing that will be necessary to perform is a count of the amount of data in the series of daily returns of our balanced portfolio.

We will do this in a simpler and faster way if we use the "CONTAR" function on the column in which this series of daily returns appears. Using this Excel function, the number of data that we have in the series of daily returns of our balanced portfolio (equivalent to the number of days minus one presented in this study) is 552. Therefore, if the confidence level established for HVaR is 95% (alpha 5%), the next step will be to multiply this amount of data number by the corresponding alpha according to the established confidence level (for a confidence level of 90%, the alpha will be 10%).

For the first case, corresponding to an alpha of 5%, the number resulting from this multiplication (552x5%) is 27'6. This means that the HVaR of our portfolio, following the definition given in section 2.2 of this study, will correspond to the smallest 27'6 return. This is a problem, since in our series of daily returns we can know which return corresponds to the smallest value 27, as well as the smallest 28 through the "K.ESIMO.MENOR" function, but not the 27'6. Faced with this small problem, what we must do is interpolate to obtain the daily return that corresponds to said value and that will correspond to the value of the HVaR for a balanced portfolio. The complete and detailed process of linear interpolation would be the one that we can observe below for the example of the balanced portfolio:

$$\frac{[-0{,}012511-(-0{,}013075)]}{(28-27)} = \frac{[X-(-0{,}013075)]}{(27{,}6-27)} \longrightarrow 0{,}6 * [-0{,}012511-(-0{,}013075)] - 0{,}013075 = X \longrightarrow$$

$$\longrightarrow -0{,}007507 + 0{,}007845 - 0{,}01307 = X \longrightarrow X = -1{,}274\%$$

For a confidence level of 90% and an alpha of 10%, the process would be similar.

Table 5: HVaR calculation

AF5 | =AF3+(AF4-AF3)*0,6

	A	B	C	D	E	F	G	H	I	J	K	L	AD	AE	AF
1		DAILY RETURNS 3/1/2018 - 28/2/2020													
2	DATE	ACCIONA	INM. COL	ENDESA	FERROVIAL	IBERDROLA	MERLÍN	CELLNEX			PORTFOLIO				
3	28/2/20	0,047436	-0,023167	-0,032609	-0,0375400	-0,0426807	-0,053480	-0,039554			-0,025942			Lowest number 27	-0,013075
4	27/2/20	-0,026234	-0,034184	-0,027939	-0,0390489	-0,0395956	-0,029661	-0,019528			-0,030884			Lowest number 28	-0,012511
5	26/2/20	0,010772	-0,001638	0,003653	-0,0042856	0,0498617	0,000790	-0,020628			0,005504			HVaR	-1,274%
6	25/2/20	-0,014337	-0,021854	-0,019731	-0,0222036	-0,0250129	-0,021893	0,009517			-0,016502				
7	24/2/20	-0,041818	-0,028415	-0,029853	-0,0376241	-0,0226152	-0,025202	-0,032200			-0,031104				
8	21/2/20	0,004275	0,004680	0,000774	0,0010076	0,0130544	0,000000	0,004950			0,004106				
9	20/2/20	-0,010230	-0,016286	-0,011552	-0,0229208	-0,0166259	-0,003012	-0,010900			-0,013075				
10	19/2/20	0,030136	-0,000769	0,015821	0,0085753	0,0094024	0,012864	0,020666			0,013814				
11	18/2/20	0,027472	0,000769	0,009771	-0,0019854	0,0049606	-0,012864	0,010074			0,005457				
12	17/2/20	0,009025	-0,012992	0,014241	0,0126415	0,0187092	-0,002253	0,000000			0,005625				
13	14/2/20	0,011856	0,006857	0,004792	0,0128034	0,0247156	0,013595	0,027370			0,014570				
14	13/2/20	-0,012762	-0,001528	-0,001600	-0,0020325	0,0152239	-0,011342	-0,001083			-0,002161				
15	12/2/20	0,011845	0,005358	-0,003591	-0,0043897	-0,0071651	0,010582	0,004558			0,002457				
16	11/2/20	0,010134	0,017809	0,004390	0,0180185	0,0119705	0,013772	0,008739			0,012119				
17	10/2/20	0,001854	0,005484	0,000400	0,0065394	0,0101671	0,005408	0,004399			0,004893				
18	7/2/20	0,022515	0,026265	-0,003196	0,0013821	-0,0101671	0,000775	0,002869			0,005778				
19	6/2/20	0,007619	0,012171	0,009217	-0,0065484	0,0174932	-0,000775	-0,007926			0,004464				
20	5/2/20	0,027133	0,000817	0,008490	0,0096653	0,0252173	0,000775	0,002195			0,010613				

We also dedicate a cell of the spreadsheet to obtain the ES of the balanced portfolio. The process for obtaining the ES is much simpler. In the case of a 95% confidence level, following the definition of this risk measure in section 2.2 of the work, we simply have to apply the formula "K.ESIMO.MENOR" on the column in which the series of daily returns of the balanced portfolio to obtain the 28 smallest returns (for simplicity of calculation) and calculate their average. For the case of a confidence level of 90%, 552x10% = 55'2 and, therefore, the ES at this confidence level of the balanced portfolio will be, for simplicity of calculation, the average of the 55 returns plus little. Obviously, for the same confidence level, the value of HVaR must be greater (closer to 0) than the value of ES.

Table 6: ES calculation

AF3 =PROMEDIO(K.ESIMO.MENOR(K3:K554;{1;2;3;4;5;6;7;8;9;10;11;12;13;14;15;16;17;18;19;20;21;22;23;24;25;26;27;28}))

	A	B	C	D	E	F	G	H	I	J	K	L	AD	AE	AF
1		DAILY RETURNS 3/1/2018 - 28/2/2020													
2	DATE	ACCIONA	INM. COL	ENDESA	FERROVIAL	IBERDROLA	MERLÍN	CELLNEX			PORTFOLIO				
3	28/2/20	0,047436	-0,023167	-0,032609	-0,0375400	-0,0426807	-0,053480	-0,039554			-0,025942			ES	-1,962%
4	27/2/20	-0,026234	-0,034184	-0,027939	-0,0390489	-0,0395956	-0,029661	-0,019528			-0,030884				
5	26/2/20	0,010772	-0,001638	0,003653	-0,0042856	0,0498617	0,000790	-0,020628			0,005504				
6	25/2/20	-0,014337	-0,021854	-0,019731	-0,0222036	-0,0250129	-0,021893	0,009517			-0,016502				
7	24/2/20	-0,041818	-0,028415	-0,029853	-0,0376241	-0,0226152	-0,025202	-0,032200			-0,031104				
8	21/2/20	0,004275	0,004680	0,000774	0,0010076	0,0130544	0,000000	0,004950			0,004106				
9	20/2/20	-0,010230	-0,016286	-0,011552	-0,0229208	-0,0166259	-0,003012	-0,010900			-0,013075				
10	19/2/20	0,030136	-0,000769	0,015821	0,0085753	0,0094024	0,012864	0,020666			0,013814				
11	18/2/20	0,027472	0,000769	0,009771	-0,0019854	0,0049606	-0,012864	0,010074			0,005457				
12	17/2/20	0,009025	-0,012992	0,014241	0,0126415	0,0187092	-0,002253	0,000000			0,005625				
13	14/2/20	0,011856	0,006857	0,004792	0,0128034	0,0247156	0,013595	0,027370			0,014570				
14	13/2/20	-0,012762	-0,001528	-0,001600	-0,0020325	0,0152239	-0,011342	-0,001083			-0,002161				
15	12/2/20	0,011845	0,005358	-0,003591	-0,0043897	-0,0071651	0,010582	0,004558			0,002457				
16	11/2/20	0,010134	0,017809	0,004390	0,0180185	0,0119705	0,013772	0,008739			0,012119				
17	10/2/20	0,001854	0,005484	0,000400	0,0065394	0,0101671	0,005408	0,004399			0,004893				
18	7/2/20	0,022515	0,026265	-0,003196	0,0013821	-0,0101671	0,000775	0,002869			0,005778				
19	6/2/20	0,007619	0,012171	0,009217	-0,0065484	0,0174932	-0,000775	-0,007926			0,004464				
20	5/2/20	0,027133	0,000817	0,008490	0,0096653	0,0252173	0,000775	0,002195			0,010613				

Once the template has been created in the spreadsheet using the balanced portfolio as a starting point, dedicating one cell to the expected daily return of the portfolio and two cells to obtaining, respectively, the HVaR and the ES of this, you can consider with Solver the optimization problem that will allow us to obtain efficient portfolios using HVaR and ES as risk measures. Specifically, we are going to create efficient portfolios using these two risk measures at the two confidence levels discussed above, 95% and 90%.

The approach followed in the optimization problem to obtain these efficient portfolios is to find the relative weights of the 7 assets in the portfolio that minimize the risk of this measured through their HVaR or ES (given the definition of these risk measures, minimizing risk is equivalent to maximizing them) for a certain level of confidence (95% or 90%), setting an expected return on the portfolio. This is the approach that we define in Solver as reflected in the following figure.

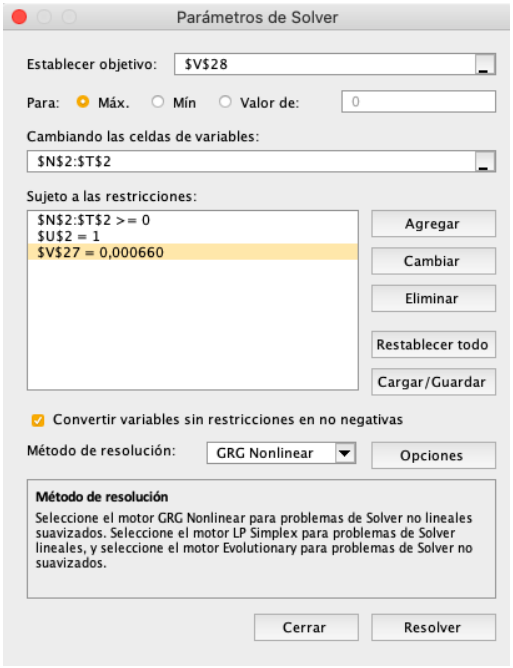

Figure 4: Introduction of Solver parameters

It is about maximizing the HVaR or ES of the portfolio for a certain level of confidence ("Set objective" section of Solver), changing the variable cells corresponding to the relative weights of the assets that make up the portfolio, setting a certain expected return of the same. In addition, it must be remembered that the following restrictions mentioned above must also be added:

- All the relative weights must add up to 1, that is, all the money is invested.
- These weights must necessarily be equal to or greater than 0.

Through solving this optimization problem through Solver, 18 efficient portfolios have been obtained for both the HVaR and the ES as a measure of risk, both at the two confidence levels mentioned above (95% and 90%).

In order for the efficient portfolios obtained with each risk measure to be as comparable as possible to each other, an attempt has been made to use the same expected returns from the portfolio in their calculation. On some occasion it has not been possible to respect this principle because for a given expected return the optimization problem gave an inconsistent result, since the result defined a portfolio with a lower risk than another with a lower expected return. Despite this computational complication, the range of expected returns considered has been respected as much as possible.

On the other hand, this range of expected returns has been defined in a coherent way with the range of expected returns considered in the works of San Félix (2019) and Camáñez (2019), whose are based on obtaining efficient portfolios using, respectively, the ES and HVaR as measures of risk and comparing them with efficient portfolios according to the Markowitz approach. The reason for proceeding in this way is that one of the objectives of the present work is to compare our results with those obtained by these two previous studies.

Finally, it should be noted that, to obtain each efficient portfolio, the optimization problem explained above has been solved through Solver starting from the balanced portfolio, considering that the results obtained through Solver were more plausible proceeding in this way.

The following tables summarize the expected returns used to obtain efficient portfolios with each risk measure (HVaR and ES for both a 95% and 90% confidence level), specifying the risk of each of them according to these measures.

Table 7: Expected return and HVaR (95%)

	EXP. RETURN	VaR (95%)
P1	0,0660%	-1,1849%
P2	0,0680%	-1,2028%
P3	0,0694%	-1,2101%
P4	0,0719%	-1,2202%
P5	0,0742%	-1,2308%
P6	0,0761%	-1,2420%
P7	0,0777%	-1,2455%
P8	0,0793%	-1,2457%
P9	0,0842%	-1,2552%
P10	0,0852%	-1,2639%
P11	0,0890%	-1,2846%
P12	0,0891%	-1,2913%
P13	0,0902%	-1,3057%
P14	0,0916%	-1,3186%
P15	0,0940%	-1,3363%
P16	0,0983%	-1,3574%
P17	0,1050%	-1,4916%
P18	0,1151%	-1,5598%

As we can see, as expected return increases, the value of HVaR decreases, that is to say that the higher the expected return, the greater the expected loss (the risk increases) that the position of a portfolio may have under normal market conditions and as long as there is no negotiation in the portfolio.

Table 8: Expected return and ES (95%)

	EXP. RETURN	ES (95%)
P1	0,0660%	-1,9207%
P2	0,0680%	-1,9213%
P3	0,0694%	-1,9222%
P4	0,0720%	-1,9256%
P5	0,0742%	-1,9287%
P6	0,0761%	-1,9311%
P7	0,0777%	-1,9337%
P8	0,0793%	-1,9357%
P9	0,0842%	-1,9476%
P10	0,0852%	-1,9514%
P11	0,0890%	-1,9617%
P12	0,0891%	-1,9619%
P13	0,0902%	-1,9669%
P14	0,0916%	-1,9734%
P15	0,0940%	-1,9855%
P16	0,0983%	-2,0152%
P17	0,1050%	-2,0725%
P18	0,1152%	-2,1754%

Logically, it is consistent to state that, for the same expected return, the value of ES is lower (more negative) than the value of HVaR. As we have previously exposed, the ES is the average of the left tail of the distribution, while the VaR is the value closest to 0 of this tail itself and, therefore, it will necessarily always be closer to 0. The following two tables show the efficient portfolios obtained using the HVaR and the ES for a confidence level of 90% as risk measures.

Table 9: Expected return and HVaR (90%)

	EXP. RETURN	VaR (90%)
P1	0,0645%	-0,7996%
P2	0,0663%	-0,8214%
P3	0,0678%	-0,8329%
P4	0,0696%	-0,8395%
P5	0,0740%	-0,8510%
P6	0,0741%	-0,8514%
P7	0,0760%	-0,8679%
P8	0,0812%	-0,8858%
P9	0,0820%	-0,8909%
P10	0,0852%	-0,9429%
P11	0,0886%	-0,9677%
P12	0,0891%	-0,9725%
P13	0,0916%	-0,9836%
P14	0,0940%	-1,0024%
P15	0,0975%	-1,0162%
P16	0,1016%	-1,0180%
P17	0,1088%	-1,1134%
P18	0,1151%	-1,1735%

Table 10: Expected return and ES (90%)

	EXP. RETURN	ES (90%)
P1	0,0644%	-1,4950%
P2	0,0663%	-1,4972%
P3	0,0678%	-1,4986%
P4	0,0696%	-1,5003%
P5	0,0740%	-1,5088%
P6	0,0741%	-1,5091%
P7	0,0760%	-1,5152%
P8	0,0812%	-1,5322%
P9	0,0820%	-1,5345%
P10	0,0852%	-1,5456%
P11	0,0886%	-1,5586%
P12	0,0891%	-1,5607%
P13	0,0916%	-1,5719%
P14	0,0940%	-1,5839%
P15	0,0976%	-1,6013%
P16	0,1016%	-1,6273%
P17	0,1088%	-1,6890%
P18	0,1152%	-1,7680%

Before ending this section, it should be noted that efficient portfolios have also been obtained according to the Markowitz approach in order to be able to compare them with the previous ones. In this sense, these efficient portfolios according to the volatility of the returns as a measure of risk have been obtained, as far as possible, for the same expected returns as the previous portfolios and respecting the range of these returns.

To obtain each portfolio using the Markowitz approach (always starting from the balanced portfolio), we dedicate a cell of the spreadsheet to the standard deviation of the daily returns of the same. In other words, we apply the “DESVEST.P” function on the column in which the series of daily returns of the balanced portfolio appears. Once this calculation is made, we use Solver in a similar way to how we have explained before, but this time the objective is to minimize the standard deviation of the portfolio's returns by setting the expected return on the portfolio. The following table summarizes the 18 expected returns used and the standard deviation of the returns of the corresponding efficient portfolio according to the Markowitz approach.

Table 11: Expected return and standard deviation

	EXP. RETURN	STANDARD. DEV
P1	0,0660%	0,7957%
P2	0,0680%	0,7983%
P3	0,0694%	0,8004%
P4	0,0720%	0,8047%
P5	0,0742%	0,8090%
P6	0,0761%	0,8133%
P7	0,0777%	0,8169%
P8	0,0793%	0,8210%
P9	0,0842%	0,8349%
P10	0,0852%	0,8380%
P11	0,0890%	0,8511%
P12	0,0891%	0,8513%
P13	0,0902%	0,8552%
P14	0,0916%	0,8608%
P15	0,0940%	0,8713%
P16	0,0983%	0,8934%
P17	0,1050%	0,9370%
P18	0,1151%	1,0273%

5. HISTORICAL VaR vs EXPECTED SHORTFALL 95%.

To perform the comparison and subsequent explanation of the evolution of the weightings of the different actions individually, we proceed to attach two summary tables. The first one represents the weights according to HVaR for the different assets according to their expected return and, the second one, exactly the same but taking ES as a risk measure.

Table 12: Weights in HVaR (95%)

		HVaR 95%						
	EXP. RETURN	ACCIONA	INM. COL	ENDESA	FERROVIAL	IBERDROLA	MERLÍN	CELLNEX
Portfolio 1	0,0660%	4,83%	13,12%	14,62%	9,87%	16,04%	24,47%	17,05%
P2	0,0680%	8,30%	14,98%	19,78%	15,67%	14,29%	13,90%	13,09%
P3	0,0694%	8,78%	13,40%	19,24%	15,87%	13,90%	14,11%	14,69%
P4	0,0719%	15,95%	23,27%	28,00%	2,17%	10,65%	7,73%	12,23%
P5	0,0742%	21,18%	15,95%	16,68%	4,94%	18,96%	11,10%	11,19%
P6	0,0761%	16,56%	13,11%	32,69%	4,24%	2,30%	9,92%	21,18%
P7	0,0777%	23,33%	14,00%	17,68%	8,81%	8,81%	10,29%	17,08%
P8	0,0793%	25,70%	16,15%	17,91%	7,36%	8,39%	7,88%	16,62%
P9	0,0842%	30,02%	17,29%	18,11%	7,30%	12,90%	0,00%	14,38%
P10	0,0852%	27,36%	15,15%	17,99%	8,63%	4,14%	4,78%	21,94%
P11	0,0890%	32,43%	20,41%	13,03%	3,88%	8,93%	1,35%	19,97%
P12	0,0891%	33,97%	21,22%	12,37%	4,70%	9,22%	0,00%	18,52%
P13	0,0902%	34,13%	16,59%	8,44%	4,60%	17,33%	1,31%	17,61%
P14	0,0916%	33,69%	15,90%	6,19%	7,99%	13,80%	1,86%	20,57%
P15	0,0940%	34,45%	18,32%	0,31%	9,76%	16,41%	0,20%	20,54%
P16	0,0983%	34,51%	20,71%	3,69%	0,75%	12,51%	1,02%	26,82%
P17	0,1050%	0,00%	6,82%	8,67%	9,91%	22,76%	4,66%	47,18%
P18	0,1151%	31,33%	4,87%	1,82%	2,09%	14,62%	0,02%	45,25%

Table 13: Weights in ES (95%)

		EXPECTED SHORTFALL 95%						
	EXP. RETURN	ACCIONA	INM. COL	ENDESA	FERROVIAL	IBERDROLA	MERLÍN	CELLNEX
Portfolio 1	0,0660%	6,39%	20,71%	7,84%	14,96%	21,70%	18,07%	10,33%
P2	0,0680%	5,80%	21,34%	8,30%	16,11%	22,36%	15,18%	10,92%
P3	0,0694%	5,33%	21,23%	8,04%	15,96%	22,63%	14,60%	12,20%
P4	0,0719%	5,06%	20,23%	7,50%	16,11%	22,82%	13,80%	14,48%
P5	0,0742%	6,24%	21,03%	5,56%	15,21%	23,68%	12,88%	15,40%
P6	0,0761%	8,45%	22,97%	2,75%	14,88%	23,64%	11,67%	15,64%
P7	0,0777%	7,41%	22,28%	2,21%	15,68%	24,85%	10,77%	16,80%
P8	0,0793%	8,46%	22,84%	1,51%	14,22%	24,79%	10,37%	17,82%
P9	0,0842%	7,61%	25,29%	2,06%	8,39%	27,83%	7,94%	20,88%
P10	0,0852%	7,92%	26,28%	2,44%	13,82%	26,75%	3,16%	19,64%
P11	0,0890%	6,80%	27,11%	3,20%	11,64%	29,76%	0,00%	21,49%
P12	0,0891%	6,79%	27,09%	3,18%	11,63%	29,77%	0,00%	21,55%
P13	0,0902%	6,82%	26,73%	2,56%	11,47%	29,81%	0,00%	22,61%
P14	0,0916%	7,27%	26,39%	1,22%	11,47%	29,84%	0,00%	23,81%
P15	0,0940%	7,18%	25,52%	0,00%	10,97%	30,04%	0,00%	26,29%
P16	0,0983%	8,48%	24,91%	0,00%	6,25%	30,02%	0,00%	30,34%
P17	0,1050%	9,51%	15,58%	0,00%	8,97%	27,68%	0,00%	38,26%
P18	0,1151%	15,25%	11,22%	0,00%	0,00%	26,10%	0,00%	47,43%

5.1 INDIVIDUALIZED ANALYSIS.

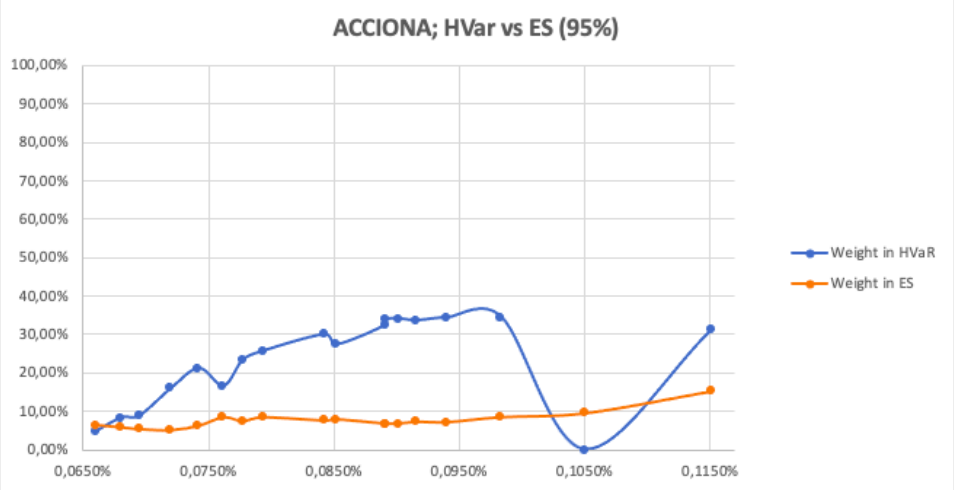

Figure 5: HVaR vs ES Acciona (95%)

In the chart above, which belongs to the company “Acciona”, we can appreciate that there is a growing trend of both weightings. That is to say, as expected return increases, the weighting of the portfolio in this specific asset also increases for both approaches, being in this case in particular, the HVaR is more increasing. However, an anomaly can be appreciated with certain clarity in the weighting corresponding to portfolio 17 for the HVaR approach, whose weight goes from being 34’51% for an expected return of 0’0983% to a weight of 0% for an expected return of 0’1050%, increasing this weight again to 31’33% for an expected return in the last portfolio of 0’1151%. In the last place, it is important to note that, except in portfolios 1 and 17, corresponding to an expected return of 0’0660% and 0’1050% respectively, the weight in HVaR is always higher than the weight in ES.

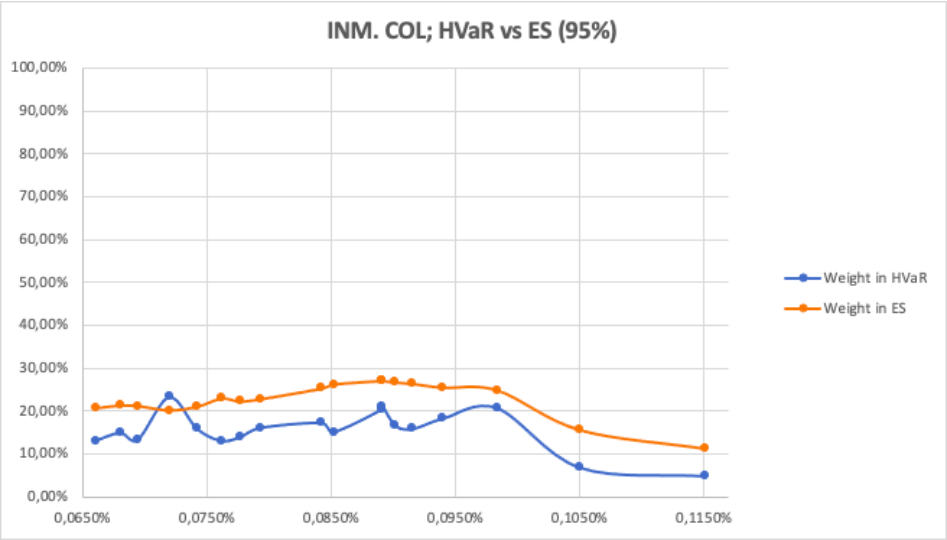

Figure 6: HVaR vs ES Inmobiliaria Colonial (95%)

Regarding the assets which correspond to the company “Inmobiliaria Colonial”, a generally decreasing trend can be observed in both weightings, although there are sections of particular increase. There is also a settled anomaly in portfolio 2 regarding to an expected return of 0'0680%, although much less bulky than in the previous case. In this case and except in the portfolio that we have mentioned above, the weight in ES is always higher than the weight in HVaR.

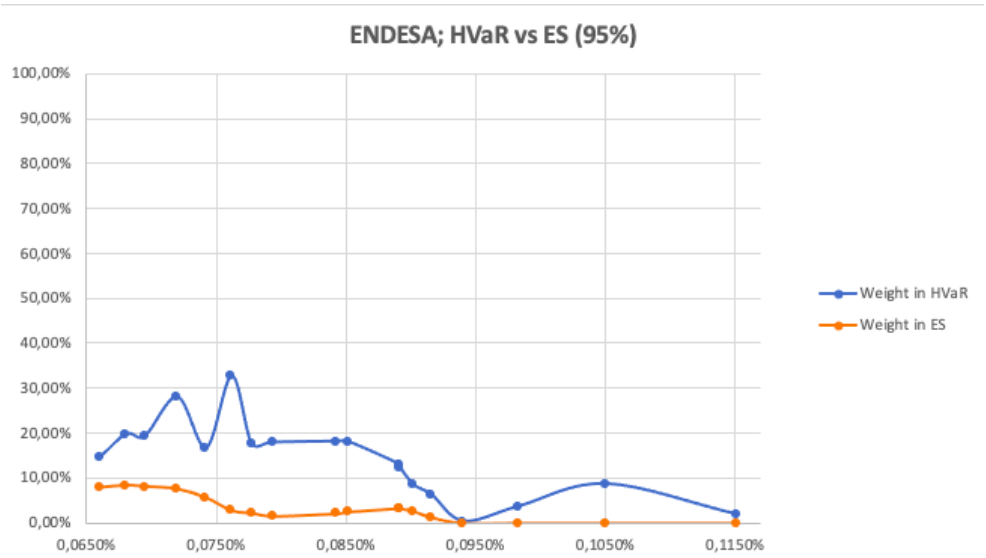

Figure 7: HVaR vs ES Endesa (95%)

In the company “Endesa” there is a clear downtrend in the percentage in the weightings of both HVaR and ES. This appreciation is sharper in the weightings of ES as we can observe in the first portfolios, the weightings according to HVaR show constant increases and decreases in weighting. As was occurring in the case of “Acciona”, the HVaR weights are higher than the weights in ES except in portfolio 15, corresponding to an expected return of 0’0940%, where both weights are practically the same. For the first time, it can be seen that, as profitability increases, both weights are getting closer.

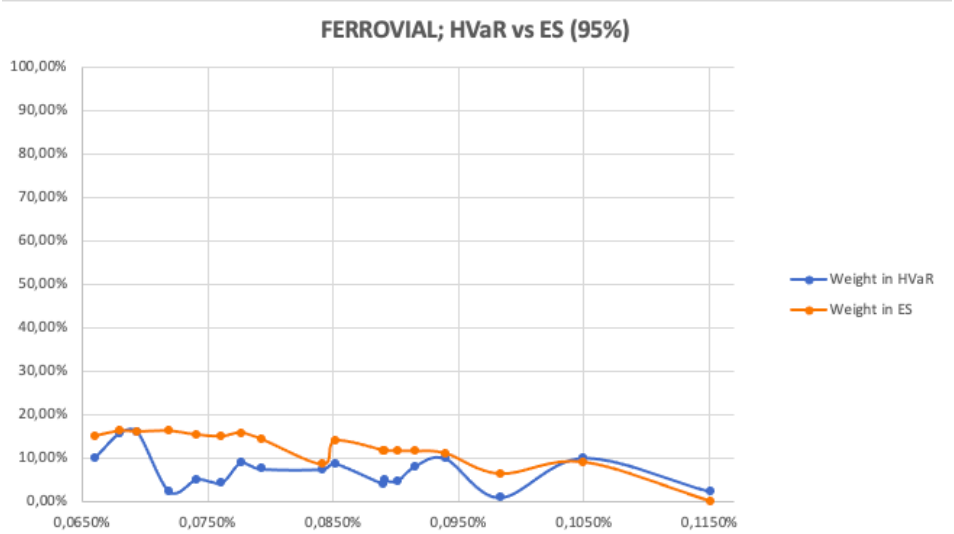

Figure 8: HVaR vs ES Ferrovial (95%)

In this case, corresponding to the company “Ferrovial”, we cannot appreciate any sort of defined pattern for the weight in HVaR, since there are constant increases and decreases in the weightings. Regarding the ES, a clear decrease in their weights can be seen as expected return increases. In most portfolios, it is perceived that the weights according to both methods are very close.

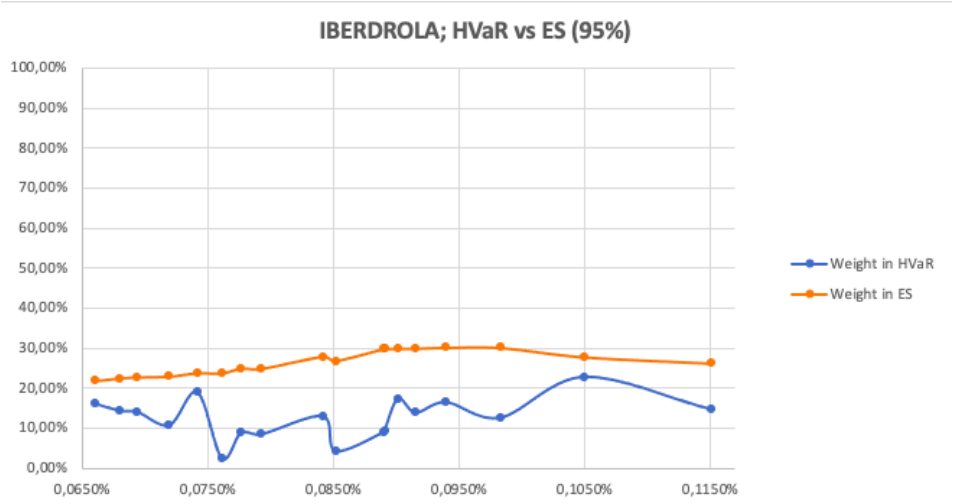

Figure 9: HVaR vs ES Iberdrola (95%)

For the enterprise “Iberdrola”, the weight in ES follows a slightly upward trend. On the other hand, we could classify the trend in HVaR as indeterminate. Furthermore, as in some of the cases seen previously, the weights in ES are always greater than the weights in HVaR. Finally, we can affirm that the weights neither approach nor move away as expected return increases.

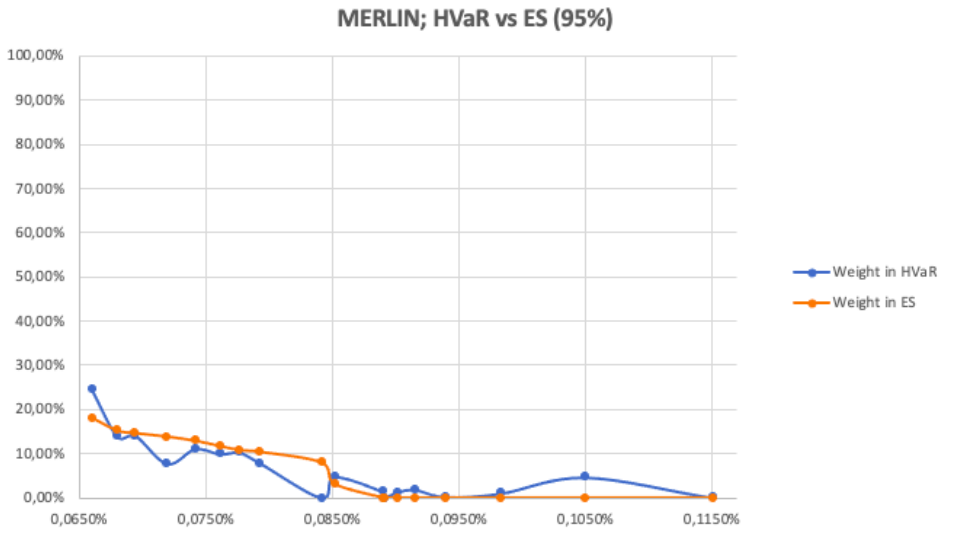

Figure 10: HVaR vs ES Merlin (95%)

This chart belonging to “Merlin Properties” shows a clear downward trend of both weights for the different approaches. We have to highlight that the weights in ES from portfolio number 11, or in other words, the weights for an expected return equal to or greater than 0'0890% are practically 0%. Moreover, in this case, the weights are always similar for the entire range of returns.

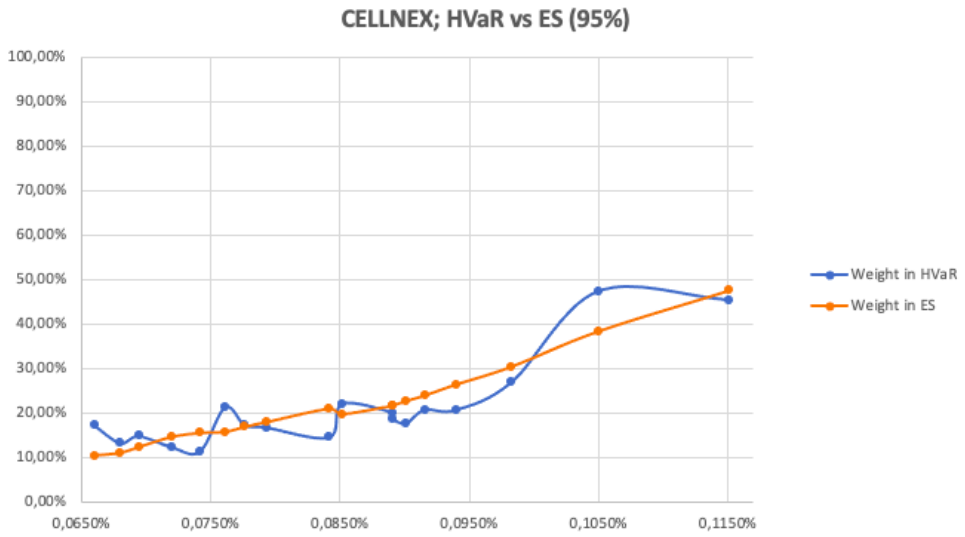

Figure 11: HVaR vs ES Cellnex (95%)

To conclude with, in the company “Cellnex”, we can observe a clear increase in both weights as expected return increases. The weights are quite similar for each and every one of the returns. In this case, it is important to take note that as expected return increases, sometimes the weight is higher in HVaR than in ES and in some other portfolios, it is just the opposite.

Once the seven companies that are part of our study have been analyzed, we can draw the conclusion that the weightings obtained for the optimal portfolios using the HVaR approach as a risk measure, show in general a more erratic behavior than if it is considered the ES approach as a measure of risk.

This general conclusion is established if we compare the weightings according to HVaR and ES with those that we would obtain for the same levels of expected return, considering the variance of yields as a measure of risk. In another words, applying the Markowitz approximation.

5.2 COMPARISION WITH MARKOWITZ.

The charts that we can find in the appendix of this study show that ES is always very close and follows the Markowitz trend. On the other hand, the weights according to HVaR are in general very different from those of Markowitz, and as we established previously, they show a quite erratic behavior.

The similarity of the weightings according to ES to the weightings according to Markowitz, has already been obtained in previous study, San Félix (2019). In the same way, the erratic behavior of the weightings according to HVaR have also been obtained in previous study, Camáñez (2019).

In this previous work, the weightings according to Markowitz for the asset corresponding to "Inmobiliaria Colonial" follow a similar trend to those obtained in the current project for the company "Merlin Properties". Following this direction, it is interesting to compare the weightings according to the HVaR (95%) in these two cases.

Source: Camáñez (2019)

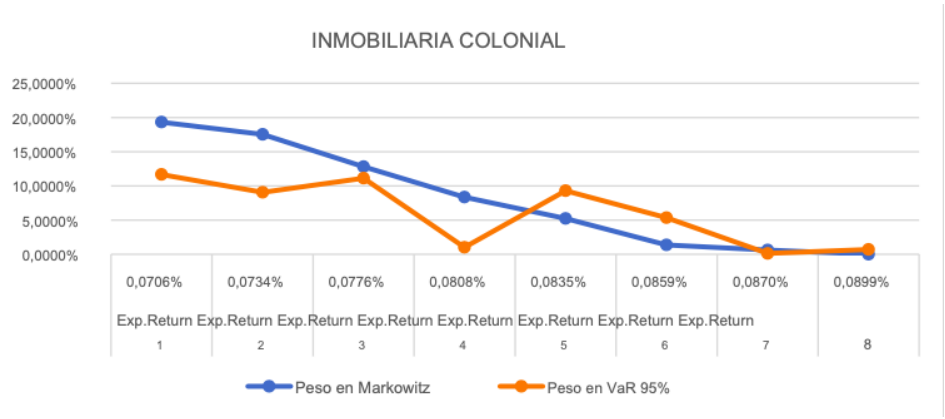

Figure 12: Markowitz vs HVaR Inmobiliaria Colonial (95%)

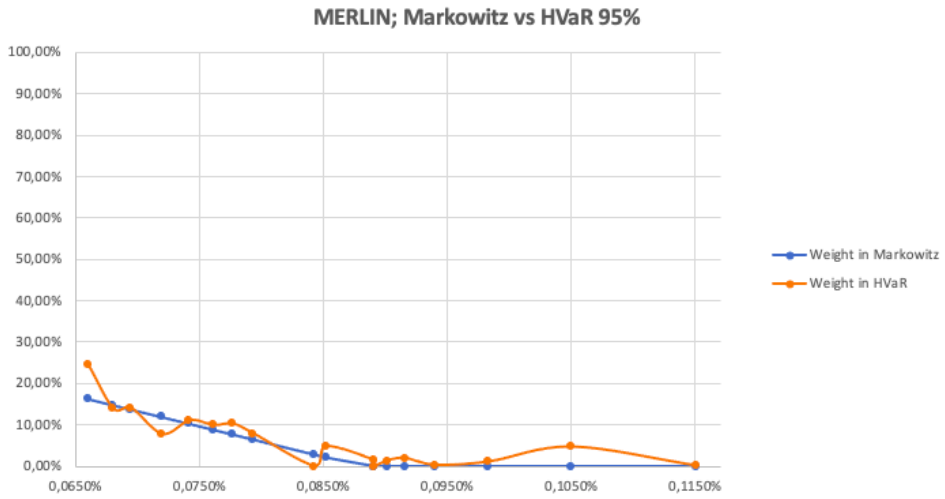

Figure 13: Markowitz vs HVaR Merlin (95%)

This comparison leads us to think that the fact that the weightings according to Markowitz are similar in both cases does not ensure that the same can happen for the weightings according to the HVaR (95%).

6. HISTORICAL VaR vs EXPECTED SHORTFALL 90%.

As in the previous section, we proceed to attach the following weighting tables according to the HVaR or ES approach and the evolution of these for the different expected return levels that can be observed in their corresponding column.

Table 14: Weights in HVaR (90%)

		HVaR 90%						
	EXP. RETURN	ACCIONA	INM. COL	ENDESA	FERROVIAL	IBERDROLA	MERLÍN	CELLNEX
P1	0,0645%	10,68%	22,92%	10,97%	8,00%	18,04%	20,04%	9,36%
P2	0,0663%	8,58%	17,88%	14,24%	14,81%	12,76%	18,29%	13,44%
P3	0,0678%	8,84%	17,58%	14,19%	13,31%	13,14%	18,22%	14,72%
P4	0,0696%	3,69%	15,73%	14,10%	17,91%	16,13%	15,87%	16,57%
P5	0,0740%	9,91%	18,15%	14,43%	18,13%	16,63%	8,30%	14,45%
P6	0,0741%	9,89%	17,63%	14,97%	18,16%	16,37%	8,26%	14,72%
P7	0,0760%	5,70%	14,16%	13,93%	18,34%	18,73%	10,31%	18,82%
P8	0,0812%	10,97%	14,90%	12,50%	14,14%	14,79%	9,68%	23,02%
P9	0,0820%	12,56%	16,57%	11,36%	11,85%	14,32%	10,04%	23,30%
P10	0,0852%	27,60%	10,14%	11,30%	10,68%	14,87%	6,39%	19,03%
P11	0,0886%	33,01%	9,52%	9,30%	9,63%	16,35%	3,73%	18,46%
P12	0,0891%	32,48%	10,48%	9,27%	9,95%	13,56%	4,04%	20,21%
P13	0,0916%	32,63%	10,61%	10,04%	5,90%	16,24%	3,13%	21,45%
P14	0,0940%	37,40%	11,99%	6,07%	5,91%	15,34%	1,90%	21,40%
P15	0,0975%	37,03%	11,06%	4,06%	8,09%	13,97%	0,79%	25,00%
P16	0,1016%	29,04%	36,25%	0,00%	0,00%	0,00%	0,00%	34,71%
P17	0,1088%	15,97%	6,53%	4,62%	5,27%	16,26%	4,67%	46,68%
P18	0,1151%	39,71%	0,00%	0,00%	0,00%	19,49%	0,19%	40,61%

Table 15: Weights in ES (90%)

		EXPECTED SHORTFALL 90%						
	EXP. RETURN	ACCIONA	INM. COL	ENDESA	FERROVIAL	IBERDROLA	MERLÍN	CELLNEX
P1	0,0645%	6,05%	26,50%	10,20%	16,21%	19,73%	14,13%	7,19%
P2	0,0663%	7,84%	26,74%	12,90%	18,34%	17,22%	9,76%	7,20%
P3	0,0678%	7,15%	29,36%	11,58%	15,60%	18,26%	9,68%	8,37%
P4	0,0696%	8,03%	27,12%	9,42%	16,18%	20,87%	9,44%	8,94%
P5	0,0740%	10,26%	26,46%	7,09%	14,15%	21,79%	8,53%	11,72%
P6	0,0741%	9,63%	27,94%	7,39%	13,06%	21,69%	8,33%	11,96%
P7	0,0760%	11,33%	25,71%	7,29%	13,32%	22,06%	7,45%	12,85%
P8	0,0812%	14,51%	27,95%	3,76%	10,13%	21,22%	6,15%	16,28%
P9	0,0820%	14,89%	26,59%	3,83%	9,83%	21,82%	6,23%	16,81%
P10	0,0852%	17,06%	28,22%	1,34%	8,75%	22,25%	4,43%	17,95%
P11	0,0886%	17,99%	28,80%	0,98%	7,73%	24,48%	1,30%	18,71%
P12	0,0891%	18,44%	28,40%	0,01%	8,01%	24,94%	1,33%	18,88%
P13	0,0916%	19,35%	27,22%	0,00%	6,28%	27,47%	0,00%	19,67%
P14	0,0940%	22,01%	24,41%	0,85%	5,23%	25,44%	0,00%	22,06%
P15	0,0975%	22,80%	22,06%	0,00%	3,54%	26,31%	0,00%	25,30%
P16	0,1016%	21,25%	19,33%	0,00%	2,60%	26,69%	0,00%	30,11%
P17	0,1088%	18,33%	15,31%	0,00%	0,00%	27,61%	0,00%	38,75%
P18	0,1151%	17,51%	7,29%	0,00%	0,00%	29,55%	0,00%	45,65%

6.1 INDIVIDUALIZED ANALYSIS.

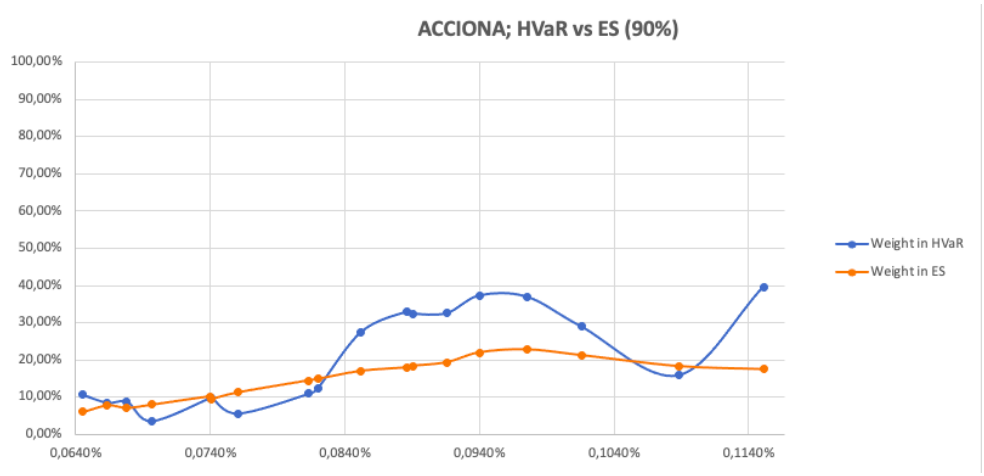

Figure 14: HVaR vs ES Acciona (90%)

Regarding to the Spanish entity which is dedicated to the promotion and management of infrastructures and renewable energies, we can appreciate a positive slope in both weights for the new range of expected returns. Another time, and as it seems in most cases, the weightings in ES are more stable, which means that they have a more horizontal slope, while in HVaR, we can observe that there is a trend as expected return increase quite indefinitely (although there is a growing trend). For an expected return of 0'0663% (portfolio 2), 0'0678% (portfolio 3), 0'0740% (portfolio 5), 0'0741% (portfolio 6), 0'0760% (portfolio 7) and 0'1088% (portfolio 17), the weights are practically the same according to one approach or another, and for the rest of the returns, there is a large difference in weightings depending on whether HVaR or ES is used as a risk measure.

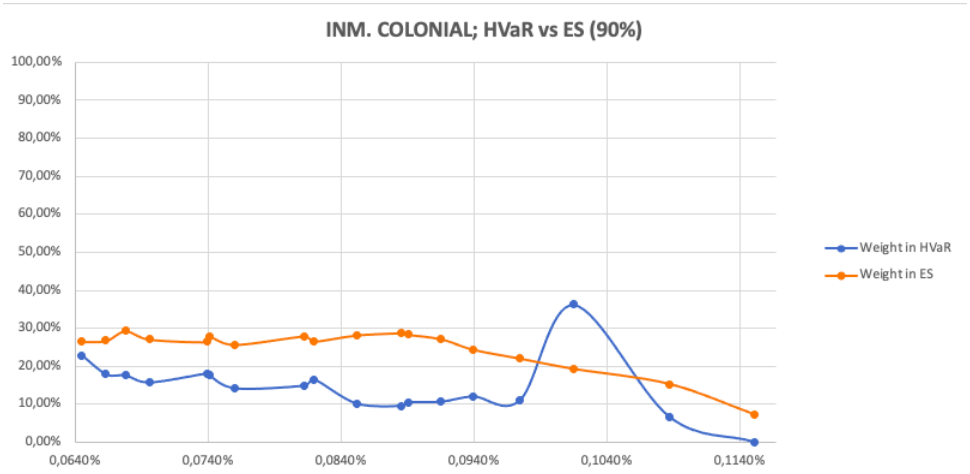

Figure 15: HVaR vs ES Inmobiliaria Colonial (90%)

In the case of a multinational enterprise like “Inmobiliaria Colonial” which is dedicated to the real estate sector we can establish that there is a clear downward trend for both weights as expected return increases. It seems that both weightings are constant downwards, that is to say that there are no fluctuations that make us think that there is some erratic behavior on the part of either of these two approaches. However, we can point out a clear anomaly in the weight according to HVaR for an expected return of 0’1016% (portfolio 16), which is the only portfolio in which the weighting for this company in HVaR exceeds that of ES for each return. For the rest of the returns, the weight in ES is always higher than the weighting through the HVaR approach.

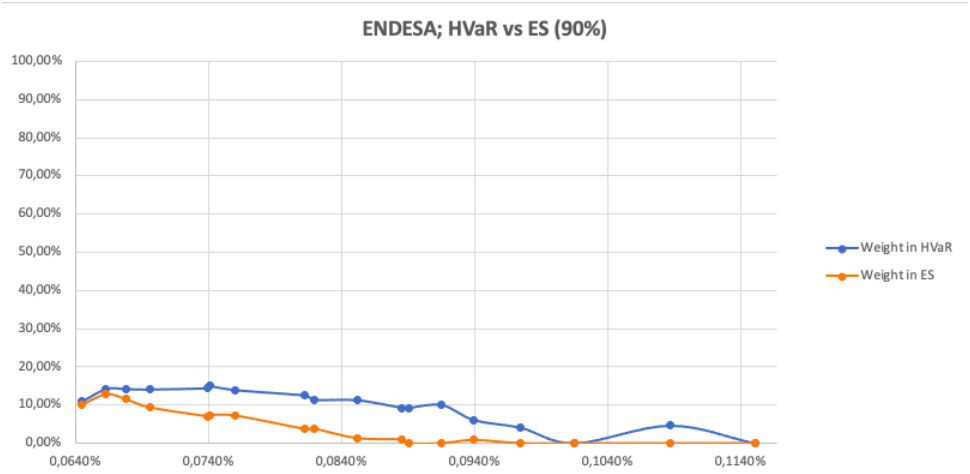

Figure 16: HVaR vs ES Endesa (90%)

The chart that belongs to the company “Endesa” shows that as expected return grows, the weights for HVaR and ES fall. In addition, they do it in a very similar way, this is to say that the weightings for the range of returns have a fairly similar evolution. Eventhough, the weights in HVaR are always higher than the weights in ES. It is important to note down that in ES and for a return equal to or greater than 0'0886%, the weights are practically or very close to 0%.

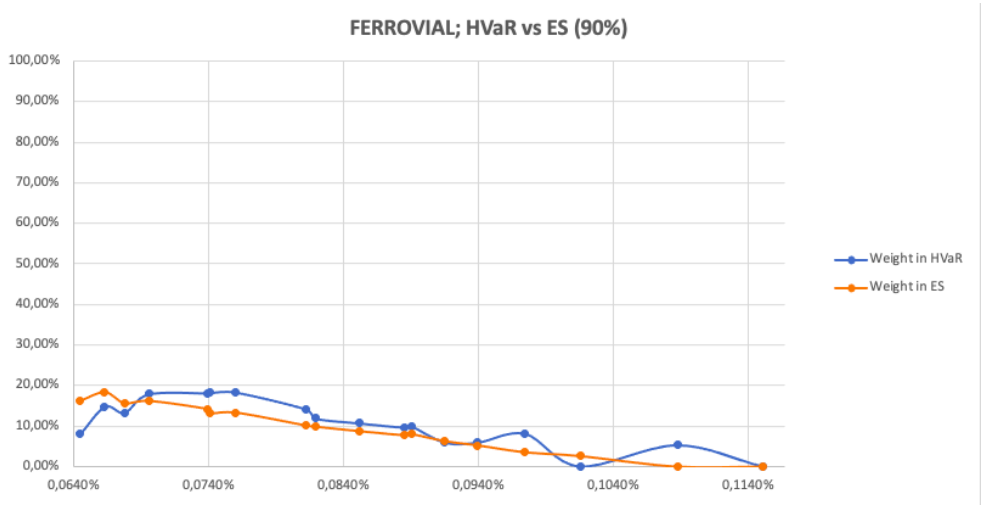

Figure 17: HVaR vs ES Ferrovial (90%)

In this new occasion we can appreciate that the behavior of the HVaR (90%) behaves more irregularly than the ES (90%). Both of them have a negative slope. However, ES (90%) is more constant in the decrease of the weights in “Ferrovial” as expected return increases (it does not show any anomalies). HVaR (90%) shows different increases in their weightings in the first portfolios, and subsequently, a constant drop is observed in their weights, even in the portfolios with the highest expected return, small increases and decreases in their weights are observed. In the portfolio with the highest yield, the weight of this asset is practically 0%, both in HVaR and in ES at 90%.

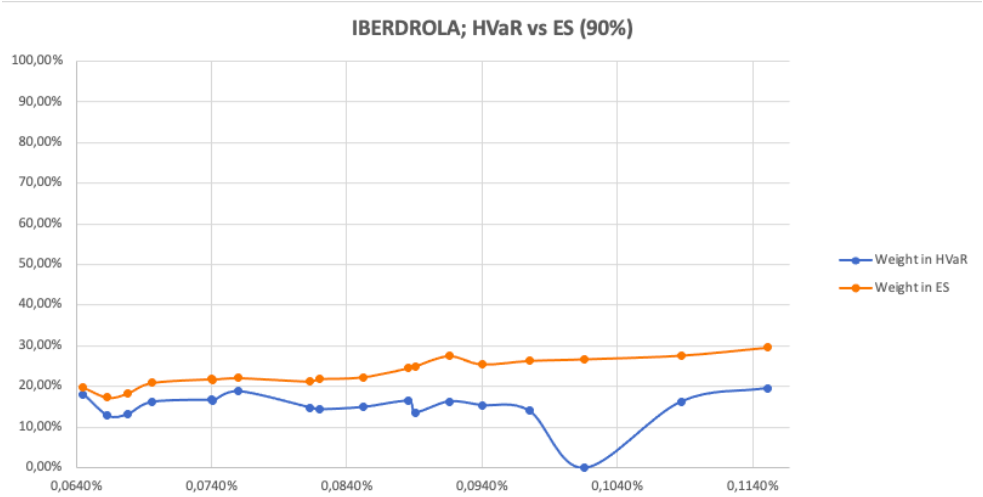

Figure 18: HVaR vs ES Iberdrola (90%)

Unusually, in the previous chart which belongs to “Iberdrola”, it can be clearly observed that as the level of expected return increases, the weightings according to both risk measures are more distant. There is a clear uptrend for the ES approach, while it is perhaps more difficult to tell if we use HVaR weightings. At the end term, it is worth highlighting an erratic behavior according to HVaR for an expected return of 0’1016%, where the weight for this is 0%, being 13’97% in the previous portfolio and 16’226% in the following portfolio. In this case, the weighting if we use a risk evaluation method based on ES, it is always higher than if we use the HVaR method for this range of returns for this company.

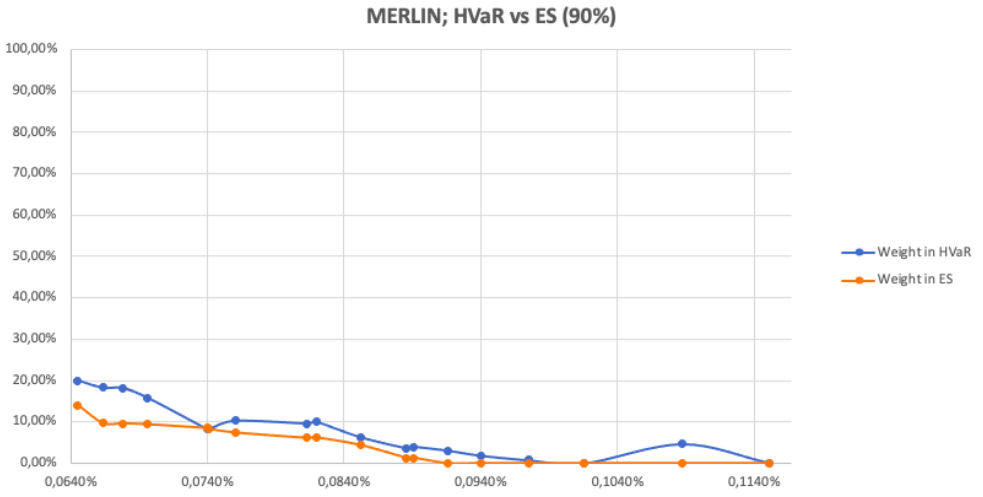

Figure 19: HVaR vs ES Merlin (90%)

In this other case, a downward trend can be observed in the “Merlin Properties” weightings for both approaches. Both risk assessment methods appear to be quite stable and do not exhibit high volatility as expected return increases. Also, both weights are very similar for this range of yields. Interestingly, for the highest returns, the weights are 0% or practically 0% except for the penultimate portfolio in HVaR, where the weight is slightly increased to get back to 0% in the last portfolio.

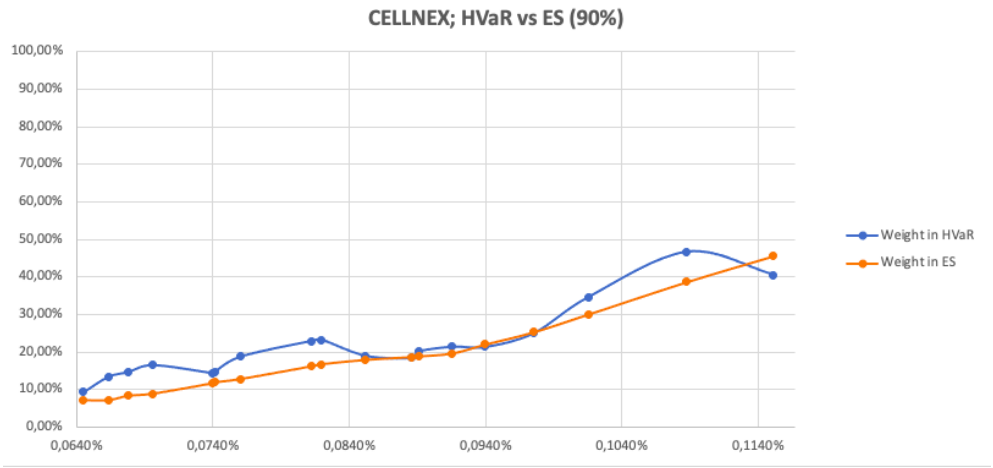

Figure 20: HVaR vs ES Cellnex (90%)

The Spanish company belonging to the telecommunications industry presents an evolution of its weightings through the two clearly bullish approaches (growing slope). The weights according to ES are once again more stable than the weights in HVaR, which present although to a lesser extent than in some of the previous cases. We can appreciate certain trend changes as expected return increases. Finally, we can affirm that the weightings are quite similar (especially in intermediate returns) for the entire range of expected returns depending on whether one method or another is used.

Once again, we can affirm that, for a confidence level of 90% and for the seven companies that make up our study, using the HVaR as a risk measure shows in general a more confusing behavior than if we used instead the ES as a risk measure.

6.2 COMPARISION WITH MARKOWITZ.

The following charts which like it was seen in point 5.2 of this study, which can be found in the appendix show as in this previous case, that ES follows a trend in the weightings very similar to if we used as a method of risk assessment the Markowitz approach. On the other hand, and with respect to the HVaR approach, the weights are in general terms very different from those of Markowitz and in the same way, they present an irregular behavior.

In Camáñez (2019), the weightings according to Markowitz for the asset corresponding to "Inmobiliaria Colonial" follow a trend very close to those obtained in the present work for the company "Merlin Properties". Once more, it is again interesting to compare the weightings according to HVaR in these two cases.

Source: Camáñez (2019)

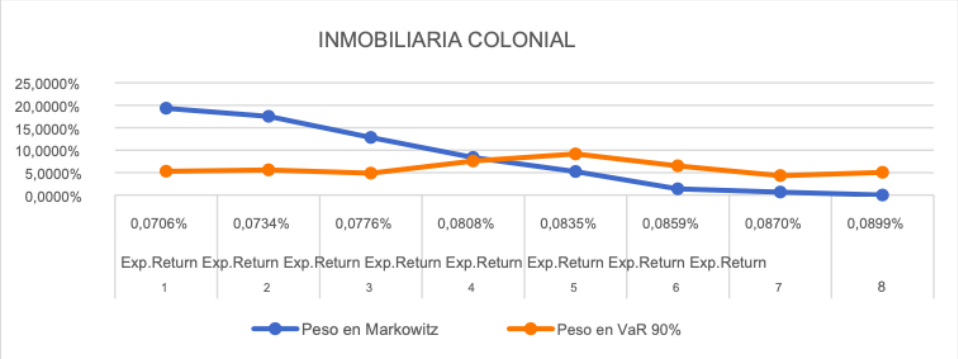

Figure 21: Markowitz vs HVaR Inmobiliaria Colonial (90%)

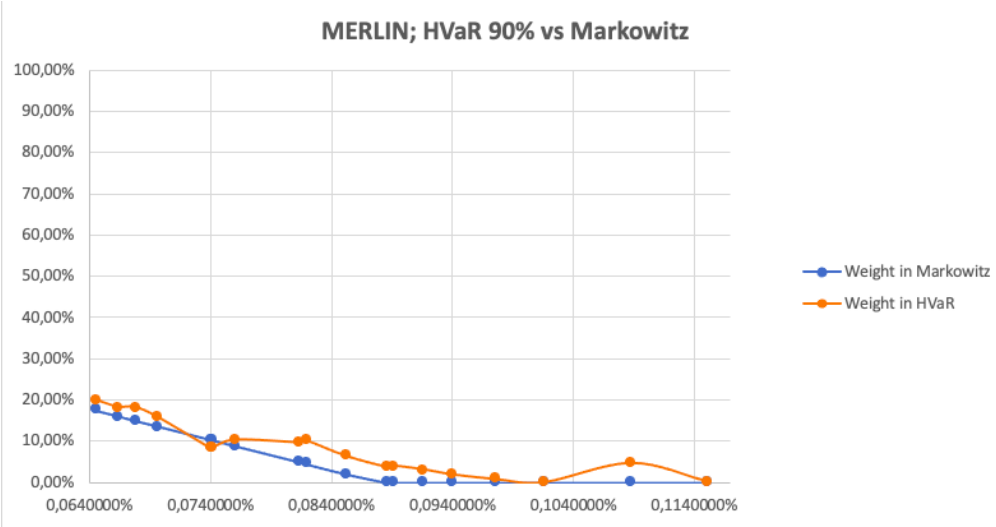

Figure 22: Markowitz vs HVaR Merlin (90%)

To conclude with, we can appreciate that the weightings according to Markowitz are very similar in both charts. Despite this, it does not mean that something similar happens for the weightings according to the HVaR (90%).

7. OVERVIEW OF THE BEHAVIOR OF THE OPTIMAL WEIGHTS.

As discussed in the previous two sections, the optimal weightings according to the ES at both 95% and 90% are in general more similar to the optimal weights using the variance of yields as a measure of risk than the optimal weightings according to HVaR, also at both 95% and 90%. Furthermore, these optimal weightings according to both 95% and 90% HVaR have been found to be broadly quite erratic and do not define a clear pattern.

To continue with, if we look at the expected return provided by the 7 assets that are part of this thesis, we can see that Cellnex and Merlin are the companies that provide the highest and lowest expected returns respectively for this period of time that is under study. In this way, it is articulate to affirm that, as expected return increases, portfolio diversification decreases and, as a result, we will invest more and more in the asset that provides the best return (Cellnex) and we will reduce this proportion in the asset with the lowest yield (Merlin). This aspect can be seen in the following table that represents the trend of assets:

Table 16: Trend evolution according to the asset weights

ORDER	ASSET	EXP. RETURN	HVAR (95%)	HVAR (90%)	ES (95%)	ES (90%)
2	ACCIONA	0,093%				
4	INM. COLONIAL	0,061%				
6	ENDESA	0,049%				
5	FERROVIAL	0,055%				
3	IBERDROLA	0,086%				
7	MERLIN	0,005%				
1	CELLNEX	0,151%				

increase
decrease

In this way, we observe that, in the case of Cellnex (1) and Acciona (2) which assets have provided higher profitability for this time interval, the trend in weights is positive as expected return increases for both HVaR and ES for both confidence levels.

Quite the opposite occurs for Endesa (6) and Merlin (7), which trend is clearly downward in the weightings for both risk measures and for both confidence levels as expected return increases for the different portfolios. It is important to highlight the case of “Iberdrola”, which trend is upward for all risk measures and for both confidence levels except for HVaR (90%).

In this sense, it is important to note that both in ES at 95% and also at 90%, the weightings of the different assets evolve in a very similar way, although not identical, as in the results of San Félix (2019).

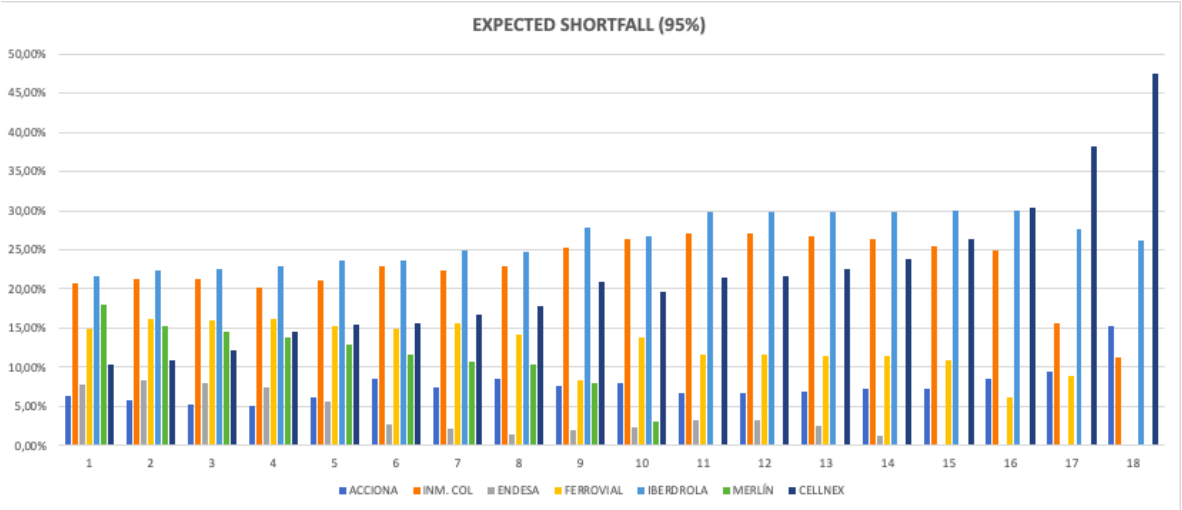

Figure 23: ES evolution (95%)

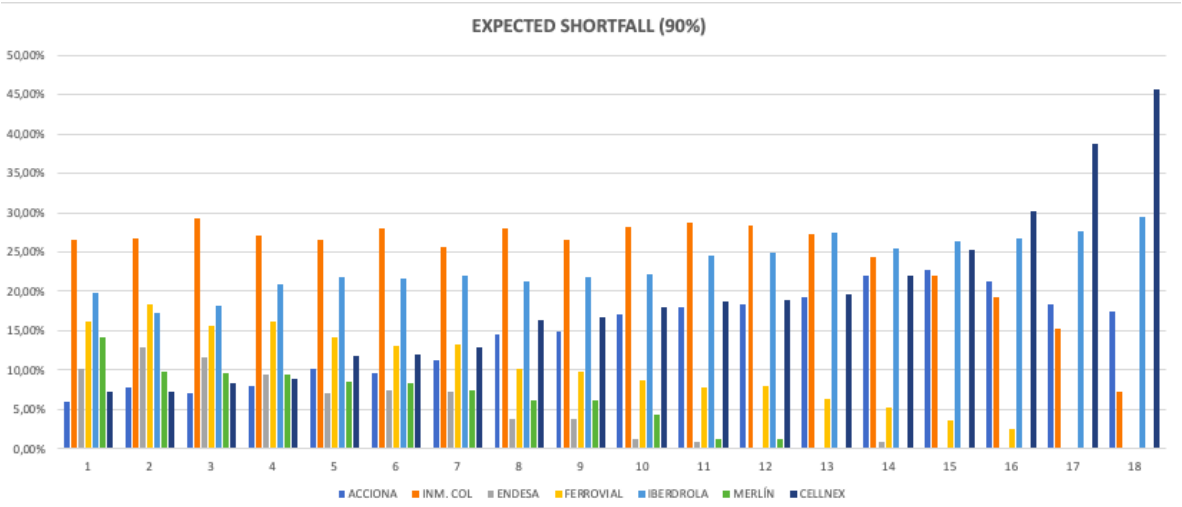

Figure 24: ES evolution (90%)

In fact, although in a very subtle way, the results seem to indicate that ES (90%) is more similar to Markowitz as can be seen in the appendix of the present study. However, this is not always the case in the results obtained by San Félix (2019).

In the same way, it is also interesting to make the comparison between HVaR at 95% and 90%, where the weightings evolve in a very similar way for some assets such as “Cellnex”, “Merlin” and also “Acciona” and others less like “Inmobiliaria Colonial”, “Endesa” and “Ferrovial”, the behavior in these last two cases being less erratic for the HVaR (90%). Similarly, Camáñez (2019) shows that the HVaR (95%) implies, in general terms, a more erratic behavior.

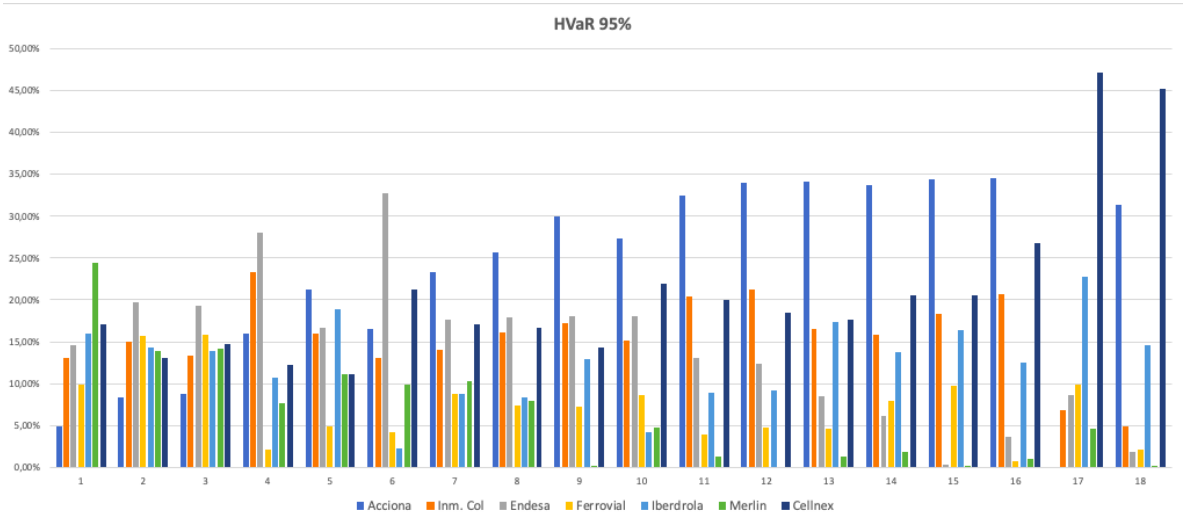

Figure 25: HVaR evolution (95%)

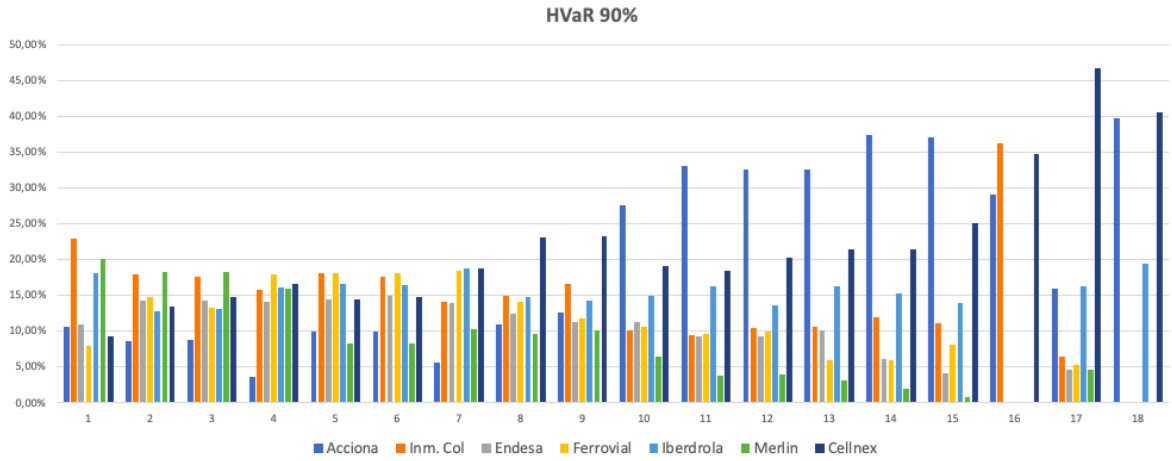

Figure 26: HVaR evolution (90%)

It can be observed that, as expected return increases, Cellnex's weight grows and on the other side, Merlin's weight falls, until in portfolio 15, the weight ends up being practically 0%. This is due to the concept of diversification that we talked about in the previous paragraphs.

It has also been seen in the previous sections that the fact that the behavior of the optimal weightings using the variance of the yields as a measure of risk, is similar for two different stocks, does not imply that the same fact happens with the behavior of the optimal weightings of these two stocks, according to HVaR at both 95% and 90%.

In spite of the mentioned fact, certain stocks have been detected that for relatively high expected returns, these would not form part of efficient portfolios if the variance of yields is used as a measure of risk. Nevertheless, the behavior of the mentioned optimal weightings according to HVaR (95%) presents the same anomaly, which can be observed in the following charts.

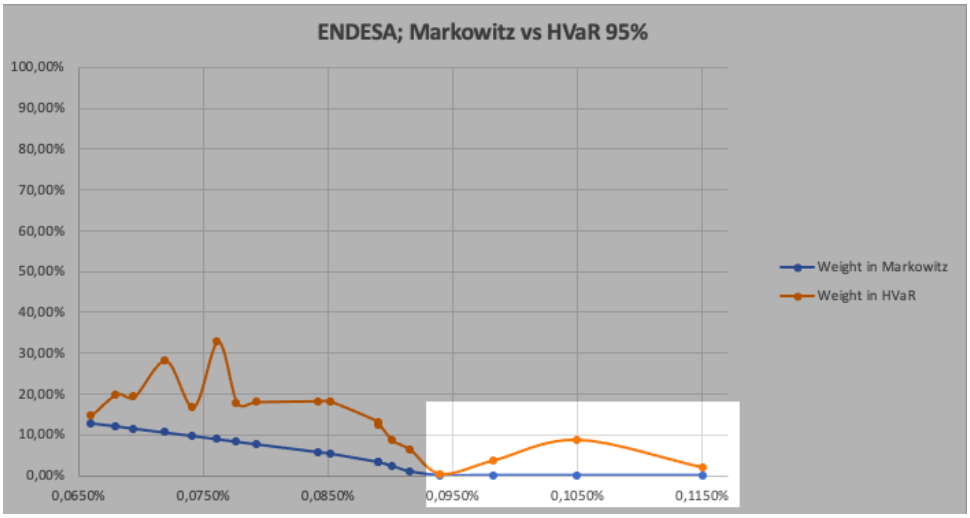

Figure 27: Anomaly 1

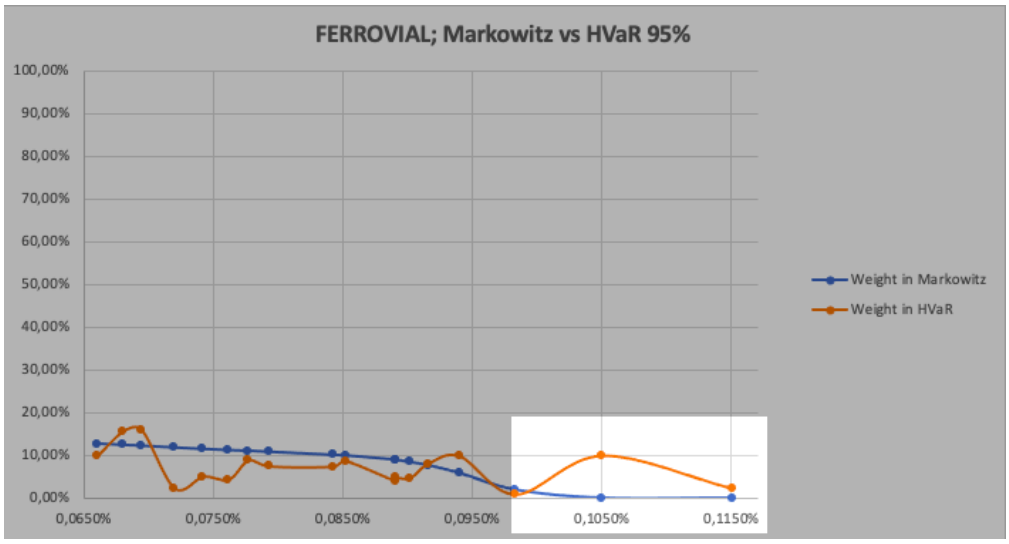

Figure 28: Anomaly 2

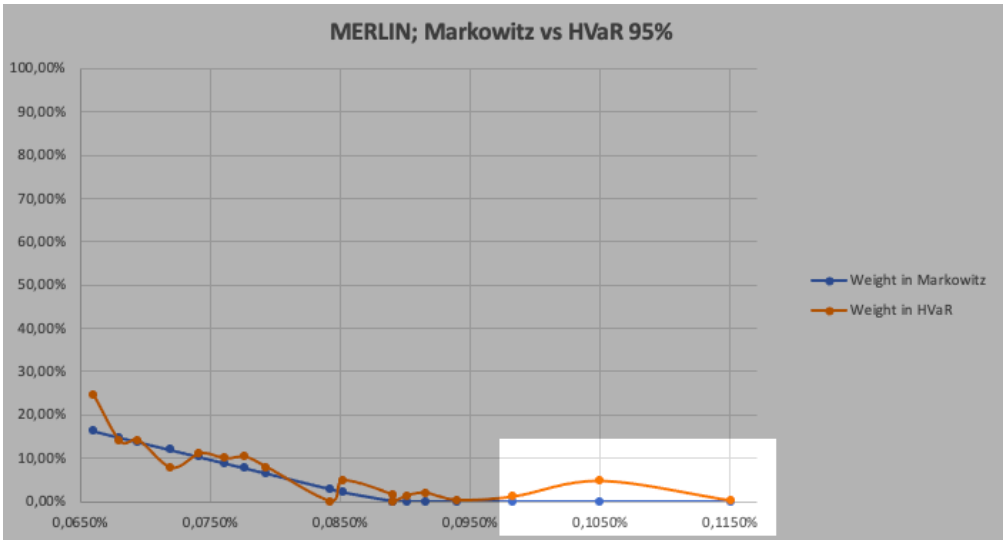

Figure 29: Anomaly 3

For relatively high expected returns, the optimal weighting according to HVaR (95%) is zero, as if the variance of yields was used as a measure of risk. Even though, the first of these ones experiences a rebound before falling back to zero. Curiously, this anomaly is already observed in the results obtained in Camáñez's (2019) thesis, as can be seen in the following charts.

Source: Camáñez (2019)

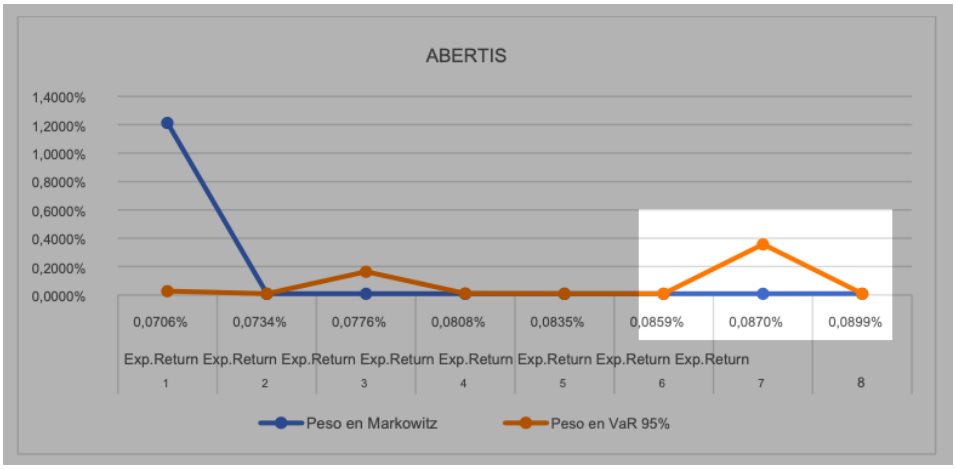

Figure 30: Anomaly 4

Source: Camáñez (2019)

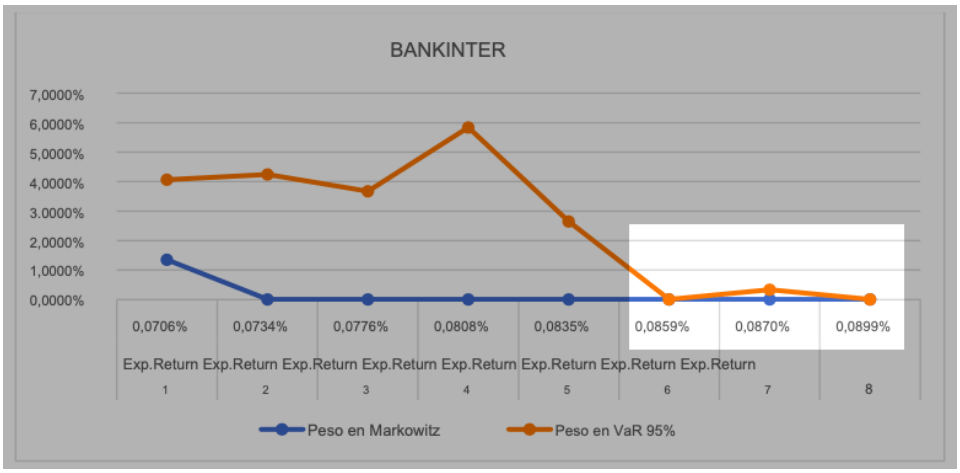

Figure 31: Anomaly 5

Source: Camáñez (2019)

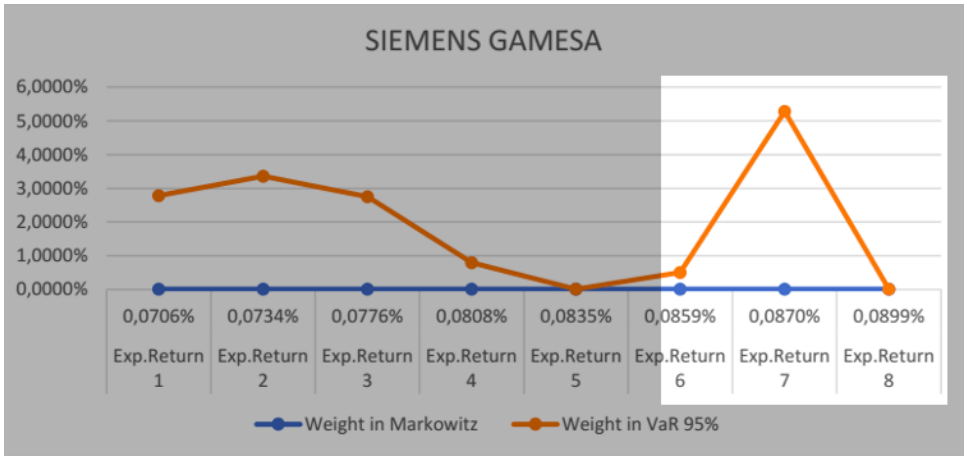

Figure 32: Anomaly 6

In the case of the optimal weightings according to HVaR (90%), the data used in the present study lead to a similar result for the case of the previous companies, as can be seen in the following graphs.

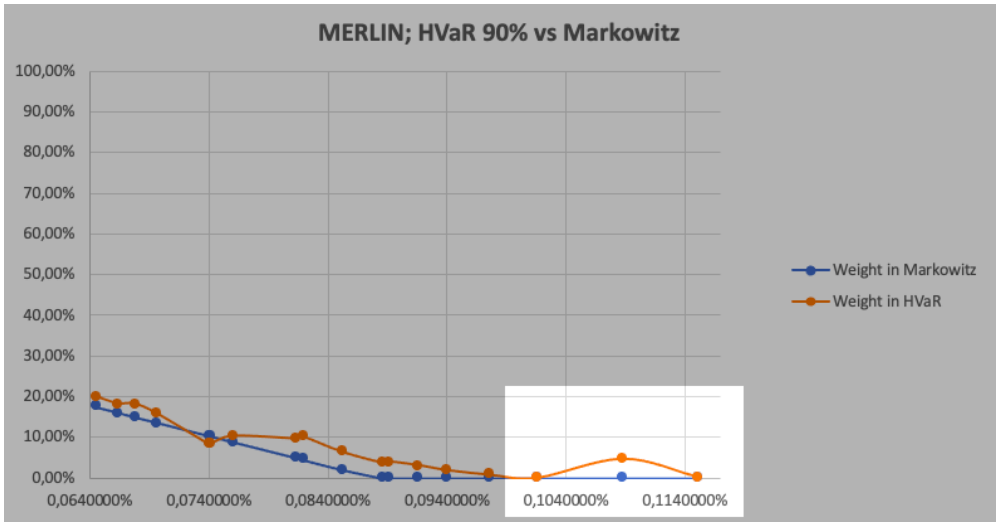

Figure 33: Anomaly 7

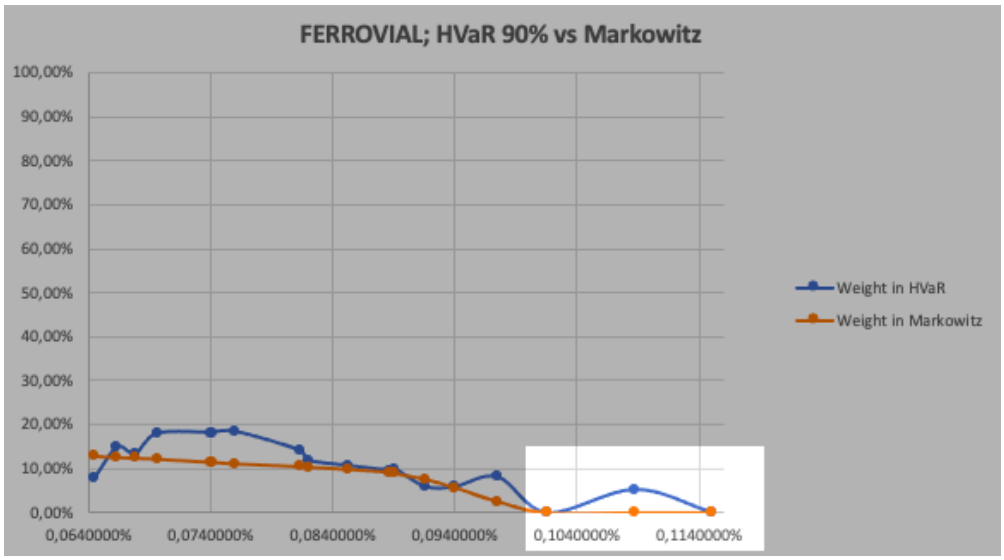

Figure 34: Anomaly 8

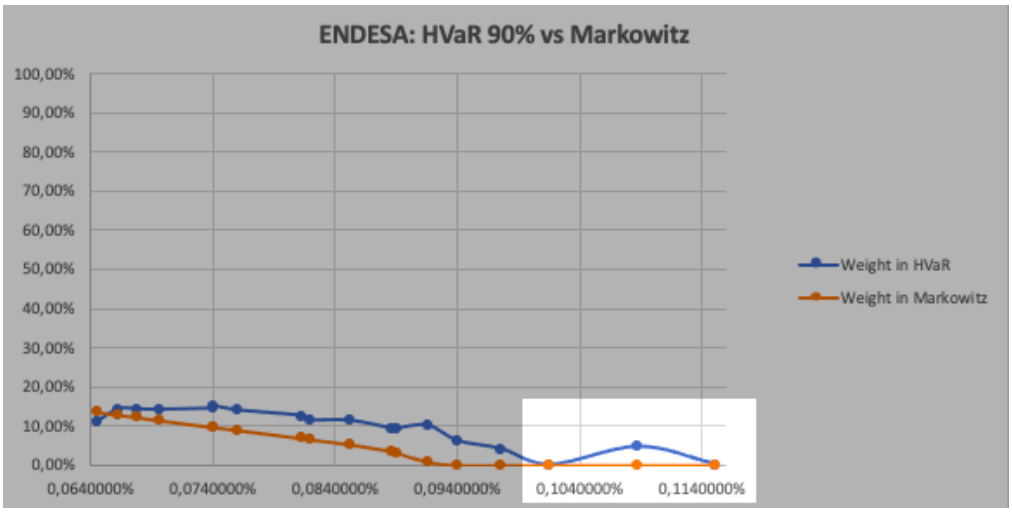

Figure 35: Anomaly 9

Notwithstanding, as we can extract from Camáñez (2019), this anomaly is not found when he uses the HVaR (90%) as a risk measure.

In respect of other anomalies, this study has been compared with the thesis of Camáñez (2019) and no other important similarities have been tracked down to highlight in this analysis.

8. CONCLUSION.

This thesis aims to compare efficient investment portfolios using two different approaches as risk measurement in asset optimization. On the one hand, the Value at Risk and, on the other hand, the Expected Shortfall, for two different confidence levels. In addition, a brief emphasis on Markowitz's approach has also been sought, which has helped us to make comparisons with studies carried out previously, corresponding to Camáñez (2019) and San Félix (2019).

In this way, we can affirm that the objective of this analysis that we have carried out is to study the evolution of the weightings of the seven assets as the expected return of the portfolio increases for the different approaches described in the previous paragraph.

In general terms, it has been observed that the behavior of the weightings according to the HVaR approach, both for a confidence level of 95% and 90%, is much more erratic than if we use the ES approach as a risk measure. That is to say, that it can be clearly observed that the ES, as the portfolio's expected return increases, its increase or decrease from the optimal weights are much more constant and follow a clear pattern with respect to the HVaR that, in most cases, does not it follow a clear pattern and also shows quite a few abnormalities. These anomalies have been seen in the same way in the thesis carried out by Camáñez (2019) that, for a confidence level of 95%, it is clearly observed that, for higher expected returns, the weights according to HVaR and according to Markowitz coincide except in the penultimate portfolio, which experiences a notable rebound. However, for HVaR at 90%, Camáñez (2019) does not find this anomaly. San Félix (2019) does not find these anomalies.

To follow with the comparison, some differences are also observed when we compare our results with those of Camáñez (2019). In this sense, it is important to note that the anomalies previously mentioned continue to appear in the present study for HVaR 90%, quite the opposite of Camáñez (2019), which anomalies disappear for this level of confidence. On the other hand, the comparison with Camáñez (2019) also shows that the similarity in trends according to Markowitz does not imply similarity in the trends according to HVaR at 95% or 90%.

Another aspect to highlight of this study is that, the evolution of the weights according to the ES approach for both levels of confidence worked throughout this thesis is, in general, very similar to the evolution of the weightings according to Markowitz, aspect which also occurs in the thesis of San Félix (2019). However, ES (90%) is subtly more similar to Markowitz than ES (95%) in this study, but not in San Félix (2019).

We have observed that the optimal weights (considering HVaR and ES as a risk measure at both 90% and 95%) of those assets that have provided a higher expected return in the study period analyzed, have a clearly upward trend. Otherwise, those assets which expected return has been lower in said period have a downward trend. It has been concluded that diversification defines the trend line of the assets, that is, as the portfolio's expected return increases, the risk increases and that is why a greater proportion is invested in the assets with the highest yield and, consequently, to a lesser extent in those with lower yield.

In conclusion, risk is a measure that should always be considered by all investors. Your level of risk aversion will be important when deciding which investment to make. This aversion will be purely subjective, which means it will be determined both by the investor's personal situation, his assets, his training and other aspects. Furthermore, it is not only necessary to have the knowledge of how to minimize the risk of an investment but, to carry it out, both experience and knowledge of financial markets and specifically of the market in which it is carried out will also be a key point to consider.

APPENDIX.

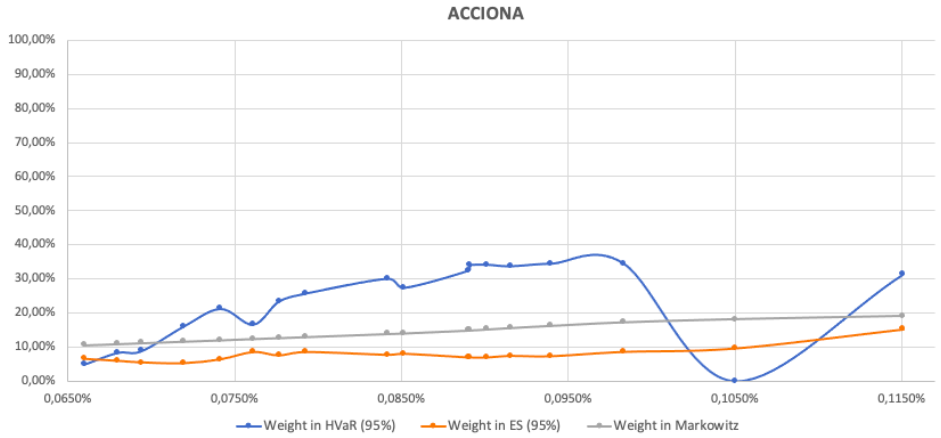

Figure 36: HVaR vs ES vs Markowitz Acciona (95%)

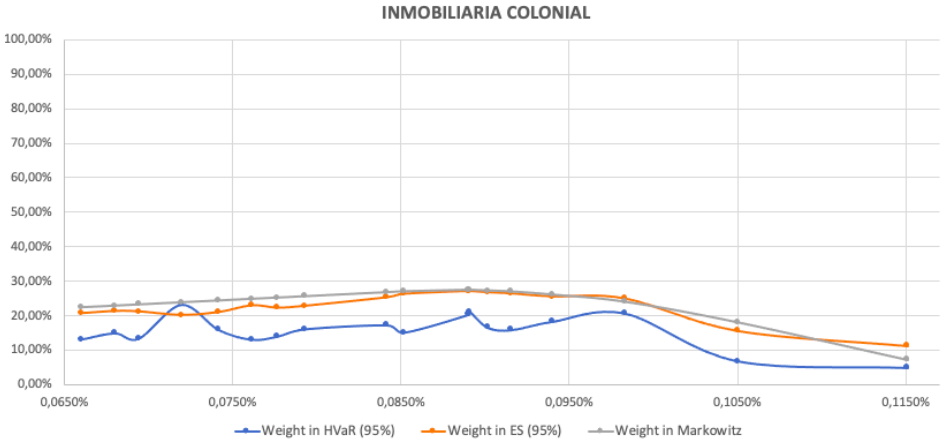

Figure 37: HVaR vs ES vs Markowitz Inmobiliaria Colonial (95%)

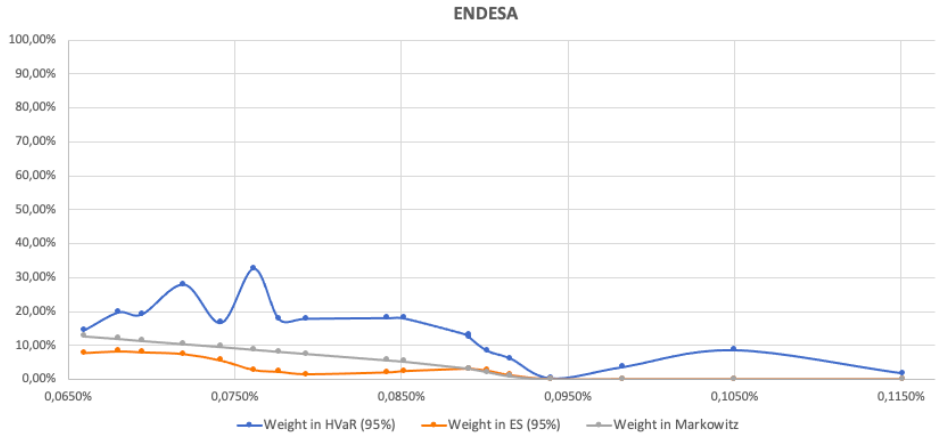

Figure 38: HVaR vs ES vs Markowitz Endesa (95%)

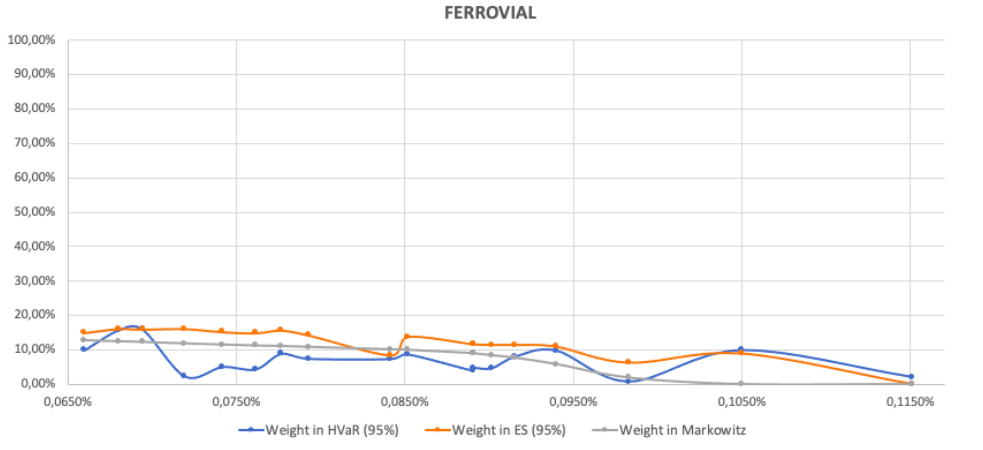

Figure 39: HVaR vs ES vs Markowitz Ferrovial (95%)

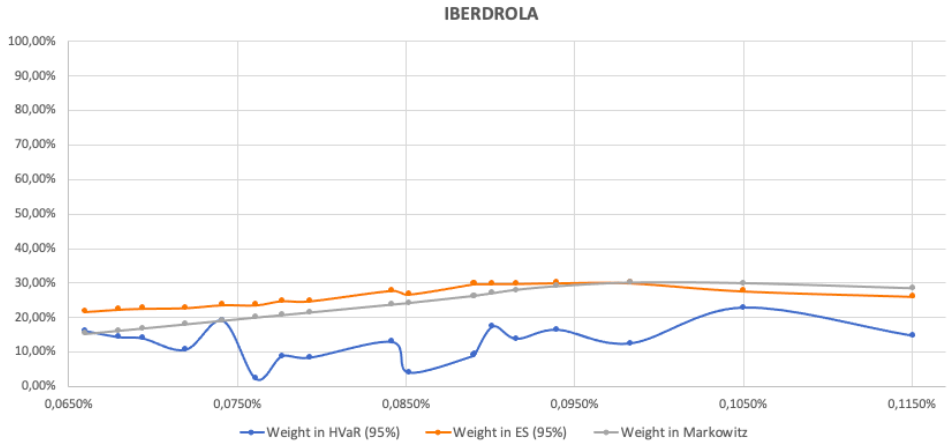

Figure 40: HVaR vs ES vs Markowitz Iberdrola (95%)

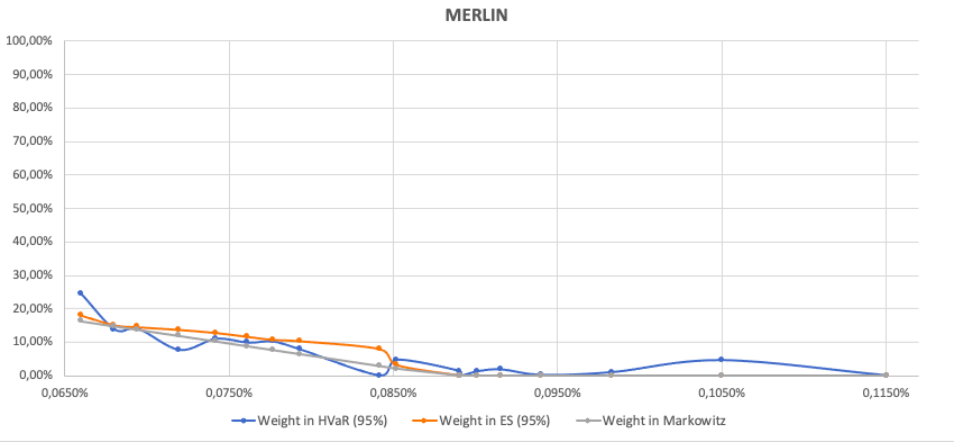

Figure 41: HVaR vs ES vs Markowitz Merlin (95%)

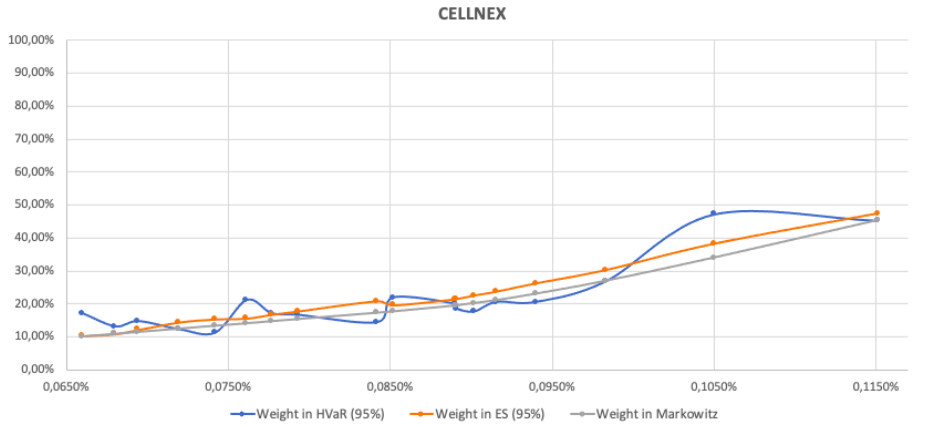

Figure 42: HVaR vs ES vs Markowitz Cellnex (95%)

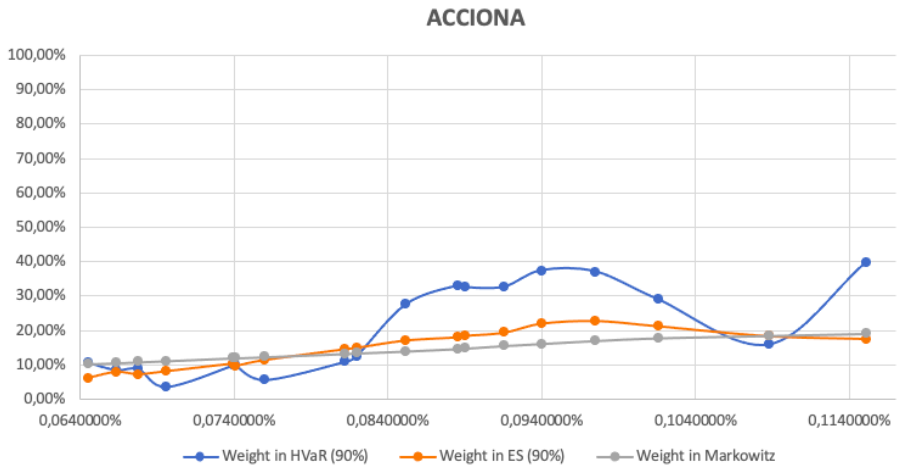

Figure 43: HVaR vs ES vs Markowitz Acciona (90%)

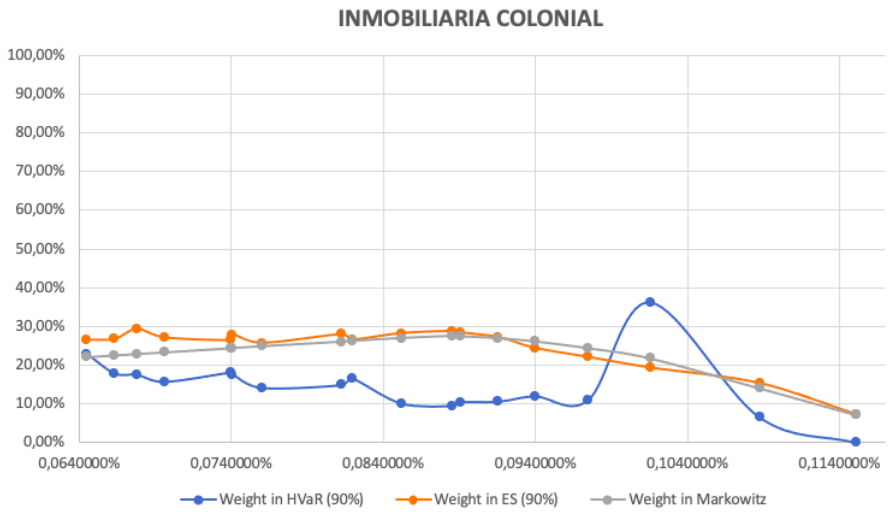

Figure 44: HVaR vs ES vs Markowitz Inmobiliaria Colonial (90%)

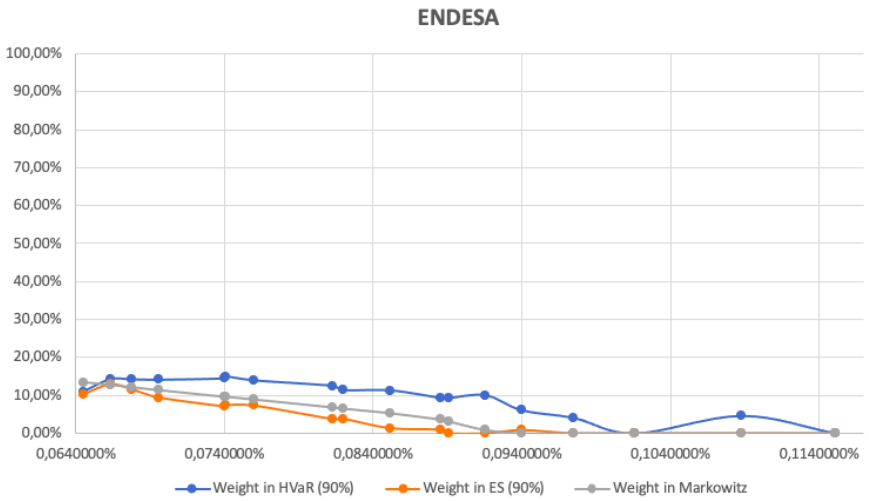

Figure 45: HVaR vs ES vs Markowitz Endesa (90%)

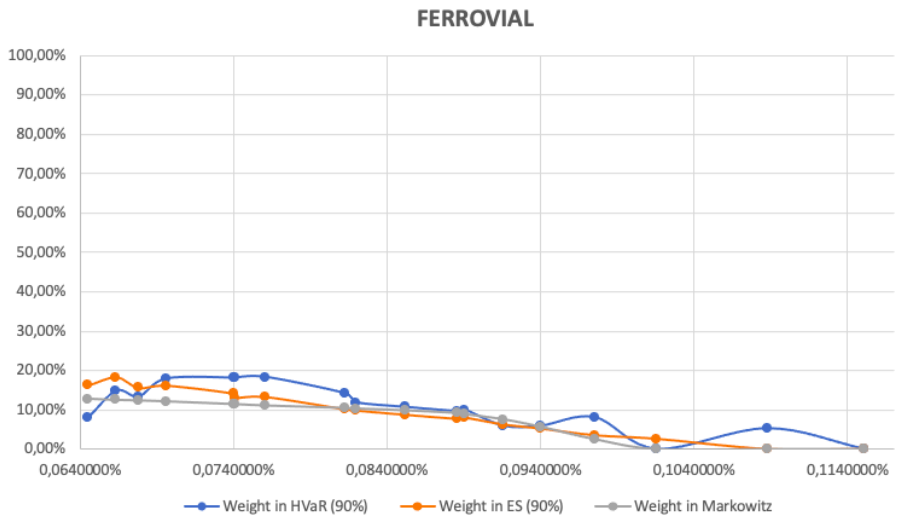

Figure 46: HVaR vs ES vs Markowitz Ferrovial (90%)

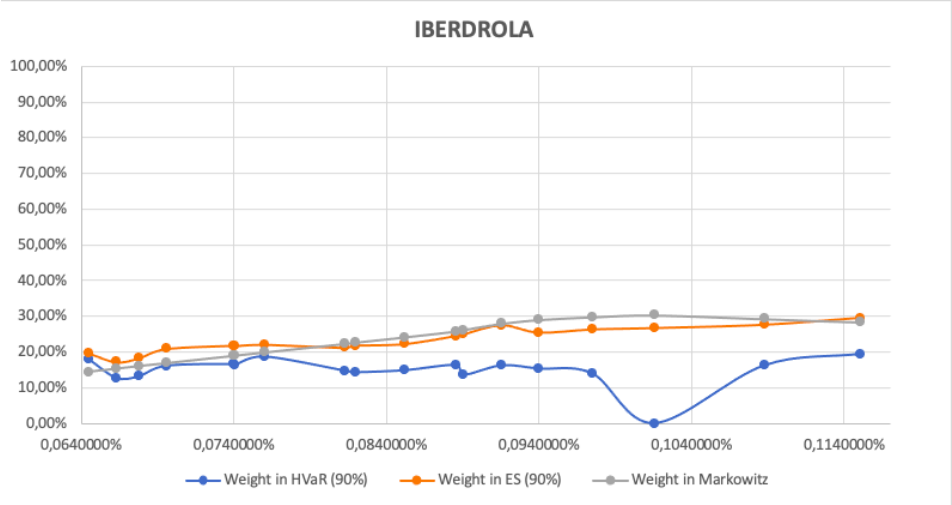

Figure 47: HVaR vs ES vs Markowitz Iberdrola (90%)

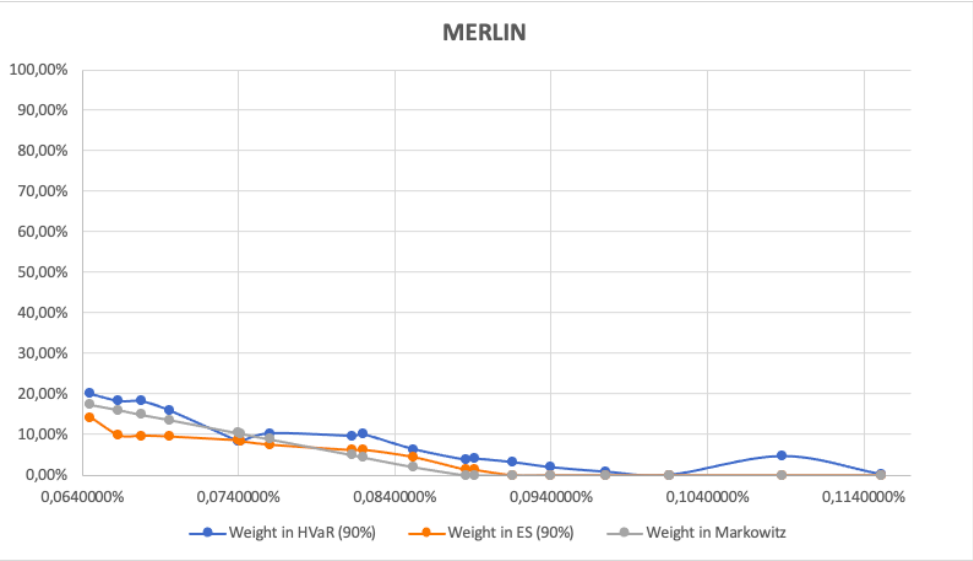

Figure 48: HVaR vs ES vs Markowitz Merlin (90%)

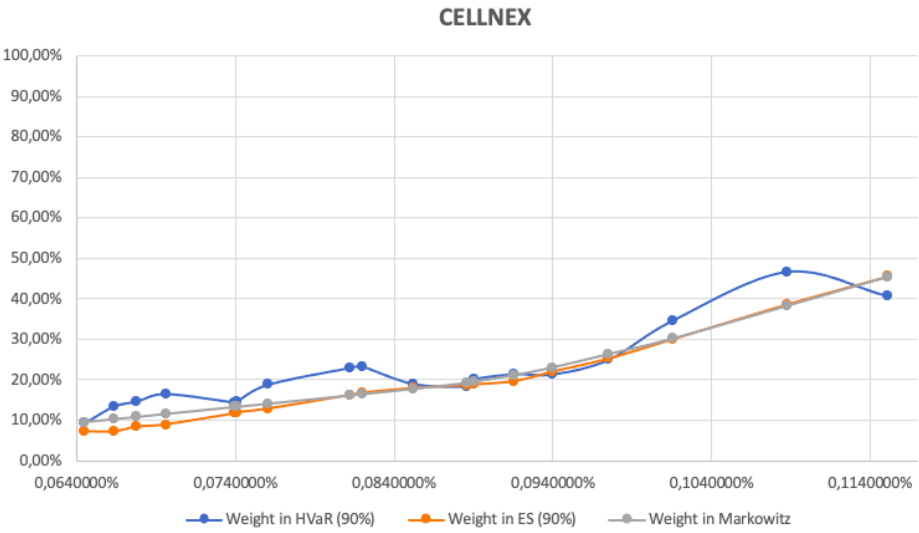

Figure 49: HVaR vs ES vs Markowitz Cellnex (90%)

BIBLIOGRAPHIC REFERENCES.

BBVA NOTICIAS. 2020. *¿Qué es el riesgo financiero? 5 consejos para evitarlo*. [online] Available at: https://www.bbva.com/es/finanzas-para-todos-el-riesgo-financiero-y-sus-tipos/

Benati, S., and Rizzi, R., 2007. A mixed integer linear programming formulation of the optimal mean/value-at-risk portfolio problem. European Journal of Operational Research, 176(1), pp. 423-434.

Cáceres, E., 2017. *Pérdida esperada o expected shortfall*. [online] Mirai Advisory. Available at: https://blog.mirai-advisory.com/perdida-esperada-o-expected-shortfall/

Camáñez., 2019. Analysis of investment in financial markets: Markowitz against Value at Risk historical approach. Available at: http://repositori.uji.es/xmlui/bitstream/handle/10234/186227/TFG_2019_CamanezMont_AlejandroRamon.pdf?sequence=1&isAllowed=y

Campbell, R., Huisman, R., and Koedijk, K., 2001. Optimal portfolio selection in a Value-at-Risk framework. Journal of Banking & Finance, 25(9), pp.1789-1804.

Isaksson, D., 2016. Robust Portfolio Optimization with Expected Shortfall. PhD. KTH Royal Institute of Technology. Available at https://www.math.kth.se/matstat/seminarier/reports/M-exjobb16/160601d.pdf

Nieto, Á., 2016. *Valor en riesgo o value at risk*. [online] Mirai Advisory. Available at: https://blog.mirai-advisory.com/valor-en-riesgo-o-value-at-risk-var-parte-i/

Nieto, Á., 2016. *Valor en riesgo o value at risk (cont.).* [online] Mirai Advisory. Available at: https://blog.mirai-advisory.com/valor-en-riesgo-o-value-at-risk-var-parte-ii/

Peiro Ucha, A., 2015. *Pérdida esperada – Expected shortfall (tail VaR) – Definición, qué es y concepto*. [online] Economipedia. Available at: https://economipedia.com/definiciones/perdida-esperada-expected-shortfall.html

San Félix., 2019. Portfolio optimization: Markowitz's approach vs Expected Shortfall as a risk measure. Available at: http://repositori.uji.es/xmlui/bitstream/handle/10234/187227/TFM_San_F%c3%a9lix_Forner%2c_Pablo.pdf?sequence=1&isAllowed=y

Vanini, P., 2003. Expected Shortfall and Value at Risk. [online] Research Gate. Available at: https://www.researchgate.net/figure/Expected-Shortfall-and-Value-at-Risk-In-case-of-Expected-Shortfall-the-tail-of-the_fig1_228183419

Yoshida, Y., 2009. An estimation model of value-at-risk portfolio under uncertainty. Fuzzy Sets and Systems, 160(22), pp. 3250-3262