UNIVERSITAT JAUME•I

Finance and Accounting Degree

DEGREE FINAL PROJECT

PORTFOLIO OPTIMIZATION: MARKOWITZ'S APPROACH VS EXPECTED SHORTFALL AS RISK MEASURE

Presented by: Pablo San Félix Forner

Mentored by: Alejandro José Barrachina Monfort

E-mail: al269248@uji.es

Castellón, July 2019

ABSTRACT

The objective of this study is to compare the optimal portfolios obtained under two risk measures. On the one hand, under the risk measure used by the Markowitz approach (1952, 1959). On the other hand, under the measure of risk through the Expected Shortfall.

To create the optimal portfolios on which the study was based, the daily quotes of seven companies listed on the IBEX35 have been used in a period of time from January 2, 2012 to March 18, 2016.

The conclusions we have obtained are that, regardless of the three levels of confidence considered for the Expected Shortfall, the weights of the assets analyzed in the corresponding optimal portfolios under the Expected Shortfall as a risk measure follow the same trend with respect to their weightings in the optimal portfolios in the sense of Markowitz (1952, 1959).

Keywords: Measure of risk; Portfolio optimization; Mean-variance approach; Expected Shortfall

JEL Code: G11

INDEX

LIST OF FIGURES

LIST OF TABLES

1. Introduction

The financial risk of an investment portfolio can be measured in different ways, but it is often desirable to measure it in currency units, which means that the risk is expressed as compensation capital that must be added to a portfolio in order to protect it from undesired results.

However, in the first mathematical formalization of the idea of diversification of risky investment portfolios represented by the works of Markowitz (1952, 1959), which gave rise to the Modern Portfolio Theory, risk is measured as the variance (or standard deviation) of the future achievements of the portfolio. Other contribution of Markowitz (1952, 1959), considering the case of rational risk-averse investors, was the concept *efficient frontier*, which would be formed by those portfolios that maximize the expected profitability for a certain level of risk, or, minimize the risk for a given expected profitability. That is, the efficient frontier is formed by optimal portfolios.

The question is that lately the variance is not considered as very good measure tool of risk in finances, since it is defined as the expected squared deviation of the average value, and, consequently, makes no difference between positive deviations, holding gains and negative deviations, and loss of securities portfolio. Moreover, standard deviation can only be considered accurate enough to translate into currency risk if the future value of the portfolio's value is distributed, approximately, in the normal manner. Frequently, this assumption is too strict and simplifies too much the registration distribution and the actual portfolio yield. By contrast, it is often desirable to use a risk measure that makes the difference between god and bad deviations from the future expected portfolio's value. In this study, the measure of risk basic theory is presented first, and after that it is specified a risk measure widely used in financial risk management.

The different risk measures have different properties. Next, there is presented a list of those mathematical properties which are considered useful or desirable according to the research that has been carried out.

1. **Translation Invariance.** That means that adding the quantity c with a R_o risk free rate, to a portfolio, reduces the risk equally.
2. **Monotonicity.** That means that, if you know, for certain, that one portfolio X_1 is bigger than one portfolio X2 in the future, in that case, the first one is considered as high risk.

3. **Convexity.** Risk measurement rewards diversification, what means that takes into consideration that it is often recommended to divide the investment in various risk positions, instead of investing all-in one.

4. **Normalization.** That means that it is acceptable not to invest in risky assets, consequently an empty portfolio will be risk-free.

5. **Positive homogeneity.** That means that, for example, to invest double in one position is twice as dangerous, in terms of risk.

6. **Subadditivity.** This property must also be interpreted as meaning that risk measure rewards diversification. A company that consists of two business units is interpreted as dangerous (in terms of risk) in comparison with those two units, considered as separated companies.

A measure of risk with invariance and monotonicity of property conversion is said to be a monetary measure of risk, and a measure of risk considered to replace the variance in Markowitz's mean-variance optimization problem should satisfy at least these two properties. A measure of risk that in addition to the invariance of translation and monotonicity, also satisfies convexity is a of convex risk measure.

The family of convex risk measures is, consequently, a subset of the family of monetary risk measures. Lastly, the third family of risk measures is Coherent risk measure, where the risk measure fulfills the following properties: the invariance of the translation, monotonicity, positive homogeneity and subadditivity.

It is easy for a risk measure that satisfies a positive homogeneity satisfy also normalization. In addition, it can be shown that positive homogeneity and convexity, together, implies subadditivity, but not reserve.

Therefore, one coherent risk measure is also a convex risk measure, but generally, the opposite is not valid, so the family of coherent risk measures, is a subset of the family of convex risk measures and, consequently, it is also a subset of the family of monetary risk measures.

By selecting an adequate risk measure for a portfolio optimization problem that replaces Markowitz’s mean variance optimization problem, can be consider that convex and coherent risk measures are, at least, as good as the monetary risk measures.

The reader is referred to the book from Hult, Lindskog, Hammarlid and Rehn (2012), in order to find a more detailed presentation about risk measures general theory and more comments on the above characteristics, as well as more information about why variance is considered a bad risk measure in finance, particularly because does not meet the properties of translation and monotonicity.

Next, are presented two risk measures that are commonly used in risk management and are considered to solve problems of variance as risk measure.

2. Possible risk measures which provide solution to Markowitz problem

2.1. Value-at-risk

The first risk measure presented is Value at Risk (VaR). This risk measure fulfills the invariance of the translation, monotonicity and positive homogeneity, and consequently, is a monetary risk measure.

VaR measure is always associated to a level of confidence, so it is known as q% VaR and denoted as VaR (q%) where $q \in (0, 100)$. A portfolio's Var (q%) provide us with portfolio's yield, in such a manner that there are only q% of probabilities of the portfolio providing yield smaller than one. May consider the following examples:

1. VaR (10%) of a portfolio give us its yield in such a way that there is only 10% of probability of the portfolio providing a yield smaller than one. Equivalently, it is said that we can be 90% sure of the fact that the portfolio's yield won't be lower than the yield represented by VaR (10%).

2. VaR (5%**)** of a portfolio give us its yield in such a way that there is only 5% of probability of the portfolio providing a yield smaller than one. Equivalently, it is said that we can be 95% sure of the fact that the portfolio's yield won't be lower than the yield represented by VaR (5%).

One direct VaR advantage over traditional variance is that can be used when the variance is not a relevant risk measure, for example, when the expected value of a distribution does not represent a fair image of the distributive appearance.

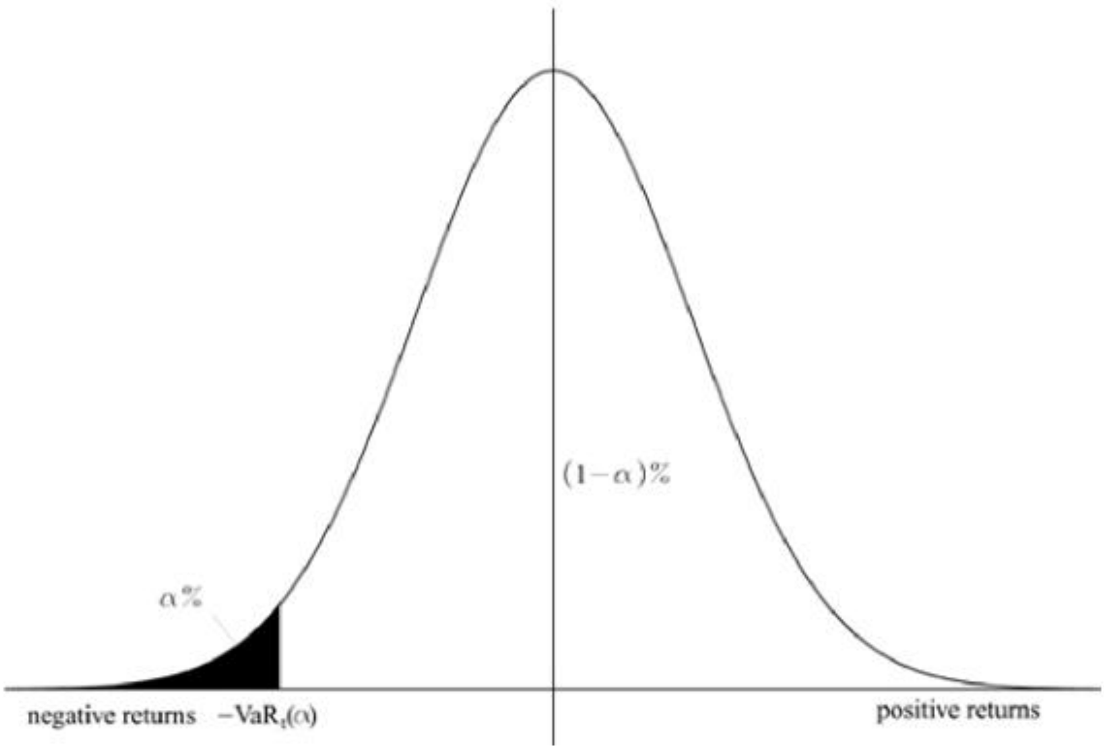
Moreover, since VaR calculates level (1-p) of quantile of discounts, it takes into account only great losses, but not big profits, consequently, VaR makes difference between positive and negative deviations of the portfolio future expected value. However, since

VaR only takes into account one particular level (1-p) of the quantiles, operators can use this to hide risky investments making the losses more extreme so that the VaR does not discover them.

With this strategy, high risk portfolios could be acceptable, otherwise, they would not have been acceptable if the risk were visible for risk managers. That could lead to companies being exposed to extremely large scenarios, although with a relativity small probability, but with the possible outcome of the company suffering a huge loss and, possibly, a bankruptcy.

An equivalent definition of the VaR would be to say that VaR (q%) is a value *u* so that *P(R<u)=q%*, that is, that the probability of getting a yield of the smallest portfolio *u* is *q%*. The following graphic shows the definition of VaR for a series of yields that follow a normal distribution.

FIGURE 1. GRAPHICAL DEFINITION OF VAR



Source: www.semanticshoclar.org

Therefore, VaR is like the "best scenario of the worst cases", which implies that sometimes underestimates the potential losses and, so, the risk of some portfolios. Which brings us to the next alternative measure of risk.

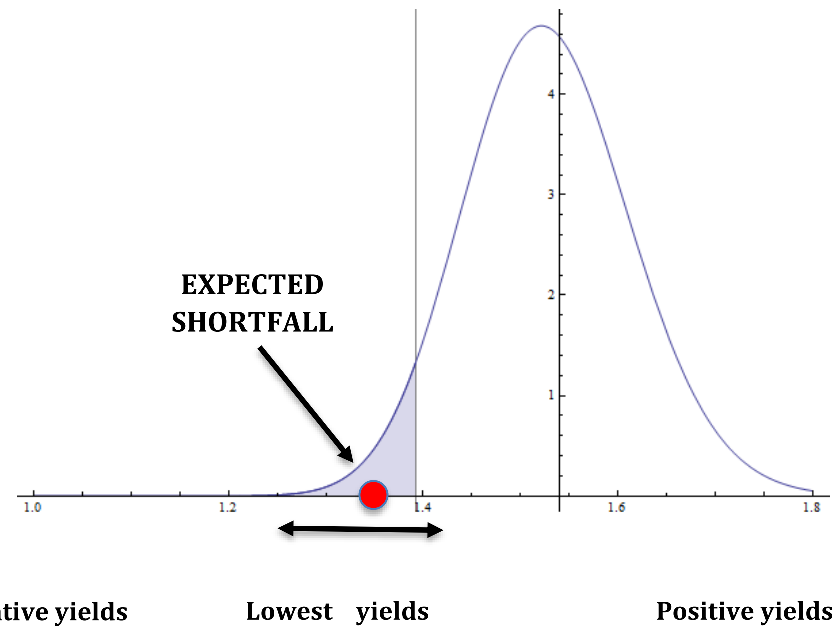
2.2. Expected Shortfall

The Expected Shortfall (ES) goes by various different names in literature and with minor changes is called Conditional Value-at-Risk (CVaR), Average Value-at-Risk (AVaR), Tail Value-at-Risk (TVaR), Expected Tail Loss (ETL) y Tail Conditional Expectation (TCE). This risk measure emerges as an attempt to overcome the VaR limitation derived from the fact that it can underestimate the potential losses of an investment. In this sense the ES is a better measure of risk since it takes into account all the losses located in the tail of the quantile of the distribution.

To overcome this limitation of VaR, Expected Shortfall at level of q% is defined as the expected yield of the portfolio at worst q% of the cases. That is, the Expected Shortfall (10%) of a portfolio gives the average of the lowest yields of 10% of the portfolio. Expected Shortfall (5%) of a portfolio gives the average of the lowest yields of 5% of the portfolio.

Considering that ES is defined through Value-at-Risk, it inherits Value-at-Risk properties, being the following ones: the invariance of the translation, monotonicity and positive homogeneity. In addition, it can be demonstrated that ES also satisfies subadditivity property, and consequently, is a coherent risk measure.

FIGURE 2. GRAPHICAL DEFINITION OF EXPECTED SHORTFALL



Source: own elaboration

The objective of this paper is to compare the optimal portfolios obtained under two of the three risk measures that have just been discussed. On the one hand, the variance (that is, the risk measure used by Markowitz's approach (1952, 1959) to the optimization of portfolios), which, as we have said, does not fulfill all the desirable properties for a coherent risk measure. On the other hand, the Expected Shortfall, which does fulfill these desirable properties. Hence, the objective of the work can be seen as an attempt to study the possible biases that introduces a measure of non-coherent risk, such as the variance of yields, into the optimization of portfolios.

Some previous studies have addressed the issue of portfolio optimization under risk measures other than variance, such as Campbell, Huisman, and Koedijk (2001), Benati and Rizzi (2007) and Yoshida (2009), but inn general they have focused on applying the VaR. Only one work has been found that deals with the optimization of portfolios under Expected Shortfall as a risk measure, it's about Isaksson (2016). He compares his results with he would get when applying the Markowitz approach (1952, 1959), but only for an expected level of rentability and for a single confidence level of Expected Shortfall. In the present work a similar exercise is carried out but for several levels of expected yield and for different confidence levels of the Expected Shortfall, to be able to study the effects of these variations about the differences in portfolio optimization according to both perspectives.

3. Data

To build the optimal portfolios in which this work will be based on, it has been used the daily quotes of seven IBEX35 companies for a period of time that goes from the 2nd of January 2012 to the 18th of March 2016. These data have been used because they were already available and processed for the calculation of optimal portfolios in the sense of Markowitz (1952, 1959) in a previous job. The seven companies that were chosen were the following ones:

1. Acciona: Its mission is to leader in creation, promotion and infrastructure, energy, water and services management; actively contributing to social welfare, sustainable development and value generation for their stakeholders.

2. Group ACS: Their mission is to become a world reference in the construction and infrastructure development industry, both civil and industrial.

3. Ferrovial: Is one of the main operating companies worldwide, speaking in terms of infrastructures and city services managing, committed with the development of sustainable solutions.

4. Bankinter: Is one of the most important banks in Spain and has been rewarded by some of the most prestigious institutions in brand's world.

5. Grifols: Founded in 1940 in Barcelona and is one the leading companies worldwide in terms of production of Plasma-derived medicinal products.

6. Iberdrola: Nowadays the electrical company is the first European in terms of market capitalization and is also world leader in renewable energies.

7. Mapfre: Mapfre is a Spanish multinational dedicated to the insurance sector, with presence in 49 countries. The group's parent is the holding company Mapfre S.A., whose shares are traded in Madrid and Barcelona Stock Exchanges.

Before going on to the next section where I will explain in detail the methodology followed in the present work, I would like to detail for each selected company, the expected daily yield, the variance of daily yields and the Expected Shortfall for three levels of confidence, 90%, 95% and 99%, that are the three levels for which the present work will consider the Expected Shortfall as a risk measure to obtain optimal portfolios. Although many readers will already know what we are talking about, I will make a brief explanation of each of the measures:

- **Expected yield:** Is a weighted arithmetic mean of all the possible results for the yields on an asset, where the weighing represents a probability of these specific results will happen. For the case of an historical series of yields, all the yields of the series are considered equally likely.

- **Variance:** is the arithmetical mean of the squared deviations with respect to its average. An elevated variance will mean that data are much more dispersed. A low value of the variance will mean that values are close to the average. The standard deviation is the square root of the variance and the interpretations that can be deduced are the same.

- **Expected Shortfall:** In the next section, we focus on its methodology. Even though, briefly, is defined as the expected loss from the portfolio/ asset within a given time horizon, having overcome VAR measured by the chosen level of confidence. In this project we have chosen three level of confidence (99%, 95% y 90%).

After this brief explanation, I proceed to reflect the obtained results in each one of the selected companies.

TABLE 1. PROFITABILITY, VARIANCE AND ES (90%, 95% AND 99%) - ACCIONA

Profitability	0,0001143149
Variance	0,000485284
ES (90%)	-4,1329351%
ES (95%)	-5,2066784%
ES (99%)	-8,2023062%

TABLE 2. PROFITABILITY, VARIANCE AND ES (90%, 95% AND 99%) OF ACS

Profitability	0,000381566
Variance	0,000451011
ES (90%)	-3,7804099%
ES (95%)	-4,7112801%
ES (99%)	-6,9443938%

TABLE 3. PROFITABILITY, VARIANCE AND ES (90%, 95% AND 99%) OF BANKINTER

Profitability	0,0007484072
Variance	0,000618522
ES (90%)	-4,3895304%
ES (95%)	-5,4109677%
ES (99%)	-7,3889431%

TABLE 4. PROFITABILITY, VARIANCE AND ES (90%, 95% AND 99%) OF FERROVIAL

Profitability	0,0008922642
Variance	0,000233785
ES (90%)	-2,6715994%
ES (95%)	-3,4184215%
ES (99%)	-5,1814140%

TABLE 5. PROFITABILITY, VARIANCE AND ES (90%, 95% AND 99%) OF GRIFOLS

Profitability	0,0001140923
Variance	0,00029575
ES (90%)	-2,8339275%
ES (95%)	-3,6162612%
ES (99%)	-6,0956329%

TABLE 6. PROFITABILITY, VARIANCE AND ES (90%, 95% AND 99%) OF IBERDROLA

Profitability	0,0004300867
Variance	0,000236824
ES (90%)	-2,802990%
ES (95%)	-3,6339278%
ES (99%)	-5,8277463%

TABLE 7. PROFITABILITY, VARIANCE AND ES (90%, 95% AND 99%) OF MAPFRE

Profitability	0,0000831302
Variance	0,000646607
ES (90%)	-4,3058014%
ES (95%)	-5,4073086%
ES (99%)	-8,2075283%

In the next section, I am going to discuss the two economic models' methodologies which this work is based on. (Markowitz y Expected Shortfall).

4. Methodology

As it has been discussed in the introduction, the objective of this study is to compare optimal portfolios according to two different approximation. On the one hand, the approximation of Markowitz (1952,1959), which is based in the variance of the yields of the portfolio as a risk measure. On the other hand, the approximation that uses Expected Shortfall as a measure risk.

To reach this objective using the datum of the seven shares described at the previous section, it has been followed a three-step methodology. At the first one, several optimal portfolios are obtained in the sense of Markowitz (1952,1959). At the second one, are obtained the portfolios that minimize the risk calculated by Expected Shortfall, with three different levels of confidence (99%, 95% y 90%), for each expected yield of the optimal portfolios obtained at the previous stage. Finally, at the third stage the obtained portfolios are compared from the two perspectives. That is, the optimal weights of each asset for each pair of portfolios are compared with the same expected yield obtained in the previous two stages to identify similarities and differences between the optimal portfolios according to the different risk measures used, the variance of the yields and the Expected Shortfall in the three confidence levels considered. The following two subsections are about the first two stages, while the third stage is about in the next stage of the work.

4.1. The variance as a measure of risk: optimal portfolios in the sense of Markowitz (1952, 1959).

As it has been commented previously (see Datum section), this work is based on a previous work, which is about the application of the Markowitz model (1952, 1959) for the obtaining of optimal portfolios. In particular, the first stage of the applied methodology at the present study, corresponds with this obtention of optimal portfolios in the sense of Markowitz (1952, 1959) that was carried out in that previous work.

As it has been commented at the introduction, Markowitz (1952, 1959) developed his model focusing on the bases of the investor rational behavior risk aversion, that is, about the idea that the investor wishes profitability and rejects risk. Consequently, for the investor, an efficient portfolio is the one that represents the lower possible risk for a determined level of profitability, or in the same way, if it provides the maximum possible profitability for a given level of risk.

To get these efficient portfolios in the sense of Markowitz (1952, 1959), it has been applied the application Excel spreadsheets and its Solver tool (that allows to solve complete optimization problems) from the perspective of maximizing profitability given different levels of risk. The details of the procedure to obtain efficient portfolios in the sense of Markowitz (1952, 1959) are generally known and is not a relevant aspect for the present study. An important aspect to note is that one of the restrictions established when calculating efficient portfolios is that the weights of the assets in each portfolio are at least 3.1% to avoid possible errors in the assets.

4.2. Optimal portfolios under Expected Shortfall as a measure of risk.

In a similar way to how the efficient portfolios have been obtained in the sense of Markowitz (1952, 1959) in the previous stage, in this second stage has also been used the application of Excel spreadsheets and its Solver tool to obtain optimal portfolios under the Expected Shortfall as a measure of risk. The following explains the Excel functions involved and the technical procedure followed to obtain the portfolios that minimize the risk measured by the Expected Shortfall.

In Excel, the function *MENOR(matriu;k)* returns *k*th value smaller in a range of values, a column or a row. (Entry *matriu* in the function).

If we want the smallest value in the range of values, we should determine *k = 1*, meaning, *MENOR(matriu;1)*; to obtain the second smallest value in the range of values, we should determine *k = 2*, meaning, *MENOR(matriu;2)*; to obtain the third smallest value in the range of values, we should determine that *k = 3*, in other words, *MENOR(matriu;3)*, etc.

This function also can take into account more than one smaller value. For example, if we write *k = {1; 2}* (meaning, *MENOR(matriu; { 1; 2})*), the function takes into account the two smaller values in the range of values. If we write *k = {1; 2; 3}* (meaning, *MENOR (matriu; {1; 2; 3})*) the function takes into account the three smaller values included in the range of values, etc.

Then, this function can be combined with the following one: *MITJANA(matriu). T*his last function yields the average of a range of values (arithmetic mean). Therefore, combining these two functions and establishing. for example, *k = {1; 2}*, meaning,

MITJANA (MENOR (matriu; {1; 2}))

we obtain the average of the two lowest values of the range of values (la *matriu*); Combining these two functions and establishing *k = {1; 2; 3},* in other words,

MITJANA (MENOR (matriu; {1; 2; 3}))

we obtain the average of the three lowest values of the range of values etc.

Consequently, if we do have, for example, the set of portfolio yields situated in a column, is quite easy to calculate portfolio Expected Shortfall. We definitely should take into consideration the number of yields in the set of yields, and also the level of confidence that we want to establish for Expected Shortfall.

If the set of yields includes 100 yields and we want to calculate *ES (10%),* we have to calculate the average of the lowest yields (10%), concretely, the average of the ten lowest yields. 10 (= 100 x 10%) meaning,

ES(10%) = MITJANA(MENOR(matriu; { 1; 2; 3; 4; 5; 6; 7; 8; 9; 10}))

If we want to calculate *ES (5%)*, we do have to also calculate the average of yields (lowest than 5%)

so, the average of 5% (= 100 x 5%) lower yields, meaning,

ES(5%) = MITJANA(MENOR(matriu; { 1; 2; 3; 4; 5}))

Given that the objective of this paper is to compare optimal portfolios in the sense of Markowitz (1952, 1959) with optimal portfolios under the Expected Shortfall as a measure of risk, and given that the variance of the yields of a portfolio as a measure of the risk of the It is not directly comparable with its Expected Shortfall, the procedure used has been to start from the expected yields of the efficient portfolios in the sense of Markowitz (1952, 1959) calculated in the previous stage and look for the portfolio that minimizes the risk measured by the Expected Shortfall (for a certain level of confidence) for each of these expected yields.

At the end of the process, the same number of portfolios that were obtained applying the Markowitz approximation (1952, 1959) in the previous stage, with the same expected yields but minimizing the risk measured by the Expected Shortfall for a certain level of trust. Therefore, for a given expected yield, the weights of the assets that minimize the risk measured by the variance of the yields can be compared with the weights that minimize the risk measured by the Expected Shortfall.

The technical procedure to obtain these optimal portfolios under Expected Shortfall (for a certain confidence level) as a risk measure in Excel and applying its Solver tool has been as follows:

- In one column, we must obtain the yield of a portfolio, as the addition of the yields of each asset, multiplied by its weigh in the portfolio.

- Calculate expected yield of the portfolio as the average of all portfolio’s yields.

- Calculate the Expected Shortfall of the portfolio.

    -This intention will depend on the level of confidence and the number of portfolios yields.

- For example, if the establish level of confidence reaches 90% and the number of portfolio yields is 531, Expected Shortfall is the average of yields lower than 10%, in other words, yields lower than 53 (531 x 10% = 53,1).

- Once, the expected yield and the Expected Shortfall is determined, the function SOLVER can be used to calculate the weight of the portfolio's assets which minimizes the risk, measured by Expected Shortfall. In respect of the parameters SOLVER, it is important to take into account the following:

- Define a goal: cell where Expected Shortfall is calculated.

- For: choose max. (Take into account that Expected Shortfall is defined as the average of the portfolio's lowest yields, so, to minimize the risk measured by Expected Shortfall, this average should be maximized).

- Changing the cells that contain the variables: range of excel cells with the weights of the assets in the portfolio.

- Subject to restrictions:

- Each asset weight in the portfolio must be zero or positive (≥ 0), in practice, we are going to consider that the weight of each asset must be as minimum 0.1% (the same thing that has been considered when calculating the optimal portfolios according to the approach of Markowitz (1952, 1959) to avoid possible errors in calculations).

- The addition of all the asset weights must be equal to one.

- The cell, in which expected yield is calculated, should be identical to the expected yields for which you want to calculate the weight of the assets that minimize the risk.

These calculations have been carried out for three different confidence levels of the Expected Shortfall, specifically 99%, 95% and 90%. Before proceeding to compare the results obtained, it is important to highlight some problems that have been obtained in the calculation process in Excel.

On one side, in average yields of portfolios, used in Markowitz approach, weigh was concentrated in very few assets. That did not help us to compare them with calculations made with Expected Shortfall method. Because of that, we started to use other average yields in order to see if, in that case, the weight would be more distributed. That would make our comparisons much more interesting.

Moreover, we realized that Solver optimization problem, including Expected Shortfall, was restricted to a concrete range of yields.

Taking that into account, we carried out different checks, until reaching the following conclusion: in order to get reliable and valid results (given by Solver) to compare, we should add a range of yields. This range goes from minimum yield of 0.0095% to 0.1135%. Using lower yields than the minimal yield or bigger yields than the maximum yield, Solver gave estrange results for Expected Shortfall that couldn't be compared to the optimal portfolios according to Markowitz's approach (1952, 1959).

Lastly, we realized that Excel gave us a much more exact result calculating Expected Shortfall, if we equated all assets' weightings before using Solver. If we maintained the percentage obtained before, Solver gave us different results.

5. Comparative analysis of results

In this section we will analyze and compare the results we have obtained when constructing the different investment portfolios taking into account the four risk measures considered, the variance of portfolio yields, and the Expected Shortfall at the 99%, 95% and 90% confidence levels.

An investment portfolio is a set of assets in which money is invested diversified in order to generate a surplus value. In other words, an investment portfolio is the set of assets with an investor or saver carrying out its financial strategy. That is to say, it is the set of financial products to which the saver uses his money in order to obtain a profitability for it. The concept of investment portfolio introduces a global view of investments, taking into account the correlations that can occur between the different assets.

Taking into account all the aspects considered in the previous section, fifty optimal portfolios have been calculated in the sense of Markowitz (1952, 1959) whose expected yields were between 0.085% and 0.1135%. Therefore, fifty portfolios have been calculated with the same expected yields, but which minimize the risk measured by the Expected Shortfall for each of the three confidence levels considered (99%, 95% and 90%).

In the following subsections, we will comment on the results obtained in each of the companies that make up the different portfolios. In the first place, we will comment on the results obtained by optimizing the portfolios taking into account the risk measures considered individually, and secondly comparing the optimal portfolios according to Markowitz (1952, 1959) with the optimal portfolios considering the Expected Shortfall as a measure of risk.

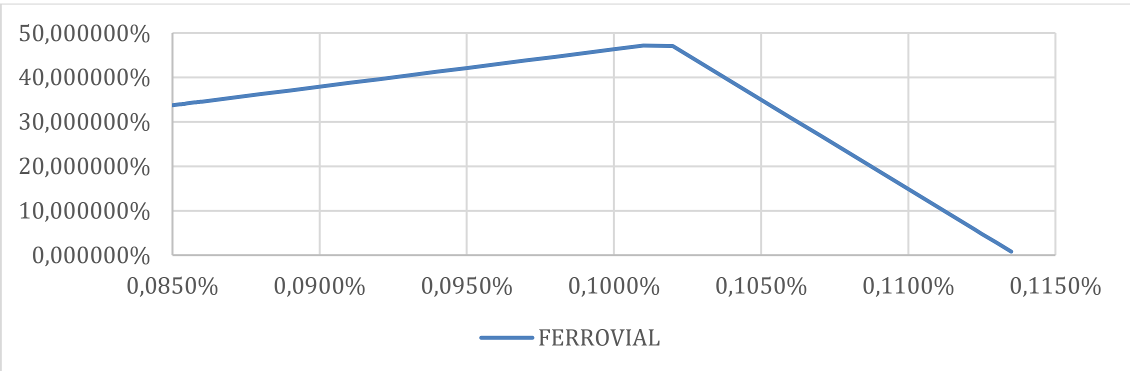
5.1. Results according to the different risk measures considered individually.

5.1.1. Optimal portfolios in the sense of Markowitz (1952, 1959): variance of yields as a measure of risk.

In the first place, it is important to note that, according to the results obtained, for the range of expected yields considered, the weights of Acciona, Grupo ACS, Bankinter and Mapfre in the optimal portfolios according to Markowitz's (1952, 1959) perspective are the minimum possible, that is to say the 0.10% established as a minimum when defining the optimization problem with Solver. This result means that, for each of the portfolios and focusing on these four companies, the weighting for these companies that makes each portfolio optimal is the minimum. Therefore, this result is telling us that these four companies are not adequate to minimize the risk (measured by the variance of the yields of the portfolio) for expected yields between 0.085% and 0.1135%.

In this way, we will now focus on the remaining companies that are: Ferrovial, Grifols and Iberdrola.

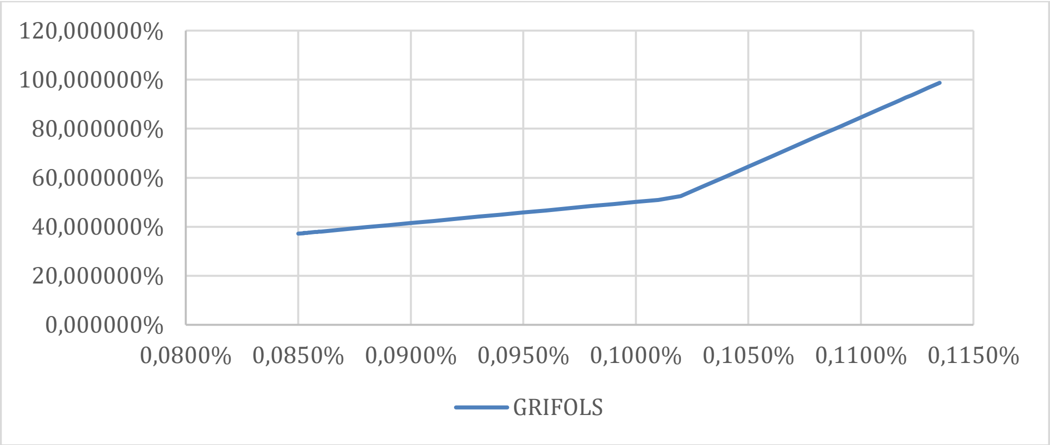
FIGURE 3. FERROVIAL WEIGHTS ACCORDING TO MARKOWITZ



Source: own elaboration

Starting with Ferrovial, we can observe that in the first portfolio (which has the lowest expected yield) Solver tell us that we should invest 3.724% of the capital. As we increase expected yield, Solver also increases the weighting of the asset, reaching the maximum weighting in portfolio number 33, with 0.1010% of expected yield. Solver gives this asset a 47.169% weighting of the total. From that portfolio on, as expected yield increases, weighting decreases to a point where Solver give us a weighting of 0.7944% for the last portfolio (the one with the higher expected yield). Finally, in respect of the progress of Ferrovial, it is essential to remark that between expected yields 0.1020% and 01120%, weighting, according to Markowitz, decreases 40.215%. That means that, in a difference of only 1% between expected yields, the weighting decreases in a significant way.

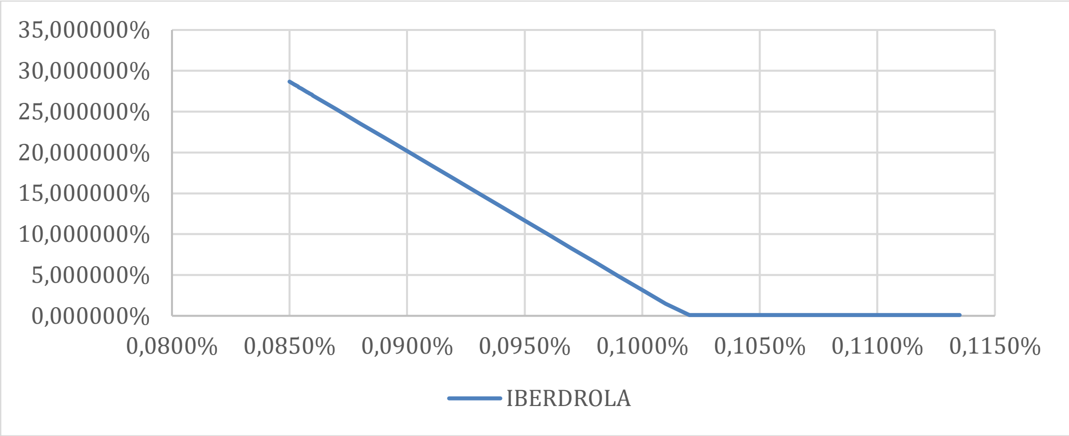
FIGURE 4. GRIFOLS WEIGHTS ACCORDING TO MARKOWITZ



Source: own elaboration

If we focus on the Grifols company evolution, we can highlight that Solver gives the bigger weighting for this particular asset. That means that it is the more attractive according Markowitz's model. In the lower expected yield (portfolio Nº1) with the weighting of 37.20%, and reaching 98.705% in the last portfolio, that is the portfolio with the higher expected yield. This implies that as we increase expected yield, Solver recommends investing more capital in this asset.

FIGURE 5. IBERDROLA WEIGHTS ACCORDING TO MARKOWITZ



Source: own elaboration

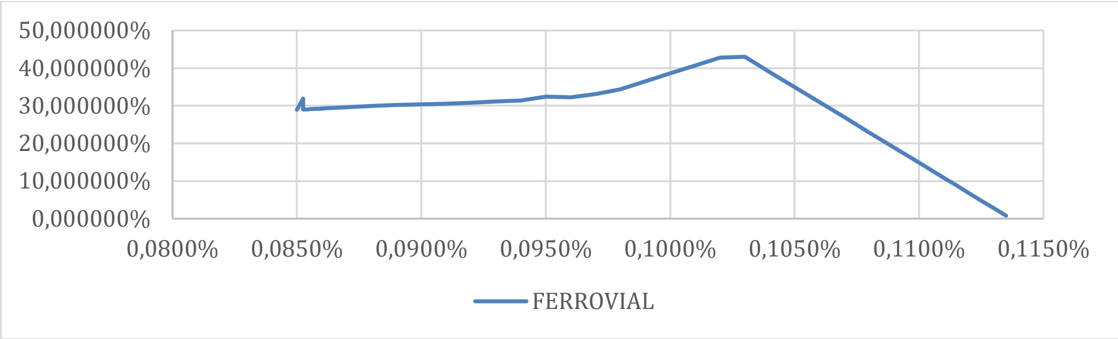
With respect to the interpretation of the results obtained according to Markowitz, we are going to discuss Iberdrola evolution. It starts with the weighting of 28.67% and decreases constantly until it reaches the minimum possible (0.10%) in portfolio nº 34.

One we have discussed obtained results following Markowitz's model (1952, 1959), we are going to repeat the process, but taking into account the results obtained according to Expected Shortfall method. It is important to remember that we have chosen three levels of confidence in order to perform calculations according to Expected Shortfall method. Results are going to be discussed and explained in parts, considering each level of confidence. For this, first of all, in a similar way to what has been done for the case of the weights of the different assets in the optimal portfolios in the sense of Markowitz (1952, 1959), the results obtained according to Expected Shortfall for each one of the chosen levels of reliability will be shown graphically and, after each graph, I will comment on the results.

5.1.2. Optimal portfolios under Expected Shortfall (99%) as a measure of risk.

First of all, it should be noted that the companies Acciona, Grupo ACS, Bankinter and Mapfre follow the same tendency as they did with Markowitz, but with a little difference. For an expected yield of 0.096%, Solver gives Bankinter a 1.438% weighting, and for a yield of 0.08525% , Solver gives Mapfre a 5.827% weighting. Except for these two cases, Solver gives for these four companies the minimal weighting possible. Therefore, the following analysis will focus on the three same companies on which the previous analysis has focused, Ferrovial, Grifols and Iberdrola.

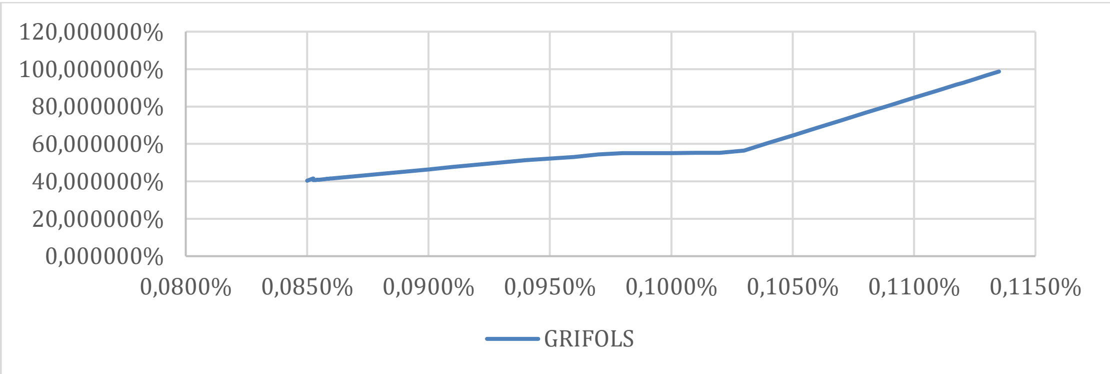
FIGURE 6. FERROVIAL WEIGHTS ACCORDING TO EXPECTED SHORTFALL (99%)



Source: own elaboration

Focusing on the three companies that we can compare, we start with Ferrovial. We notice its first portfolio with a 28.889% weighting, in portfolio nº3, it decreases, but in the following ones it increases until reaching the maximum level, in portfolio nº 35 (expected yield = 0.1030%) From that point it begins to decrease again until the last portfolio with a 0.7947% weighting.

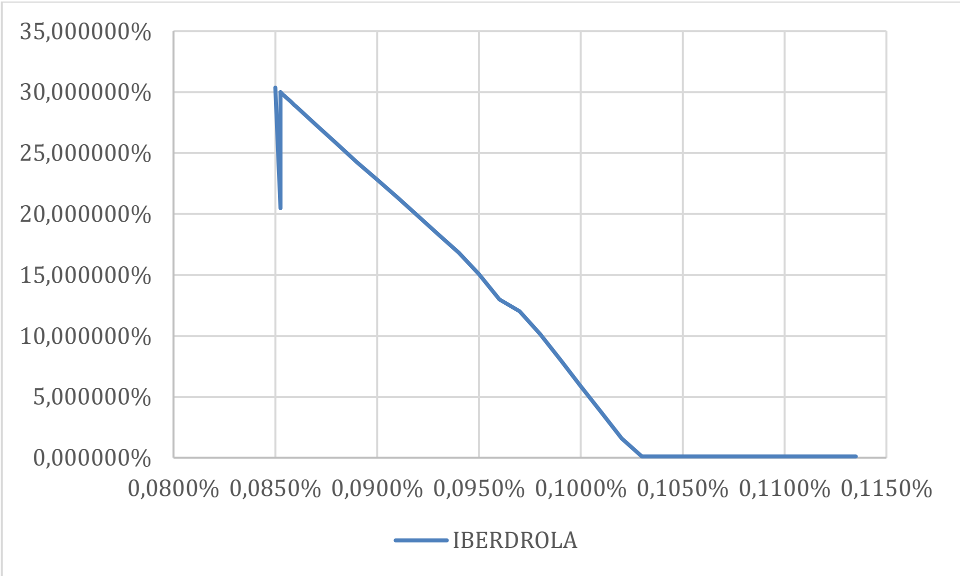
FIGURE 7. FERROVIAL WEIGHTS ACCORDING TO EXPECTED SHORTFALL (99%)



Source: own elaboration

Focusing on Grifols evolution, following Expected Shortfall method and with a level of confidence that reaches 99%, Solver gives the higher weightings in all the portfolios of this asset. Whereby, Grifols is the most attractive asset of each portfolio. It can also be observed the first portfolio with a 40.34% weighting and that, in spite of the fact that in portfolio nº 3 it decreases a little, in the following it increases until the last portfolio where it has a weighting of 98.70%.

FIGURE 8. FERROVIAL WEIGHTS ACCORDING TO EXPECTED SHORTFALL (99%)



Source: own elaboration

Lastly, it is interesting to observe Iberdrola evolution. We are able to see how its weighting decreases as the expected yield increases until the portfolio that has an expected yield of 0.1030%, Solver gives the minimum weighting (0.10%) that it is maintained until the last portfolio's performance.

Next, the evolution of different assets according to Expected Shortfall with a level of confidence of 95% is discussed.

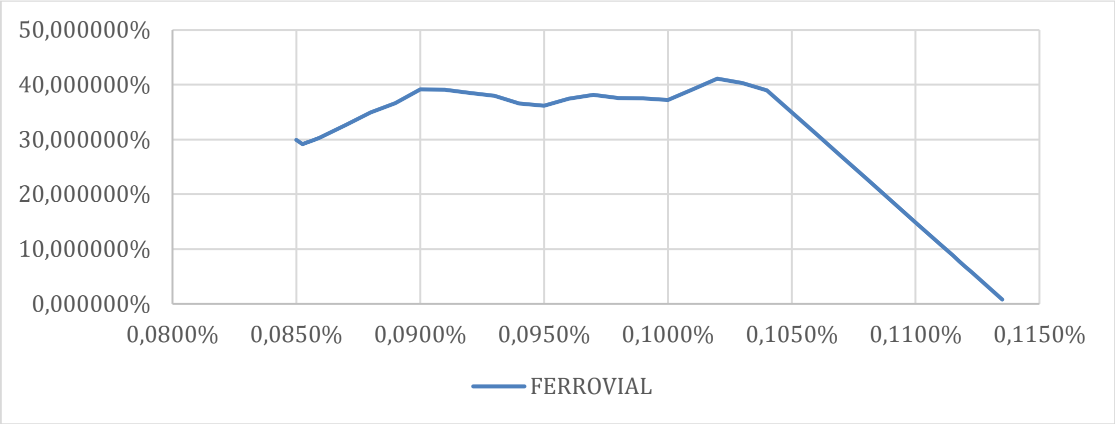
5.1.3. Optimal portfolios under Expected Shortfall (95%) as a measure of risk.

This analysis of the weights of the different companies in the optimal portfolios under the Expected Shortfall for a level of confidence of 95% as a measure of risk will also focus on Ferrovial, Grifols and Iberdrola because, as in the optimal portfolios according to the outlook of Markowitz, the weights of the companies Acciona, Grupo ACS, Bankinter and

Mapfre is the minimum for all the portfolios in the expected profitability range considered. That is, a weighting of 0.10%.

As it is going to show, the results are very similar to those obtained according to Expected Shortfall with a level of reliability of 99%. However, some differences can be drawn.

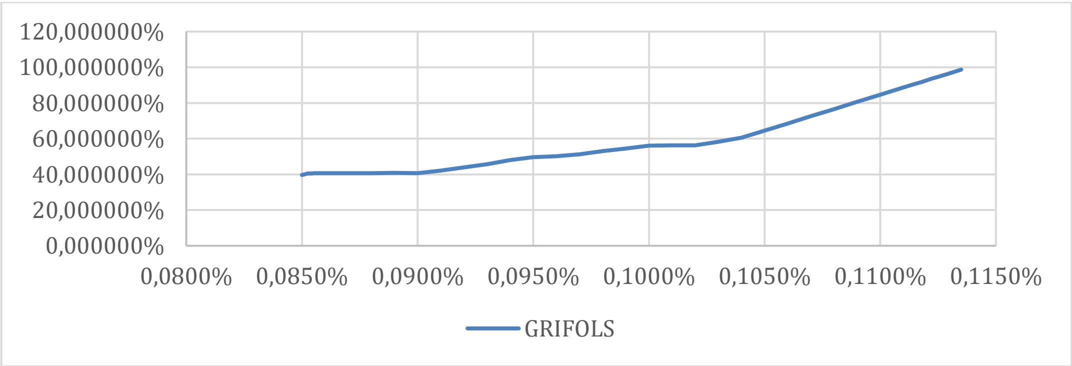
FIGURE 9. FERROVIAL WEIGHTS ACCORDING TO EXPECTED SHORTFALL (95%)



Source: own elaboration

As can be observed, presented data are very similar to obtained data according to Expected Shortfall with a confidence level of 99%. However, there are some differences. On one hand, Ferrovial asset has the higher weighting level with confidence level of 95%. Its maximum weighting is in portfolio nº 34, and with a confidence level of 99% it is in portfolio nº 35. It is observed that as expected yield increases, its weightings increase or decrease, although the trend is upwards. From its maximum weighting situated in portfolio nº 34, its weighting decreases until reaching the last portfolio with 0.7942%.

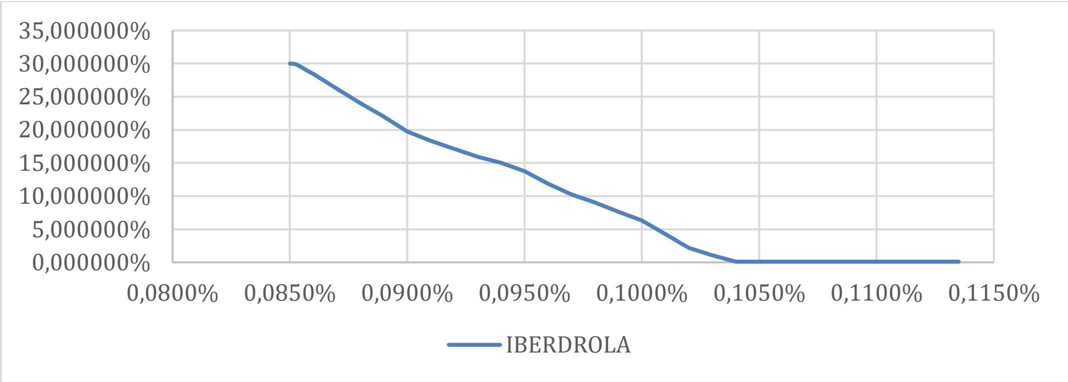
FIGURE 10. GRIFOLS WEIGHTS ACCORDING TO EXPECTED SHORTFALL (95%)



Source: own elaboration

Grifols results are very similar, and, even though, portfolio nº 1 has the lowest weighting level with regard to the confidence level of 99%. This asset remains the more attractive, according to SOLVER results. Moreover, in the last portfolio, obtains a 98.705% weighting. As we have explained in the Ferrovial case, Grifols is more unstable with a confidence level of 95% and its weightings fluctuate up and down as expected yield increases. Although, it is from expected yield of 0.090%, its weighting increases until the last portfolio.

FIGURE 11. IBERDROLA WEIGHTS ACCORDING TO EXPECTED SHORTFALL (95%)



Source: own elaboration

We have already said that Grifols obtains similar results, so Iberdrola follows the same path. It is observed how it follows the same tendency and the only remarkable point is that with a confidence level of 99%, this asset obtains the minimum weighting with an expected yield of 0.1030% and with a confidence level of 95% it obtains the minimum weighting with an expected yield that reaches 0.1040%.

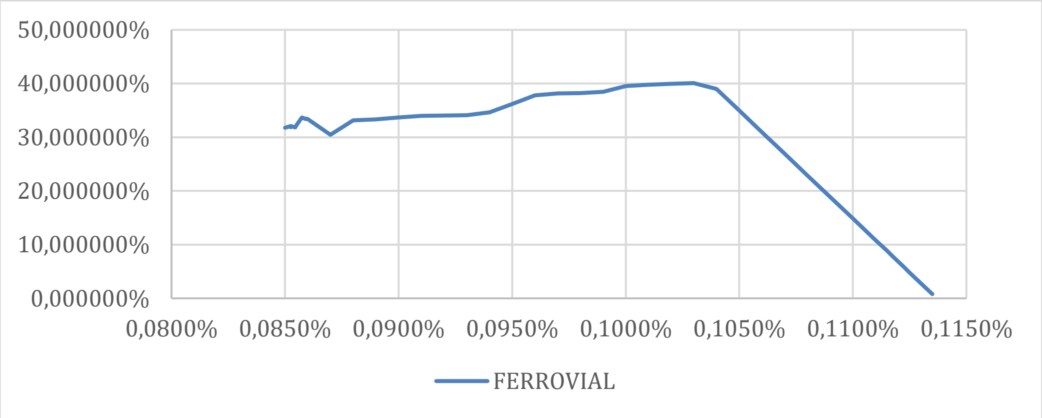
Next, the evolution of the different assets according to Expected Shortfall and with a level of reliability of 90% is commented.

5.1.4. Optimal portfolios under Expected Shortfall (90%) as a measure of risk.

The optimal portfolios obtained under the Expected Shortfall for a confidence level of 90% as a measure of risk are still very similar if compared with the Expected Shortfall for a confidence level of 99% as compared to the Expected Shortfall for a 95% confidence level, although they are more similar to the results obtained with the Expected Shortfall for a confidence level of 95%.

In the first place, it should be noted that, as in the optimal portfolios in the sense of Markowitz (1952, 1959) and in the optimal low portfolios, the ES (95%) as a risk measure, the Acciona companies, ACS Group, Bankinter and Mapfre continue to have a weighting of 0.10% in all portfolios for the range of expected yields considered. For this reason, the following analysis, like the previous ones, focuses on Ferrovial, Grifols and Iberdrola.

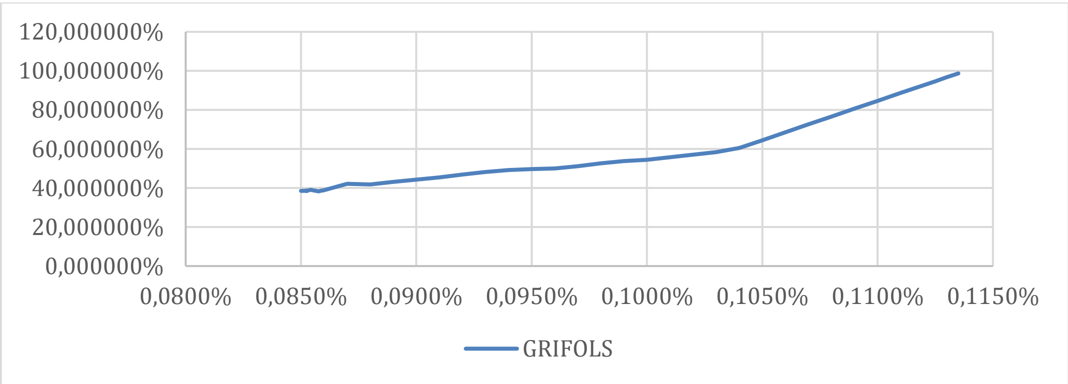
FIGURE 12. FERROVIAL WEIGHTS ACCORDING TO EXPECTED SHORTFALL (90%)



Source: own elaboration

The company Ferrovial obtains in the first portfolio, its highest weighting (31.77%) in comparison with the first portfolios with confidence levels of 99% and 95% (28.88% y 29.90% respectively). Moreover, it obtains its maximum weighting in portfolio nº 35 (40.077%) and from that portfolio on, its weighting decreases until 0.7947%.

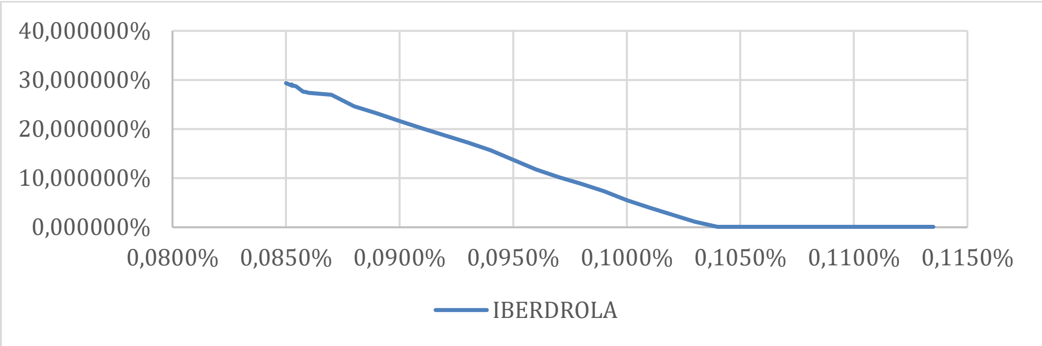
FIGURE 13. FERROVIAL WEIGHTS ACCORDING TO EXPECTED SHORTFALL (90%)



Source: own elaboration

About Grifols, Solver give us a 38.46% weighting in the first portfolio, and as expected yield increases, weightings increase and decrease until portfolio nº 20. From there, weightings increase until the last portfolio, in which Solver gives a 98.705% weighting. So, it can be said that with a confidence level of 90%, Grifols still remains the most attractive according to Solver results.

FIGURE 14. IBERDROLA WEIGHTS ACCORDING TO EXPECTED SHORTFALL (90%)



Source: own elaboration

Lastly, Iberdrola obtains a 29.35% weighting in the first portfolio and a 0.10% weighting from portfolio nº 36 to the last one portfolio.

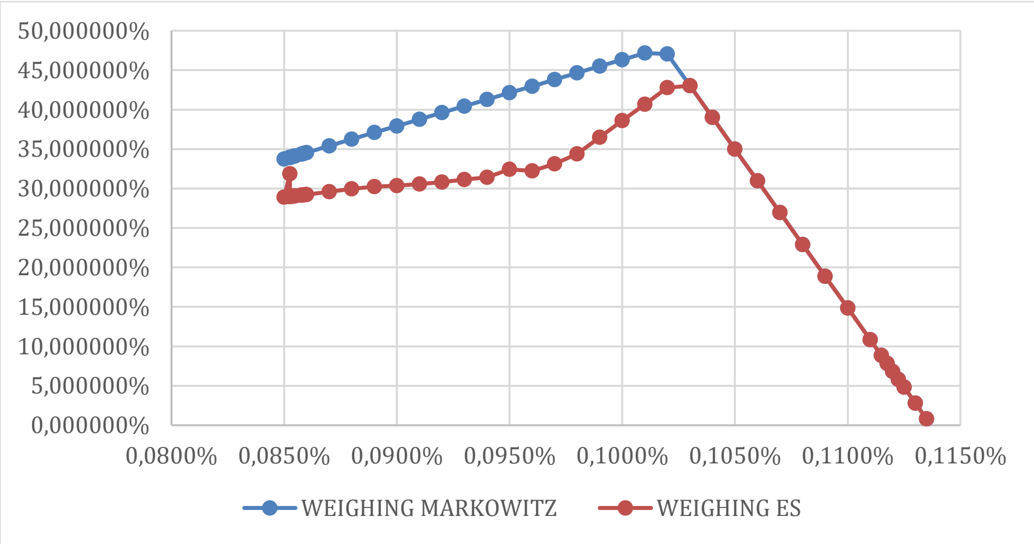
5.2. Joint analysis of the results.

Once the results have been analyzed individually, we want to discuss them collectively. In other words, we are going to compare the obtained results according to Markowitz's method with the results obtained according Expected Shortfall, having in mind the different levels of confidence. After having compared them individually, it has been seen that, the companies Acciona, Grupo ACS, Bankinter y Mapfre in minimal levels. Solver weights to the minimum. Therefore, in his global comparison we are not going to take into account these companies, since, we have obtained the same results with both, Markowitz's method and Expected Shortfall. To compare the remaining companies (Ferrovial, Grifols and Iberdrola); we have made graphics for the comparison to be more visual.

5.2.1. Variance of the yield vs Expected Shortfall (99%) as risk measures.

In the first place, we compare obtained results according to Markowitz's method with obtained results according Expected Shortfall with a confidence level of 99%.

FIGURE 15. FERROVIAL - MARKOWITZ VS EXPECTED SHORTFALL (99%)

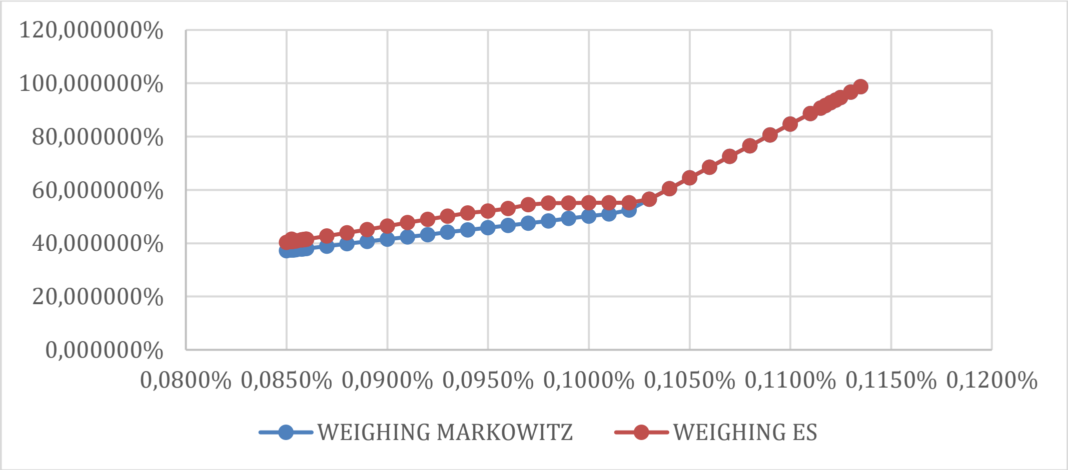


Source: own elaboration

In the case of Ferrovial, it can be observed that, it has higher weightings according to Markowitz's method than according to Expected Shortfall. In both models the trend is the following one: as expected yield increases, weightings also increase. According to Expected Shortfall weightings don't follow a clear trend, since they increase and decrease according to the expected yield increasement, although the trend is upward.

From the moment that the expected profitability reaches 0.1030%, values are identical and remain unchanged until the last portfolio, considering that from that specific expected yield, weightings decrease.

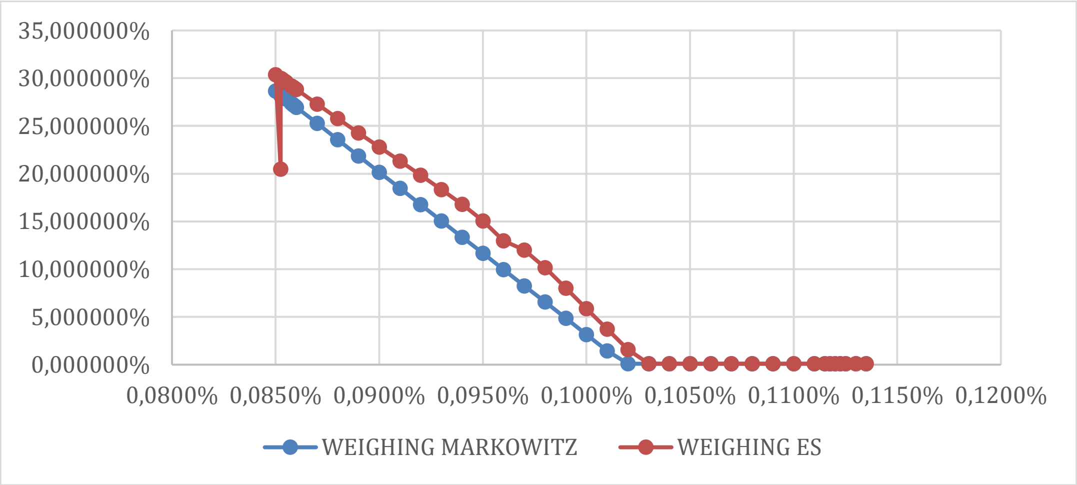
FIGURE 16. GRIFOLS - MARKOWITZ VS EXPECTED SHORTFALL (99%)



Source: own elaboration

In Grifols, data are so similar, even though according to Expected Shortfall, Solver gives us higher weightings than according to Markowitz's method the trend is the following one: as expected yield increase, weightings increase, being the most attractive asset of all portfolios. As in the case of Ferrovial, datum equalize to the expected yield of 0.1030% and it remains unchanged until the last portfolio.

FIGURE 17. IBERDROLA - MARKOWITZ VS EXPECTED SHORTFALL (99%)



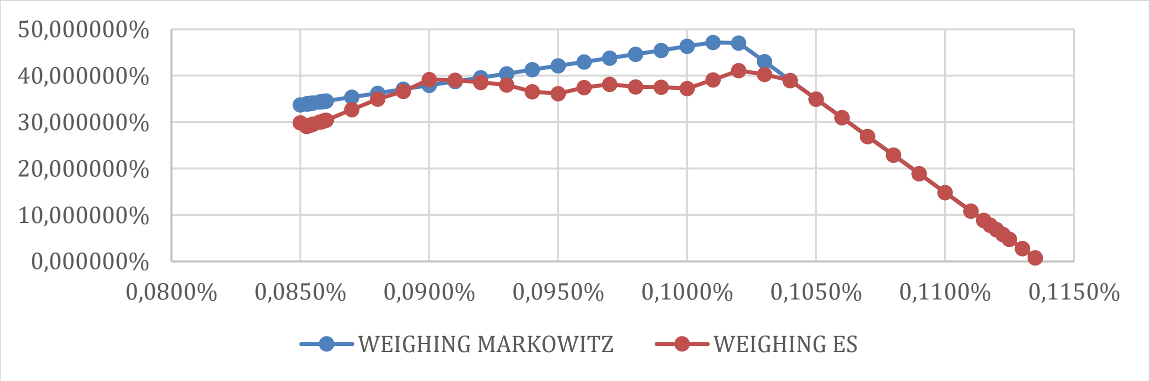
Source: own elaboration

Regarding Iberdrola, according to Expected Shortfall, Solver gives us the higher weightings. Moreover, as expected yield increases, weightings decrease. Just like in the other two companies, results according to Markowitz and ES, both results converge when expected yield reaches 0.1030%, and remain unchanged until the end, although with a particularity, for that yield, the weighting is 0.10%.

5.2.2. Variance of the yield vs Expected Shortfall (95%) as risk measures.

After this comparison, now we compare both results according to Markowitz and Expected Shortfall with a confidence level of 95%.

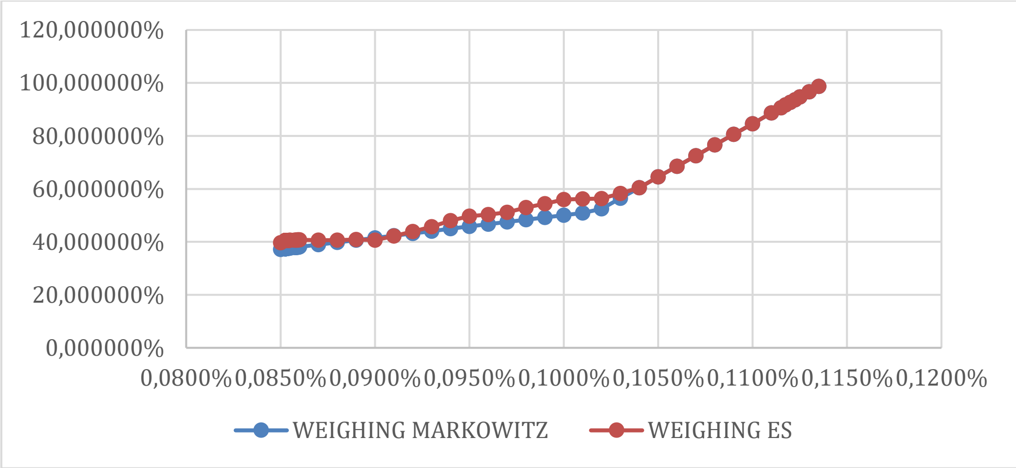
FIGURE 18. FERROVIAL - MARKOWITZ VS EXPECTED SHORTFALL (95%)



Source: own elaboration

In the case of Ferrovial, we noticed that, according to Expected Shortfall results are very varied and don't really follow a fix trajectory. According to Markowitz's method, we obtain higher weightings, and, it has a clear tendency: weightings increase as expected yield increase. From expected yield 0.1020% weightings of both models start to decrease, and in the expected 0.1040% results equal and remain unchanged until the last portfolio.

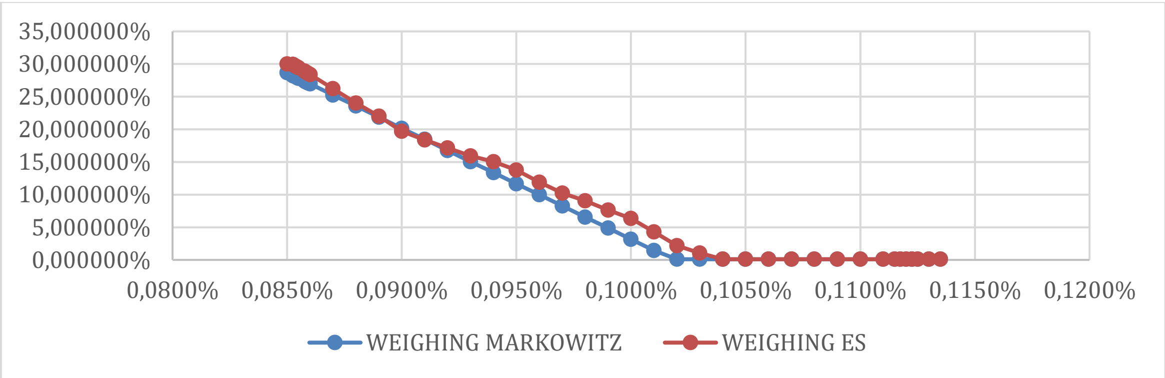
FIGURE 19. GRIFOLS - MARKOWITZ VS EXPECTED SHORTFALL (95%)



Source: own elaboration

In Grifols, results are very similar, although, using Expected Shortfall method, we obtain higher weightings. It can also be observed that the tendency followed is: weightings increase as expected yield increase. From the value 0.1040% of expected yield, weightings are identical in the two models until the last portfolio.

FIGURE 20. IBERDROLA - MARKOWITZ VS EXPECTED SHORTFALL (95%)



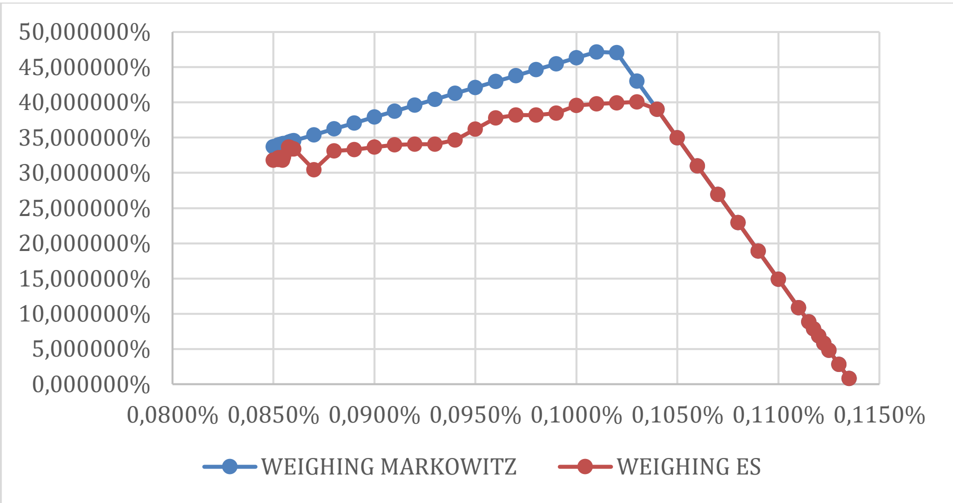
Source: own elaboration

Expected Shortfall also gives us higher weightings in the case of Iberdrola. The tendency followed by both models is the following one: weightings decrease, as expected yield increases. As it happens in Ferrovial y Grifols, results start to be identical when the expected yield reaches 0.1040% and remain unchanged until the end. I would like to mention that, as it can be observed, in the point where expected yield reaches 0.1020%, according to Markowitz, the weighting is the minimal however, according to Expected Shortfall, it is not. It is in the expected return 0.1040% when according to Expected Shortfall the weighting it is equalized, also being the minimum.

5.2.3. Variance of the yield vs Expected Shortfall (90%) as risk measures.

To end with this subsection in which results obtained using both methods have been analyzed and compared, I am going to make the last comparison between the obtained results using Markowitz's method and the obtained results using Expected Shortfall with a confidence level of 90%.

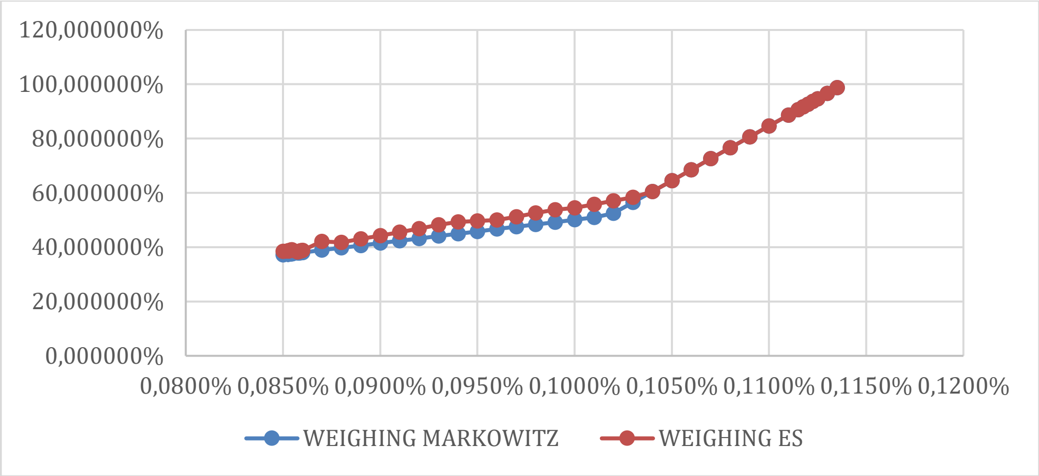
FIGURE 21. FERROVIAL - MARKOWITZ VS EXPECTED SHORTFALL (90%)



Source: own elaboration

As can be observed, for Ferrovial, Solver gives us higher weightings according to Markowitz. According to Expected Shortfall weightings are very varied and there is not a determined evolution line. In this sense, Markowitz is more stable and follows a steady path. For expected yield 0.1040%, the weightings equalize and remain unchanged until the last portfolio.

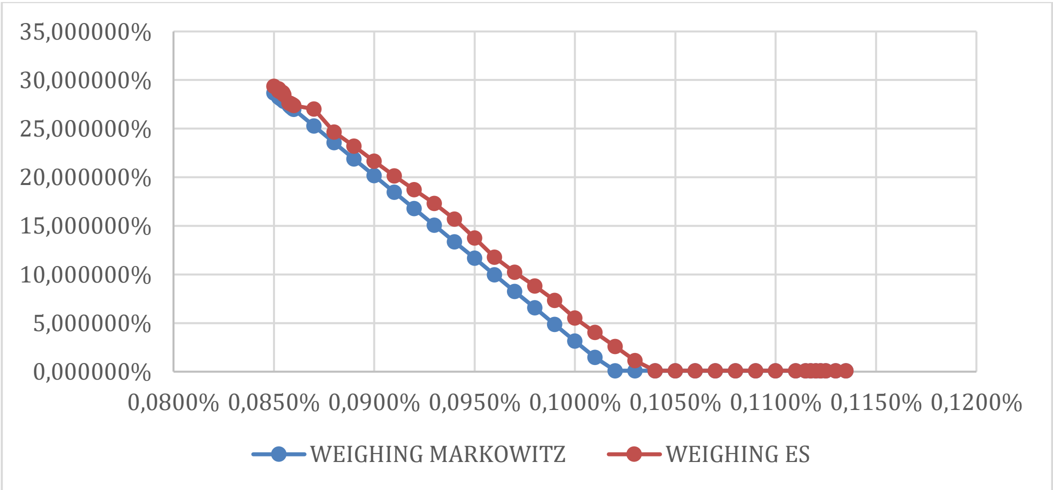
FIGURE 22. GRIFOLS - MARKOWITZ VS EXPECTED SHORTFALL (90%)



Source: own elaboration

In respect to Grifols, obtained data are very similar to the previous ones, although, according to Expected Shortfall, the weightings are a little bit higher. As in Ferrovial case, both weightings (from both methods) are equalized in expected yield 0.1040% and remain unchanged until the end. Moreover, in both models, as expected yield increase, the weightings increase too.

FIGURE 23. IBERDROLA - MARKOWITZ VS EXPECTED SHORTFALL (90%)



Source: own elaboration

Finally, we can observe Iberdrola. Its results, according to Expected Shortfall, are better, giving higher weightings than according to Markowitz. In both models, the trend is the following one: as expected yield increases, weightings decrease. As in the two previous companies, weightings are equalized when expected yield reaches 0.1040% and remains unchanged until the end.

6. Discussion of the results

Once analyzed the results obtained in the optimization of portfolios according to Markowitz (1952, 1959) and under the Expected Shortfall (for the three levels of confidence considered) as a measure of risk, and once these results are compared with each other, in this section we will try to synthesize these results in the observed general trends, which will allow to extract the most important conclusions of the present study.

First, it should be emphasized that, regardless of the three levels of confidence considered for the ES, the weights of the three analyzed assets in the corresponding optimal portfolios under the ES as a measure of risk follow the same trend with respect to their weightings in the optimal portfolios in the sense of Markowitz (1952, 1959).

In the case of Ferrovial, for low expected yield levels of the portfolio, its optimal weights under the ES as a measure of risk are lower than its weights under the variance of the portfolios yield as a measure of risk. However, in both cases the weights show an increasing trend with respect to the expected yield until the optimal weights under both risk measures end up coinciding with and expected yield of the portfolio.

With respect to Grifols, the trend of its optimal weights under both risk measures is also increasing for low expected portfolio yields, but unlike Ferrovial, under the ES the Grifols optimal weights are relatively greater than if the variance of the yields of the portfolio is considered as a measure of risk. In this case, the optimal weights of the asset under both risk measures end up coinciding, but in this case presenting a more pronounced upward trend than when they not coincide.

Iberdrola’s optimal weights under the ES as a measure of risk for low expected levels of the portfolio are also relatively higher than in the case where the variance of the portfolios yields is considered as a measure of risk, but its trend with respect to the expected yield is decreasing under both risk measures. After a certain level of expected profitability of the portfolio, the optimal weights of Iberdrola under both irrigation measures end up coinciding, as with Ferrovial and Grifols. However, in the case of Iberdrola, this coincidence corresponds to the fact that the asset is no longer relevant to minimize the risk of the portfolio for the levels of expected profitability.

Another curious general result is that the optimal weights of the three assets under the ES for a certain level of confidence and under the variance of the yields of the portfolio as measures of the risk coincide from the same level of expected yield of the portfolio. In particular, this coincidence in the optimal weights is given from an expected yield of the portfolio equal to 0.1030% when the ES for a confidence level of 99% is considered as a measure of the risk. When the ES for a 95% confidence level is considered as a measure of risk, the coincidence between the optimal weights is given from an expected yield of 0.1040%. Curiously enough, this expected level of profitability, based on the coincidence between the optimal weights, is maintained when the ES is considered as a risk measure for a confidence level of 90%.

Finally, it should be noted that, in general, the optimal weights of the three assets considered under the ES for a confidence level of 95% as a measure of risk are the most similar to their weightings under portfolio optimization in the sense of Markowitz (1952, 1959) although weights under both perspectives is given for an expected level of profitability higher than when considering the ES for a level of confidence of 99% as a measure of risk. In addition, the behavior of the optimal weights under the ES for a 95% confidence level as a measure of risk is generally more erratic (especially that of Ferrovial weights) than when measuring the risk though the ES for confidence levels of 99% and 90%.

Therefore, according to the general results obtained, it seems clear that although the variance of portfolio yields, and ES are very different measures of risk, of which only the last one fulfils all the properties that are considered adequate for a good measure of risk, in the context of portfolio optimization, both measures of risk lead to the same result from a certain level of expected yield on the portfolio. Furthermore, within this general trend, the ES for a 95% confidence level is the measure of risk under which the optimal portfolios are more similar to those obtained according to the approach of Markowitz (1952, 1959), even though the level of profitability for which there is convergence of results in the optimal portfolios according to both perspectives (the one that considers the ES as a measure and the one that considers the variance of the yield of the assets) seems to be decreasing with respect to the level of reliability of the ES.

7. Conclusion

This study compares the optimal portfolios obtained, on the one hand, according to the approach of Markowitz (1952, 1959) and, on the other hand, under the expected deficit (for the three confidence levels considered) as risk measures to optimize asset portfolios.

Therefore, the objective of the study can be seen as an attempt to study the possible biases that introduce a measure of non-coherent risk, such as the variance of returns, in the optimization of portfolios.

I would like to warn the reader that, from this study, conclusions can not be drawn in broad strokes, since the time to carry it out is limited and, in the same vein, the conclusions drawn. Even so, and focusing on the fifty portfolios analyzed, the conclusions are valid and can be used for a more complete future work. In addition, this work can introduce the reader into the great world of asset portfolios and in calculating the risk of these portfolios given an expected return.

As discussed in the previous section and in the context of portfolio optimization, both risk measures analyzed lead to the same result based on a certain level of expected profitability from the portfolio. Furthermore, within this general trend, the Expected Shortfall for a 95% confidence level is the measure of the risk under which the optimal portfolios are more similar to those obtained according to Markowitz's approach (1952, 1959). Even so, regardless of the three confidence levels considered for the Expected Shortfall, the weights of the three assets analyzed in the corresponding optimal portfolios under the Expected Shortfall as a measure of risk follow the same trend with respect to their weights in the optimal portfolios in the sense of Markowitz (1952, 1959).

Another conclusion that, indirectly, can be drawn from this study is that no rational investor will place their capital in a single asset. The appropriate strategy consists in distributing the funds between two or more assets, in proportions that each investor will have to establish, according to the yield and / or risk that they intend to obtain or assume, respectively, in their investment. The reason for this is that the yields of the securities are not perfectly correlated with each other, and the investor can use these small asymmetries to mitigate the risk more than proportional to the performance. In the case of this study, we have focused on calculating the risk of different portfolios of assets given expected yields.

The risk is always, or must be present, in the mind of the investor, and the aversion to risk that each investor has will depend, to a large extent, on the type of investment he makes, his training and the time horizon of his investment, in definitive, of the investor profile.

Finally, I would like to emphasize the large losses that can be obtained when investing in asset portfolios. Not only is it worth to have good mathematical knowledge of how to minimize risk, it also takes years of experience and a high level of knowledge about the market in which you want to invest.

Bibliography

Acciona.com. (2019). ACCIONA: Infraestructuras Sostenibles y Energías Renovables. [online] Available at: https://www.acciona.com/es/ [Accessed 6 Mar. 2019].

Bankinter. (2019). Banca online y servicios financieros | Particulares | Bankinter. [online] Available at: https://www.bankinter.com/banca/inicio [Accessed 29 Apr. 2019].

Benati, S. and Rizzi, R., 2007. A mixed integer linear programming formulation of the optimal mean/value-at-risk portfolio problem. European Journal of Operational Research, 176(1), pp.423-424.

Campbell, R., Huisman, R., and Koedijk, K., 2001. Optimal portfolio selection in a Value-at-Risk framework. Journal of Banking & Finance, 25(9), pp.1789-1804.

Corporativo MAPFRE. (2019). Grupo MAPFRE Corporativo - Acerca de MAPFRE. [online] Available at: https://www.mapfre.com/corporativo-es/ [Accessed 5 Mar. 2019].

Díaz Cáceres, A. (2014). *Una Comparación de Medidas de Riesgo Financiero*. Grado. Universidad de Chile.

Ferrovial. (2019). Operador global de infraestructuras y servicios - Ferrovial. [online] Available at: https://www.ferrovial.com/es/ [Accessed 11 May 2019].

Grifols. (2019). Grifols – Home Page Oficial. [online] Available at: https://www.grifols.com/es/home [Accessed 6 Apr. 2019].

Grupoacs.com. (2019). Grupo ACS. [online] Available at: https://www.grupoacs.com/ [Accessed 3 Apr. 2019].

Hult, H., Lindskog, F., Hammarlid, O. and Rehn, C. (2012). *Risk and portfolio analysis*. New York: Springer.

Iberdrola.es. (2019). Ofertas, planes y tarifas para luz, gas y electricidad - IBERDROLA. [online] Available at: https://www.iberdrola.es/ [Accessed 6 Apr. 2019].

Isaksson, D., 2016. Robust Portfolio Optimization with Expected Shortfall. PhD. KTH Royal Institute of Technology. Available at https://www.math.kth.se/matstat/seminarier/reports/M-exjobb16/160601d.pdf

Lehikoinen, K., Salo, A. and Jespersen, M. (2019). Development of Systematic Backtesting Processes of Value-at-Risk. [online] Semanticscholar.org. Available at: https://www.semanticscholar.org/paper/Development-of-Systematic-Backtesting-Processes-of-Lehikoinen-Salo/0c6c64126a2d5108e47c8823e995f381d3112753 [Accessed 12 May 2019].

Markowitz, H.M., 1952. Portfolio Selection. *The Journal of Finance*, 7(1), pp.77-91.

Markowitz, H.M., 1959. Portfolio Selection: *Efficient Diversification of Investments*. New York: John Wiley & Sons, Inc.

Pérez Hernández, I. (2013). Teoría de la Cartera (Portafolio) y el Análisis de Riesgo Financiero. GestioPolis. Available at: https://www.gestiopolis.com/teoria-de-la-cartera-portafolio-y-el-analisis-de-riesgo-financiero/ [Accessed 16 Mar. 2019].

Villaizán Castro, J. (2017). VaR: *Comparación de Tres Metodologías para la Medición del Riesgo de Mercado de Posiciones de Cambio*. Máster. Universidad del Pacífico.

Yoshida, Y., 2009. *An estimation model of value-at-risk portfolio under uncertainty*. Fuzzy Sets and Systems, 160(22), pp.3250-3262.