Metrics of human perception of vanishing points in perspective sketches

Raquel Plumed

Departament of Mechanical Engineering and Construction Universitat Jaume I Avda. Sos Baynat, s/n Spain (E-12080), Castellón de la Plana

plumed@uji.es

Pedro Company

Ins. of New Imaging Technology, Universitat Jaume I Avda. Sos Baynat, s/n Spain (E-12080), Castellón de la Plana

pcompany@uji.es

Peter A.C. Varley

Ins. of New Imaging Technology, Universitat Jaume I Avda. Sos Baynat, s/n Spain (E-12080), Castellón de la Plana

varley@emc.uji.es

ABSTRACT

This paper describes an experiment aimed at discovering how humans perceive vanishing points depicted in perspective sketches of engineering shapes. The goal is to find criteria and metrics for an algorithmic approach to replicate human perception of vanishing points. A new approach is required for Sketch-Based Modelling, since most current image analysis approaches take 2D camera images as their input, so do not solve satisfactorily the problem of geometrical imperfections inherent in sketches.

We have conducted a pilot experiment to determine which vanishing points are perceived by people, and under what circumstances they are perceived. We test the hypotheses that (i) people are able to detect and locate vanishing points in sketches in spite of their inherent imperfections, and (ii) factors such as distance of vanishing points from the sketch and number and lengths of lines converging at the vanishing points influence their perception.

Keywords
Sketch-Based Modelling, Perspective projection, Vanishing points.

1. INTRODUCTION
Our goal is to assist designers to interact in a friendly way with computers. The advantages of this are well-documented [Joh09], but we believe that this interaction must not come at the cost of unexpected behavior. People will only trust the computer if they feel that it interprets things more or less as they do. To this end, we intend to develop algorithmic approaches which replicate human perception when reconstructing models depicted in perspective sketches of engineering designs.

Hence, we should know how humans interpret design sketches. Here, we describe an experiment aimed at discovering: which vanishing points people perceive, where they are located; and what geometrical flexibility in their locations can be tolerated.

After analysing the results, we obtain criteria and metrics which will help to create algorithms which mimic human behaviour.

We first revisit the background of central projection and vanishing points. We then describe the design of our experiment and analyse our results.

2. DEFINITION OF TERMS
In the field of sketch-based geometric reconstruction, one approach is to tackle the issue as an artificial perception problem [Lip96], [Var03], [Com04], [Yua08], [Tia09]. Human beings have an intrinsic or learnt capability to mentally reconstruct three-dimensional objects from 2D images by means of pictorial clues [Gol99], [Hof00]. Here, we are interested in a specific pictorial clue, the vanishing point (VP).

In *perspective* projection, parallel lines not parallel to the image plane converge to a *vanishing point* (VP). The fundamentals of perspective were first codified in Durer's Four Books on Measurement in 1522, and their effects on how we see and draw are well-known [Pal99], [Wri83].

The number of VPs in an image depends on the orientation of the depicted object relative to the projection plane. A normalon polyhedron (one with all its edges parallel to one of the three main Cartesian axes) may produce three distinct situations: a) two axes parallel to the image plane and just one VP where the lines parallel to the third axis converge; b) one axis parallel to the image plane and two VPs where the lines parallel to the other two axes converge, and c) no axis parallel to the image plane and three VPs where the lines parallel to the three axes converge. The three varieties, known as one, two and three vanishing point perspectives, are shown in Figure 1.
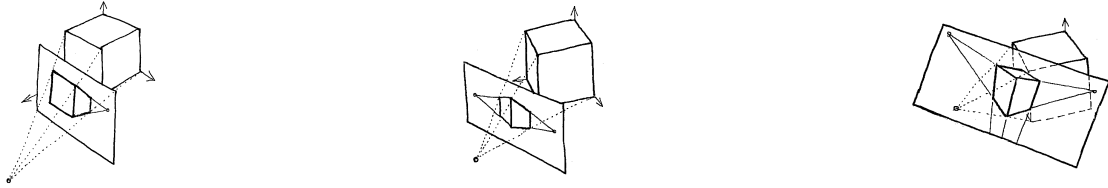
Figure 1. Linear perspective with one (left), two (middle) and three (right) vanishing points.

Another important distinction is between *main* and *oblique* VP. For a general polyhedron with n different sets of parallel edges, the varieties of linear perspective become n, $n-1$, $n-2$, ..., depending on the number of groups of parallel edges in the model which are parallel to the image plane. Figure 2 shows an example with only two main VP: the vertical axis is parallel to the image plane, so vertical lines do not converge; however, an additional oblique VP results from the convergence of the lateral edges of the wedge.
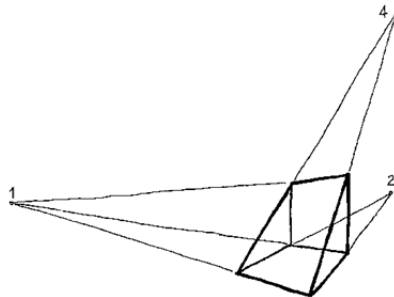

Figure 2. Wedge with two main vanishing points plus one oblique vanishing point.

There is no theoretical distinction between different locations of VPs relative to the object. But there is a useful practical distinction between VPs located *inside* and *outside* the object (Figure 3). Internal VPs are typical in indoor architectural scenes, but are rarely used to depict engineering products. Hence, we do not study them here.
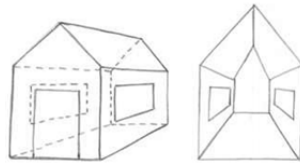

Figure 3. Linear perspectives of a prismatic shape with one external (left) and internal (right) vanishing point.
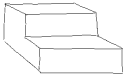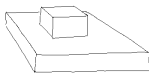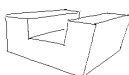
3. HYPOTHESES

The purpose of our experiment is to obtain criteria and metrics for algorithms which mimic human perception in detecting vanishing points in a sketch. Here, we propose the hypotheses to be tested:

1. Human beings perceive the existence of intended vanishing points in sketches of 3D polyhedral shapes, in spite of their inherent imperfections.

2. Humans beings are able to locate quite precisely those vanishing points which are neither too close to nor too far away from the drawing.

3. The lengths of lines influence convergence detection. The longer the lines, the easier it is to detect a vanishing point.

4. Design of the experiment

We designed our pilot experiment as follows. First, we selected a set of sketches. Then, we asked a group of subjects to determine the approximate number and location of vanishing points implied by a sketch, and also to label the different sets of parallel edges. Finally, we analysed the results to determine to what extent people agree in perceiving the same vanishing points, and what are the most influential factors in this perception process.
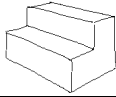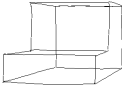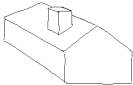
1		2	
3		4	
5		6	
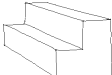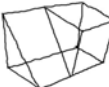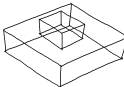
7		8	
9		10	
11		12	
13		14	
15		16	
17		18	

Figure 4.   Set of sketches used in the experiment

Here, we have not considered the scale as a main factor, because we guess that it only affects human perception for too small or too big drawings, where some lines may be perceived with difficulty. But this is not the case for design drawings sketched on pen input devices.

Set of sketches

Our set of test sketches is derived from typical Engineering Design training exercises (Figure 4).

The sketches were selected to meet the following criteria:

- They should be simple, containing no unnecessary features or details which could divert the attention
- They should be representative of shapes usually sketched in engineering design processes
- They should represent polyhedral shapes, in both natural and wireframe styles.
- They should be tidied line drawings in central projection style.
- They should represent different varieties of central projection (one, two, three vanishing points).
- Some of them should contain vanishing points corresponding to oblique directions.
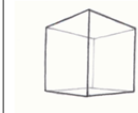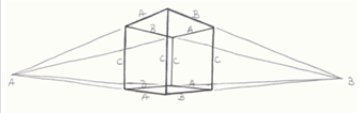
The result was the set of 18 sketches shown in Figure 4.

We then circulated these sketches to our test subjects, who marked them up as requested.

Finding vanishing points

The first task for the test subjects was to find and mark all the VPs for a given sketch. We gave them standardised A4 questionnaire containing a short explanation of the task, a visual example (Figure 5), and two sketches selected randomly from the test set.

Figure 5.   Questionnaire.

Finding non-convergent groups of lines

We also wanted to know why possible VPs were left unmarked: were they dubious, or did they corresponding to perceived parallel (i.e. non-converging) edges? Hence, as second task, the subjects were asked to mark all those groups of lines representing parallel edges, and label them as separate sets.

Participants

The bulk of the subjects who participated in the experiment were drawn from diverse departments of the same university, and included mechanical, electric and industrial engineers, architects, designers and artists. The level of experience ranged from undergraduate students to professors. Of the 149 participants, 92 (61.7%) were engineers, 23 (15.4%) architects, 20 (13.4%) had artistic knowledge, and 14 (9.4%) were school-age (17-18 years) students whose studies included technical drawing.

We found no systematic differences in the results between subjects from different backgrounds. In the analysis below, we treat the subjects as a single homogeneous group.

5. Results

We issued 298 questionnaires, of which 291 were returned. At this stage, we removed from the study those questionnaires which did not give coherent results. The most common mistakes were due: (i) to misunderstanding the concept of vanishing points, marking erroneous VPs which did not correspond to intersections of lines of the drawing (Figure 6a); or (ii) failing to understand the drawing as a representation of a 3D shape with parallel edges, marking erroneous VPs at the intersection of lines of the drawing which did not represent parallel edges of the depicted 3D shape (Figure 6b).

(a)

(b)

(c)
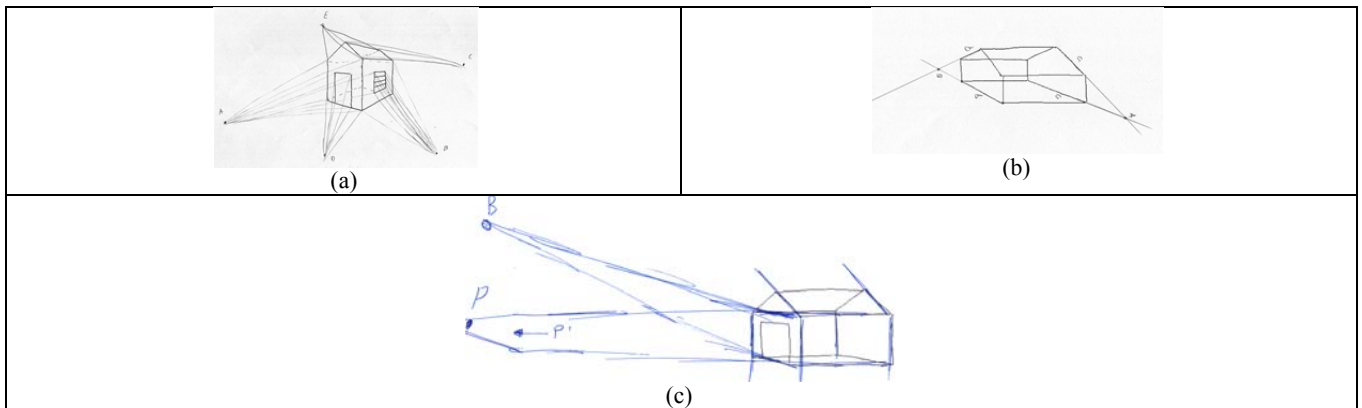
Figure 6.   Erroneous vanishing points.

Some other questionnaires were also dismissed or only partially considered as it was difficult to interpret them objectively. Most of these dubious questionnaires were of sketch 15 (Figure 6c).

From the total of 291 collected questionnaires, we were left with 266 coherent responses. Of these, 7 were explicitly marked as not containing any VPs (four of these were sketch 18, two were sketch 15 and one was sketch 11).

Qualitative validation of the first hypothesis

We grouped the lines of each sketch by the parallel edges they belong to in the 3D object. Since all of the sketches depict polyhedral models, they always have at least three main axes; we labelled these three main axes as X, Y and Z (or 1st, 2nd and 3rd), as in Figure 7; verticality is always labelled as Z (axis 3). Objects 3, 7, 10, 11, 12 and 15 include one or two additional oblique axes, labelled as axes 4 and 5.
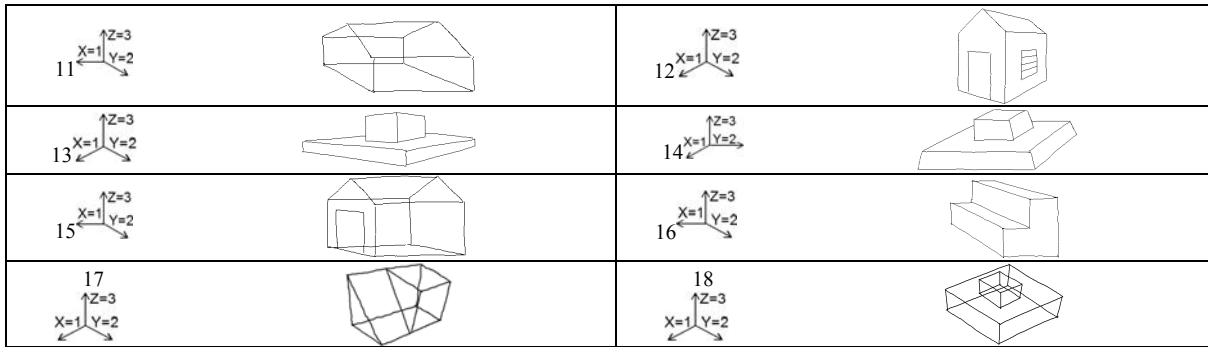
Figure 7. Main axis in the set of sketches used in the experiment

We then analysed the perception of VPs for each sketch and each candidate axis. Table 1 lists the percentage of polled people who perceived convergence for each sketch and axis.

These results show that people perceive the existence of a VP for axis 1 in sketches 3, 4, 5, 6, 9, 12, 13, 14 and 17. The existence of VP 1 is uncertain for sketches 7 and 8. Lines parallel to axis 1 are perceived as non-convergent for sketches 1, 2, 10, 11, 15, 16 and 18.

Similarly, people perceive the existence of a VP for axis 2 in all sketches except 3 and 15 (where the vanishing point is uncertain), and 9, 14, 17 and 18, which are perceived as non-convergent.

Sketch	Axis 1 (X)	Axis 2 (Y)	Axis 3 (Z)	Axis 4	Axis 5
1	46.67	93.33	33.33		
2	17.65	94.12	23.53		
3	100.00	72.22	5.56	66.67	
4	100.00	76.92	0.00		
5	100.00	84.21	5.26		
6	100.00	85.71	78.57		
7	68.75	93.75	6.25	75.00	
8	72.22	94.44	0.00		
9	100.00	7.14	0.00		
10	7.14	92.86	0.00	71.43	
11	22.22	77.78	0.00	44.44	33.33
12	100.00	76.47	5.88	35.29	5.88
13	100.00	92.31	0.00		
14	83.33	8.33	75.00		
15	28.57	57.14	28.57	28.57	42.86
16	6.67	100.00	0.00		
17	94.74	31.58	84.21	68.42	
18	6.25	12.5	68.75		

Table I. Perceived vanishing points

Axis 3 is perceived as convergent in sketches 6, 14, 17 and, with less certainty, in sketch 18. No- one perceived convergence in sketches 4, 8, 9, 10, 11, 13 and 16. It appears that engineering designers are less used to sketching convergence in the vertical direction, and subjects seem to be less willing to perceive convergence for this axis.

Convergence of oblique lines is only perceived with certainty for sketch 7; it is uncertain for sketches 3, 10 and 17. It appears that (i) humans do not readily perceive oblique convergence, but (ii) a large number of lines (as in sketch 7) help humans to identify convergence to an oblique VP.

In summary, convergence of the main axes seems to be readily and generally perceived, regardless of sketching imperfections. It is somewhat more difficult to perceive VPs for oblique axes. In Section 6, we shall attempt to determine the minimum threshold of angle of convergence and number of lines which guarantee a general perception of VPs.

Qualitative validation of the second hypothesis

For each sketch, we superimposed all VPs located by the subjects. For example, the red points in Figure 8 are the locations of the VP of lines aligned with axis 1, the blue points those aligned with axis 2, and the green points those aligned with axis 4. The results clearly show that people agree about the orientation angle along which the VP is located, but fail to agree about the position of the VP along this line.

It also appears that clouds of clearly-perceived VPs are shorter (and may be bounded by an ellipse), while clouds of uncertain VPs tend to be longer and resemble a straight line.

It also appears that the dispersion in the location increases when a) the VP is distant from the drawing, and b) when the group of lines is small and/or the lines are short.

5
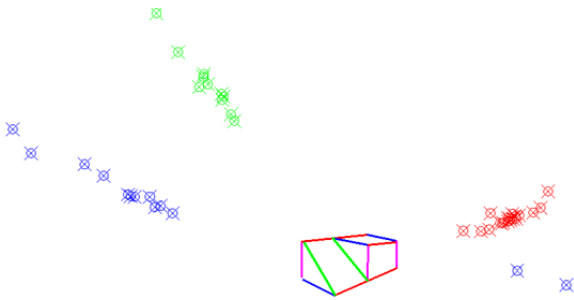
Figure 8.   Superimposition of all VPs located by the subjects for sketch 3.

We note in passing that in 4 of the questionnaires where objects are depicted by natural drawings, subjects drew the hidden lines in order to find the VP locations. This demonstrates that they knew how to interpret the sketches, but felt unable to fix VP location with precision. Figure 9 (sketch 12) shows how one subject even used hidden lines to locate an additional VP which corresponds just to a single line of the original sketch.
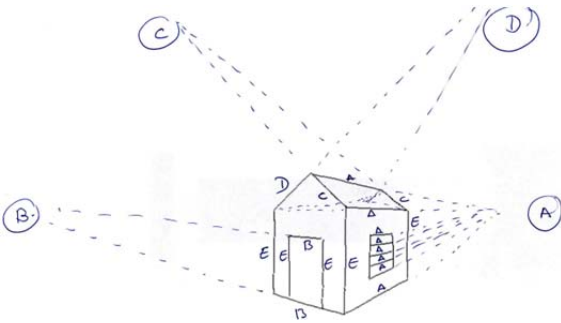
Figure 9.   Hidden lines for VPs location in sketch 12.

Analysing the questionnaires, we noticed that the subjects used one of three strategies to overcome the imperfections of the sketches and find the most likely location for VPs.
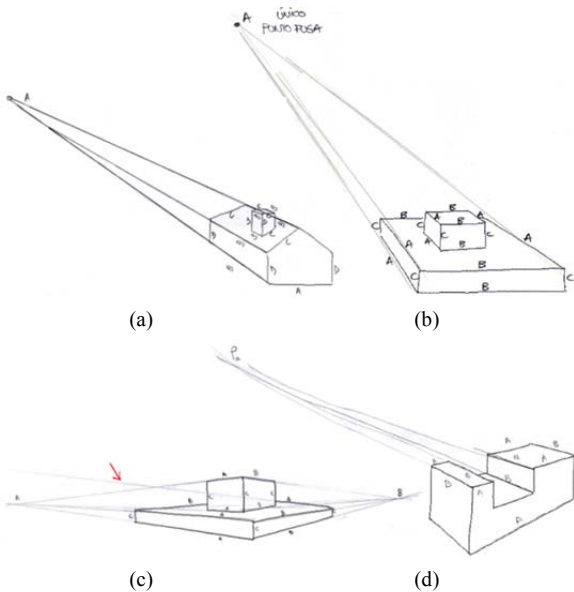
Figure 10. Strategies to select the most likely VP location

Firstly, some subjects selected the most likely lines in the group and use them to find the VP. Subjects in this group used different strategies to select the most representative lines: (a) select the outer lines (those which encompass the whole group, as in Figure 10a), (b) to discard the shortest lines (Figure 10b) and the most erratic lines (Figure 10c); or (c) simply estimate a rough VP location using a random subset of the lines (Figure 10d). This strategy was most commonly used for sketches 2 (Fig. 10b), 3, 5 (Fig. 10d), 7, 10 (Fig. 10a), 13 (Fig. 10c) and 14.

Secondly, some subjects calculated more than one location for the same VP. These subjects identified the convergence in two ways: (a) by means of a point cloud formed by intersections of the lines of the same group (Figure 11a), or 2) defining different groups of lines which belong to the same axis (Figure 11b). This strategy was most commonly used for sketches 9 (Figure 11b), 12 (Figure 11c), 13 (Figure 11d) and 18 (Figure 11e).
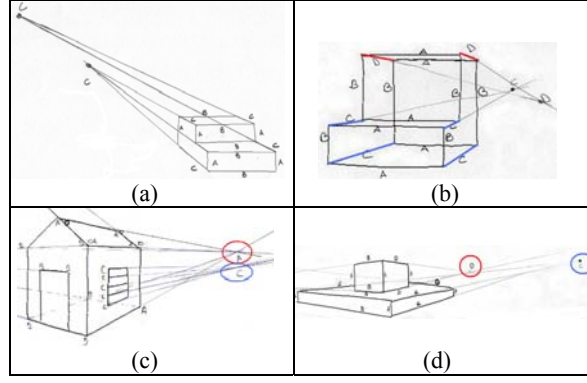
Figure 11. Point clouds for VPs definition

Thirdly, some subjects seemed to detect convergence but did not locate any VPs.

Thus, our second hypothesis should be rejected: humans do not seem to be able to locate VPs precisely: they agree about the orientation angle of the VP, but not about its position along this line.

Qualitative validation of the third hypothesis

We note that the uncertain cases in Table I (those in the range 50-75%) are typically those formed from either a) groups of lines which contain the shortest lines of the drawing, or b) groups with low density of lines. This perception supports our third hypothesis: that length of lines influences the convergence detection. However, in the light of the data, we must also take into account the density of the group of lines.

6. Numerical measurement

Having evaluated our hypotheses qualitatively, we now search for metrics which can help to tune automatic algorithms for finding VPs in engineering sketches. We define and study some geometric parameters.

Firstly, since our goal is finding parameters which influence in the perception success, we shall compare our geometric parameters with the parameter used in Table I:

Perception degree (Det) is the percentage of the subjects who identified a VP for a specific sketch and axis.

$$\text{Det} = (\text{No. detections} / \text{No. Questionnaires})*100 \quad (1)$$

Some geometric parameters must be normalised to avoid the influence of the drawing size. We use the radius of the sketch's bounding circle. We calculate this as the maximum distance between the centre of mass of the sketch and any of its vertices. First, the centre of mass of the sketch is calculated (x_{com}, y_{com}).

$$x_{com}= (\textstyle\sum x_i)/n_v), \ y_{com}= (\textstyle\sum y_i)/n_v \quad \forall i \in \mathbf{V_S} \quad (2)$$

where n_v represents the number of vertices of the sketch S and $\mathbf{V_S}$ is the set of vertices of sketch S.

$$\text{size} = \max(((x_i\text{-}x_{com})^2 + (y_i\text{-}y_{com})^2)^{1/2}) \quad \forall i \in \mathbf{V_S} \quad (3)$$

In order to evaluate metrics related to the first hypothesis we analysed the convergence by means of the following parameter:

Angular dispersion (AD) is the maximum aperture angle between pairs of lines in a group of lines which represent parallel edges in space:

$$\text{AD} = \max (|\alpha_i\text{ -}\alpha_j|) \quad \forall i, j \in \mathbf{L_{SA}} \quad\quad (4)$$

where α_i and α_j represent, respectively, the angles of edges i and j relative to the same origin; and $\mathbf{L_{SA}}$ is the group of lines of sketch S and axis A.

In order to evaluate metrics related to the second hypothesis we define the following parameters:

Dispersion (Disp) measures the density of the point cloud of VPs located by the subjects (such as the blue, green and red clouds in Figure 8).

First, the centroid of the cloud is calculated:

$$x_{centroid}= (\textstyle\sum x_i)/n_{vp}), \ y_{centroid}= (\textstyle\sum y_i)/ n_{vp} \quad \forall i \in \mathbf{VP_{SA}}(5)$$

where $\mathbf{VP_{SA}}$ is the cloud of VPs for sketch S and axis A, and n_{vp} is the size of this cloud. Next we calculate the Euclidean distances d_i between each point (x_i, y_i) in the cloud and its centroid ($x_{centroid}$, $y_{centroid}$).

$$d_i= [(x_i\text{-}x_{centroid})^2 + (y_i\text{-}y_{centroid})^2]^{\frac{1}{2}} \ \forall i \in \mathbf{VP_{SA}} \quad (6)$$

Then, the standard deviation of these distances (dev) is calculated.

$$\text{Av} = (\textstyle\sum d_i)/n \quad \forall i \in \mathbf{VP_{SA}} \quad\quad (7)$$

$$\text{dev} = [(1/(n\text{-}1))*\textstyle\sum(d_i\text{–Av})^2]^{\frac{1}{2}} \quad \forall i \in \mathbf{VP_{SA}} \quad (8)$$

Finally, to avoid the influence of the drawing size, the parameter is normalised.

$$Disp = (dev/size)*100 \qquad (9)$$

Distance ratio between centroids (DRC) measures how far away the point cloud is from the sketch. It is calculated as the Euclidian distance between the centre of mass of the sketch and the centroid of the point cloud:

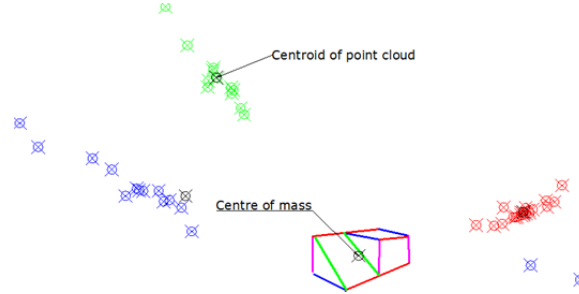$$Dist = ((x_{centroid} - x_{com})^2 + (y_{centroid} - y_{com})^2)^{1/2} \qquad (10)$$

Figure 12. DRC calculation for Sketch 3, Axis 4

Finally, to avoid the influence of the drawing size, the parameter is normalised:

$$DRC = Dist / size \qquad (11)$$

In order to evaluate metrics related to the third hypothesis we define the following parameters:

Length Dispersion (LeD) is a standard deviation which measures the dispersion of the lengths of the lines (l_i) which belong to the same sketch and axis: the more the lengths of lines differ, the higher its value. First the average length L_{aver} is calculated. Taking l_i as the length of each line and n_l as the number of lines:

$$L_{aver} = (\textstyle\sum l_i)/n_l \quad \forall\, i \in \mathbf{L_{SA}} \qquad (12)$$

$$LeD = [(1/(n_{SA}-1))*\textstyle\sum(l_i - L_{aver})^2]^{1/2} \quad \forall\, i \in \mathbf{L_{SA}} \qquad (13)$$

where n_{SA} is the number of lines in the group of parallel edges of sketch S, axis A.

Number of lines (NL) which belong to the same sketch and axis.

$$NL = n_{SD} \qquad (14)$$

Location Dispersion (LoD) is a standard deviation which measures the influence of dispersion of the locations of the midpoints of lines which belong to the same sketch and axis) and the normalised length of each line. The more the locations of lines differ, and the shorter the lines, the higher the value.

We first compute the location of each midpoint, by way of the head (xh_i, yh_i) and tail (xt_i, yt_i) coordinates of each line:

$$loc_i = ((xh_i + xt_i)/2 , (yh_i + yt_i)/2) \quad \forall\, i \in \mathbf{L_{SA}} \qquad (15)$$

$$Loc_{aver} = (\textstyle\sum loc_i)/n_l \quad \forall\, i \in \mathbf{L_{SA}} \qquad (16)$$

Next we calculate the standard deviation normalised with the relative length:

$$LoD = [(1/(n_{SD}-1))*(size/l_i)*\textstyle\sum(loc_i - Loc_{aver})^2]^{1/2}$$

$$\forall\, i \in \mathbf{L_{SA}} \qquad (17)$$

We calculated all of these parameters for each sketch and axis. The results are shown in the Table II, which is arranged in decreasing order of values of convergence detection.

Sketch	Axis	Det	AD	Disp	DRC	LeD	NL	LoD
3	1	100.00	16.99	27.54	3.15	12.61	3	7.23
4	1	100.00	27.95	51.92	2.19	1.91	6	4.76
5	1	100.00	15.25	45.26	3.46	14.46	7	37.17
6	1	100.00	31.67	35.54	2.03	3.34	6	6.85
9	1	100.00	51.51	50.89	1.69	7.44	6	50.18
12	1	100.00	41.24	22.31	1.91	8.98	8	10.69
13	1	100.00	23.73	32.11	1.72	12.19	5	20.11
16	2	100.00	17.42	23.30	3.18	1.67	5	7.04
17	1	94.74	16.12	44.48	3.66	9.33	4	9.99
8	2	94.44	15.23	46.23	4.70	1.58	5	5.06
2	2	94.12	24.65	26.51	2.16	12.09	6	22.61
7	2	93.75	16.85	39.22	3.70	11.47	3	7.66
1	2	93.33	16.35	39.05	3.46	4.39	5	14.87
10	2	92.86	16.32	96.48	4.00	13.19	7	26.21
13	2	92.31	14.21	36.53	2.11	20.59	5	25.36

6	2	85.71	15.12	30.89	2.93	8.95	6	22.06
17	3	84.21	21.27	200.23	3.53	2.43	4	12.89
5	2	84.21	26.42	78.50	2.82	0.95	6	14.84
14	1	83.33	13.57	40.21	3.02	8.37	6	16.51
6	3	78.57	21.04	86.08	4.18	6.30	5	29.35
11	2	77.78	7.24	25.93	3.11	5.15	2	4.82
4	2	76.92	27.29	194.10	1.64	5.11	6	27.50
12	2	76.47	12.20	33.43	3.14	11.17	2	13.92
7	4	75.00	17.07	62.57	3.92	4.18	4	5.38
14	3	75.00	44.09	20.41	1.75	0.94	6	90.77
3	2	72.22	15.31	224.17	3.39	1.67	3	14.74
8	1	72.22	14.05	161.48	5.94	4.84	5	14.88
10	4	71.43	16.45	26.16	2.38	6.73	3	7.32
7	1	68.75	33.87	12.41	2.33	2.73	4	5.26
18	3	68.75	19.70	63.05	3.49	4.63	8	61.74
17	4	68.42	5.95	68.31	4.82	3.57	2	0.67
3	4	66.67	8.52	44.61	4.22	2.27	2	0.60
15	2	57.14	6.23	86.23	4.19	6.14	3	14.47
1	1	46.67	1.55	545.13	8.02	2.67	5	6.09
11	4	44.44	8.93	20.18	5.63	2.44	2	5.33
15	5	42.86	3.90	164.41	7.14	1.41	2	4.62
12	4	35.29	6.10	15.12	3.31	4.26	2	7.94
1	3	33.33	6.36	197.15	4.61	2.12	5	18.98
11	5	33.33	1.62	113.02	7.52	2.60	2	9.45
17	2	31.58	4.60	479.19	1.99	2.61	4	14.12
15	1	28.57	1.06	0.00	18.07	1.47	5	2.73
15	3	28.57	2.12	1429.85	20.07	2.38	6	21.68
15	4	28.57	2.29	0.00	16.70	0.61	2	1.91
2	3	23.53	7.03	144.20	2.52	1.46	6	92.01
11	1	22.22	1.92	0.00	1.79	4.09	5	2.43
2	1	17.65	3.82	745.68	12.49	21.37	6	83.56
18	2	12.50	4.18	0.00	1.78	24.38	8	9.63
14	2	8.33	1.54		3.38	19.12	6	19.98
9	2	7.14	1.17		4.15	5.73	6	8.66
10	1	7.14	5.29		2.31	13.76	3	30.75
16	1	6.67	3.73		5.76	7.29	5	23.03
7	3	6.25	3.26		12.89	4.67	4	10.96
18	1	6.25	5.89		2.41	29.39	8	16.92
12	3	5.88	1.61		2.18	9.51	7	17.04
12	5	5.88	0.00		3.26		1	
3	3	5.56	1.71		10.14	2.60	3	19.86
5	3	5.26	4.08		4.71	6.71	6	17.25
4	3	0.00	1.88			8.98	6	43.72
8	3	0.00	1.91			4.49	5	17.59
9	3	0.00	1.76			8.11	6	26.04
10	3	0.00	2.82			2.52	6	30.36
10	5	0.00	0.00				1	
11	3	0.00	1.76			1.53	4	35.24
13	3	0.00	3.27			4.88	6	252.70
16	3	0	2.05			8.16	5	28.28

Table II. Parameters which influence perception success

7. Analysis

A statistic analysis based on Pearson correlation shows the mutual influence of each pair of parameters. For this study we omitted the cases where the group was a single line (NL= 1), as they give no useful information (specifically, we omitted sketch 10 axis 5 and sketch 12 axis 5, although, as it was showed in Fig. 9, some people included hidden lines for sketch 12 axis 5 to get the information they needed to locate the VP).

		Det	AD	Disp	DRC	LeD	NL	LoD
Det	Pear	1	.757**	-.378**	-.403**	-.054	-.044	-.222
	Sig.		.000	.009	.002	.672	.733	.080
	N	63	63	47	56	63	63	63
AD	Pear		1	-.295*	-.460**	-.064	.229	.018
	Sig.			.044	.000	.621	.071	.888
	N		63	47	56	63	63	63
Disp	Pear			1	.581**	-.028	.117	.166
	Sig.				.000	.854	.433	.266
	N			47	47	47	47	47
DRC	Pear				1	-.219	-.181	-.064
	Sig.					.104	.181	.642
	N				56	56	56	56
LeD	Pear					1	.379**	.048
	Sig.						.002	.710
	N					63	63	63
NL	Pear						1	.301*
	Sig.							.017
	N						63	63
								1

Table III. Pearson correlation among parameters

We analyse our hypotheses in the light of these results.

Hypothesis 1

Table III shows that detection degree (Det) correlates best with AD (as was qualitatively deduced in Section 5.2). From Table II, we can see that, for directions where Det > 75%, the minimum AD value is 7.24º (sketch 11 axis 2, or figure 13a). This could be used as a minimum threshold, particularly since it is close to the maximum threshold of 8º proposed in [Plu10] for considering a bundle of lines as parallel. However, our qualitative analyses found sketch 11 to be uncertain. If we exclude this result, we get a more conservative minimum value of AD = 12.2º (sketch 12 axis 2, or figure 13b).
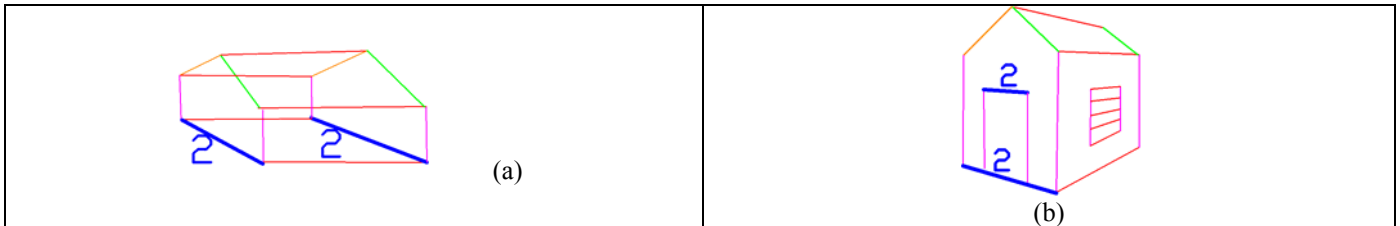

Figure 13. Sketches 11 and 12

We conclude that people clearly and consistently perceive convergence in spite of sketch imperfection if the lines span an angle of at least 12º. Between 12º and 8º the perception is uncertain. Hence, an algorithm for finding VPs should give a high probability to bundles spanning at least 12º; the probability should decrease between 12º and 8º, and should be close to zero below 8º.

Considering the uncertain cases of table II (those detected between 50% and 75%), three of them (sketch 17 axis 4, sketch 3 axis 4 and sketch 15 axis 2) have AD values close to 8º, as predicted by the above criterion. However, other cases (sketch 3 axis 2 (Fig. 14a), sketch 8 axis 1(Fig. 14b), sketch 10 axis 4 (Fig. 14c), sketch 7 axis 1 (Fig. 14d) and sketch 18 axis 3 (Fig. 14e) have AD higher than 12º, which, according to our criterion, should encourage subjects to perceive them as unambiguously convergent. We note that all of these contain the shortest lines of their sketches (sometimes alongside medium-length lines), and in addition they have a low line density (i.e. not only is the number of lines low—3 or 4 lines, except sketch 8, which has 5 lines for axis 1—but they are dispersed around the sketch rather than clustered together). This seems to support our third hypothesis, that the lengths of lines will influence perception of their convergence.

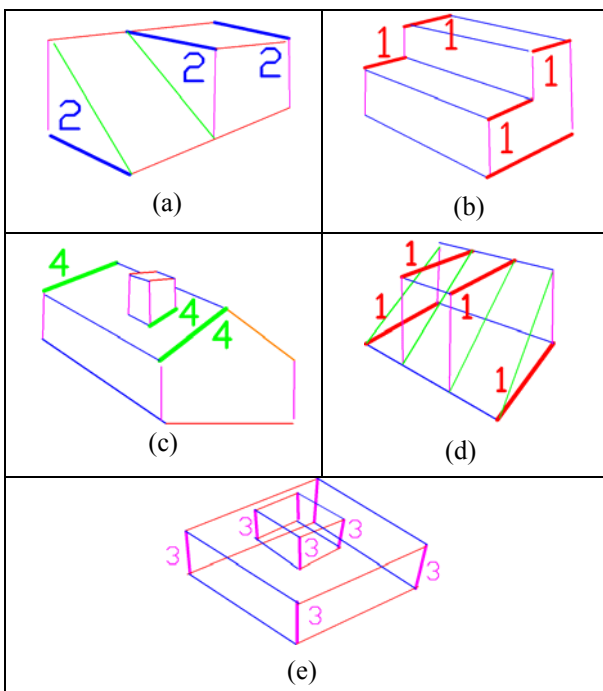

Figure 14. Sketches and axis

Hypothesis 2

As argued in section 5.2, humans are able to agree about the orientation angle of a VP but not about the precise location of the VP along this line. This pattern can be quantified by means of DRC and its dispersion measure Disp.

Table III shows that detection degree (Det) correlates with small values of distance ratio between centroids (DRC), and usually with small dispersion between the intersection points (Disp).

From Table II, we see that, for those VPs perceived by all subjects (Det=100), the value of DRC varies between 1.6 and 3, except in those cases where the group of lines includes the largest lines of the drawings (sketch 3 axis 1, sketch 5 axis 1, sketch 16 axis 2), in which cases the length of the lines seems to encourage subjects to locate the VP further away, and the value of DRC rises to 3.46. These locations correspond to small or medium values of the dispersion point cloud (with Disp between 22.31% and 51.92%), which means that people agree to locate the VPs within a small area.
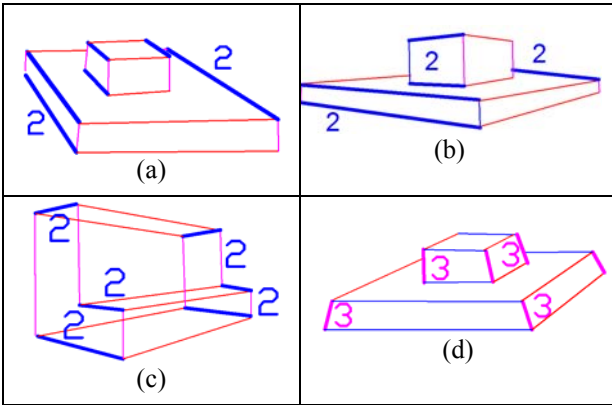

Figure 15. Sketches and axis

For Det in the range 99–75, the value of DRC is generally in the range 3 to 5. The exceptions are those examples which are uncertain or have a DRC value slightly lower than 3. In both Sketch 2 axis 2 (Fig. 15a) and sketch 13 axis 2 (Fig. 15b), the difference of line lengths is evident, and the shortest lines form a visual group distinct from the longest lines. The other exceptions are sketch 4 axis 2 (Fig. 15c), sketch 14 axis 3 (Fig. 15d), sketch 6 axis 2 and sketch 5 axis 2 in which short or medium lines are dispersed around the drawing.

Cases with detection under 50% generally have values of DRC higher than 5, except sketch 12 axis 4 (two lines in an oblique direction, Fig. 16a) and sketch 1 axis 3 (Fig. 16b), sketch 17 axis 2 (Fig. 16c) and sketch 2 axis 3 (Fig. 16d), which contain short and medium lines, far apart in the drawing.

Cases with detection under 50% also generally have values of "Disp" over 100%, except for two cases of oblique axes (sketch 11 axis 4 and sketch 12 axis 4, Fig. 16a) where subjects seem to reach agreement about the locations.
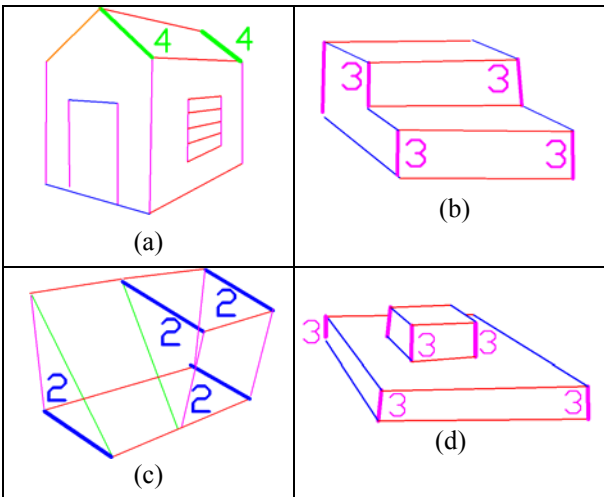

Figure 16. Sketches and axis

We do not take into account those cases with very low point cloud density, since the information is insufficient to extract any significant conclusion. For example, the closeness of the centroid of the point cloud to the sketch for sketch 11 axis 1 (Fig. 17) is clearly irrelevant.
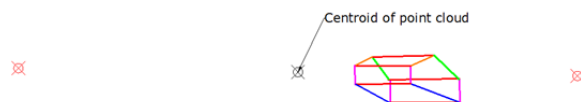

Figure 17. Point cloud for sketch 11 axis 1.

In summary, we found that DRC between 1.6 and 3 a VP is likely to be perceived. With DRC between 3 and 5 a VP is somewhat less likely to be perceived. With DRC higher than 5, lines are not perceived as convergent.

Hence, an algorithm for finding VPs should assign a high probability to candidate VPs located in a ring whose minimum radius is 1.6 times the radius of the bounding circle, and whose maximum radius is 3 times the radius of the bounding circle. The probability should decrease outside this ring, and be close to zero outside an outer ring whose radius is 5 times the radius of the bounding circle.

In cases where the lines are short and visually apart from one another, people's behaviour changes. This is considered under the third hypothesis.

Hypothesis 3

To evaluate our third hypothesis, we focus on the standard deviation between lengths LeD. Table III shows no relationship between the detection of convergence (Det) or the answers dispersion (Disp) and LeD. Should we then simply reject hypothesis 3? We cannot deny the influence of differences in line length, since we have already noticed that subjects changed their answer patterns under specific situations:

a) When the group of lines contains lines of very different lengths.

b) When the group of lines includes only short lines and they are dispersed through drawing.

c) When number of lines in the group is low.

Thus we should consider LoD as a secondary parameter, whose influence appears only in certain cases.

According to table II, for values of LoD higher than 70 (short lines widely dispersed around the sketch), our subjects generally followed the criteria described in previous hypotheses, and parameters AD and DRC reflect the response of human perception.

LoD values below 5 may result from long lines with a homogeneous location in the drawing (as in Fig. 18). However, a low value may also be due to low number of lines, which distorts the standard deviation measure. For groups of fewer than 4 lines, this parameter may not give reliable information.
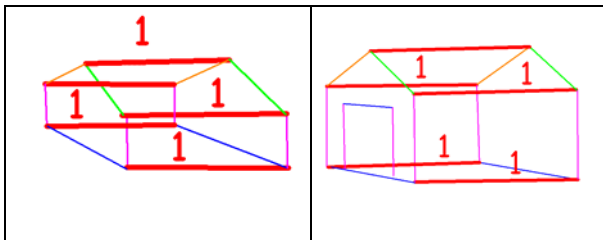

Figure 18.   Sketches 11 and 15 with low value of LoD.

LoD in the range 5-70 seems to influence human perception in two different ways: (i) sometimes the detection degree is slightly different from that predicted by parameters AD and DRC—for example, in sketch 18 axis 3(Fig. 14e), it seems that in spite of having an AD higher than 12, the separation between the lines, which are also the shortest in the sketch, prevents humans from perceiving the convergence (25% of subjects said it was an axonometric drawing); (ii) at other times the location agreement decreases which results in higher values of Disp.

Sketch	Axis	Det	AD	Disp	DRC	LeD	NL	LoD
15	3	28.57	2.12	1429.85	20.07	2.38	6	21.68
2	1	17.65	3.82	745.68	12.49	21.37	6	83.56
1	1	46.67	1.55	545.13	8.02	2.67	5	6.09
17	2	31.58	4.60	479.19	1.99	2.61	4	14.12
3	2	72.22	15.31	224.17	3.39	1.67	3	14.74
17	3	84.21	21.27	200.23	3.53	2.43	4	12.89
1	3	33.33	6.36	197.15	4.61	2.12	5	18.98
4	2	76.92	27.29	194.10	1.64	5.11	6	27.50
15	5	42.86	3.90	164.41	7.14	1.41	2	4.62
8	1	72.22	14.05	161.48	5.94	4.84	5	14.88
2	3	23.53	7.03	144.20	2.52	1.46	6	92.01
11	5	33.33	1.62	113.02	7.52	2.60	2	9.45
10	2	92.86	16.32	96.48	4.00	13.19	7	26.21

Table IV. Perceived vanishing directions

Table IV is a sub-table of table II rearranged in descending order of Disp. From this table, it appears that very high values of Disp (Disp>500) seem to correspond to nearly parallel groups of edges (AD<4). Values of Disp in the range (100 to 500) seem to appear as a combined effect of small number of lines (NL<=3), and/or high dispersion of the location of lines LoD $\in$ (5, 70).

Thus, dispersion is high for uncertain cases which have small values of AD, or when the set of lines contains short and medium lines dispersed through the sketch. However, we have previously noted that when the group of lines contains lines of very different lengths people usually apply the strategies illustrated in figure 10 (using outermost lines, ignoring short or erratic

lines) - the most common is ignoring the shorter lines, and using only the longer lines to locate the vanishing point - and in such cases it seems that the dispersion decreases.

This strategy is easy to replicate algorithmically and it would model human perception well. The most representative cases (sketches 2, 5, 10, 13, illustrated in Fig. 10, and sketches 3, 7 and 14) have values of Led higher than 10. Thus Led > 10 could be used as a threshold for signalling to the algorithm that it should use only long lines to locate the VP.

8. Conclusions

Current image analysis approaches take 2D camera images as their input, so do not solve satisfactorily the problem of geometrical imperfections inherent in sketches. At this end we have conducted a pilot experiment which gives us preliminary criteria and metrics for implementing algorithms which mimic human perception in detecting vanishing points in design sketches.

Human beings can perceive the existence of intended vanishing points in sketches of 3D polyhedral shapes, in spite of their inherent imperfections. Humans generally perceive vanishing points for sets of lines spanning 12 or more degrees.

Humans agree about the orientation angle of the VP relative to the sketch. They often do not agree about the distance of the VP from the sketch. VPs are easiest to perceive and to locate if they are neither too close nor too far away from the sketch: ideally, at distances not much more than the size of the sketch. (We can hypothesise that sketches are produced according to these expectations.) Algorithms should follow these perceptual criteria: enforcement of the acceptance criteria should be tolerant to imperfections inside the main region (1.6x to 3x), and stricter outside (3x to 5x).

A number of secondary parameters combine to influence the perception and location of VPs, including line length, lines location and density of lines. The influence of these "distractions" can be subtle, and remain a matter for future research.

9. Acknowledgments

10. REFERENCES

[Com04] Company, P., Contero, M., Conesa, J. and Piquer, A. An optimisation-based reconstruction engine for 3D modelling by sketching. Computer and Graphics 28, 955-979, 2004.

[Gol99] Goldstein, E.B. Sensation & Perception. Wadsworth Publishing Company, 1999, ISBN 0-534-34680-4

[Hof00] Hoffmann; D. Visual Intelligence. How we create what we see, New York: WW Norton & Company, 2000.

[Joh09] Johnson, G., Gross, M.D., Hong, J. and Yi-Luen Do, E., Computational Support for Sketching in Design: A Review, Foundations and Trends in Human–Computer Interaction, 2(1), 1-93, 2009.

[Lip96] Lipson, H. and Shpitalni, M. Optimization-based reconstruction of a 3D object from a single freehand line drawing. Computer-Aided Design; 28(8):651–63, 1996.

[Pal99] Palmer, SE., Vision science, Photons to phenomenology, Cambridge, MA: The MIT Press, 1999.

[Plu10] Plumed, R., Company, P., Piquer, A. and Varley, P.A.C. Do engineers use convergence to a vanishing point when sketching? Proc. Int. Symposium on Distributed Computing and Artificial Intelligence 2010, (DCAI'10), pp.241-250, 2010.

[Tia09] Tian, C., Masry, M. and, Lipson, H. Physical sketching: Reconstruction and análisis of 3D objects from freehand sketches. Computer aided design; 41: 147-158, 2009.

[Var03] Varley, P.A.C. Automatic Creation of Boundary-Representation Models from Single Line Drawings, PhD Thesis, University of Wales, 2003.

[Wri83] Wright, L., Perspective in Perspective, Routledge & Kegan Paul, London, 1983.

[Yua08] Yuan S., Tsui L.Y. and Jie S. Regularity selection for effective 3D object reconstruction from a single line drawing. Pattern Recognition Letters 29 (10), 1486-1495, 2008.