To appear in the *International Journal of Systems Science*
Vol. 00, No. 00, Month 20XX, 1–25

Co-design of H-infinity jump observers for event-based measurements over networks

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(*Received 00 Month 20XX; final version received 00 Month 20XX*)

This work presents a strategy to minimize the network usage and the energy consumption of wireless battery-powered sensors in the observer problem over networks. The sensor nodes implement a periodic send-on-delta approach, sending new measurements when a measure deviates considerably from the previous sent one. The estimator node implements a jump observer whose gains are computed off-line and depend on the combination of available new measurements. We bound the estimator performance as a function of the sending policies and then state the design procedure of the observer under fixed sending thresholds as a semidefinite programming problem. We address this problem first in a deterministic way and, to reduce conservativeness, in a stochastic one after obtaining bounds on the probabilities of having new measurements and applying robust optimization problem over the possible probabilities using sum of squares decomposition. We relate the network usage with the sending thresholds and propose an iterative procedure for the design of those thresholds, minimizing the network usage while guaranteeing a prescribed estimation performance. Simulation results and experimental analysis show the validity of the proposal and the reduction of network resources that can be achieved with the stochastic approach.

Keywords: State estimation; Networked control systems; Wireless sensor network; event-based sampling; send-on-delta; Co-design.

1. Introduction

With the increasing use of network technologies for process control, researchers focus on the reduction of the network data flow to increase flexibility under the addition of new devices (see Chen et al. (2011); Nagahara et al. (2013)). The sensor nodes can help by reducing their data transmissions with an event-based sending strategy (see Lunze and Lehmann (2010)), what furthermore helps to decrease maintenance costs if they are wireless and self-powered, as stated in Stark et al. (2002); Ploennigs et al. (2010). Some examples of using an energy-efficient sampling strategy in real-world applications are Beschi et al. (2014a,b); Ruiz et al. (2014)

State estimation plays a key role in networked control systems as the state of the plant is rarely directly measured for control purposes and because the output measurements are irregularly available due to communication constraints or packet dropouts (see Chen et al. (2011); Qiu et al. (2012)). The approaches found in the literature to address the state estimation problem with event-based sampling can be classified depending on the sending policy, and on the communication or computational resources required on the sensor nodes. The authors in Nguyen and Suh (2007); Suh et al. (2007) use a send-on-delta (SOD) strategy where the sensor node decides whether to send a new measurement if the actual acquired one differs more than a given threshold with respect to the last sent one. With those measurements, the estimator node implements a modified Kalman filter that uses the last acquired data and modifies the update equation to

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account the lack of data by means of including a virtual noise. In the work Nguyen and Suh (2008) each node uses the integral of the difference between the last acquired measurement and the last sent one to decide whether sending a new sample (send-on-area), while the authors in Sijs and Lazar (2012) combine SOD and time-triggered strategies in the sensor nodes. In other works like Battistelli et al. (2012); Millán et al. (2013) the authors include an state estimator in each sensor node to decide the sending of new data (output or state estimation), while in Wu et al. (2013) the authors impose the sensor node to receive and process several information to decide whether it should send the measurement.

Under the motivation of reducing the computational effort of the estimator and the sensor nodes, we use a jump linear estimator that at each instant uses a precomputed gain that depends on the availability of new measurements, and the nodes implement a send-on-delta strategy with fixed thresholds. With the aim of extending the approaches found in the literature to a wider class of disturbances, we obtain the gains that guarantee an H_∞ attenuation level based on a linear matrix inequalities (LMI) problem. With the aim of having less conservative results, we also obtain the range of probabilities of having new measurements with the send-on-delta mechanism. In this case we bound the H_∞ attenuation level for all the possible probabilities in the range with sum of squares (SOS) techniques Chesi (2010).

The use of a jump linear estimation instead of a time varying one, and the use of LMI formulation of the problem would allow to easily extend the proposal of this work to face, for instance, model uncertainties, models depending on time-varying parameters or sector-bounded nonlinearities, time-delays, packet dropouts or quantization (see, for instance, Qiu et al. (2010, 2015, 2009)). The LMI formulation of the problem results in the calculation of a bound on the state estimation error and the possibility of extending the observer design presented in this work to the design of inferential controllers or fault diagnosis systems.

Some works have shown that there is a trade-off between communication rate and estimation quality Wu et al. (2013). The authors in Wang and Lemmon (2009); Dai et al. (2010); Gaid et al. (2006); Irwin et al. (2010) named the problem of optimizing the network usage while assuring some performance measurement as co-design problem. The works Sijs and Lazar (2012); Nguyen and Suh (2009) addressed this problem with the time-triggering condition, and Suh et al. (2007) addressed it deciding the threshold levels of sensors implementing a SOD strategy. In the last work the authors modeled the network usage with a Gaussian probability distribution of the system outputs.

Motivated by extending the applicability of the co-design procedure to more general cases, we use the bounds on the probability of having new transmissions to measure the network usage, and to guarantee tight bounds of the achievable performance of the estimator.

We consider the value of the threshold Δ in each sensor node as a trade-off parameter between the network usage and the estimation performance. When the thresholds are fixed, we obtain a set of constant estimator gains that maximizes the estimator performance following different strategies. First, we assume that no information about the outputs is known and develop a deterministic approach that guarantees poly-quadratic stability and a bound on the RMS norm of the state estimation error. For the second strategy, we assume some information about the outputs distribution and develop different stochastic approaches formulated in terms of the probabilities of output transmissions, that guarantees mean square stability and a tighter bound on the RMS norm. Then, we address a co-design strategy with an iterative optimization problem that returns both the estimator gains and the value of Δ that leads to the lowest data transmission for a given bound on the RMS norm.

The main contributions of this work are that we propose three alternatives for observer design over send-on-delta measurements that can tighten the bound on the estimation error depending on the knowledge on the outputs distribution, and that we use those design formulations to address the co-design problem. The use of the network can also be alleviated depending on the assumptions that can be made for the measurable outputs. The results in this paper have the advantage of explicitly showing several tuning parameters that can help tightening the bounds for the estimation error and the network requirements.

Notation : Let A and B be some matrices. $A \prec B$ means that matrix $A-B$ is negative definite. Similar applies to $\succeq$. $\text{diag}\{A, B\}$ is a block diagonal matrix with A and B on its diagonal. Let $x[t] \in \mathrm{R}^n$ be a stochastic process. Expected value and probability are denoted by $\mathcal{E}\{\cdot\}$ and $P\{\cdot\}$. We write $\|x[t]\|_{RMS}^2 \triangleq \lim_{T\to\infty} \sum_{t=0}^{T-1} \frac{1}{T}\|x[t]\|_2^2$ for the RMS norm of $x[t]$.

2. Problem Statement

Consider a networked control system that updates the control action synchronously with the output measurement and the plant model

$$x[t+1] = A\,x[t] + B_u\,u[t-1] + B\,w[t], \tag{1a}$$
$$y[t] = C\,x[t] + v[t], \tag{1b}$$
$$z[t] = C_z\,x[t], \tag{1c}$$

where $x \in \mathrm{R}^n$ is the state, $u \in \mathrm{R}^{n_u}$ is the known input vector, $w \in \mathrm{R}^{n_w}$ is the unmeasurable state disturbance vector, $y \in \mathrm{R}^{n_y}$ is the measured output, $v \in \mathrm{R}^{n_y}$ is the measurement noise, and $z[t] \in \mathrm{R}^{n_z}$ the signal of interest. Throughout this work we assume that the control input is causally available at all times. This can be achieved when the controller and estimator are collocated, and the control action is transmitted through a reliable network (without dropouts), see Fig. 1. We assume that each measurable output uses a sensor node that acquires the measurement and decides whether to send it to the estimator node.

Let us assume that the sensor node i has sent a measured plant output to the estimation node through the communication network at period $t = t_{k_i}$ and we call it $y_i[t_{k_i}] = y_i[t_{k_i}]$ (where k_i enumerates the sent data from sensor i). Then, a new measurement will be sent if the following condition holds

$$|y_i[t] - y_i[t_{k_i}]| \geq \Delta_i, \quad \Delta_i > 0, \quad t > t_{k_i}. \tag{2}$$

In that case, the sensor sends the (k_i+1)-th measurement, and $y_i[t]$ becomes $y_i[t_{k_i+1}]$ for future reference.

We assume that there is a central state estimator node that uses the received messages from the sensor nodes to perform the estimation using the equations

$$\hat{x}[t^-] = A\,\hat{x}[t-1] + B_u\,u[t-1], \tag{3a}$$
$$\hat{x}[t] = \hat{x}[t^-] + L[t](m[t] - C\,\hat{x}[t^-]), \tag{3b}$$
$$\hat{z}[t] = C_z\hat{x}[t], \tag{3c}$$

where $m[t]$ is the estimated measured output vector, and $L[t]$ is the observer gain. $m[t]$ includes both the information of the received output values ($y[t_{k_i}]$) and the information of the measurement uncertainty. The i-th component of $m[t]$ remains constant while there is no new measurement from sensor i, i.e.,

$$m_i[t] = y_i[t_{k_i}], \qquad t_{k_i} \leq t < t_{k_i+1},$$

and we model its relation with the actual state as

$$m_i[t] = \begin{cases} C_i\,x[t] + v_i[t], & t = t_{k_i}, \\ C_i\,x[t] + v_i[t] + \delta_i[t], & t_{k_i} < t < t_{k_i+1}, \end{cases} \tag{4}$$

being C_i the i-th row of matrix C, and where $\delta_i[t]$ is a virtual noise fulfilling $\|\delta_i[t]\|_\infty < \Delta_i$, as we have $|y_i[t] - y_i[t_{k_i}]| < \Delta_i$ for $t_{k_i} < t < t_{k_i+1}$.

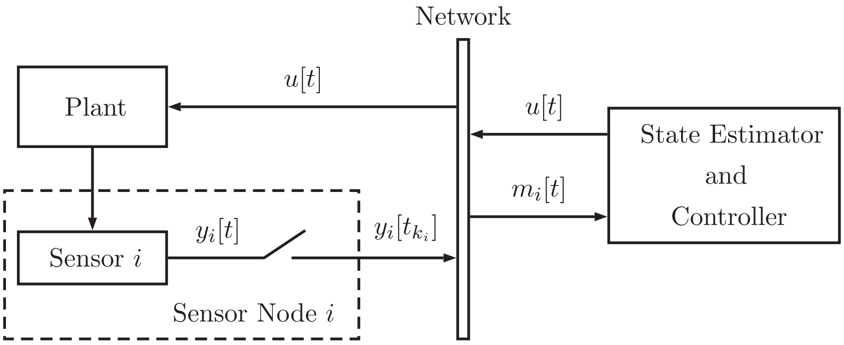

Figure 1. send-on-delta based networked state estimator.

Remark 1. *While $t \in \mathrm{N}$ refers to each time instant, t_{k_i} (with $k_i \in \mathrm{N}$) enumerates only the instant when the k_i-th measurement from the i-th sensor is received. For instance, if we receive the k_2-th and (k_2+1)-th measurements from sensor 2 at instants $t_{k_2} = 8$ and $t_{k_2+1} = 11$, then, instants $t_{k_2} + 1 = 9$ or $t_{k_2} + 2 = 10$ refer to instants when no measurements from sensor 2 are received.*

Let us define $\alpha_i[t]$ as the availability factor for each sensor i, that is a binary variable that takes a value of 1 if there is a new measurement received from the sensor node i and 0 otherwise. We define the availability matrix as a diagonal one including the $\alpha_i[t]$ factor of each sensor, i.e.,

$$\alpha[t] = \operatorname{diag}\{\alpha_1[t], \dots, \alpha_{n_y}[t]\}.$$

We then model the available measurements of the outputs as

$$m[t] = C\,x[t] + v[t] + (I - \alpha[t])\,\delta[t], \tag{5}$$

with $\delta[t] = [\delta_1[t] \ \cdots \ \delta_{n_y}[t]]^T, \ \ \delta_i[t] \in (-\Delta_i, \Delta_i)$.

Matrix $\alpha[t]$ can take different values depending on the measurements successful transmission possibilities and they belong to a known set

$$\alpha[t] \in \Xi = \{\eta_0, \eta_1, \dots, \eta_q\}, \tag{6}$$

where η_i denotes a possible combination of available measurements at each control period. We recall those combinations as sampling scenarios. Matrix η_0 denotes the scenario with unavailable measurements and q the number of different scenarios with available measurements. In the general case, any combination of obtainable sensor measurements is possible, leading to $q = 2^{n_y} - 1$.

The first of our goals is to define a centralized observer that uses the scarcely received distributed data and the uncertainty knowledge. We propose the observer equation (3) and define the gain observer law $L[t]$ as

$$L[t] = L(\alpha[t]), \tag{7a}$$

$$L(\alpha[t]) = L_j, \quad \text{if } \alpha[t] = \eta_j, \tag{7b}$$

what leads to a jump observer. The gains take, in general, $q+1$ different values within a predefined set, i.e.,

$$L(\alpha[t]) \in \mathcal{L} = \{L_0, \dots, L_q\}. \tag{8}$$

The gains are computed off-line once, and the centralized observer chooses the applicable gain depending on the availability of new measurements (see Smith and Seiler (2003); Dolz et al. (2014) for other jump observers applicable on networked control systems).

With the estimator defined by (3) and (7), we obtain the state estimation error dynamics given by

$$\tilde{x}[t] = \mathcal{A}(\alpha[t])\tilde{x}[t-1] + \mathcal{B}(\alpha[t])\xi[t], \tag{9}$$

$$\xi[t] = \left[w[t-1]^T \; v[t]^T \; \delta[t]^T\right]^T,$$

$$\tilde{z}[t] = C_z\tilde{x}[t], \tag{10}$$

with $\tilde{x}[t] = x[t] - \hat{x}[t]$, $\tilde{z}[t] = z[t] - \hat{z}[t]$, and

$$\mathcal{A}(\alpha[t]) = (I - L(\alpha[t])C)A,$$

$$\mathcal{B}(\alpha[t]) = \left[(I - L(\alpha[t])C)B \quad -L(\alpha[t]) \quad -L(\alpha[t])(I - \alpha[t])\right].$$

As we restrict $L(\alpha[t])$ to take $q+1$ different values depending on the value of matrix $\alpha[t]$, we get a jump linear system with discrete state $\alpha[t]$ and with a finite number of modes.

Remark 2. *The only condition to find a stabilizing observer is that the system (A, CA) is detectable. Note that if we restrict the gains to be constant (i.e., $L(\alpha[t]) = L$), the dynamics of the observer is given by the constant matrix $(I - LC)A$, what leads to the aforementioned condition. The idea of using virtual measurements when the real ones are not available is to assure the detectability of the system at each sampling instant and thus, the stability of the observer, while the idea of adapting the gain to the sampling scenario $\alpha[t]$ is to avoid the propagation of the virtual noise.*

The second of our goals is to jointly design the observer gains and the thresholds Δ_i that minimize the network usage while guaranteeing a predefined estimation performance. The network usage is proportional to the rate in which (2) occurs, so we achieve this goal by minimizing a cost function related to the sending thresholds Δ_i. In this work, we present alternatives to bound the estimator performance and the network usage depending on Δ_i. For each of them we calculate the minimum probability of receiving a measurement and the maximum variance of the resulting virtual noise $\delta[t]$.

We reformulate the main objective of this paper as the simultaneous design of the $q+1$ gains L_j and the n_y thresholds Δ_i that minimize the network usage, at the same time that guarantee a given bound on the estimation error.

3. Observer design

We present two jump observer design approaches for SOD policy with fixed Δ_i that assure stability and $\mathcal{H}_\infty$ attenuation level. We propose first a deterministic strategy that does not require any assumption on the output statistics. Then, we propose different assumptions about the statistical information of the output, and then develop a stochastic strategy that allows us to relax the bound on the achievable performance.

3.1 *Deterministic approach*

Theorem 1. *Consider that observer (3) with gain (7) estimates the state of system (1) that sends its outputs with the SOD policy. If there exist matrices P_j, Q_j, X_j ($j = 0, 1, \ldots, q$), and positive values γ_w, γ_{v_i} and γ_{δ_i} ($i = 1, \ldots, n_y$) such that $P_j = P_j^T \succ 0$, and for all $j, k \in \{0, \ldots, q\} \times \{0, \ldots, q\}$*[1]

$$\Phi_{j,k} = \begin{bmatrix} Q_j + Q_j^T - P_j & \star & \star & \star & \star \\ ((Q_j - X_j C)A)^T & P_k - C_z^T C_z & \star & \star & \star \\ ((Q_j - X_j C)B)^T & 0 & \gamma_w I & \star & \star \\ -X_j^T & 0 & 0 & \Gamma_v & \star \\ -(I - \eta_j) X_j^T & 0 & 0 & 0 & \Gamma_\delta \end{bmatrix} \succ 0, \tag{11}$$

being $\Gamma_v = \operatorname{diag}\{\gamma_{v_1}, \ldots, \gamma_{v_{n_y}}\}$, $\Gamma_\delta = \operatorname{diag}\{\gamma_{\delta_1}, \ldots, \gamma_{\delta_{n_y}}\}$, then, defining the observer gains as $L_j = Q_j^{-1} X_j$ ($j = 0, \ldots, q$), the following conditions are fulfilled: under null disturbances, the system is asymptotically stable, and, under null initial conditions, the state estimation error is bounded by

$$\|\tilde{z}[t]\|_{RMS}^2 < \gamma_w \|w[t]\|_{RMS}^2 + \sum_{i=1}^{n_y} \gamma_{v_i} \|v_i[t]\|_{RMS}^2 + \sum_{i=1}^{n_y} \gamma_{\delta_i} \|\delta_i[t]\|_{RMS}^2. \tag{12}$$

Proof 1. *If (11) holds, then $Q_j + Q_j^T - P_j \succ 0$ and Q_j is a nonsingular matrix. If P_j is a positive definite matrix, then $(P_j - Q_j)^T P_j^{-1} (P_j - Q_j) \succeq 0$, implying that $Q_j + Q_j^T - P_j \succeq Q_j^T P_j^{-1} Q_j$. If we replace X_j by $Q_j L_j$, in (11), perform congruence transformation by matrix $\operatorname{diag}\{Q_j, I, I, I, I\}$ and apply Schur complements, we obtain that*

$$\begin{bmatrix} P_k - C_z^T C_z & \star & \star & \star \\ 0 & \gamma_w I & \star & \star \\ 0 & 0 & \Gamma_v & \star \\ 0 & 0 & 0 & \Gamma_\delta \end{bmatrix} - \underbrace{\begin{bmatrix} ((I - L_j C)A)^T \\ ((I - L_j C)B)^T \\ -(L_j)^T \\ -(I - \eta_j) \cdot (L_j)^T \end{bmatrix}}_{\star} P_j (\star)^T \succ 0. \tag{13}$$

Consider a Lyapunov function depending on the sampling scenario as

$$V[t] = V(\tilde{x}[t], \alpha[t]) = \tilde{x}[t]^T P(\alpha[t]) \tilde{x}[t],$$

with $P(\alpha[t])$ taking values on the set $\{P_0, \ldots, P_q\}$ depending on the value $\alpha[t]$ as

$$P(\alpha[t]) = P_j, \quad \text{if } \alpha[t] = \eta_j, \ \forall\, j \in [0, \ldots, q].$$

Multiplying expression (13) by $[\tilde{x}[t]^T, w[t]^t, v[t]^T, \delta[t]^T]$ on the left, and by its transpose on the right, and assuming $\alpha[t+1] = j$ and $\alpha[t] = k$, it leads

$$\begin{aligned} &\tilde{x}[t+1]^T P_j \tilde{x}[t+1] - \tilde{x}[t]^T P_k \tilde{x}[t] + \tilde{z}[t]^T \tilde{z}[t] < \\ &< \gamma_w w[t]^T w[t] + v[t]^T \Gamma_v v[t] + \delta[t]^T \Gamma_\delta \delta[t] \end{aligned} \tag{14}$$

for any pair j, k in $\{0, \ldots, q\} \times \{0, \ldots, q\}$. If we consider null disturbances, then $V[t+1] < V[t]$, demonstrating the asymptotic stability of the observer. If we assume null initial state estimation

[1] the symbol $\star$ refers to the required element to make the matrix symmetric.

error ($\tilde{x}[0] = 0$, $V[0] = 0$) *and we add expression* (14) *from* $t = 0$ *to* T, *we obtain*

$$V[T+1] + \sum_{t=0}^{T} \tilde{z}[t]^T \tilde{z}[t] < \tag{15}$$

$$< \sum_{t=0}^{T} \left(\gamma_w w[t]^T w[t] + v[t]^T \Gamma_v v[t] + \delta[t]^T \Gamma_\delta \delta[t] \right)$$

As $V[T+1] > 0$, *if we divide by* T *and take the limit when* T *tends to infinity, we obtain* (12).

Remark 3. *As* A *and* C *are constant matrices, the only requirement to find a stabilizing observer is that the pair* (A, AC) *is detectable.*

3.2 *Stochastic approach*

The previous theorem leads to conservative results due to the consideration of all the possible sequences of new data reception with the same probability. For instance, it can respond satisfactorily to the situation of acquiring just a first measurement at the start-up of the observer and then working indefinitely with that unique measurement. If the disturbances and noises are not negligible, we can assume that there is a small probability of acquiring new data, and that is the key in the stochastic approach to reduce the conservativeness. The probability of having available new data at a given sampling instant is

$$\beta_i = P\{\alpha_i[t] = 1\} = P\{|y_i[t] - y_i[t_{k_i}]| \geq \Delta_i\}, \quad t > t_{k_i}.$$

The difference $y_i[t] - y_i[t_{k_i}]$ depends on the achieved state $x[t_{k_i}]$ during the last sent measurement, the inputs, disturbances and number of elapsed periods from t_{k_i} (let us call it N) as

$$y_i[t] - y_i[t_{k_i}] = y_i[t_{k_i} + N] - y_i[t_{k_i}] = \tag{16}$$

$$C_i \left(A^N x[t_{k_i}] + \sum_{j=0}^{N-1} A^{N-1-j} (B_u u[t_{k_i} + j - 1] + B w[t_{k_i} + j]) \right) + v[t_{k_i} + N] - v[t_{k_i}].$$

The dependency of that difference on the inputs leads us to a non stationary probability that can change at every sampling instant, i.e.,

$$\beta_i[t] = P\{|y_i[t] - y_i[t_{k_i}]| \geq \Delta_i\}, \quad t > t_{k_i}.$$

As the difference include the stochastic values $w[t]$ and $v[t]$, we assume that the probability belongs to the set

$$\beta_i[t] \in \mathcal{S}_i = \{\beta_i[t] : \beta_i' \leq \beta_i[t] \leq 1\}. \tag{17}$$

$\beta[t] = 1$ applies when the control action or the state evolution are sufficiently high to assure a new measurement transmission. $\beta[t] = \beta_i'$ applies during the less excited periods (with $x[t_{k_i}] = 0$ and $u[t] = 0$ for $t \geq t_{k_i}$) that leads to the less favorable scenario to acquire new data, when only the disturbance and noise excite the send-on-delta mechanism. If we choose $\beta_i' = 0$ we face again the deterministic approach, but choosing $\beta_i' > 0$ implies assuming that there is at least a small probability of acquiring new data, thus reducing conservatism.

The probability of obtaining a sampling scenario η_j $(j = 1, \ldots, q)$ is also non stationary and is given by

$$p_j[t] = P\{\alpha[t] = \eta_j\} = \prod_{\substack{i=1\\ \forall \eta_{j,i}=0}}^{n_y} (1 - \beta_i[t]) \prod_{\substack{i=1\\ \forall \eta_{j,i}=1}}^{n_y} \beta_i[t], \tag{18}$$

where $\eta_{j,i}$ refers to the i-th diagonal entry of η_j. The probability of having no measurement available at a control period is given by

$$p_0[t] = P\{\alpha[t] = \eta_0\} = \prod_{i=1}^{n_y}(1 - \beta_i[t]), \tag{19}$$

and the probability of sending some measurement is $1 - p_0[t]$.

Remark 4. *In the stochastic approach, the probabilities $\beta_i[t]$ $(i = 1, \ldots, n_y)$ are assumed to vary within two bounds. We will study in Section 3.3.2 how to obtain the lower bound on $\beta_i[t]$ (see* (29) *and* (33)*). The upper bound is the natural one $\beta_i[t] \leq 1$ that is achieved when the control action is sufficiently exciting to make the outputs cross the thresholds continuously. Therefore, each probability $\beta_i[t]$ is contained in the set $\mathcal{S}_i$ defined in* (17)*. With these bounds on $\beta_i[t]$ we can derive bounds on the probabilities of the sampling scenarios $p_j[t]$ $(j = 0, \ldots, q)$.*

With the probabilities of the sampling scenarios (18) we can obtain the set of gains that assure an attenuation level for any probability within the bounds. In the following theorem we omit the dependency on time of the probabilities for brevity.

Theorem 2. *Consider that observer* (7) *estimates the state of system* (1) *that sends its outputs with the SOD policy. Consider that there exist matrices $P = P^T \succ 0$, Q_j, X_j $(j = 0, \ldots, q)$, and positive values γ_w, γ_{v_i} and γ_{δ_i} $(i = 1, \ldots, n_y)$ such that for any $\{\beta_1, \ldots, \beta_{n_y}\} \in \mathcal{S}_1 \times \mathcal{S}_2 \cdots \times \mathcal{S}_{n_y}$*

$$\Psi(\beta) = \begin{bmatrix} M_1 & \star & \star & \star & \star \\ M_2 & P - C_z^T C_z & \star & \star & \star \\ M_3 & 0 & \gamma_w I & \star & \star \\ M_4 & 0 & 0 & \Gamma_v & \star \\ M_5 & 0 & 0 & 0 & \Gamma_\delta \end{bmatrix} \succ 0, \tag{20}$$

where

$$\begin{aligned} M_1 &= \operatorname{diag}\{p_0(Q_0 + Q_0^T - P), \ldots, p_q(Q_q + Q_q^T - P)\}, \\ M_2 &= [p_0 \bar{\mathcal{A}}_0^T \ \cdots \ p_q \bar{\mathcal{A}}_q^T], \quad M_3 = [p_0 \bar{\mathcal{B}}_0^T \ \cdots \ p_q \bar{\mathcal{B}}_q^T], \\ M_4 &= [-p_0 X_0^T \ \cdots \ -p_q X_q^T], \\ M_5 &= [-p_0(I - \eta_0) X_0^T \ \cdots \ -p_q (I - \eta_q) X_q^T], \\ \bar{\mathcal{A}}_j &= (Q_j - X_j C) A, \quad \bar{\mathcal{B}}_j = (Q_j - X_j C) B, \ j = 0, \ldots, q, \\ \Gamma_v &= \operatorname{diag}\{\gamma_{v_1}, \ldots, \gamma_{v_{n_y}}\}, \quad \Gamma_\delta = \operatorname{diag}\{\gamma_{\delta_1}, \ldots, \gamma_{\delta_{n_y}}\}, \end{aligned}$$

and p_j is a short notation for the following expression

$$p_j = \prod_{\substack{i=1\\ \forall \eta_{j,i}=0}}^{n_y} (1 - \beta_i) \prod_{\substack{i=1\\ \forall \eta_{j,i}=1}}^{n_y} \beta_i. \tag{21}$$

Then, if the observer gains are defined as $L_j = Q_j^{-1}X_j$ $(j = 0, \dots, q)$, the system is mean square stable and, under null initial conditions, the estimation error is bounded by

$$\|\tilde{z}[t]\|_{RMS}^2 < \gamma_w \|w[t]\|_{RMS}^2 + \sum_{i=1}^{n_y} \gamma_{v_i} \|v_i[t]\|_{RMS}^2 + \sum_{i=1}^{n_y} \gamma_{\delta_i} \|\delta_i[t]\|_{RMS}^2 \tag{22}$$

Proof 2. *Following similar steps than those of proof 1, inequalities* (20) *imply*

$$\mathcal{E}\{V[t+1]\} - V[t] + \tilde{z}[t]^T\tilde{z}[t] < \gamma_w w[t]^T w[t] + v[t]^T\Gamma_v v[t] + \delta[t]^T\Gamma_\delta \delta[t], \tag{23}$$

where $\mathcal{E}\{V[t+1]\}$ is the expected value of the Lyapunov function $V[t] = \tilde{x}[t]^T P\tilde{x}[t]$ at the next period over the possible modes of the system ($\alpha[t] = \{\eta_0, \dots, \eta_q\}$ in (9)*). Assuming null disturbances we obtain $\mathcal{E}\{V[t+1]\} < V[t]$, assuring the mean square stability of the observer. Assuming initial state estimation error ($\tilde{x}[0] = 0$, $V[0] = 0$) and adding expression* (23) *from $t = 0$ to T, we obtain*

$$\mathcal{E}\{V[T+1]\} + \sum_{t=0}^{T} \tilde{z}[t]^T\tilde{z}[t] < \sum_{t=0}^{T} \left(\gamma_w w[t]^T w[t] + v[t]^T\Gamma_v v[t] + \delta[t]^T\Gamma_\delta \delta[t]\right). \tag{24}$$

As $\mathcal{E}\{V[T+1]\} > 0$, dividing by T and taking the limit when T tends to infinity, one finally obtains (22)*.*

Remark 5. *The only condition to find a solution for the previous LMI problem is that the system is detectable, as one can always choose a constant L and then use the fact that $\sum_{i=0}^{q} p_i = 1$, what would lead to detectability condition of the pair (A, AC).*

The previous problem is an infinite dimensional one that must be assured for any possible combination of the values β_i within the sets $\mathcal{S}_i$ $(i = 1, \dots n_y)$. In order to make the problem numerically tractable we use the sum of squares (SOS) decomposition (Chesi (2010); Peñarrocha et al. (2013, 2014); Dolz et al. (2015)) to define sufficient conditions to accomplish with the previous guaranteed performance. The idea is to consider the probabilities $\beta_i[t]$ on the previous LMI constraint as new variables of the problem and thus transform it into a polynomial matrix inequality (PMI). Then we express $\beta_i \in \mathcal{S}_i$ with a polynomial expression of the form $\pi_i(\beta_i) \geq 0$. Finally we check the positivity of the PMI for all values of β_i fulfilling $\pi_i(\beta_i) \geq 0$ using the tools shown in the Appendix, that allow us to handle a PMI problem as a LMI one and, therefore, can be faced with standard LMI solvers.

Theorem 3. *Let us assume that there exist matrices $P = P^T \succ 0$, Q_j, X_j $(j = 0, \dots, q)$, positive values γ_w, γ_{v_i} and γ_{δ_i} $(i = 1, \dots, n_y)$ and SOS polynomials $s_i(z, \beta)$ of fixed degree (with z a vector of proper dimensions) such that*

$$z^T\Psi(\beta)z - \sum_{i=1}^{n_y} s_i(z,\beta)\pi_i(\beta_i) \text{ is SOS}, \tag{25}$$

with $\pi_i(\beta_i) = (\beta_i - \beta_i')(1 - \beta_i)$ and $\beta = [\beta_1 \cdots \beta_{n_y}]$. Then, conditions of Theorem 2 are fulfilled.

Proof 3. *First note that each of the sets $\mathcal{S}_i$ $(i = 1, \dots, n_y)$ can be rewritten with its corresponding polynomial π_i as $\mathcal{S}_i = \{\beta_i : \pi(\beta_i) \geq 0\}$. Then, applying Lemmas 4 and 5 in the Appendix, it follows that the conditions on Theorem 2 are fulfilled for any $\beta_i = \beta_i[t]$.*

Remark 6. *In the previous theorem, variables in vectors β and z are used to construct the polynomials from which the LMI problem is derived but they are not decision variables. The determining variables are P, Q_j, X_j $(j = 0, \dots, q)$, γ_w, γ_{v_i}, γ_{δ_i} $(i = 1, \dots, n_y)$ and the scalar* *coefficients used to construct the n_y polynomials $s_i(z,\beta)$.*

In any of the previous design approaches we can reduce the computational cost of the observer implementation by means of imposing some restrictions on the gain matrices. We achieve the lower computational cost when the matrices are forced to be equal, thus $L_i = L_j$ for all $i, j = 1, \dots, q$. This can be achieved imposing equality constraints over matrices Q_j and matrices X_j in problems (11) and (20).

3.3 *Optimization design procedure*

If we know the values of $\|\delta_i[t]\|_{RMS}$, $\|v_i[t]\|_{RMS}$ and $\|w[t]\|_{RMS}$, the optimization problem

$$\begin{aligned} \min \quad & \gamma_w \|w[t]\|_{RMS}^2 + \sum_{i=1}^{n_y} \gamma_{v_i} \|v_i[t]\|_{RMS}^2 + \sum_{i=1}^{n_y} \gamma_{\delta_i} \|\delta_i[t]\|_{RMS}^2 \\ s.t. \quad & \Theta \succ 0, \end{aligned} \tag{26}$$

leads to the jump observer that minimizes the RMS value of the state estimation error for that assumption, where $\Theta = \Phi_{j,k}$ (11) for the deterministic approach and $\Theta = \Psi(\beta)$ (20) for all $\beta_i \in \mathcal{S}_i$ for the stochastic one. If the RMS values of the disturbances are unavailable, they can be used as tuning parameters to achieve a desired behavior.

The previous optimization procedure also applies when we can only bound the disturbances and sensor noises by the norms $\|w[t]\|_\infty$ and $\|v_i[t]\|_\infty$, as the RMS norm is bounded by the l_∞ norm: $\|w[t]\|_{RMS} < \|w[t]\|_\infty$ and $\|v_i[t]\|_{RMS} < \|v_i[t]\|_\infty$. In this case, we substitute the RMS norm of the previous optimization procedure by its corresponding l_∞ norm.

The optimization in both approaches needs a bound for $\|\delta_i[t]\|_{RMS}^2$, while in the stochastic approach, a lower probability bound β_i' is also needed. Furthermore, in order to proceed with the co-design problem in the next section, we need to express both bounds as explicit functions of Δ_i. We discuss now how to obtain those bounding functions for each of the approaches.

3.3.1 Deterministic approach design procedure

In the deterministic approach, we have the bound $\|\delta_i[t]\|_{RMS} < \|\delta_i[t]\|_\infty < \Delta_i$ from the definition of the virtual noise signal. However, if a uniform distribution of $\delta_i[t]$ is assumed, this leads to $\|\delta_i[t]\|_{RMS} < \Delta_i/\sqrt{3}$, that relaxes the optimization problem. This assumption on the virtual noise distribution is commonly used in the literature, e.g. Suh et al. (2007).

3.3.2 Stochastic approach design procedure

In the stochastic approach, we must obtain relationships showing the increase of β_i' (in (17)) with lower values of Δ_i as well as the increase of $\|\delta_i[t]\|_{RMS}$ with higher values of Δ_i. In order to obtain those bounding relationships, we first note that (from (16)) during the less excited periods we have the difference

$$y_i[t] - y_i[t_{k_i}] = C_i \sum_{j=0}^{N-1} A^{N-1-j} B w[t_{k_i} + j] + v[t_{k_i} + N] - v[t_{k_i}]. \tag{27}$$

The smallest change in the output corresponds to $t = t_{k_i} + 1$, and hence, to obtain a lower bound on the probability, $N = 1$ is taken. Therefore, we must first obtain the probability density function for the difference

$$y_i[t] - y_i[t_{k_i}] = C_i B w[t_{k_i}] + v[t_{k_i} + 1] - v[t_{k_i}],$$

and use it to obtain both the lower bound of the probability of having a new sample and the corresponding expected RMS value of the virtual noise. This probability density function is tedious to obtain as it requires recovering the density function of the sum of several signals with different distribution laws. For this reason, we present a simplification of its computation that allows us to obtain tractable expressions relating β_i' and $\|\delta_i[t]\|_{RMS}^2$ with Δ_i by using two different assumptions on the outputs. In order to improve the readability of this section, we have included in the Appendix 6.1 the necessary but straightforward auxiliary results used to obtain the expressions.

3.3.2.1 Uniform assumption. If we assume symmetrically bounded disturbances and noises, we can bound the difference $y_i[t] - y_i[t_{k_i}]$ in (16) (for $N = 1$ and in the lees excited scenario) within $[-r_i, r_i]$, being r_i such as

$$r_i = \|C_i B w[t]\|_\infty + 2\|v[t]\|_\infty, \tag{28}$$

where $\|C_i B w[t]\|_\infty$ $(i = 1, \dots, n_y)$ can be computed as

$$\|C_i B w[t]\|_\infty = \sum_{j=1}^{n_w} \left(\sum_{k=1}^{n} |C_{i,k} B_{k,j}| \right) \|w_j[t]\|_\infty,$$

w_j is the j-th element of vector w, and $\|w_j[t]\|_\infty$ and $\|v[t]\|_\infty$ are assumed to be known. If the outputs are uniformly distributed and fulfill $y_i[t] - y_i[t_{k_i}] \in [-r_i, r_i]$ and $\Delta_i \leq r_i$, the probability of having a new measurement is lower bounded by

$$\beta_i' = \frac{1}{r_i^2}(r_i - \Delta_i)^2. \tag{29}$$

In this case, the RMS norm is bounded by

$$\|\delta_i[t]\|_{RMS}^2 < \sigma_{\delta_i}^2 = \frac{2\Delta_i^3}{r_i^2} \left(\frac{r_i}{3} - \frac{\Delta_i}{4} \right). \tag{30}$$

See Lemma 1 in the Appendix for the details. If a sensor uses a threshold $\Delta_i > r_i$, it will never send a measurement during the less excited scenario, and, therefore in that case $\beta_i' = 0$ and $\sigma_{\delta_i}^2 = r_i^2/6$.

3.3.2.2 Gaussian assumption. If the disturbances and noises are distributed with covariances W and V_i and zero mean, in the less excited scenario we have that the difference $y_i[t] - y_i[t_{k_i}]$ is distributed with variance

$$\sigma_i^2 = C_i B W B^T C_i^T + 2V_i. \tag{31}$$

If our knowledge is the RMS norm of vector w and noises v_i, we can bound σ_i^2 as

$$\sigma_i^2 \leq \mathrm{tr}(B^T C_i^T C_i B)\|w[t]\|_{RMS}^2 + 2\|v_i[t]\|_{RMS}^2. \tag{32}$$

Assuming that the difference between two consecutive samples follow a normal distribution with zero mean and variance σ_i^2, the probability of having a new measurement is bounded by

$$\beta_i' = 1 - \mathrm{erf}\left(\frac{\Delta_i}{\sqrt{2}\sigma_i} \right), \tag{33}$$

being $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ the error function. In this case, the RMS norm is bounded by

$$\|\delta_i[t]\|_{RMS}^2 < \sigma_{\delta_i}^2 = \sigma_i^2 \text{erf}\left(\frac{\Delta_i}{\sqrt{2}\sigma_i}\right) - \frac{\sqrt{2}\Delta_i\sigma_i}{\sqrt{\pi}} e^{\frac{-\Delta_i^2}{2\sigma_i^2}}. \tag{34}$$

See Lemma 2 in the Appendix for the details.

Remark 7. *If the system outputs do not exactly follow the previous distributions we can use the values r_i and σ_i in (29)-(34) as tuning parameters. In that case, we must choose sufficiently small values r_i and σ_i to assure that the computed probability of having new measurements is below the real one, and such that the computed variance for the virtual noise is higher than the real one. With that choice, we can at least compute a less conservative upper bound of the state estimation error than the one obtained with the deterministic approach. One of the advantages of having those bounding relationships is that allows to face the co-design problem (explained next), consisting on looking for the values of Δ_i that fulfill some estimation error and network usage. In that sense, one could know in advance some maximum values $\Delta_{i,\max}$ below which the search is carried out (e.g., some fraction of the output sensor range). In that case, the lowest values for r_i and σ_i that assure that the co-design problem is sensitive to Δ_i within all its range are $r_i = \Delta_{i,\max}$ and $\sigma_i = \frac{\Delta_{i,\max}}{3}$ (following the 3σ criterion).*

4. Observer co-design

Once we have developed the design procedure to minimize the estimation error for a given SOD policy, we now address the minimization of the network usage guaranteeing a desired estimation error. We first propose the cost indexes to measure the network usage.

For the deterministic approach, without statistical information of the outputs, we propose the index

$$J(\Delta_{1:n_y}) = \sum_{i=1}^{n_y} \frac{g_i}{\Delta_i} \tag{35}$$

where g_i are some free weighting factors, that can be used to account for the different range of variation of the different sensors, and $\Delta_{1:n_y} = [\Delta_1 \ \cdots \ \Delta_{n_y}]$.

For the stochastic approach, we propose to use as the cost index, the probability of network usage in the lowest excitation case, that is:

$$J(\Delta_{1:n_y}) = 1 - p_0 = 1 - \prod_{i=1}^{n_y} (1 - \beta_i'(\Delta_i)) \tag{36}$$

where $\beta_i'(\Delta_i)$ $(i = 1, \ldots, n_y)$ depends on Δ_i by means of (29) or (33).

The actual probability of network usage will be close to this cost index only in the case of the lowest excitation, i.e., when the change of the output is minimum. When it is larger (for example when the input u changes), the probability of network usage will be higher. However, this usage will be proportional to the cost index, and hence, minimizing the cost index results in reducing the network usage for the desired estimation error in any case.

We then obtain the observer that assures a prescribed bound $\tilde{z}_{rms}$ in the estimation error, i.e. $\|\tilde{z}[t]\|_{RMS}^2 \leq \tilde{z}_{rms}$, and minimizes the network usage $J(\Delta_{1:n_y})$ by solving the following optimization problem:

$$\begin{aligned}
\min \quad & J(\Delta_{1:n_y}) \\
s.t. \quad & \Theta(\Delta_{1:n_y}) \succ 0, \\
& \gamma_w \left\|w[t]\right\|^2_{RMS} + \sum_{i=1}^{n_y} \left(\gamma_{v_i} \left\|v_i[t]\right\|^2_{RMS} + \gamma_{\delta_i} \sigma^2_{\delta_i}(\Delta_{1:n_y})\right) \leq \tilde{z}_{rms}.
\end{aligned} \tag{37}$$

The new decision variable Δ_i appears both on the cost index and in the definition of $\sigma^2_{\delta_i}$ used to bound $\|\delta_i[t]\|^2_{RMS}$. In the deterministic approach, we express $J(\Delta_{1:n_y})$ as (35), $\Theta(\Delta_{1:n_y}) = \Phi_{j,k}$ as in (11), and we use the bound $\sigma^2_{\delta_i}(\Delta_{1:n_y}) = \Delta_i^2$. Under the assumption of uniform distribution of $\delta_i[t]$, we can relax the problem using the bound $\sigma^2_{\delta_i}(\Delta_{1:n_y}) = \Delta_i^2/3$.

In the stochastic approach, we express $J(\Delta_{1:n_y})$ as (36), $\Theta(\Delta_{1:n_y}) = \Psi(\Delta_{1:n_y})$ as (20), and we express $\sigma^2_{\delta_i}(\Delta_{1:n_y})$ as (30) or (34), depending on the output assumption. In this case Δ_i appears in the bound of the probabilities β_i for which $\Psi(\beta)$ in (20) must be positive definite.

Remark 8. *In the previous co-design problem, one must choose the desired bound for the estimation error $\tilde{z}_{rms}$. This desired bound should be higher than the achievable one with standard sampling (i.e. when $\Delta_{1:n_y} = 0$) in order to have a solvable problem. A reasonable option is to express the desired bound in relative terms with respect to the achievable one for $\Delta_{1:n_y} = 0$, that can be obtained with* (26) *and* (11) *with $\|\delta_i[t]\|^2_{RMS} = 0$. In that case, the set of LMIs* (11) *could be simplified eliminating the last row and column matrix blocks and using just the case $j = k = q$ where $\eta_j = I$ (standard sampling). If we call that performance index $\tilde{z}^0$, then the desired bound in* (37) *can be expressed as $\tilde{z}_{rms} = \mu\, \tilde{z}^0$ with $\mu > 1$.*

The optimization problem (37) is non-linear in the variables Δ_i, but reduces to a LMI problem if we fix the values of Δ_i. Some approaches to solve this non-linear optimization are brute force with a grid approach over Δ_i, greedy algorithms and heuristic optimization with genetic algorithms. If we use the latter one and the stochastic approach, the optimization problem can be written as

$$\begin{aligned}
\min_{\Delta_i} \quad & J(\Delta_{1:n_y}) \\
s.t. \quad & z^\star(\Delta_{1:n_y}) - \tilde{z}_{rms} \leq 0 \\
& z^\star(\Delta_{1:n_y}) = \begin{cases} \min \gamma_w \left\|w[t]\right\|^2_{RMS} + \sum_{i=1}^{n_y} (\gamma_{v_i} \left\|v_i[t]\right\|^2_{RMS} + \gamma_{\delta_i}\sigma^2_{\delta_i}(\Delta_i)) \\ s.t.\ \Theta(\Delta_{1:n_y}) \succ 0 \end{cases}
\end{aligned} \tag{38}$$

In this work, we propose a greedy algorithm as an alternative to the previous optimization problem. A greedy algorithm is a tree search where at each step we only explore the branch that locally optimizes the problem in the hope that this choice leads to a globally optimal solution (see Cormen et al. (2001)). This kind of algorithm never comes back to previous solutions to change the search path and hence, global solutions are not guaranteed. The advantage is the lower computational cost. We propose now the following greedy algorithm to solve the previous co-design problems.

Step 1 Take a small $\epsilon > 0$. Take some initial small $\Delta_{i,0} \gtrsim 0$ ($i = 1, \ldots, n_y$) such that $\|\tilde{z}[t]\|_{RMS} < \tilde{z}_{rms}$ is achievable. Set $k = 0$ and $\Delta_{i,k} = \Delta_{i,0}$ and $J_0 = J(\Delta_{1:n_y,0})$.
Step 2 Set $k = k + 1$ and $J_k = J_{k-1} - \epsilon$.
Step 3 For $i = 1$ to n_y repeat:
Set $\Delta_j = \Delta_{j,k-1}$, $j \neq i$.
Set $\Delta_i = \Delta_{i,k} = \arg\{J_k = J(\Delta_{1:n_y})\}$
Compute $\sigma^2_{\delta_j} = \sigma^2_{\delta_j}(\Delta_j)$, $j = 1, \ldots, n_y$.

Compute[2] $\beta'_j = \beta'_j(\Delta_j)$, $j = 1, \dots, n_y$, and p_l, $l = 0, \dots, q$.
Solve optimization problem (26).
Store $z_i^\star = \gamma_w \|w[t]\|^2_{RMS} + \sum_{j=1}^{n_y}(\gamma_{v_j} \|v_j[t]\|^2_{RMS} + \gamma_{\delta_j}\sigma^2_{\delta_j}(\Delta_j))$.

Step 4 Set $i = \arg\min_i \; z_i^\star$.

If $z_i^\star < \tilde{z}_{rms}$, then
set $\Delta_{i,k} = \arg\{J_k = J(\Delta_{1:n_y}), \Delta_j = \Delta_{j,k-1}, j \neq i\}$,
$\Delta_{j,k} = \Delta_{j,k-1}$, $j = 1, \dots, n_y$, $j \neq i$,
and go to step 2.
Else, exit.

The algorithm starts considering small values of Δ_i and $\beta'_j \lesssim 1$, what leads to the standard periodic sampling case. Then it reduces iteratively the communication cost index while possible. At each step, it calculates the n_y new sets $\Delta_{1:n_y}$ that lead to the new cost, changing one of the Δ_i in each set. Then, it selects the set that led to the lowest $z_i^\star$, i.e., the solution allowing a larger future search before the algorithm ends. It changes only one value Δ_i at each step.

The previous iterative algorithm could be run for different values of maximum allowed estimation error, leading to a set of soft functions that should express the thresholds and the gains as a function of the associated network usage. Those functions could be implemented in the estimator node, allowing the change of the parameters (thresholds and gains) when the state of the network requires it (for example increasing the thresholds to reduce the network usage to avoid congestion). If the estimator node had high computing capabilities, an alternative could be to directly compute the thresholds and gains through running the full iterative optimization algorithm at that node when required.

5. Examples

In this section we show two different examples. In the first one we explore the achievable trade-offs between estimation error and network usage for the approaches presented in this work and compare them with other strategies existing in the literature. In the second example we apply the observer design based on send-on-delta measurements to control the velocity of a real DC motor using an inferential control approach. In both examples we aim to show the performance of the proposed approaches when neither of the considered output distribution assumptions hold, i.e., when the output distribution is not uniform or normal. For brevity, we will only explore the deterministic approach and the uniform output distribution assumption one.

5.1 *Simulation example*

In this example we aim to show the performance of the proposed approaches. For brevity, we will only explore the deterministic approach and the uniform output distribution assumption one. We consider the following discrete-time process (randomly chosen)

$$A = \begin{bmatrix} 1.005 & 0.221 & 0.171 \\ -0.031 & 1.008 & 0.136 \\ 0.049 & 0.038 & 1.028 \end{bmatrix}, \; B = \begin{bmatrix} -0.229 & 0.023 \\ 0.231 & 0.211 \\ -0.186 & 0.245 \end{bmatrix},$$

$$B_u = \begin{bmatrix} 0.658 & 0.919 \\ 0.342 & 0.584 \\ 0.481 & 0.845 \end{bmatrix}, \; C = \begin{bmatrix} 0.519 & -0.233 & 0.095 \\ 0.569 & 0 & 0 \end{bmatrix}, \; C_z = I.$$

[2] Only in the stochastic approach.

The measurement noises are assumed Gaussian signals with zero mean and $\|v_i[t]\|_{RMS} = 0.032$ (for $i = 1, 2$) while the disturbances are given by

$$w[t] = \left(0.3 + 0.2\sin(10^{-4}\pi\, t)\right) \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

with $\|w[t]\|_{RMS} = 0.468$. The control input is generated by a relay-based control with dead zone such as

$$u_i[t] = \begin{cases} -8, \text{ if } y_i[t] > 8 \\ 8, \quad \text{if } y_i[t] < -8 \\ 0, \quad \text{if } -8 \leq y_i[t] \leq 8 \end{cases}$$

The aim of this example is to show the performance of the co-design approach from Section 4, i.e., minimize the network usage while guaranteeing that the estimation error is lower than a prescribed one. For this purpose, the following four approaches are analyzed:

C1 Deterministic approach with jump observer (see Section 3.1).
C2 Deterministic approach with constant gain.
C3 Stochastic approach based on uniform distribution assumption for a jump observer (see Section 3.3.2).
C4 Stochastic approach based on uniform distribution assumption for constant gain.

We choose the parameters r_i that define the uniform distribution assumption for each output using expression (28). The maximum value of the disturbances and noises are $\|w[t]\|_\infty = [0.8\ 0.7]$ and $\|v[t]\|_\infty = [0.15\ 0.15]$. Then, we obtain $r = [0.9\ 0.75]$.

We quantify the network usage with the two cost functions presented in Section 4. For the deterministic cases C1 and C2 we use $J = \frac{r_1}{\Delta_1} + \frac{r_2}{\Delta_2}$ (see (35)). However, when we characterize the measurement transmission by its probability (cases C3 and C4), we use $J = 1-p_0$ that is the probability of having any successful data transmission in the lowest excitation case (see (36)).

We denote by $\tilde{z}^0$ the error $\|\tilde{z}[t]\|^2_{RMS}$ resulting from the standard measurement transmission (i.e. $\Delta = 0$), which turns to be $\tilde{z}^0 = 0.225$. In this example we analyze the results of the co-design procedures when fixing different values of $\tilde{z}^2_{rms}$ in (38). We denote by μ the ratio between the desired performance and $\tilde{z}^0$, i.e., $\mu = \frac{\tilde{z}^2_{rms}}{\tilde{z}^0}$.

Figure 1 compares the thresholds Δ_i resulting from conducting the co-design procedure (see Section 4), by imposing a ratio in the range $1 \leq \mu \leq 3$. The deterministic approaches C1 and C2 are both conservative and lead to the lowest thresholds, while the stochastic approaches C3 and C4 lead to the highest thresholds, and therefore, to the lowest network usage. The thresholds in C1 and C2 remain equal, what implies that using a jump observer in the deterministic approach does not improve the co-design with a constant gain. However, when we have some knowledge about the probability of the different sampling scenarios (stochastic approach), the use of a jump observer (case C3) enlarges Δ_i at the expense of a higher computational complexity with respect to C4.

Figure 2 shows the time-average probability of having a new measurement from a given sensor β_i and its virtual noise RMS norm $\sigma^2_{\delta_i}$ as a function of the Δ_i presented in Figure 1 resulting from a Monte Carlo simulation. It also displays the obtained results of assuming uniform distributed outputs (see (29) and (30)) and the use of the criterion in Suh et al. (2007) ($\sigma^2_{\delta_i} = \Delta_i^2/3$). The choice of $r = [0.9\ 0.75]$ results in lower probabilities and higher variances than in simulation. Therefore the stochastic design will be conservative, but will guarantee the prescribed bound on the estimation error (see Section 3.3.2). The result proposed in Suh et al. (2007) for bounding the virtual noise RMS norm assuming it as a uniform variable (i.e. where $\|\delta_i[t]\|^2_{RMS} < \Delta_i^2/3$) is more conservative than the one resulting from the difference of uniform output signals assumption that we propose in this work.

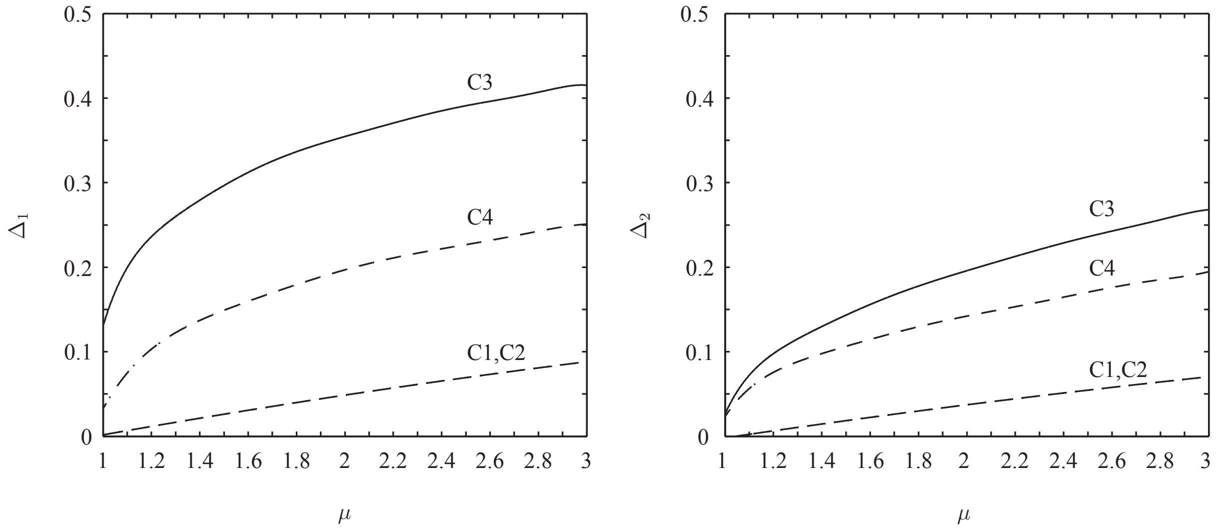

Figure 1. Thresholds Δ_i obtained for the co-design approach as a function of the ratio μ.

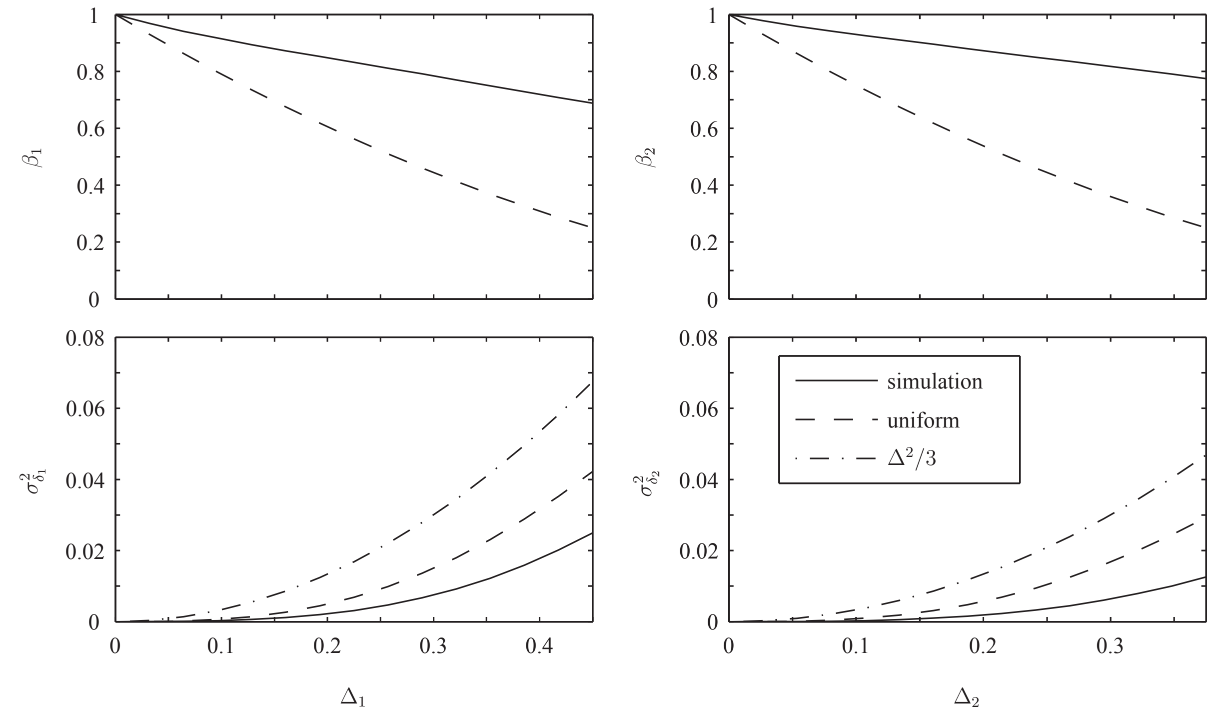

Figure 2. Probability of having a new measurement β and variance (RMS norm) of the virtual noise σ^2_δ as a function of Δ; 'simulation': time-average probability and virtual noise RMS norm obtained from simulation; 'uniform': probability and virtual noise variance bounds from the uniform output distribution assumption; '$\Delta^2/3$': bound of the virtual noise RMS norm proposed in Suh et al. (2007).

Simulating the estimation algorithm with the send-on-delta procedure for the thresholds in Figure 1 we obtain the number of sent measurements and the performances indicated in Figures 3 and 4, respectively.

Figure 3 reasserts the conclusions extracted from Figure 1. The case C3 leads to the lowest network consumption, while case C4 improves the usage of the deterministic approaches requiring less computational requirements than case C3.

Figure 4 shows whether the imposed bound on the estimation error in the co-design procedure is fulfilled in simulation. The deterministic approaches C1 and C2 are far below the maximum

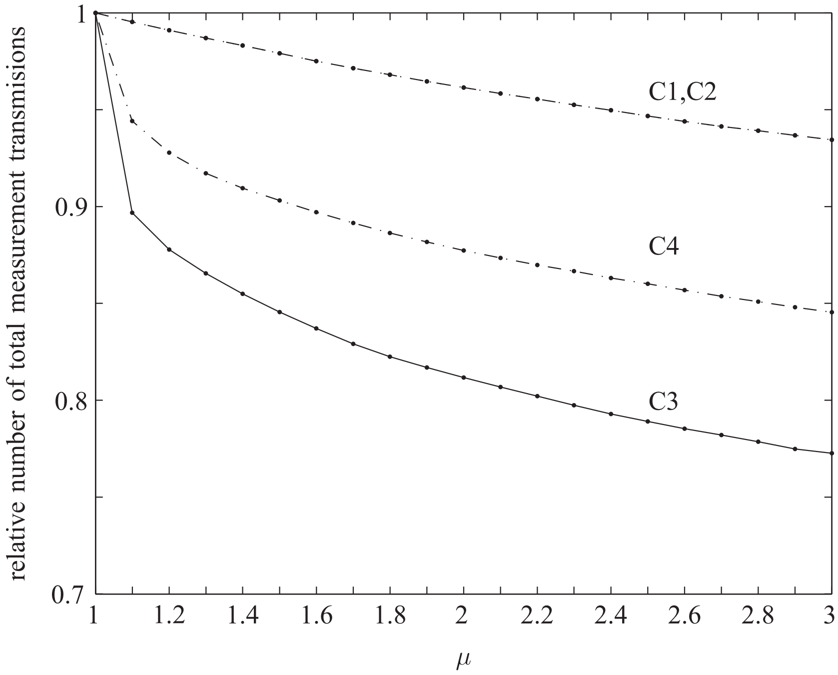

Figure 3. Number of total measurement transmission divided by the number of simulation periods as a function of the ratio μ.

allowed estimation error. This is due to the conservativeness introduced by the virtual noise variance estimation proposed by Suh et al. (2007). The stochastic approaches C3 and C4 are also below the maximum allowed estimation error, but closer to it. The conservativeness in the stochastic design is introduced by the choice of the parameter r (see Figure 2). Note that the use of a jump observer (C1 and C3) leads to less conservative results (estimation errors closer to the allowed one) than the use of a constant gain observer (C2 and C4). This rapprochement to the allowed error is what allows the jump observer to reach higher thresholds and to reduce the network usage.

In order to show the order of magnitude of the computational complexity of the design, in this example, one LMI is solved on about 1.7 seconds in a Pentium i7-3770 computer, and a full co-design procedure could take about 70 seconds in average.

Let us now compare the aforementioned achieved performances with the co-design method using the modified Kalman filter from Suh et al. (2007) for the case $\mu = 1.2$. As our disturbance is not Gaussian, there is not a systematic way of chosing the covariance to be used in the Kalman filter. However, in order to compare both approaches we will test this proposal for a covariance matrix of the form $W = wI$, for different values of w. For a value of $w = 0.1$ we obtain a pair of thresholds $\Delta = [0.9\, 0.5]$ that leads to a much lower network usage than our approach. However, the obtained performance after simulation with the indicated disturbance, is $\|\tilde{z}[t]\|^2_{RMS} = 0.4174$, thus violating the design constraint $\|\tilde{z}[t]\|^2_{RMS} < 1.2 \cdot \tilde{z}^0 = 0.267$. Now we focus on the design using a value of $w = 0.02$, obtained after computing the covariance of the generated disturbance for simulation. Applying the Kalman filter for $\Delta = [0.236\ 0.098]$ (the same values that the obtained with C3 and $\mu = 1.2$), we obtain after simulation $\|\tilde{z}[t]\|^2_{RMS} = 1.5 \cdot \tilde{z}^0$, what indicates that the performance is deteriorated with respect to the H_∞ approach.

This proofs that the proposed framework guarantees robustness against a larger type of disturbances and measurement noises, which are not necessarily uncorrelated Gaussian signals.

In conclusion, this example shows that if no information about the output is known, the deterministic approach is the only option. However, making some assumption on the output distribution, we can use a stochastic approach during the co-design procedure, that reduces the resulting network usage. We have shown that if we use a jump observer the measurement

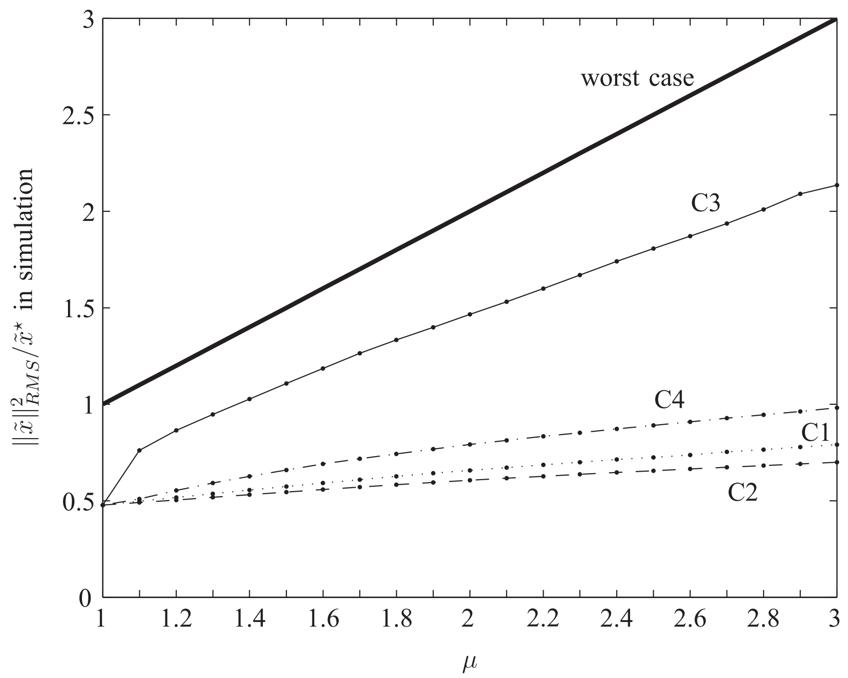

Figure 4. Ratio from the $\|\tilde{z}[t]\|^2_{RMS}$ obtained in simulation to the the standard $\mathcal{H}_\infty$ observer performance bound as a function of the ratio μ.

transmissions can be reduced at the expense of more computational complexity, with respect the use of a constant gain.

5.2 *Experimental application example*

In this example we show the behavior of the proposed observer in a real application, see Figure 5. The process under study consists on a DC motor with an incremental encoder and an H-bridge driver module based on a L298. The plant has three nodes connected through a CAN network (as shown in Figure 5). Two of the nodes are Texas Instruments TMS320LF2407 microcontrollers. One of them reads the encoder signal to compute the shaft speed and implements periodically the send-on-delta mechanism deciding whether to send the measurement after comparing it with the last sent. The other node receives messages with the voltage to be applied on the motor and periodically generates accordingly a pulse width modulation signal and digital signals to apply on the H bridge. The third node connected to the network is implemented in an industrial computer with a CAN card running xPC from Mathworks. The computer reads the message containing the shaft speed, uses that measurement to observe the system state and runs a speed controller sending the resulting control action to the actuator node at each instant. The sensor, actuator and observer/controller nodes update the required values for the velocity control each 5ms.

After performing an identification experiment, we obtain the following transfer function from voltage to angular velocity in the motor

$$G(s) = \frac{5.1e^{-0.01s}}{(1+0.033s)(1+0.0032s)}\,\frac{rad/s}{V}.$$

Note that we have an additional delay of 10ms due to the network behavior (in addition to the inherent one sample delay in digital control). A discretized PI controller with $K_p = 0.25$ and $K_i = 7.78$ and reference weighting $b = 0.7$ is applied at 5ms period. The reference is generated as an square wave between 40 and 50rad/s with a period of 1s. Our aim is to compare the difference of applying the PI controller with the available measurements (using repeatedly the same measurement while no new one is available), with respect of applying a PI controller that uses the estimated output with the proposed observer (inferential PI controller).

A zero-order-hold discrete equivalent model is obtained from the continuous model in order to apply the methodology in this work to design the observer, giving us matrices A, B, and C, and, as we are interested in estimating the output, we fix $C_z = C$. With respect the disturbances in the system we consider the following issues: we assume a disturbance entering in the input channel bounded by $\|w[t]\|_{RMS} = 0.1$V and a measurement noise bounded by $\|v[t]\|_{RMS} = 0.5$rad/s due to the encoder accuracy.

We fix the desired output estimation error as $\tilde{z}_{rms} = 0.5$rad/s and perform the codesign procedure with the stochastic approach with uniform assumption taking $r = 5$rad/s, obtaining both the gains of the observer and the threshold $\Delta = 2$rad/s for the sensor node. With these considerations, we implement the SOD mechanism on the sensor node and the PI controller and observer on the industrial computer.

Figure 6 shows in the left higher part the behavior of the controlled motor when applying the PI controller directly with the received measurements (that are constant between new arrivals), while in the right part we show the outcome when the PI uses as feedback signal the output of the observer. We show the measurement in the sensor node with thin lines, the received one in the controller node with circles, and with thick lines the estimated output. The lower figure shows the resulting control actions for both approaches. We observe that using the observer and the estimated output in the PI controller leads to a tracking error of 5%, but avoids the limit cycle produced by the PI controller with SOD measurements observed in the left figure (see Beschi et al. (2012) for further details in this issue). Furthermore, as an indirect effect, the number of transmissions from the sensor node is reduced.

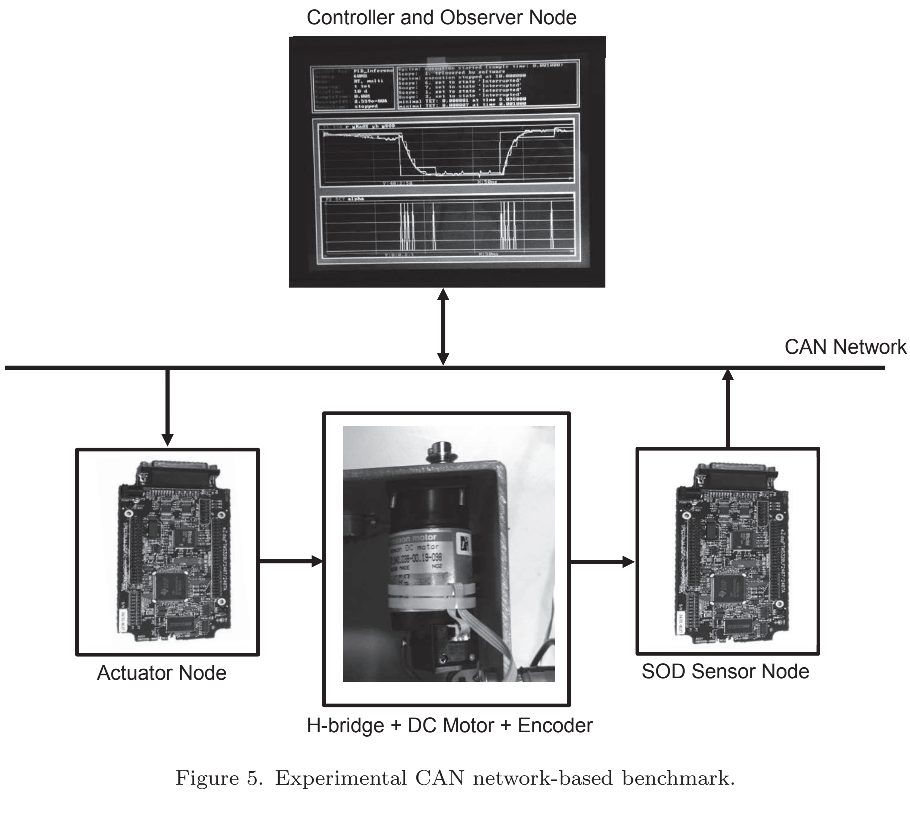

Figure 5. Experimental CAN network-based benchmark.

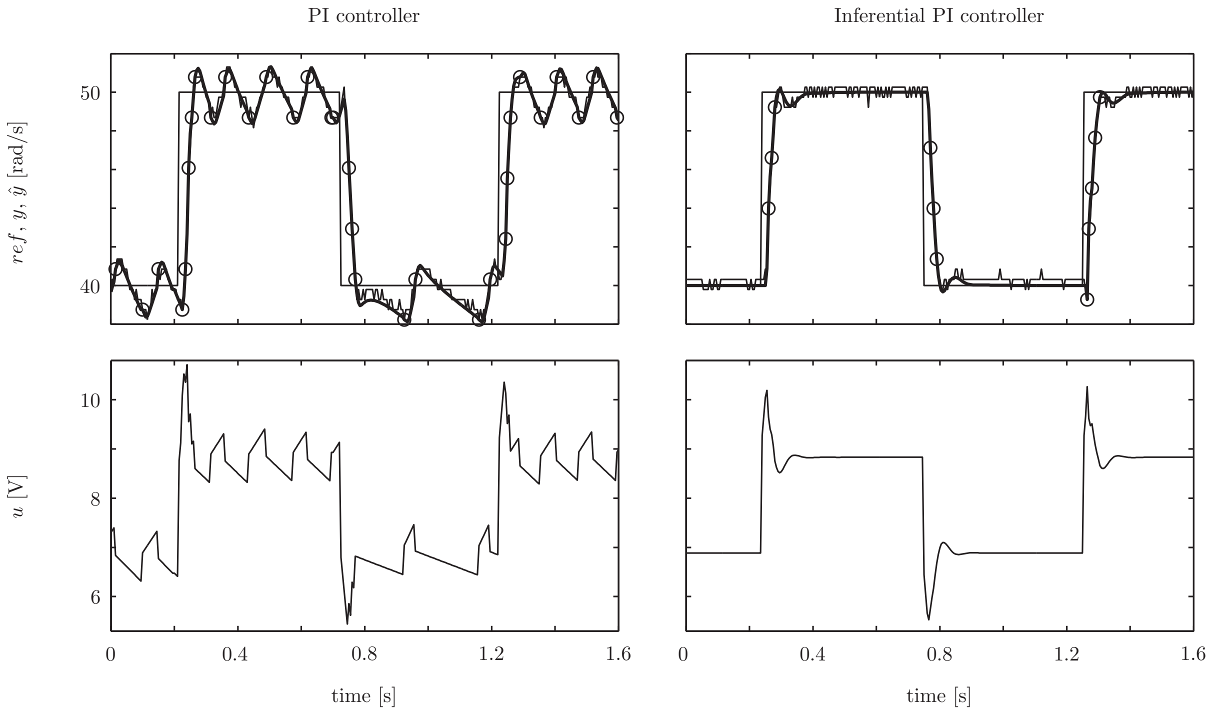

Figure 6. Comparison under a SOD measurement transmission policy of an output feedback PI controller versus an inferential PI one. '–': estimated output; '-': current measurement, reference and control action; '○': new measurement arrival.

6. Conclusions

In this work, we addressed an observer co-design procedure for state estimation over networks when using event-based measurements. We used a low computational cost estimation strategy that consists on using a simple send-on-delta strategy on the sensor nodes, and a $\mathcal{H}_\infty$ jump linear estimator that uses a gain within a predefined set depending on the combination of available measurements at each instant. We included a virtual noise to update the state estimation when new measurements are not available. We developed a strategy based on linear matrix inequalities to obtain the observer gains when the thresholds of the sensor nodes are fixed. To reduce conservativeness, we derived a lower bound on the probability of receiving a measurement and an upper bound on the RMS norm of the resulting virtual noise. In this case, we addressed the design of the jump observer by using optimization over polynomial techniques to include the uncertainty on the measurement receiving probability.

We then defined two characterizations of the network usage and used them to derive the co-design problem, consisting on finding the thresholds of the sensor nodes and the corresponding observer gains that led to the lowest network usage allowing to reach a prescribed performance on the state estimation error.

As future research works, we will address the co-design in inferential control and fault diagnosis problems based on observers that use send-on-delta measurements.

ACKNOWLEDGEMENTS

This work has been funded by MICINN project DPI2011-27845-C02-02 from Spanish Government and by project P1·1B2013-51 and grant PREDOC/2011/37 from Universitat Jaume I of Castelló, Spain.

Appendix

In this appendix we present the necessary auxiliary results that gives us the probabilities and RMS norm of the virtual noises in the stochastic approach, as well as the auxiliary tools (sum of squares decomposition) that allow us to present the stochastic approach robust against probability uncertainties.

6.1 *Probabilities and variances computation*

We derive here the expressions to compute the probability of sending new measurements and the associated variance of the virtual noise during the less excited periods.

Lemma 1. *Let us assume that each measured output of sensor i is independent and uniformly distributed within the range $\bar{y}_{i,\min} \leq y_i \leq \bar{y}_{i,\max}$, being $r_i = \bar{y}_{i,\max} - \bar{y}_{i,\min}$ as given in (28), for the periods t_{k_i} and $t_{k_i}+1$, thus fulfilling $y_i[t] - y_i[t_{k_i}] \in [-r_i, r_i]$. Then, the probability of having a new measurement from sensor i at period $t = t_{k_i}+1$ with law (2) and $\Delta_i \leq r_i$ is given by*

$$\beta_i = P\{|y_i[t] - y_i[t_{k_i}]| \geq \Delta_i\} = \frac{1}{r_i^2}(r_i - \Delta_i)^2. \tag{39}$$

The variance of the virtual noise related to this Δ_i is given by

$$\sigma_{\delta_i}^2 = Var\{\delta_i[t]\} = E\{\delta_i[t]^2\} = \frac{2\Delta_i^3}{r_i^2}\left(\frac{r_i}{3} - \frac{\Delta_i}{4}\right). \tag{40}$$

Proof 4. *Let us call the random variable $y_i[t_{k_i}]$ as u, denoting the measurement value at t_{k_i}, the last time it was sent from sensor to observer node. Then, the probability density function of u is*

$$f_U(u) = \begin{cases} \frac{1}{r_i}, & u \in [\bar{y}_{i,\min}, \bar{y}_{i,\max}] \\ 0, & \text{otherwise} \end{cases}$$

Let us also call the random variable $y[t]$ as v, denoting the posterior measured output at $t = t_{k_i}+1$, with a probability density function

$$f_V(v) = \begin{cases} \frac{1}{r_i}, & v \in [\bar{y}_{i,\min}, \bar{y}_{i,\max}] \\ 0, & \text{otherwise} \end{cases}$$

The probability density function of the random variable $w = y_i[t] - y_i[t_{k_i}] = u - v$ is given by the convolution

$$f_W(w) = \int_{-\infty}^{\infty} f_U(v+w) f_V(v) dv.$$

Since $f_V(v) = 1/r_i$ if $\bar{y}_{i,\min} \leq v \leq \bar{y}_{i,\max}$ and 0 otherwise, this becomes

$$f_W(w) = \frac{1}{r_i} \int_{\bar{y}_{i,\min}}^{\bar{y}_{i,\max}} f_U(v+w) dv.$$

The integrand is 0 unless $\bar{y}_{i,\min} \leq v+w \leq \bar{y}_{i,\max}$ (i.e., unless $\bar{y}_{i,\min} - w \leq v \leq \bar{y}_{i,\max} - w$) and *in that case, it is* $\frac{1}{r_i}$. *So it leads*

$$f_W(w) = \begin{cases} \frac{1}{r_i}\int_{\bar{y}_{i,\min}-w}^{\bar{y}_{i,\max}} \frac{1}{r_i} dv = \frac{r_i+w}{r_i^2}, & -r_i \le w \le 0, \\ \frac{1}{r_i}\int_{\bar{y}_{i,\min}}^{\bar{y}_{i,\max}-w} \frac{1}{r_i} dv = \frac{r_i-w}{r_i^2}, & 0 \le w \le r_i, \\ 0, & w^2 > r_i. \end{cases}$$

The probability of having two consecutive measurements with an absolute difference greater than Δ_i *is*

$$\beta_i = P\{|y_i[t] - y_i[t_{k_i}]| \ge \Delta_i\} = 1 - P\{-\Delta_i < w < \Delta_i\}.$$

Using the above probability density function this can be computed by

$$\beta_i = 1 - \int_{-\Delta_i}^{\Delta_i} f_W(w) = \frac{(r_i - \Delta_i)^2}{r_i^2}$$

The virtual noise signal can be obtained as a function of the previous random variable w *as*

$$\delta[t] = g(w) = \begin{cases} w, & -\Delta_i < w < \Delta_i \\ 0, & \text{otherwise} \end{cases}$$

Its expected value is given by

$$E\{\delta[t]\} = \int_{-\infty}^{\infty} g(w) f_W(w) dw = 0,$$

and the variance is given by

$$\begin{aligned} Var\{\delta[t]\} = E\{\delta[t]^2\} &= \int_{-\infty}^{\infty} g(w)^2 f_W(w) dw \\ &= \frac{2\Delta_i^3}{r_i^2}\left(\frac{r_i}{3} - \frac{\Delta_i}{4}\right), \end{aligned}$$

Lemma 2. *Let us assume that the difference* $y_i[t] - y_i[t_{k_i}]$ *is normally distributed with zero mean and variance* σ_i^2 *given by (31) for* $t = t_{k_i} + 1$. *Then, the probability of having a new measurement from sensor* i *at each period* $t = t_{k_i} + 1$ *with law (2) is given by*

$$\beta_i = P\{|y_i[t] - y_i[t_{k_i}]| \ge \Delta_i\} = 1 - \text{erf}\left(\frac{\Delta_i}{\sqrt{2}\sigma_i}\right), \tag{41}$$

being $\text{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2} dt$ *the error function. The variance of the virtual noise related to this* Δ_i *is given by*

$$\sigma_{\delta_i}^2 = Var\{\delta[t]\} = E\{\delta[t]^2\} = \sigma_i^2 \text{erf}\left(\frac{\Delta_i}{\sqrt{2}\sigma_i}\right) - \frac{\sqrt{2}\Delta_i\sigma_i}{\sqrt{\pi}} e^{\frac{-\Delta_i^2}{2\sigma_i^2}}. \tag{42}$$

Proof 5. *The variable* $w = y_i[t] - y_i[t_{k_i}]$ *has the density function*

$$f_W(w) = \frac{1}{\sigma_i\sqrt{2\pi}} e^{\frac{-w^2}{2\sigma_i^2}}.$$

The probability of having a new measurement available is

$$\beta_i = 1 - \int_{-\Delta_i}^{\Delta_i} f_W(w)dw = 1 - \frac{2}{\sqrt{\pi}} \int_0^{\Delta_i} h'(w)e^{-(h(w))^2}dt$$

with $h(w) = \frac{w}{\sqrt{2}\sigma_i}$ *and* $h'(w) = \frac{1}{\sqrt{2}\sigma_i}$*, and where we accounted that* $f_W(w)$ *is symmetric with respect to* $w = 0$*. Applying the definition of the error function, it leads* (33)*.*

We express the virtual noise signal as a function of the previous random variable w *as*

$$\delta[t] = g(w) = \begin{cases} w, \ -\Delta_i < w < \Delta_i \\ 0, \ \text{otherwise} \end{cases}$$

Its expected value is given by

$$E\{\delta[t]\} = \int_{-\infty}^{\infty} g(w)f_W(w)dw = 0$$

and the variance is given by (with $E\{\delta[t]^2\} = 0$*)*

$$Var\{\delta[t]\} = \int_{-\infty}^{\infty} g(w)^2 f_W(w)dw = \int_{-\Delta_i}^{\Delta_i} w^2 \frac{1}{\sigma_i\sqrt{2\pi}} e^{\frac{-w^2}{2\sigma_i^2}}dw.$$

Integrating this expression, it follows straightforwardly (34)*.*

6.2 *Optimization over polynomials*

We state in this section some necessary results about optimization over polynomials that are needed for the robust formulation of the stochastic approach against the probability uncertainties.

Lemma 3. *Let* $p(x)$ *be a polynomial in* $x \in R^n$ *of degree* $2d$*. Let* $Z(x)$ *be a column vector whose entries are all monomials in* x *with degree* $\leq d$*. Then* $f(x)$ *is said to be SOS if and only if there exists a positive semidefinite matrix* Q *such that* $p(x) = Z(x)^T QZ(x)$*.*

We denote the SOS polynomials in variables x by $f(x) \in \Sigma(x)$. The following results can be derived from the called Positivstellensatz result Chesi (2010) which states that feasibility conditions over polynomials can be dealt searching for some SOS polynomials.

Lemma 4. *Let* $f(x)$ *be a polynomial in* $x \in R^n$*, and let* $X = \{x \in R^n : g_j(x) \geq 0, j = 1, \ldots, m\}$*. Suppose there exist SOS polynomials* $s_j(x) \in \Sigma(x)$ *(*$j = 1, \ldots, m$*,* $x \in R^n$*) such that* $f(x) - \sum_{j=1}^m s_j(x)\, g_j(x) \in \Sigma(x)$*, then, the following condition holds:* $f(x) \geq 0, \ \forall\, x \in X$*.*

Lemma 5. *Let* $F(x)$ *be a* $N \times N$ *symmetric polynomial matrix in* $x \in R^n$ *and let* $X = \{x \in R^n : g_j(x) \geq 0, \ j = 1, \ldots, m\}$*,* $v \in R^N$*. Suppose there exist SOS polynomials* $s_j(x, v) \in \Sigma(x, v)$ *(*$j = 1, \ldots, m$*) such that* $v^T F(x)v - \sum_{j=1}^m s_j(x, v)\, g_j(x) \in \Sigma(x, v)$*, then, the following condition holds:* $F(x) \succeq 0, \ \forall\, x \in X$*.*

The previous Lemmas show that verifying that a polynomial matrix is non-negative over polynomial constraints can be formulated as a LMI problem. This can be implemented with several LMI parsers as the one in Löfberg (2009).

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