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A model of the topology of the bank-firm credit network and its role as channel of contagion

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Title: A Model of the Topology of the Bank-Firm Credit Network and its Role as Channel of Contagion

by Thomas Lux

No. 1950 | August 2014

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A Model of the Topology of the Bank - Firm Credit Network and Its Role as Channel of Contagion*

Thomas Lux†

Abstract

This paper proposes a stochastic model of a bipartite credit network between banks and the non-bank corporate sector that encapsulates basic stylized facts found in comprehensive data sets for bank-firm loans for a number of countries. When performing computational experiments with this model, we find that it shows a pronounced non-linear behavior under shocks: The default of a single unit will mostly have practically no knock-on effects, but might lead to an almost full-scale collapse of the entire system in a certain number of cases. The dependency of the overall outcome on firm characteristics like size or number of loans seems fuzzy. Distinguishing between contagion due to interbank credit and due to joint exposures to counterparty risk via loans to firms, the later channel appears more important for contagious spread of defaults.

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1 Introduction

The last few years have seen a surge of interest in the properties of the interbank market and formal modelling of interbank connections via models and methods of network theory. The major motivation for the emergence of this new research area have been the events unfolding in 2008 and beyond after the default of Lehman Brothers. As it appeared (and the same danger was lurking around the corner in minor local crises like the Greek and Cyprian debt crises), the breakdown of one major (or even minor) component of the international financial system could have led to hitherto unexpected domino effects via a variety of contagion channels culminating in the complete collapse of the worldwide financial system. In the political arena, this has led to the awareness of the importance of systemic risk and a shift from the micro prudential framework of former Basle Accords to attempts to include systemic risk factors in a new macro-prudential regulatory framework. Academic scientists, policymakers and the public realised that the inner life of the banking sector had been very much of a blank spot in many respects: neither had there been much data available on the size and distribution of activity within the banking sector, nor had economists shown much interest in this component of the economic system, or had developed particular models to explain its activity.

As of 2008, the pertinent literature consisted of a handful of empirically oriented, data-analytical papers driven by natural scientists' interest to study large data sets from a network perspective (Boss et al., 2004; Inaoka et al., 2007; Soramäki et al., 2007) and the theoretical literature on contagion effects in stylized banking networks initiated by Allen and Gale (2000). A certain confluence of both approaches happened with the first computational models of contagion effects in the interbank market (Nier et al., 2008; Haldane and May, 2011; among others). However, these first-generation interbank models have focused their attention exclusively on the role of direct interbank credit as a contagion channel. While they revealed important insights on the trade-off between risk sharing and risk propagation (the later aspect absent in traditional theorizing about the financial sector), other channels of contagion of stress have only been allowed for very recently. A number of authors have recently introduced additional layers of network structure alongside direct credit into models of the interbank market: Halaj and Kok (2013) consider funding risk together with credit risk of interbank loans, whereas both Huang et al. (2013) and Montagna and Kok (2013) consider the contagion effects induced by portfolio overlaps and changing market valuation of portfolio components. Interestingly, they observe a nonlinear interplay between the contribution of interbank credit and joint portfolio exposures to overall systemic risk. In particular, they document, how for a sample of the largest European banks, systemic risk under joint modelling of both channels often turns out to be more than the sum of both components considered in isolation. In the light of this literature, an overall aim in this paper is to provide a foundation for the addition of another layer of network structure, namely, joint exposures via loans extended to non-financial firms. While this is somehow similar to portfolio overlaps, the network structure generated by such joint credits might be different as the majority of firms typically does not have credit connections to more than two banks, while portfolio synchronisation might be much more common. However, as we will see, the resulting sparseness of the bipartite network characteristic of bank-firm loans does not imply that it is of less relevance from the viewpoint of contagious spread of stress.

The plan of the remainder of this paper is the following: In sec.2 we will review basic stylized facts inferred from comprehensive data sets of bank-firm loans for economies like the Italian and the Japanese. We will then propose a stochastic model for generating a bipartite network that shares these stylized facts and also is in line with the typically very heterogeneous, right-tailed distribution of firm and bank sizes. In sec.3, we will introduce joint credit exposure towards the non-bank corporate sector along with interbank credit into a model of the banking sector with fully articulated balance sheet structures. In sec.4 we use this multi-layer network model to study the effect of defaults of single business firms on the overall system, and compare the relative contribution of bank-firm and bank-bank credits to the spread of defaults through the system. Thereby, interbank contagion works via the standard mechanism of defaults of interbank loans causing losses in the creditor banks, while we assume that a bank default spills over to the company sector via the loss of funding for the banks' borrowers. Sec. 5 concludes. A technical Appendix details the condition for emergence of a large connected component in our network of bank-firm credit connections that is crucial for potentially system-wide contagious effects.

2 Construction of a Realistic Bipartite Credit Network between Banks and Firms

In order to capture another important layer of contagion effects in the financial sector, joint exposures via credit to the same set of borrowers should be included into standard contagion models that so far lack this important channel. To this end, a consistent and realistic bipartite network structure for loans from banks to firms of the real sector of the economy needs to be designed. Some basic plausible stylized facts can be inferred from the analysis of a comprehensive Italian data set by de Masi and Gallegati (2012):

1. The distribution of degrees in this bipartite network is much wider for banks than for firms. For instance, they find the mean for firms to be 1.8 and for banks to be 149, respectively. The maximum number of links of firms as borrowers is 15 while on the lending side the most active bank reports 6899 credit relationships.

2. The number of links, while heterogeneous, is size dependent for both banks and firms: there are small banks that basically provide credit to mostly such companies that themselves have few lenders while also large hubs exist among banks with a multitude of lending relationships to the corporate sector.

Similar results are reported for Japanese data in de Masi et al. (2011). In particular, the average number of degrees for both banks and firms are very close to the Italian case, while at the extremes of the degree distribution a larger maximum degree for firms and a smaller maximum degree for banks is found.

In the following, we develop a simple simulation algorithm that is designed to capture the basic design principles of this heterogeneous bipartite network. As in Montagna and Lux (2013) we assume a Pareto distribution for the balance sheet size of the banking sector and also, following well-known insights in industrial economics, suppose a similar Pareto distribution of firms' size.

To start with, we fix the number of banks and firms in our system (all numbers used in the following might be changed over relatively wide intervals without changing the basic structural features of the network). Typically, the number of corporate firms in the real sector exceeds the number of banks by at least one order of magnitude. De Masi and Gallegati (2012), for instance investigate bank-firm relations for about 40000 firms and 500 banks.

We denote the number of banks and firms by N_b and N_f and the average number of links of the firm sector by λ_f. Consequently, the average number of links per bank is $\lambda_b = \lambda_f \cdot \frac{N_f}{N_b}$. However, we also assume that these average degrees do not apply uniformly, but are the means of heterogeneous linking probabilities both across banks and firms that are increasing with their balance sheet size.

For banks, the balance sheet sizes are assumed to be random draws from a truncated Pareto distribution, i.e. total assets A_i, $i = 1, 2, \ldots, N_b$ are distributed as:

$$A_i \sim \frac{\alpha \cdot L^{\alpha} A_i^{-\alpha-1}}{1 - \left(\frac{L}{H}\right)^{\alpha}} \tag{1}$$

with L and H the minimum and maximum of the support and α the Pareto index.

The simplest way to allow for a dependency of the number of links of a unit on its size is to assume that the degrees are distributed proportionally to the balance sheets sizes, i.e. bank i ($i = 1, 2, \ldots, N_b$) has an expected degree:

$$\lambda_i = \bar{\lambda}_b A_i. \tag{2}$$

This can be achieved in the following way: let us assume that the degrees are random draws from a Poisson distribution. Then, the average expected degree across the population of banks could be written as:

$$\int_L^H \bar{\lambda}_b A_i f(A_i) dA_i \equiv \lambda_b. \tag{3}$$

We see that the constant $\bar{\lambda}_b$ is simply obtained as:

$$\bar{\lambda}_b = \frac{\lambda_b}{\bar{A}_i}, \tag{4}$$

the given (observed) average degree λ_b divided by the mean balance sheet size $\bar{A}_i$ across the system. Given the ensemble of banks with their balance sheets, for each of them, its specific number of degrees is consequently obtained as a draw from a Poisson distribution with specific parameter $\bar{\lambda}_b A_i$ i.e. a Poisson-Pareto mixture distribution[1] (other relationships could be used as well, as, for example, logarithmic dependence of degrees on size, etc.).

We now turn to the firm sector and construct its distribution of loan sizes and degrees so as to be consistent with those obtained for banks and the stylized facts reported above. First of all, the mean loan size of firms can be obtained as follows:

$$\bar{f}_j = \theta \bar{A}_i \cdot \frac{N_b}{N_f} \tag{5}$$

where θ is the (average) fraction of external assets (here only loans to firms) in banks' balance sheets.

Assuming that the firm size distribution (and with it the distribution of loans) follows a Pareto distribution with the same shape parameter α as for

[1] This type of mixture distribution is well-known in the actuarial literature, cf. Albrecht (1984).

banks, we can obtain a truncated Pareto distribution for firms by using this mean together with a lower threshold l and upper threshold h obtained in the same way:

$$l = \theta L \frac{N_b}{N_f},\ h = \theta H \frac{N_b}{N_f}. \tag{6}$$

Our assumption of a larger number of firms than banks guarantees that the truncated distribution of loans by firms lies to the left of the one we had used for banks.

Having obtained the parameters of the loan size distribution, l, h, and α, we randomly draw the individual realizations for the ensemble of firms, and, again similar to the previous approach for banks, determine the number of links of each firm j, $j = 1, 2, ..., N_f$, by Poisson draws with parameters:

$$\lambda_j = \bar{\lambda}_f B_i \text{ , with } \bar{\lambda}_f = \frac{\lambda_f}{\bar{B}_i} \tag{7}$$

where λ_f is the average degree across all firms (1.8 in de Masi and Gallegati) and $\bar{B}_i$ is the average (total) loan size across firms.

Both the resulting numbers for the sum of degrees over all banks and firms, respectively, as well as the total amount of loans from the perspective of both lenders and borrowers should be in rough agreement as their expectations are the same in our stochastic framework. Since both are realizations of stochastic numbers, they will, of course, not be exactly identical in any realization of this system. We provide for consistency by taking the minimum of the aggregate links of banks and firms, and add connections one after the other by following the approach of the so-called static model for scale-free networks (Goh *et al.*, 2001): we first assign to each node (bank or firm) a weight according to the realization of its degree. We, then, choose randomly one node from the weighted ensemble of banks and a second one from the weighted ensemble of firms and connect these. The pool of potential links is then reduced by the two stumps that we have used from the distribution of degrees of banks and firms to form this link, and we proceed in the same fashion with all remaining links until the minimum of available links from either banks' or firms' side is exhausted. Finally, we divide the total loan amount of a bank to its multiple borrowers proportionally to their loan sizes obtained from the Pareto distribution of firms. This preserves the relative increase of degrees with loan sizes across firms while the exact numbers would, of course, not exactly add up to the pertinent loan positions of banks that have been generated by independent random draws.

Figs. 1 through 3 illustrate the resulting universe of connections between the financial and corporate sector. In Fig. 1 we have aligned banks (red) and firms (blue) in two lines ordered by size, grey lines indicating an existing credit relationship between bank i and company j. Figs. 2 and 3 show the network structures within both the banking and corporate sectors that are created via joint credit and lending relationships. The pertinent adjacency matrices are created as follows: Denote by M the incidence matrix with dimension $N_b \times N_f$ that lists simply the (valued) credit links from any bank i to company j. The adjacency matrix for banks is then obtained by the one-mode projection $B = MM^\top$,while the one for firms results from the operation $F = M^\top M$. The network structure that comes about indirectly via joint credit relationships is surprisingly dense. Fig. 2 shows that the network of banks forms a complete graph, while the network of companies of Fig. 3 exhibits a few singlets together with a large connected component containing the majority of links. However, these singlets are simply those firms whose random Poisson draws were equal to zero, i.e., they have not been assigned any credit relationships from the outset. If we would modify the model in a way to impose at least one loan for each firm, then they would likely also belong to the large connected component. Note that the average number of loans per firms is $\lambda_f = 2$. Despite this very restricted average number of connections from firms to the banking sector, we find a network structure that practically amounts to a complete graph for both sectors of the economy. The Appendix shows that one could expect the existence of a giant connected component in our bipartite network generation algorithm with the chosen parameter values, and indeed for any typical combination of

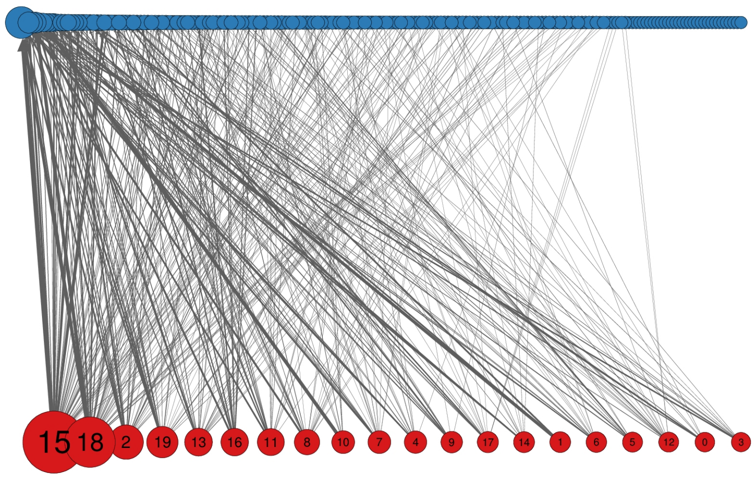

Figure 1: Connections between banking (red) and corporate (blue) sectors in a network with $N_b = 20$, $N_f = 200$, $\lambda_f = 2$, $\lambda_f = 20$ and Pareto distributed sizes of banks and firms. Banks and firms are ordered with respect to their balance sheet and total loan sizes, respectively. Numbers are assigned consecutively for firms and banks upon initialisation of the system.

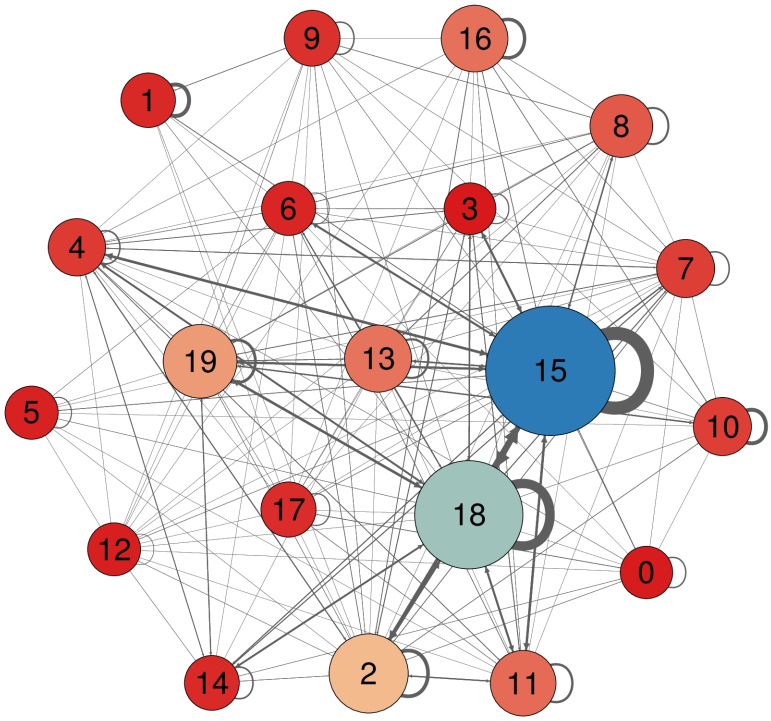

Figure 2: The network of banks' interlocks obtained from the one-mode projection of the bipartite network.

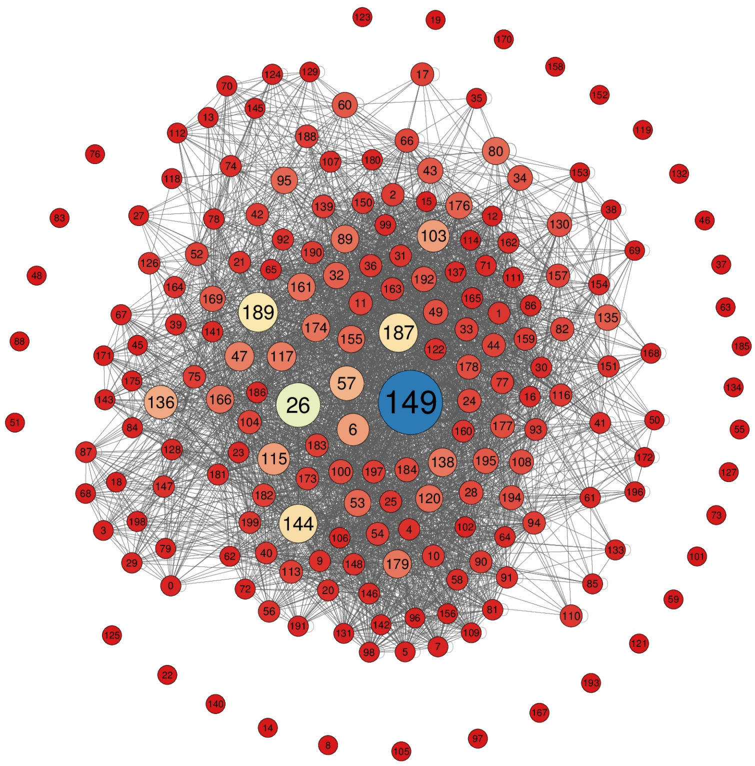

Figure 3: The network of firms' interlocks obtained from the one-mode projection of the bipartite network.

the average degrees λ_b and λ_f as well as the Pareto shape parameter α of the size distributions.

3 Embedding the Bank-Firm Network in a Model of the Interbank Market

We now use the realistic topology of credit relationships between the banking sector and the real sector of the economy to add an additional layer of connectivity between banks. As our workhorse interbank market model, we adopt the framework of Montagna and Lux (2013) that is similarly motivated in its structural assumpions by stylized facts of interbank credit. Their approach is based on a simplified representation of the balance sheet structure of banks as depicted in Fig. 4.

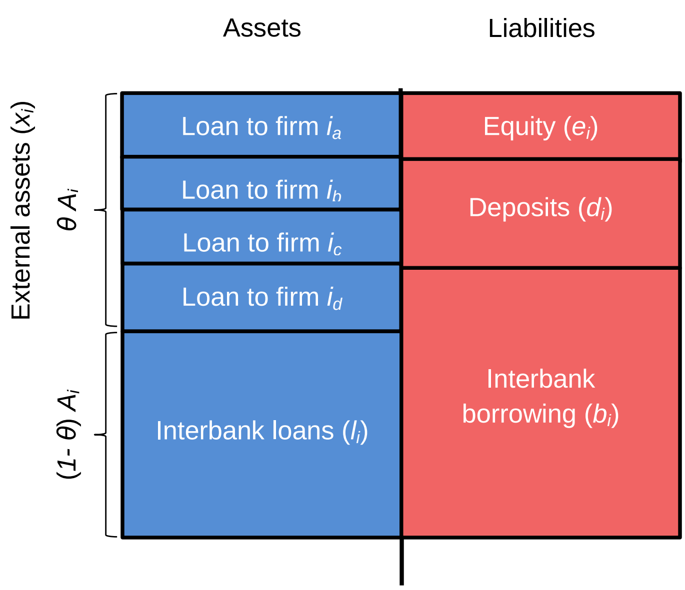

Figure 4: Balance sheet structure of banks. The number of loans to firms depends on the size of the bank and is determined via the stochastic model presented in sec.2.

This model allows for the following aggregate categories of assets and liabilities: On the liability side, it distingushes between equity (e_i), deposits (d_i) and interbank borrowing (b_i) so that total liabilities I_i of bank i can be written as:

$$I_i = e_i + d_i + b_i. \tag{8}$$

Assets are simply broken down into interbank loans (l_i) and external assets (x_i) so that total assets A_i are given as:

$$A_i = l_i + x_i. \tag{9}$$

In the approach of Montagna and Lux (2013) as well as in similar interbank models, external assets are simply a constant book-keeping entity while the focus is on contagion processes evolving via interbank loans. Here we implement external assets by substituting for this position the structure of firm-bank loans established in sec. 2. Since in the interbank market model of Montagna and Lux (2013) balance sheet sizes of banks are also drawn from a Pareto distribution, the generating mechanisms for the banking sector's structure and for the loans extended from banks to firms are fully consistent. With the addition of a fully specified banking sector, the complete model exhibits a multi-layered network structure of the financial sector: The traditional contagion channel of interbank loans becomes intertwined with the new potential contagion mechanism of joint exposures to the same counterparties. Both are designed to replicate the stylized facts of empirical data. We have detailed the generating mechanism of bank-firm loans above. Interbank loans are generated via one of the probability generating functions used by Montagna and Lux (2013). Given balance sheet sizes A_i and A_k of banks i and k, a draw from:

$$p_{ik} = P(A_i, A_k) = d \left(\frac{A_i}{A_{max}} \right)^{a_1} \left(\frac{A_k}{A_{max}} \right)^{\alpha_2}. \tag{10}$$

will determine the probability of an interbank credit link existing between i and k (a_1, a_2, d being parameters, and A_{max} the balance sheet size of the largest bank). A second binary draw of $a_{ik} \in \{0,1\}$ with probabilities $1 - p_{ik}$ and p_{ik}, respectively, will then determine whether the link will be created or not in the simulation.

We assume that the relative size of external assets and interbank loans are fixed at proportions θ and $1-\theta$ of the total balance sheet size:

$$l_i = (1-\theta)A_i \ , \ x_i = \theta A_i, \tag{11}$$

and that, given the realization of the link structure a_{ik} of the interbank network, the breakdown of interbank loans is determined via:

$$l_{ik} = \frac{l_i p_{ik} A_k}{\sum_k p_{ik} A_k} \tag{12}$$

under the constraint that $\sum_k l_{ik} = l_i$, where l_{ik} is the credit extended from bank i to k. Eq. (12) assumes that banks with a higher balance sheet size will also exchange a higher volume of credit. The overall allocation of interbank credits, of course, also defines the liability side of the interbank market, b_i, and the system is closed by deposits as the residual of the book-keeping operations while equity is fixed as a certain percentage of the overall balance sheet size. The most important characteristic of this generating mechanism is that the resulting interbank network shares the stylized fact of *disassortativity*, i.e. a negative correlation between the degrees of connected banks i and k. Hence, there will be typically nodes with high degrees connected with partners with a relatively low degree while the association of similarly active banks will be observed less frequently. This is in conformity with some banks (mostly large banks) assuming the role of money center banks, and the disassortative mixing also seems in harmony with recent findings of a core-periphery structure of the banking network (Craig and von Peter, 2014; Fricke and Lux, 2014).

4 Simulation Details

We now conduct a series of computational experiments to scrutinize the stability or vulnerability of the system defined in Sec. 2 and 3 in the presence of external shocks. While we have run various simulations with alternative settings without qualitative change of the results, we confine the presentation to the following baseline set of parameters: The number of banks and firms are set equal to $N_s = 250$ and $N_f = 10,000$, respectively. The bank size distribution is determined by a truncated Pareto distribution with shape parameter $\alpha = 1.2$, and the boundaries of the support are set equal to $L = 5$ and $H = 100$.[2] Following the empirical evidence, the average number of links is assumed to be $\lambda_\alpha = 2$ for firms and $\lambda_b = 80$ for banks. Simulation of the incidence matrix and the loan sizes for the bank-firm network then proceeds in the following steps:

1. We first determine the balance sheet size of an ensemble of banks via draws from the truncated Pareto distribution,
2. Given the overall size distribution, and the aggregate characteristics of the bank-firm network, we determine the number of outgoing links (loans to firms) of banks via their specific Poisson distributions,
3. Assuming that firms' size distribution follows a Pareto law as well, and that this is reflected in the distribution of individually aggregated loans, the size distribution of loans of firms is determined via the rescaled truncated Pareto distribution with parameters α, l and h for consistency with the size (and loan) distribution already drawn for banks,
4. Given loan sizes of each firm, realisations of their size-dependent Poisson distribution can be drawn to determine the respective number of incoming links, i.e. the number of loans taken from different banks,

[2] Using a truncated rather than the original Pareto distribution helps facilitate interpretation of the results. While it preserves to a large extend the observed degree of size heterogeneity of the banking sector, the truncated Pareto law has finite moments of all orders. In contrast many statistics would have infinite expectations under a pure Pareto law, so that the statistics of different simulations would not converge to any limiting value.

5. Given the full distribution of in-degrees and out-degrees of firms and banks, these can be connected along the lines of typical network generating mechanisms as explained in sec.2. When the allocation of all links is completed, we make sure that each firm and each bank has at least one connection to the other category of actors, and that each bank is participating in the interbank credit market. If necessary, additional links to randomly determined partners are added for hitherto isolated banks or firms.

Finally, in the algorithm generating the interbank linkages and complete balance sheets of banks, we assume that external assets (loans to firms) amount to a fraction θ of overall assets (the remainder being interbank loans), and that all banks have an equity share of 3 %. Parameters of the link-generating mechanism in the interbank market have been set equal to $d = 0.5$, $a_1 = 0.25$ and $a_2 = 1$.

5 Contagion through Joint Exposures

We are now ready to study the relevance of the firm-bank network as a channel for the propagation of shocks. While previous studies have typically studied the effect of a complete or percentage failure of the external assets of one or more banks, in our framework we can distinguish more specifically between different sources of such a shock. In particular, it suggests itself to led a shock to the system consist in the default of a single firm in the network. We will go through the whole corporate sector and consider the default of each firm and its aftereffects one after the other. Note that in doing so we also allow for interbank contagion if one of the creditor banks of the defaulted firm does not survive the shock itself. This will actually be a rare result in our network because of the different numbers of banks and firms, and, therefore, the relatively minor importance of any borrower from the corporate sector for any bank.

Nevertheless, a small fraction of induced defaults of banks is happening[3]. We allow then for aftereffects not only in the banking sector, but also for a feedback to the corporate sector in that the borrowers of defaulted banks may default themselves because of lack of funding. We assume that firms will fail, if their available funding drops below a certain percentage of their ex-ante credit volume due to defaults of one or more creditor banks. Like interbank contagion this second channel might unfold over a number of rounds. For instance, a firm might survive the first round of knock-on effects even if one of its creditors defaults, but the missing funding is relatively small. However, as the shock spreads through the system, another creditor bank of the same firm might go out of business, and the cumulative loss of funding components could become too high for the firm so that it eventually is driven into default. In this way, multiple rounds of domino effects are possible in principle.

While defaults of firms due to discontinuance of funding are modelled here in a relatively simple fashion, any alternation of it (default with a certain probability, positive recovery value) would just change the quantitative outcome, not the qualitative effects. Fig. 5, 6 and 7 exhibit the typical outcome of such simulations. First, Fig. 5 gives the average (over all $10,000$ firms as candidates for an external shock) and maximum number of defaults of banks when varying the fraction of external loans from 0 to 100 percent of banks' balance sheet with the minimum required level of maintained funding being set equal to 80 percent. As can be seen, it needs a certain minimum size (about 20% in our example) of external loans to induce any subsequent defaults at all. Beyond this threshold the average number of bank defaults increases slightly to reach about 8 percent of all banks when loans to firms are the only category of assets. What is surprising is, however, not the mean of the number of defaults, but its maximum (across the hypothesized default of any of the $10,000$ firms). The maximum jumps sharply from zero to a full breakdown of the banking sector in certain

[3]One might object that regulatory provision should prevent that one single borrower could jeopardize the solvency of a bank. However, we could easily widen our interpretation of the corporate sector by viewing 'firms' as slightly more general sectoral or regional aggregates than in any legal definition of these entities.

cases as soon as any defaults are recorded at all. Fig. 6 and 7 show defaults for the case $\theta = 0.6$. Both display a bimodal distribution of the contagious bank defaults. Apparently, either no aftereffects happen at all, or if the initial firm default leads to knock-on effects in the banking sector the combined contagion channels of joint exposures and interbank credit lead to a complete breakdown of the banking sector.

No intermediate cases are actually observed. Fig. 6 shows results in comparison to the size of the defaulting firm, Fig. 7 in comparison to the number of links, i.e. the number of its loans and, therefore, the number of initially affected banks. In both cases, there is no strong relationship between the firms' attributes and the number of bank defaults eventually caused by the firm's defaults. While the largest and most connected firms are among the 'disastrous' candidates, there are also both large firms and firms with many loans where default remains without subsequent effects as well as relatively small firms and firms with few loans that could cause a system-wide collapse. Hence, it depends on the exact positioning of firms in the network and a prediction of each firm's contribution to systemic vulnerability appears at least not straight forward. Estimation of a Binary Probit model underscores this point: Regressing the binary variable "collapse/no collapse"[4] on firm's size and degree we find both highly significant coefficients and marginal effects of both variables (all probabilities are < 0.0001) with McFadden's pseudo R square being 0.65. However, the predictions derived from the model are very unsatisfactory: Out of 191 cases of a systemic collapse only about half (100) are correctly predicted using the firm-specific input data. The adjusted percentage of correctly predicted outcomes is disappointingly low at 0.32.

[4]Cases with a few contagious defaults only (displayed on the left-bank side of Fig. 6 and 7) are classified as "no collapse".

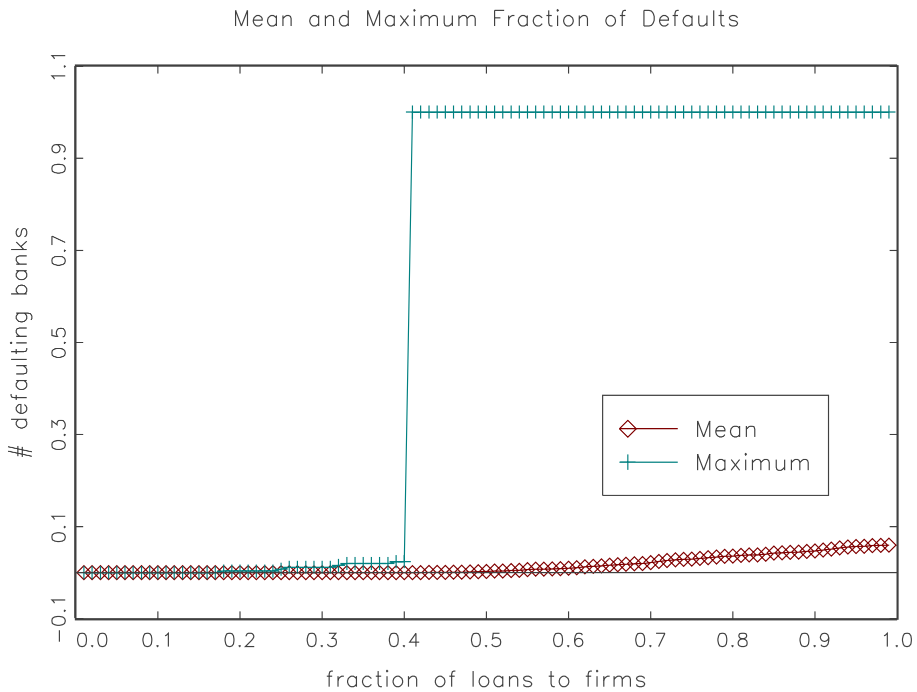

Figure 5: Dependency of bank defaults on the fraction of external assets (loans to firms) in banks' balance sheet.

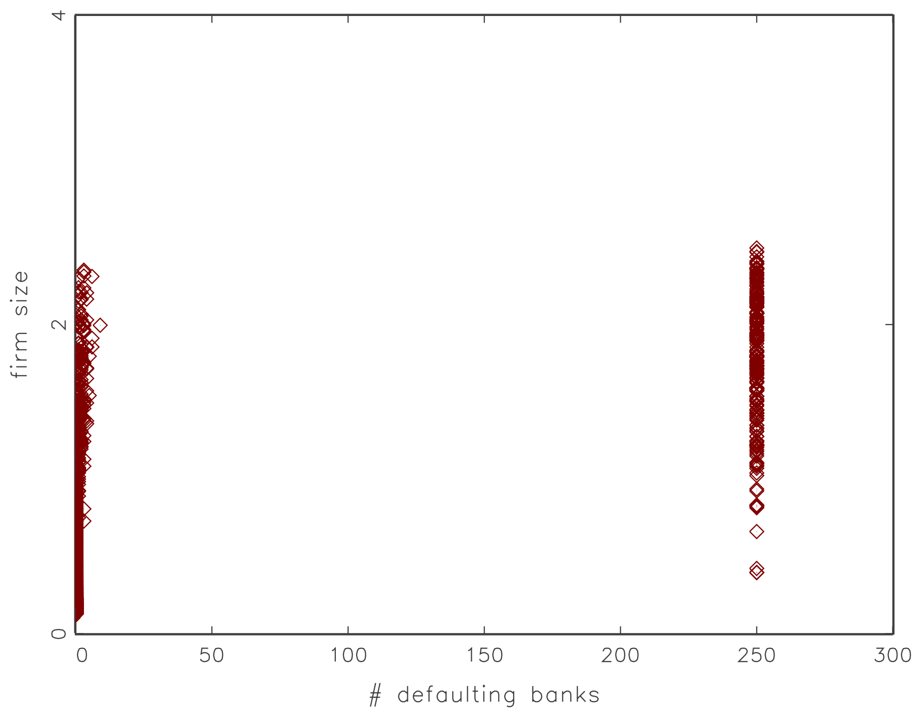

Figure 6: Number of bank defaults versus balance sheet size of defaulting firms. The fraction of external assets is $\theta = 0.6$. Note that the right-hand side cluster of full system-wide breakdown covers only about 2 percent of all cases.

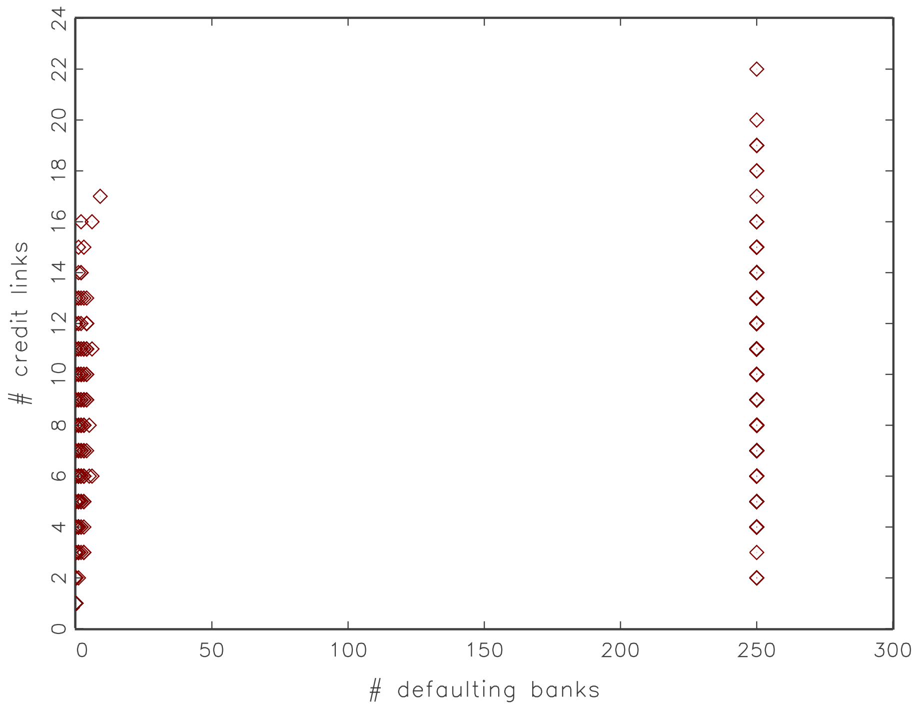

Figure 7: Number of bank defaults vs. degree of defaulting firms (i.e., number of loans). Again, $\theta = 0.6$ and the right-hand cluster only represents about 2 percent of all cases.

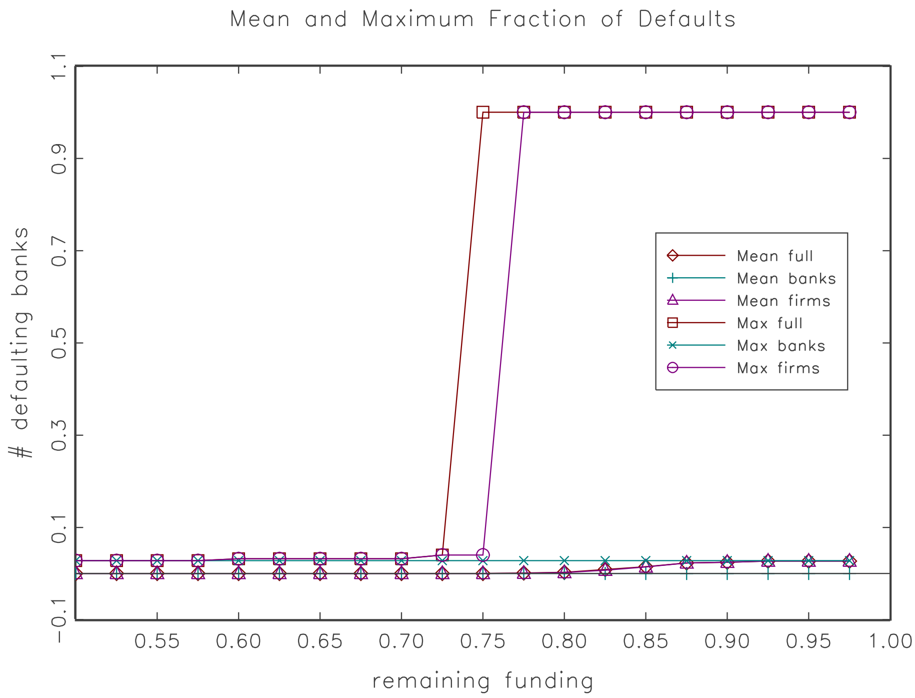

Figure 8: Dependency of bank defaults on the required fraction of continuing funding. ”Mean full” and ”Max full” denote the average and maximum aftereffects to single firm defaults in the complete system. The labels “banks” and “firms” identify those cases in which after the initial shock only interbank contagion or bank-firm contagion is considered, respectively.

Fig. 8 shows how default cascades depend on the minimum fraction of funding that firms need to maintain (considering first only the lines labelled ”mean full” and ”max full” for the entire system defined above). Similarly as with Fig. 5 we see a sharp transition at about 75 percent (or the loss of a quarter of its credit lines) at which the system bifurcates from a completely stable to a bimodal outcome.

How much does interbank contagion and how much contagion through the bank-firm network contribute to this outcome? Fig. 9 shows the same scenario as in Fig. 5 together with the outcome of the contagious process with either only the interbank network or the bank-firm network activated.

As it turns out, it is the bank-firm funding channel that basically determines the results. The standard bank-bank contagion contributes a small number of defaults and it seems unable in our setting to trigger a full-scale collapse. The same can be inferred under variation of the required fraction of continuing funding in Fig. 8. This is in contrast to the results of Montagna and Lux (2013). The reason is that mostly any borrower's default wipes out only a small portion of a bank's equity (while in Montagna and Lux, the shock consists in a complete loss of all external assets). An initial bank default happens when one borrower makes up a large portion of a bank's portfolio. This default spreads more easily via the funding channel (since most firms rely on few creditors), while losses in the interbank market are much more broadly distributed and are more easily covered by the other banks.

Fig. 10 performs a similar analysis varying the capitalization of banks, assuming again $\theta = 0.6$ and a benchmark of continuing funding by bank credit of 80 percent. The interesting outcome in this case is that higher capitalisation can provide a cushion against a full-scale collapse. In the present setting, an equity ratio of about 7.4 percent will be needed to make the system safe against a complete breakdown, if only firm-bank connections were considered. Again, interbank credit does not add much to systemic risk in the present setting.

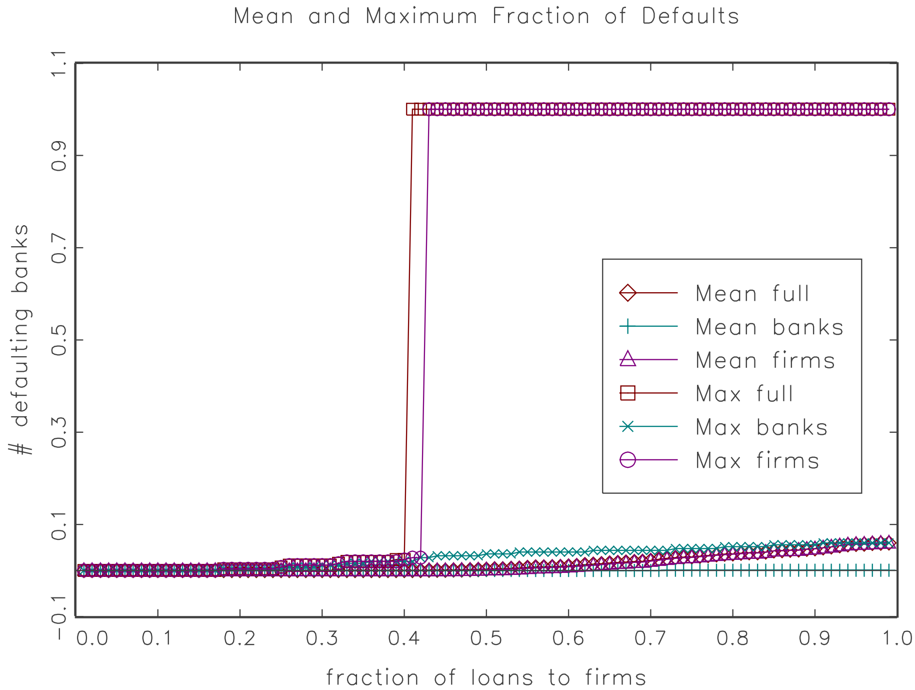

Figure 9: Dependency of bank defaults on the fraction of external assets in banks' balance sheets. "Mean full" and "Max full" denote the average and maximum aftereffects to single firm defaults in the complete system (same as Fig. 5). The labels "banks" and "firms" identify those cases in which after the initial shock only interbank contagion or bank-firm contagion is considered, respectively.

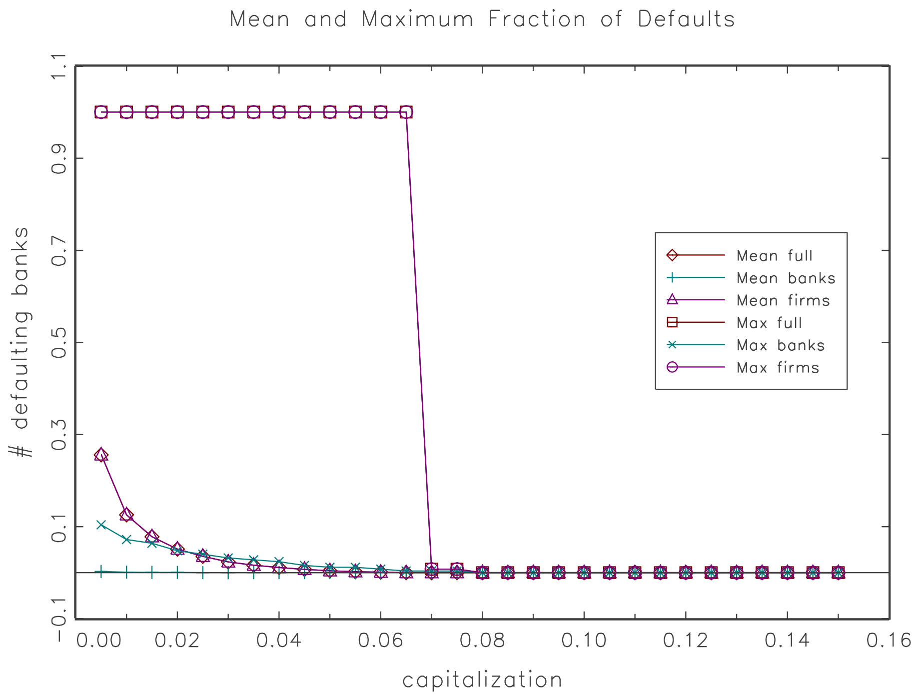

Figure 10: Dependency of bank defaults on the equity ratio of banks. Again, the effects in the complete system and in systems with one contagion channel only are displayed. Labels have the same meaning as in Fig. 9.

The last figure show the sensitivity of the system behaviour towards some of its deep parameters. Fig. 11 considers the simultaneous variation of both the Pareto indices of the bank size and firm size distribution. As can be seen, contagion eventually dies out when the heterogeneity of the sizes of the entities in both sectors is reduced. While this effect is completely monotonic for firms, a slight hump can be observed for banks when moving from a very heterogeneous system ($\alpha = 1$) to higher values. This non-monotonicity might be due to the probability distribution for the generation of interbank links which leads to a sparse matrix when applied to a very heterogeneous distribution of bank sizes (cf. Montagna und Lux, 2013). Overall, however, a system with more equally sized units appears more favourable for the stability of the system. The reason is very likely that with more homogeneous sizes, all mutual exposures become so small that knock-on effects become more and more improbable, and hence the detrimental activation of cumulative contagion channels can be avoided.

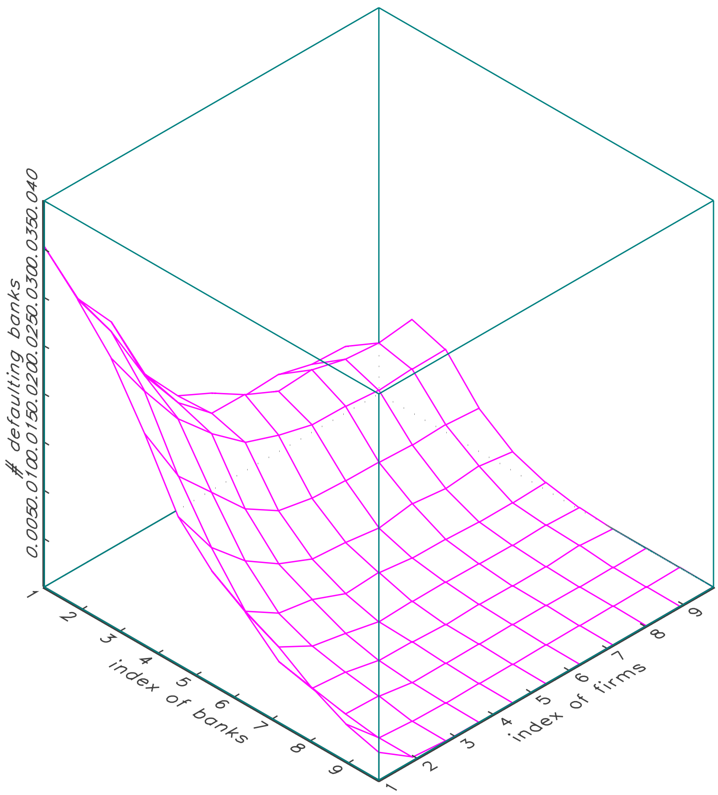

Figure 11: Dependency of bank default rates on the distribution of firm and bank sizes. Both size distributions are assumed to to follow a truncated Pareto law with shape parameters allowed to vary between 1 and 10.

6 Conclusion

We have provided a stochastic model of network generation for credit linkages between the financial sector and non-financial firms based upon well-known and plausible stylized facts confirmed by recent studies using comprehensive data sets for a number of indistrialized countries. We have embedded this network into a model of interbank credit similarly based upon the stylized facts of pertinent data, and have explored the potential of this multi-layer system to give rise to contagious domino effects spreading throughout the entire system.

The results show a distinctively dichotomic behaviour of this system: After the default of a single unit from the real economy, the system either shows practically no repercussions at all, or, if some domino stones are starting to jiggle they will drag all others with them. Note that this happens beyond a certain threshold defined by the parameters of the system (low equity ratio in particular) beyond which the system appears completely safe. But if it is in a vulnerable state, its fragility expresses itself in the lurking danger of a system-wide breakdown occurring with small probability, rather than disruptions of different magnitudes. Interbank contagion slightly aggravates the problem while the bank-firm connections are by themselves sufficiently contagious to cause systemic events for a large range of settings. Our results are similar to some extend to these reported by Huang et al. (2013). These authors study the contagious potential of portfolio overlaps and also find a similar dichotomic structure like the present paper. They do not, however, consider interbank contagion, and have a more schematic model of the distribution of portfolio components across banks. Our network formation algorithm, in contrast, is closely geared towards replication of well-established stylized facts.

The analysis in the Appendix shows that given what we know about the empirical firm size distribution and and the distribution of credit links between banks and non-financial firms, the resulting network very likely is characterized by a large connected component. In fact, all empirical estimates (e.g. in the shape of the size distribution) or mere numbers observed in the data (λ_b and λ_f which are empirical averages) are located far away from the bifurcation line where a giant component first emerges. Taking these numbers as given, the credit markets of modern economies do link practically all actors via a net of joint exposures and joint credits. A possible (but perhaps unlikely) rescue could be provided through higher-order characteristics like strong clustering within this network (as also identified by de Masi et al, 2011; de Masi and Gallegatti, 2012 for the cases of Japan and Italy). Clustering amounts to more links being concentrated in certain regions of the network which implies that some other parts would be more sparse in their connectivity. This structure might separate some parts of the overall network from others and could in principle prevent or postpone (in relation to some other parameters) the appearance of a large connected component. Since clustering coefficients are known for the empirical counterparts of a network, the change of results brought about by these additional micro-structural features needs to be explored in future research. However, even if clustering breaks up the large component into a few strongly connected clusters, the potential for contagious spread of defaults caused by small disturbances may still be relatively high. Given the difficulty of identifying the critical nodes in the firm sector, the present analysis speaks in favor of appropriately increasing the cushions (equity ratios) to prevent a full-fledged crisis to evolve from small origins. In principle, with an even closer calibration to empirical data the present framework could provide an estimate for what a sufficient cushion would be for a system where basic properties (number of banks and firms, size and degree distributions) are known.

Appendix

Here we show how we can obtain some insights into the qualitative properties of the bipartite bank-firm credit network via application of the generating function formalism developed by, among others, Newman et al. (2001).
To set the stage, note that the degree distribution of both banks and firms in our framework are both given by a Poisson-Pareto mixture distribution. Disregarding the upper boundaries used in our simulations, and defining the lower boundaries for the Pareto distribution of the Poisson parameters λ_1 and λ_2 for banks and firms by $\underline{\lambda_1}$ and $\underline{\lambda_2}$, the degree distributions can be written as:

$$P_1(k) = \int_{\underline{\lambda_1}}^{\infty} \frac{e^{-\lambda_1}\lambda_1^k}{k!}\ \alpha\ \frac{\underline{\lambda_1}^{\alpha}}{\lambda_1^{\alpha+1}}\ d\lambda_1. \tag{13}$$

$$P_2(k) = \int_{\underline{\lambda_2}}^{\infty} \frac{e^{-\lambda_2}\lambda_2^k}{k!}\ \alpha\ \frac{\underline{\lambda_2}^{\alpha}}{\lambda_2^{\alpha+1}}\ d\lambda_2. \tag{14}$$

assuming that the distribution of both λ_1 and λ_2 (and, hence, the bank and firm size distributions) are characterized by the same shape parameter α.
Since the number of links of all banks and firms have to be the same, we have the consistency requirement that:

$$E[\lambda_1]\ N_b = E[\lambda_2]\ N_f \Longleftrightarrow \frac{\alpha}{\alpha-1}\ \underline{\lambda_1}\ N_b = \frac{\alpha}{\alpha-1}\ \underline{\lambda_2}\ N_f \Longleftrightarrow \underline{\lambda_2} = \underline{\lambda_1}\ \frac{N_b}{N_f}. \tag{15}$$

Many interesting properties of networks can be derived from the probability generating function of the degree distributions. Here, we can take stock of the fact that, in general, the probability generating function of Poisson mixture models is well-known. Namely, denoting the mixing distribution by $\varphi(\lambda)$, the generating function of the mixture model can be written (cf. Karlis and Xekalaki, 2005):

$$H(s) = E[s^k] = \int_0^{\infty} e^{\lambda(s-1)}\varphi(\lambda)\ d\lambda. \tag{16}$$

In our case of Pareto mixing distributions, we obtain the generating functions

$f_0(s)$ and $g_0(s)$ for banks' and firms' degree distributions, respectively:

$$f_0(s) = \int_{\underline{\lambda_1}}^{\infty} e^{\lambda_1 (s-1)} \ \alpha \ \frac{\underline{\lambda_1}^{\alpha}}{\lambda_1^{\alpha-1}} \ d\lambda_1, \tag{17}$$

$$g_0(s) = \int_{\underline{\lambda_2}}^{\infty} e^{\lambda_2 (s-1)} \ \alpha \ \frac{\underline{\lambda_2}^{\alpha}}{\lambda_2^{-\alpha-1}} \ d\lambda_2. \tag{18}$$

We can immediately verify that the distribution is correctly normalized since $f_0(1) = g_0(1) = 1$ holds. As another property of the generating function it should hold that $f_0'(1)$ and $g_0'(1)$ are equal to the average degrees of banks and firms. This can be verified as well:

$$f_0'(s) = \alpha \ \underline{\lambda_1}^{\alpha} \ \int_{\underline{\lambda_1}}^{\infty} \lambda_1 \ e^{\lambda_1 (s-1)} \ \lambda_1^{-\alpha-1} \ d\lambda_1. \tag{19}$$

We see that for $s = 1$, this equation boils down to the formula for the mean of the Pareto distribution, and hence:

$$f_0'(1) = \frac{\alpha}{\alpha - 1} \ \underline{\lambda_1} = E[\lambda_1], \tag{20}$$

and similar for $g_0'(1)$ where both mean values are finite only if $\alpha > 1$ holds.

Randomly choosing one edge (connection), the distribution of the remaining edges of the vertex that belongs to it is:

$$f_1(s) = \frac{f_0'(s)}{f_0'(1)} = \frac{\alpha \underline{\lambda_1}^{\alpha}}{\frac{\alpha}{\alpha-1} \underline{\lambda_1}} \int_{\underline{\lambda_1}}^{\infty} e^{\lambda_1 (s-1)} \ \lambda_1^{-\alpha} \ d\lambda_1 = (\alpha-1) \underline{\lambda_1}^{\alpha-1} \int_{\underline{\lambda_1}}^{\infty} e^{\lambda_1 (s-1)} \ \lambda_1^{-\alpha} \ d\lambda_1 \tag{21}$$

and similarly for $g_1(s)$.

Connecting both parts of the bipartite network, the degree distribution of the banks to which the firms from a randomly drawn bank-firm link are connected, is given by (cf. Newman et al., 2001):

$$G_1(s) = f_1(g_1(s)). \tag{22}$$

With the help of this function, one can determine whether, given the parameters, connectivity in the system is high enough to give rise to a so-called *giant component*, i.e. a cluster that includes via indirect linkages most of the two categories of vertices (i.e., banks and firms). In this case, distress could easily expand from local origins to the whole system and lead to a system-wide breakdown of activity.

A joint component forms first when $G_1'(1) = 1$ holds, and prevails for $G_1'(1) > 1$ in general. To elaborate on this condition, we apply the chain rule of differentiation:

$$G_1'(s) = f_1'(g_1(s)) \cdot g_1'(s). \tag{23}$$

We easily obtain:

$$f_1'(s) = (\alpha - 1)\ \underline{\lambda_1}^{\alpha-1} \int_{\underline{\lambda_1}}^{\infty} e^{\lambda_1(s-1)}\ \lambda_1^{1-\alpha}\ d\lambda_1 \tag{24}$$

$$g_1'(s) = (\alpha - 1)\ \underline{\lambda_2}^{\alpha-1} \int_{\underline{\lambda_2}}^{\infty} e^{\lambda_2(s-1)}\ \lambda_2^{1-\alpha}\ d\lambda_2 \tag{25}$$

$$g_1(1) = (\alpha - 1)\ \underline{\lambda_2}^{\alpha-1} \int_{\underline{\lambda_2}}^{\infty} \lambda_2^{-\alpha}\ d\lambda_2 = 1. \tag{26}$$

We, then, obtain:

$$G_1'(1) = \left\{(\alpha - 1)\ \underline{\lambda_1}^{\alpha-1} \int_{\underline{\lambda_1}}^{\infty} \lambda_1^{1-\alpha}\ d\lambda_1\right\} \left\{(\alpha - 1)\ \underline{\lambda_2}^{\alpha-1} \int_{\underline{\lambda_2}}^{\infty} \lambda_2^{1-\alpha}\ d\lambda_2\right\}. \tag{27}$$

With appropriate normalisation, the integrals define the second moment of the Pareto distribution so that we arrive at:

$$G_1'(1) = \frac{\alpha - 1}{\alpha}\ \underline{\lambda_1}^{-1}\ E[\lambda_1^2] \cdot \frac{\alpha - 1}{\alpha}\ \underline{\lambda_2}^{-1}\ E[\lambda_2^2]. \tag{28}$$

Since $E[\lambda_1^2] = \frac{\alpha}{\alpha-2}\underline{\lambda_1}^2$, $E[\lambda_2^2] = \frac{\alpha}{\alpha-2}\underline{\lambda_2}^2$, we end up with

$$G_1'(1) = \left(\frac{\alpha - 1}{\alpha - 2}\right)^2 \ \underline{\lambda_1}\ \underline{\lambda_2} = \left(\frac{\alpha - 1}{\alpha - 2}\right)^2 \ \underline{\lambda_1}^2 \ \frac{N_b}{N_f}. \tag{29}$$

Hence, a giant duster forms if $\left(\frac{\alpha-1}{\alpha-2}\right)^2 \ \underline{\lambda}_1^2 \ \frac{N_b}{N_f} > 1$. Local containment of disturbances thus requires that $\alpha > 2$ as a necessary condition and that additionally, if this condition is met, $\underline{\lambda_1} < \frac{\alpha-2}{\alpha-1} \left(\sqrt{\frac{N_f}{N_b}}\right)$ *or* $E[\lambda_1] < \frac{\alpha-2}{\alpha}\sqrt{\frac{N_f}{N_b}}$ holds. Unfortunately, most empirical evidence for firm (and bank size) distributions speaks plainly against the first condition (Zipf's law amounts to $\alpha = 1$), and even if this were fulfilled, the typical empirical numbers would require an unrealistically low left-hand border for the banks' degree distribution, cf. Fig. A1.

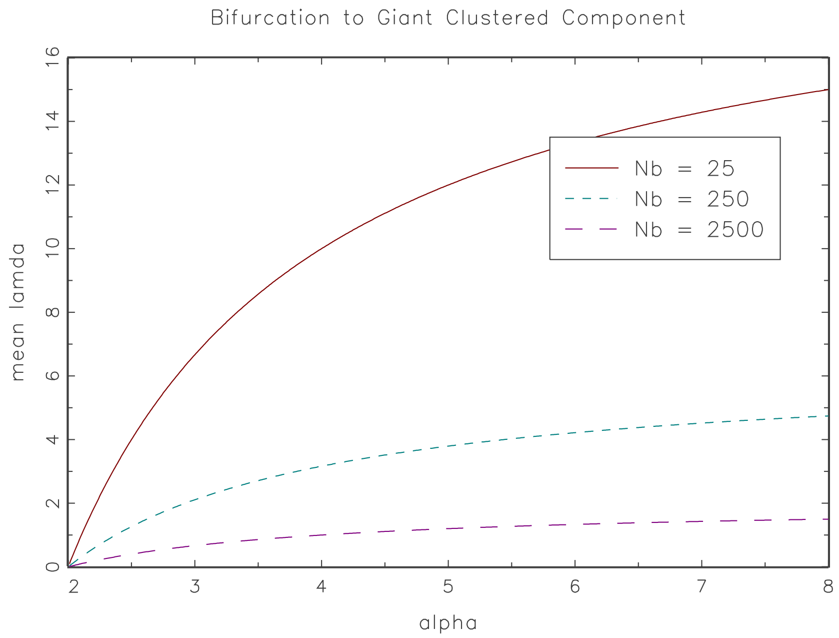

Figure A1: Bifurcation to giant clustered component. The figure shows the expectation $E[\lambda_1]$ for given α and numbers of banks and firms at which a transition to a giant connected component occurs. $N_f = 10,000$ in all cases.

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